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Notes and Comment

Resolution in one dimension with random variations in background dimensions

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In the usual one-dimensional resolution (discrimination or identification) experiment, the stimuli vary along one dimension, the *target* dimension, and the observer is required to respond differentially on the basis of the stimulus value along that dimension. In this note, we focus on experiments in which the observer's task remains the same, but in which the other parameters that define the stimuli, the *background* parameters, are varied randomly over specified ranges of these parameters. Thus, for example, the stimulus set might consist of tone pulses of fixed duration, the target parameter T might be the carrier frequency of the tone pulse, and the background parameter B that is randomly varied might be the amplitude of the tone pulse. The terms *FIXED* or *CERTAIN* are used for situations of the first type, whereas *ROVED* or *UNCERTAIN* are used for situations of the second type. Obviously, paradigms of the second type converge to those of the first type as the range of variation for the background parameters goes to zero. In this sense, FIXED paradigms can be regarded as a special case of ROVED paradigms.

ROVED paradigms have been used for a number of purposes. For example, they have been used to assess perceptual interaction between dimensions. If uncertainty in a background dimension does not affect performance in the target dimension, the dimensions are called *separable*; otherwise, they are *integral* (e.g., see Ashby & Townsend, 1986; Garner, 1974). Similarly, such paradigms have been used to eliminate artifactual cues in the measurement of resolution for subjective parameters. Thus, continuing with the above example, if one wants to measure the ability to discriminate tone pitch (as opposed to tone frequency), it is important to eliminate loudness cues. Generally speaking, this can be achieved much more reliably by randomly varying the amplitudes of the tone pulses than by attempting to precisely equate their loudness. Similarly, overall amplitude is usually roved in experiments on the discrimination of spectral shape (e.g., see Durlach, Braida, & Ito, 1986; Farrar et al., 1987; Green, 1983, 1988; Kidd & Mason, 1988; Kidd, Mason, & Green, 1986), and fundamental frequency is sometimes roved in experiments that focus on the phonetic characteristics of speech segments (e.g., see Macmillan, Goldberg, & Braida, 1988).

With the above distinction between FIXED and ROVED paradigms in mind, we can describe the purposes of the present note as follows: (1) to extend our previously developed model of unidimensional resolution (Durlach & Braida, 1969) to ROVED paradigms, and (2) to consider the possibility that resolution for multidimensional identification can always be approximated by the sum of resolution measures for the component unidimensional cases, provided only that the unidimensional tests involve the appropriate roving of background parameters. These issues are considered further in sections 1 and 2, respectively.

1. Extension of model to one-dimensional resolution experiments with roved parameters

According to our initial unidimensional model for FIXED paradigms, the sensitivity index d' for discrimination and identification is given by

$$d'(T_1,T_2) = \frac{\alpha(T_2)-\alpha(T_1)}{\beta} \quad \text{for FIXED discrimination} \tag{1}$$

and

$$d'(T_1,T_2) = \frac{\alpha(T_2)-\alpha(T_1)}{\sqrt{\beta^2+G^2R^2}} \quad \text{for FIXED identification,} \tag{2}$$

where T_1 and T_2 are stimulus values along the given dimension: $\alpha(T)$ is the sensation mean at T; β^2 is the variance due to sensation noise; $(GR)^2$ is the variance due to memory noise in the context-coding mode; R is the range $\alpha(T_{\max})-\alpha(T_{\min})$; and G, like β, is a constant. (For simplicity, we employ the early version of our context-coding model, as described in Durlach & Braida, 1969, rather than the revised version based on perceptual anchors, described in Braida et al., 1984.)

This model, originally proposed in connection with the study of intensity perception in audition, has also been successfully applied to the azimuth dimension in auditory localization (Searle, Colburn, Davis, & Braida, 1976), to several dimensions that distinguish speech sounds (Macmillan, 1987), and to length resolution in manual manipulation of objects (Durlach et al., 1989).

In the proposed extension of the model, we assume that any reduction in T resolution caused by roving a background parameter B arises from either or both of two

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sources: (1) an increase in the sensation variance β^2 or (2) an increase in the context-coding factor G. Denoting the enlarged sensation variance by $\tilde{\beta}^2$ and the enlarged context-coding variance by $(\tilde{G}R)^2$, we can describe the effect of B uncertainty merely by rewriting Equations 1 and 2, with $\tilde{\beta}$ substituted for β and $\tilde{G}R$ substituted for GR. The model then provides a concise summary of uncertainty effects in terms of the ratios $\tilde{\beta}/\beta$ and $\tilde{G}/G$ (both greater than or equal to unity). If $\tilde{\beta}/\beta$ equals one, the dimensions may be said to be *sensation separable*; otherwise, they are *sensation integral*. If $\tilde{G}/G$ equals one, they are *context separable*; otherwise they are *context integral*.

Direct experimental evaluation could be achieved by conducting the following sequence of experiments:

1. Discrimination and identification of T using a variety of T ranges R_T in the identification experiments but FIXED B (to test the original model for the variable T and evaluate the parameters β and G).

2. The same set of experiments as in (1), but with ROVED B (to test the model for the case of ROVED B and to evaluate $\tilde{\beta}$ and $\tilde{G}$ for various ranges R_B of the B rove).

These experiments would not only allow one to check the general applