

## Note on Unitary Symmetry in Strong Interaction. II

—Excited States of Baryons—

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A classification of the baryon isobars has been investigated on basis of the unitary symmetry model which has been developed in a previous paper under the same title.

The purpose of this note is to investigate problems of baryon isobars from the viewpoint of the unitary symmetry model.<sup>1)</sup> In this model, the mass differences among mesons and among baryons are neglected. As the result, one may wonder if such a model can be applicable to the study of the baryon isobars which appear in the meson-baryon scattering, where these mass differences are certainly not negligible. It is almost probable that our model will present a very poor approximation for this problem if compared quantitatively. However, it might be possible that many of qualitative features could be roughly explained by our model. It is due to this hope that this work has been undertaken. So all results given in this paper should not be taken in its face value, but only in a qualitative sense. In this paper, we shall concern ourselves with the case of studies of Yamaguchi-Gell-Mann scheme,<sup>2)</sup> since the case of the Sakata scheme has been treated already<sup>1),3)</sup> and would not produce any new results. We may note that our results here could be applied also for study of meson-meson resonances or for baryon-baryon scattering resonances, with small changes.

As has been noted in the previous paper,<sup>1),2)</sup> the baryon octet ( $N, \bar{E}, \Sigma, \Lambda$ ) and the meson octet ( $K, \bar{K}, \pi, \pi_0'$ ) belong to irreducible representations  $U_3(1, 0, -1)$  of the 3-dimensional unitary group  $U_3$ , and they are represented by two traceless tensors  $N_\nu^\mu$  and  $f_\nu^\mu$ , respectively, as follows:

$$\begin{aligned} \pi_+ &= f_1^2, \quad \pi_- = f_2^1, \quad \pi_0 = \frac{1}{\sqrt{2}}(f_1^1 - f_2^2), \quad \pi_0' = -\frac{3}{\sqrt{6}}f_3^3, \\ K_+ &= f_1^3, \quad K_0 = f_2^3, \quad \bar{K}_+ = f_3^1, \quad \bar{K}_0 = f_3^2, \\ \Sigma_+ &= N_1^2, \quad \Sigma_- = N_2^1, \quad \Sigma_0 = \frac{1}{\sqrt{2}}(N_1^1 - N_2^2), \quad \Lambda = -\frac{3}{\sqrt{6}}N_3^3, \end{aligned} \quad (1)$$

$$p = N_1^3, n = N_2^3, \bar{E}_- = N_3^1, \bar{E}_0 = N_3^2.$$

We may note that the same representation Eq. (1) has been given by many others<sup>4)</sup> in matrix notations. Now, as has been remarked in (I), the baryon isobars  $N^*$ ,  $N^{**}$ ,  $Y_0^*$  and  $Y_1^*$  have to belong to some of the following irreducible representations in the right-hand side of the next equation.

$$U_3(1, 0, -1) \times U_3(1, 0, -1) = 2U_3(1, 0, -1) + U_3(0, 0, 0) + U_3(2, 0, -2) \\ + U_3(2, -1, -1) + U_3(1, 1, -2). \quad (2)$$

The same is also true for meson-meson scattering isobars or for baryon-baryon scattering resonances, since both the mesons and the baryons belong to the same irreducible representations  $U_3(1, 0, -1)$ , and therefore their scattering states to the product representation  $U_3(1, 0, -1) \times U_3(1, 0, -1)$ . Thus all results given in this paper can be immediately translated from our baryon isobar case into the meson-meson and baryon-baryon scattering cases, but here we study only in the case of meson-baryon scattering problem. Below, we list a classification of particles contained in each of these irreducible representations. This can be easily done by applying the technique developed previously.<sup>1)</sup>

- (a)  $U_3(1, 0, -1)$   
( $I=1/2, Y=1$ ), ( $I=1/2, Y=-1$ ), ( $I=1, Y=0$ ), ( $I=0, Y=0$ ).
- (b)  $U_3(0, 0, 0)$   
( $I=0, Y=0$ ).
- (c)  $U_3(2, 0, -2)$   
( $I=2, Y=0$ ), ( $I=3/2, Y=+1$ ), ( $I=3/2, Y=-1$ ),  
( $I=1, Y=2$ ), ( $I=1, Y=-2$ ), ( $I=1, Y=0$ ),  
( $I=1/2, Y=1$ ), ( $I=1/2, Y=-1$ ), ( $I=0, Y=0$ ).
- (d)  $U_3(2, -1, -1)$   
( $I=3/2, Y=1$ ), ( $I=1, Y=0$ ), ( $I=1/2, Y=-1$ ) ( $I=0, Y=-2$ ).
- (e)  $U_3(1, 1, -2)$   
( $I=3/2, Y=-1$ ), ( $I=1, Y=0$ ), ( $I=1/2, Y=+1$ ) ( $I=0, Y=+2$ ),

where  $Y$  stands for hypercharge, so that  $Y=S+1$  in terms of the strangeness  $S$  in the present case. First of all, we note that a particle with  $I=1$  and  $Y=0$  is contained in all representations except in  $U_3(0, 0, 0)$ . Thus, we cannot identify the representation to which  $Y_1^*$  belongs. We shall investigate all of these in turn.

Case (a) :  $U_3(1, 0, -1)$ .

If  $Y_1^*$  belongs to this representation, we have to identify other three particles

in this representation. Obviously, we can identify the particle with  $(I=0, Y=0)$  as  $Y_0^*$ , and the one with  $(I=1/2, Y=1)$  as the second pion-nucleon resonance  $N^{**}$ , while the one with  $(I=1/2, Y=-1)$  can be considered as an excited state of  $\mathcal{E}$ . Now, the second resonance  $N^{**}$  of the pion-nucleon system is to be considered likely to have the character of a  $d_{3/2}$  resonance.<sup>5)</sup> Accordingly, we have to assign the same  $d_{3/2}$  resonances for all  $Y_1^*$ ,  $Y_0^*$  and  $\mathcal{E}^*$  in this case. This is not so bad, because the spin<sup>6)</sup> of  $Y_1^*$  appears to be  $3/2$ . However, we should remark that it is unnecessary to identify the  $(I=1/2, Y=1)$  state as  $N^{**}$ . As has been stated in the beginning, our approximation is quite poor, and as the result the state with  $(I=1/2, Y=1)$  might disappear when we take account of the mass differences among meson octet and among the baryon octet. The above statement is meant to indicate the following: "When we neglect these mass differences, the state with  $(I=1/2, Y=1)$  then certainly exists because of  $U_3$  symmetry. Now, we have to change the masses of the pion and the kaon and of the nucleon and the  $\mathcal{E}$ -particle from the common values. We may suppose that we can take such a procedure continuously with respect to these masses. Then, in course of these operations, the state with  $(I=1/2, Y=1)$  may cease to represent a resonance state." If such thing could ever happen, then we cannot say anything about the spin of  $Y_1^*$  and  $Y_0^*$ . But we do not adopt such a view here.

The irreducible representation  $U_3(1, 0, -1)$  can be characterized by a traceless tensor  $T_\nu^\mu$  whose identifications with the real isobar states can be expressed exactly in the same way as Eq. (1). Let us consider the decay of these isobars into one baryon and one meson states. We can form the following two invariant expressions for these processes:

$$\begin{aligned} S_1 &= M_\nu^\mu f_\lambda^\nu T_\mu^\lambda, \\ S_2 &= M_\nu^\mu f_\mu^\lambda T_\lambda^\nu, \end{aligned} \quad (3)$$

where we have put  $M_\nu^\mu = (N_\mu^\nu)^\dagger$  for creation operators of baryons. This occurrence of two independent forms corresponds to the double appearance of  $U_3(1, 0, -1)$  representation in the product  $U_3(1, 0, -1) \times U_3(1, 0, -1)$  as we can see from Eq. (2), and thus the same situation does not happen to other representations in the right-hand side of Eq. (2). At any rate, we cannot determine the branching ratio of  $Y_1^* \rightarrow \Sigma' + \pi$  against  $Y_1^* \rightarrow \Lambda + \pi$  in our case, unless we make some additional assumptions. One tempting hypothesis is to assume the invariance of our theory under the transpose operation; i.e. we assume the invariance under interchanges of lower and upper suffixes. By this operation, a tensor  $F_\nu^\mu$  is changed into  $F_\mu^\nu$ , so that  $S_1 \leftrightarrow S_2$  in Eq. (3) and we have the following from Eq. (1).

$$\begin{aligned} \pi_+ \leftrightarrow \pi_-, \quad \pi_0 \leftrightarrow \pi_0, \quad K_+ \leftrightarrow \bar{K}_+, \quad K_0 \leftrightarrow \bar{K}_0, \quad \pi_0' \leftrightarrow \pi_0', \\ \Sigma'_+ \leftrightarrow \Sigma'_-, \quad \Sigma'_0 \leftrightarrow \Sigma'_0, \quad p \leftrightarrow \bar{E}_-, \quad n \leftrightarrow \bar{E}_0, \quad \Lambda \leftrightarrow \Lambda. \end{aligned} \quad (4)$$

We may note that similar transformations have already been proposed by many authors.<sup>7)</sup> Then, we can compute the kinematical weights for the various processes, since the only invariant expression is now  $S_1 + S_2$  instead of an arbitrary linear combination of  $S_1$  and  $S_2$  of Eq. (3). We list our results obtained in this fashion in the following tables. In Table I the relative weights have their origin in numerical coefficients due to generalized Clebsch-Gordon coefficients. If we could neglect the mass differences among baryons, then the widths for these processes are proportional to the relative weights. However, we should take account of the baryon mass differences at least for the calculation of the phase-volume. Thus, for evaluations of relative widths, we should multiply to these weights the  $d$ -wave phase volume which is given by

$$\frac{1}{M^2} \cdot k^5 \tag{5}$$

where  $M$  is the mass of the mother isobar, and  $k$  is the magnitude of the spatial momentum of the meson in the rest system of the isobar.

Table I. Relative weights and widths for decays in case (a).

type of process	relative weight	relative width
$(N^{**})_{+} \rightarrow \begin{cases} n + \pi_{+} \\ p + \pi_{0} \end{cases}$	1	1
$(Y_1^{*})_{+} \rightarrow \Lambda + \pi_{+}$	4/9	0.014
$(Y_1^{*})_{+} \rightarrow \Sigma_{+,0} + \pi_{0,+}$	0	0
$(Y_0^{*})_{0} \rightarrow \Sigma_{\pm,0} + \pi_{\mp,0}$	4/3	0.008
$(\Xi)_{-}^{*} \rightarrow \Xi_{0,-} + \pi_{-,0}$	1	?

One interesting aspect is that  $Y_1^{*}$  does not decay into a pion and a  $\Sigma'$ , in agreement with experiment. However, this is not characteristic only of the present scheme, since the representation  $U_3(2, 0, -2)$  also forbids  $Y_1^{*} \rightarrow \Sigma' + \pi$ . Actually, it is a natural consequence of the invariance of theory under the transpose operation Eq. (4), as has been shown by Sakurai.<sup>7)</sup> As we shall see shortly, the representation  $U_3(2, 0, -2)$  is also invariant under this operation.

Now, we will investigate the case (c), since the case (b) is quite trivial.

Case (c) :  $U_3(2, 0, -2)$

This is a 27-dimensional representation, which is characterized by a tensor  $T_{\alpha\beta}^{\mu\nu}$  having the following properties :

$$T_{\alpha\beta}^{\mu\nu} = T_{\alpha\beta}^{\nu\mu} = T_{\beta\alpha}^{\mu\nu}, \quad T_{\mu\beta}^{\mu\nu} = 0. \tag{6}$$

We can form a base of the unitary representation  $U_3(2, 0, -2)$  from this  $T_{\alpha\beta}^{\mu\nu}$ , which is given by

(i)  $(I=2, Y=0)$

$$T_{11}^{22}, T_{12}^{22} - T_{11}^{12}, \frac{1}{\sqrt{6}}(T_{11}^{11} + T_{22}^{22} - 4T_{12}^{12}), T_{12}^{11} - T_{22}^{12}, T_{22}^{11}.$$

$$(ii) \quad (I=3/2, Y=1)$$

$$\sqrt{2} T_{11}^{23}, \sqrt{\frac{2}{3}}(2T_{12}^{23} - T_{11}^{13}), -\sqrt{\frac{2}{3}}(2T_{12}^{13} - T_{22}^{23}), -\sqrt{2} T_{22}^{13}.$$

$$(iii) \quad (I=3/2, Y=-1)$$

$$\sqrt{2} T_{13}^{22}, -\sqrt{\frac{2}{3}}(2T_{13}^{12} - T_{23}^{22}), -\sqrt{\frac{2}{3}}(2T_{23}^{12} - T_{13}^{11}), \sqrt{2} T_{23}^{11}.$$

$$(iv) \quad (I=1, Y=2)$$

$$T_{11}^{33}, \sqrt{2} T_{12}^{33}, T_{22}^{33}.$$

$$(v) \quad (I=1, Y=-2)$$

$$T_{33}^{22}, -\sqrt{2} T_{33}^{12}, T_{33}^{11}.$$

$$(vi) \quad (I=1, Y=0)$$

$$\sqrt{5} T_{31}^{32}, \sqrt{\frac{5}{2}}(T_{23}^{23} - T_{13}^{13}), -\sqrt{5} T_{23}^{13}.$$

$$(vii) \quad (I=1/2, Y=1)$$

$$\sqrt{\frac{10}{3}} T_{31}^{33}, \sqrt{\frac{10}{3}} T_{32}^{33}.$$

$$(viii) \quad (I=1/2, Y=-1)$$

$$\sqrt{\frac{10}{3}} T_{33}^{23}, -\sqrt{\frac{10}{3}} T_{33}^{13}.$$

$$(ix) \quad (I=0, Y=0)$$

$$\sqrt{\frac{10}{3}} T_{33}^{33}.$$

In the table listed in the above, all terms in a given sub-classification as  $(I, Y)$  have the same transformation properties as spherical harmonics  $Y_M^{(I)}$  ( $M=I, I-1, \dots, -I$ ) in the decreasing order from the left to the right. The relative numerical coefficients belonging to different sub-classifications with different  $(I, Y)$  have been determined from a requirement that

$$\sum_{\mu, \nu, \alpha, \beta} (T_{\alpha\beta}^{\mu\nu})^* T_{\alpha\beta}^{\mu\nu} = \sum_{A=1}^{27} (X_A)^* X_A \quad (7)$$

where  $X_A$  ( $A=1, \dots, 27$ ) represents each term listed in the above. The condition Eq. (7) shows that these 27  $X_A$ 's form the desired unitary base of our representation  $U_3(2, 0, -2)$ . Thus, we can identify each  $X_A$  with each isobar states appearing in  $U_3(2, 0, -2)$  as in Eq. (1).

In this case, we have an undesired isobar with  $(I=1, Y=2)$ , which could be detected in kaon-nucleon scattering but so far not found. However, we may

suppose again that such state would not appear if we take the mass differences among mesons and among baryons. We may identify  $(I=1, Y=0)$ ,  $(I=0, Y=0)$  and  $(I=3/2, Y=1)$  with  $Y_1^*$ ,  $Y_0^*$  and  $N^*$ , respectively, where  $N^*$  represents for the first pion-nucleon resonance. Then, all these have to be resonances in the  $p_{3/2}$  states, since the last one is known to be so. Then, the state with  $(I=1/2, Y=1)$  in our representation must be a resonance in  $p_{3/2}$  state also and thus this state is difficult<sup>5)</sup> to be identified with  $N^{**}$ , though we cannot completely rule out a possibility of the  $p_{3/2}$  resonance for  $N^{**}$  at the moment. The possible existence of other states in  $U_3(2, 0, -2)$  does not lead to any disagreement with the experimental data.

Now, let us consider the decay matrix element of isobars into mesons and baryons. In this case, there is only one invariant form under  $U_3$ .

$$S = M_{\alpha\mu} f_{\beta\nu} T_{\mu\nu}^{\alpha\beta}. \tag{7'}$$

We may note that Eq. (7') is invariant under the transpose operation as has been mentioned already. We can reduce this in terms of  $X_A$  and of meson and baryon components by using the table listed in the above and by Eq. (1).

Table II. Relative weights and widths for decays in the case (c).

type of the decay	relative weight	relative width
$(N^*)_{++} \rightarrow p + \pi_+$	1	1
$(Y_1^*)_+ \rightarrow \Lambda + \pi_+$	3/5	0.36
$(Y_1^*)_+ \rightarrow \Sigma_{+,0} + \pi_{0,+}$	0	0
$(Y_0^*) \rightarrow \Sigma_{\pm,0} + \pi_{\mp 0}$	1/20	0.01
$(N^{**})_+ \rightarrow \begin{cases} p + \pi_0 \\ n + \pi_+ \end{cases}$	1/10	0.45
$(Y_2^*)_{++} \rightarrow \Sigma_+ + \pi_+$	2	?
$(Z)_{++} \rightarrow p + K$	2	?

Then, we can compute the kinematical weight factors for the decay as before. For the calculation of the relative widths in the above table, we have multiplied the  $p$ -wave phase volume factor:

$$k^3/M^2. \tag{8}$$

Again,  $Y_1^*$  does not decay into  $\Sigma + \pi$ , because of the transpose invariance of  $U_3(2, 0, -2)$  as has been mentioned already. In the table,  $Y_2^*$  means the state with  $(I=2, Y=0)$ , and  $Z$  represents the state with  $(I=1, Y=2)$ .

We should note that appearance of  $Y_2^*$ ,  $Y_1^*$  and  $Y_0^*$  could be easily understood<sup>8)</sup> in terms of the static  $p$ -wave pion-hyperon interactions, if we assume that  $f_{\Lambda\Sigma} \gg f_{\Sigma\Sigma}$ . Indeed, this is the case if we take the  $D$ -type interaction<sup>1),2)</sup> in Gell-Mann's notation, which is also invariant under the transpose transformation.

Case (d):  $U_3(2, -1, -1)$

This is a 10-dimensional representation, and can be specified by a tensor

$F_{\alpha\beta}^{\mu\nu}$  having the following properties :

$$F_{\alpha\beta}^{\mu\nu} = F_{\beta\alpha}^{\nu\mu} = -F_{\alpha\beta}^{\nu\mu}, \quad F_{\mu\beta}^{\mu\nu} = 0. \quad (9)$$

The unitary base  $X_A (A=1, \dots, 10)$  of  $U_3(2, -1, -1)$  can be formed from  $F_{\alpha\beta}^{\mu\nu}$  in the same way as in the previous case, giving that

(i)  $(I=3/2, Y=1)$

$$F_{11}^{23}, \sqrt{3} F_{12}^{23} (\equiv -\sqrt{3} F_{11}^{13}), -\sqrt{3} F_{12}^{13} (\equiv \sqrt{3} F_{22}^{23}), -F_{22}^{13}.$$

(ii)  $(I=1, Y=0).$

$$\sqrt{3} F_{13}^{23} (\equiv \sqrt{3} F_{11}^{12}), \sqrt{6} F_{12}^{12} (\equiv -\sqrt{6} F_{13}^{13} \equiv \sqrt{6} F_{23}^{23}), \\ -\sqrt{3} F_{23}^{13} (\equiv \sqrt{3} F_{22}^{12}).$$

(iii)  $(I=1/2, Y=-1)$

$$\sqrt{3} F_{13}^{12} (\equiv \sqrt{3} F_{33}^{22}), -\sqrt{3} F_{33}^{13} (\equiv \sqrt{3} F_{23}^{12}).$$

(iv)  $(I=0, Y=-2)$

$$F_{33}^{12}.$$

It is interesting to note that we have a particle with the strangeness  $-3$ . The decay matrix element is again unique and has the same form as Eq. (7') when we replace  $T_{\alpha\beta}^{\mu\nu}$  by  $F_{\alpha\beta}^{\mu\nu}$ . Then, once again we can compute the weights and the relative widths. Now, we have the decay  $Y_1^* \rightarrow \pi + \Sigma$  in this case.

Table III. Relative weights and widths for decays in the case (d).

type of decay	relative weight	relative width
$(N^*)_{++} \rightarrow p + \pi_+$	1	1
$(Y_1^*)_{+} \rightarrow A + \pi_+$	1/2	0.30
$(Y_1^*)_{+} \rightarrow \Sigma_{0,+} + \pi_{+,0}$	1/3	0.043

Case (e) :  $U_3(1, 1, -2)$

This is the contragradient representation of  $U_3(2, -1, -1)$ ; i.e. the one which can be obtained from  $U_3(2, -1, -1)$  by the transpose operation. Thus, it is specified by a tensor  $G_{\alpha\beta}^{\mu\nu}$  satisfying the following conditions.

$$G_{\alpha\beta}^{\mu\nu} = G_{\beta\alpha}^{\nu\mu} = -G_{\alpha\beta}^{\nu\mu}, \quad G_{\mu\beta}^{\mu\nu} = 0. \quad (10)$$

Similarly, we can construct the unitary base by

(i)  $(I=3/2, Y=-1)$

$$G_{13}^{22}, -\sqrt{3} G_{13}^{12} (\equiv \sqrt{3} G_{23}^{22}), -\sqrt{3} G_{23}^{12} (\equiv \sqrt{3} G_{13}^{11}), G_{23}^{11}.$$

(ii)  $(I=1, Y=0)$

$$-\sqrt{3} G_{13}^{23} (\equiv \sqrt{3} G_{12}^{22}), -\sqrt{6} G_{12}^{12} (\equiv \sqrt{6} G_{13}^{13} \equiv -\sqrt{6} G_{23}^{23}), \\ \sqrt{3} G_{23}^{13} (\equiv \sqrt{3} G_{12}^{11}).$$

(iii)  $(I=1/2, Y=1)$

$$\sqrt{3} G_{13}^{33} (\equiv -\sqrt{3} G_{12}^{33}), \sqrt{3} G_{12}^{13} (\equiv \sqrt{3} G_{23}^{33}).$$

(iv)  $(I=1/2, Y=+2)$

$$G_{12}^{33}.$$

We may identify  $(I=1, Y=0)$  and  $(I=1/2, Y=1)$  with  $Y_1^*$  and  $N^{**}$ , respectively, and we can compute the widths in a similar fashion.

Table IV. Relative weights and widths for decays in the case (e).

type of decay	relative weight	relative width
$(N^{**})_+ \rightarrow N + \pi$	1	1
$(Y_1^*)_+ \rightarrow \Sigma_{+,0} + \pi_{0,+}$	2/3	$1.9 \times 10^{-3}$
$(Y_1^*)_+ \rightarrow \Lambda + \pi_+$	1	$3.2 \times 10^{-2}$

Finally, we shall give an application of the mass formula, which has been derived in (I). For particles belonging to the same irreducible representation, we have a relation among masses of these particles. It is given by

$$M = a + b \cdot Y + c \cdot [1/4 Y^2 - I(I+1)] \tag{11}$$

where  $a$ ,  $b$  and  $c$  are some constants. This relation has been proved in the lowest order perturbation of a certain type of interactions causing the mass-differences, but in all orders of the  $U_3$ -conserving interactions. As has been stated in the beginning, this would not be a good approximation for the meson-baryon scattering problem, where the mass differences between the pion and the kaon is quite important. Thus, we should not expect that our results to be given in the below have some quantitative meanings. At any rate, Eq. (11) has three unknown constants,  $a$ ,  $b$  and  $c$ . Thus, we have six relations among masses of particles contained in  $U_3(2, 0, -2)$ . If we use the experimental masses of  $Y_1^*$ ,  $Y_0^*$  and  $N^*$ , then the masses of six other particles in  $U_3(2, 0, -2)$  can be computed in terms of these three masses. In this way, we have

$$\begin{aligned} M(I=2, Y=0) &\simeq 1345 \text{ Mev}, \\ M(I=3/2, Y=-1) &\simeq 1505 \text{ Mev}, \\ M(I=1, Y=2) &\simeq 1125 \text{ Mev}, \\ M(I=1, Y=-2) &\simeq 1665 \text{ Mev}, \\ M(I=1/2, Y=1) &\simeq 1265 \text{ Mev}, \\ M(I=1/2, Y=-1) &\simeq 1535 \text{ Mev}. \end{aligned} \tag{12}$$

A serious trouble is that the mass of the particle with  $(I=1, Y=2)$  is so low that it is stable against the decay into a nucleon and a kaon. However, this difficulty may not be so serious, since such state may disappear as remarked



already. We may note that we have a similar trouble in the case of the Sakata scheme.<sup>1)</sup> It is also interesting to compare Eqs. (11) and (12) to those obtained in the case of the global symmetry model,<sup>9)</sup> and to those of the Sakata scheme.<sup>1),10)</sup>

We have made a group-theoretical classification of isobar states. As has been mentioned in the beginning, almost all of the results given in this paper are also immediately applicable to the study of the meson-meson resonances or of the baryon-baryon scatterings, with small modifications. However, we would not go into details for these cases. From our analysis on baryon isobars, it seems to be difficult to identify the best irreducible representation for these at the moment. One interesting problem is to determine the parity of the resonances so as to enable us to distinguish whether the resonances are of the  $p_{3/2}$  or  $d_{3/2}$  character.

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### References

- 1) S. Okubo, Prog. Theor. Phys. **27**(1962), 949. This paper will hereafter be referred to as (I).
- 2) Y. Yamaguchi, private communication, 1960.  
M. Gell-Mann, Phys. Rev. **125** (1962), 1067.
- 3) M. Ikeda, S. Ogawa and Y. Ohnuki, Prog. Theor. Phys. **22** (1959), 715; **23** (1960), 1073.
- 4) For example, Y. Ne'eman, Nuclear Phys. **26** (1961), 222.  
A. Salam and J. C. Ward, Nuovo Cimento **20** (1961), 419.
- 5) J. O. Maloy et al., Phys. Rev. **122** (1961), 1338.  
B. J. Moyer, Rev. Mod. Phys. **33** (1961), 367.
- 6) R. P. Ely et al., Phys. Rev. Letters **7** (1961), 461.
- 7) For example, G. Feinberg and R. E. Behrends, Phys. Rev. **115** (1959), 745.  
J. J. Sakurai, Phys. Rev. Letters **7** (1961), 426.
- 8) M. Uehara, talk given at a seminar held in Kyoto, Oct., 1961.  
It appears that J. Franklin also has done a similar analysis. This was pointed out by J. J. Sakurai in a footnote of his paper which has appeared in Phys. Rev. Letters **7** (1961), 355.
- 9) R. E. Marshak and S. Okubo, Prog. Theor. Phys. **26** (1961), 380.  
T. D. Lee and C. N. Yang, Phys. Rev. **122** (1961), 1954.
- 10) K. Matumoto, Prog. Theor. Phys. **16** (1956), 583.  
S. Sawada and M. Yonezawa, Prog. Theor. Phys. **23** (1960), 662.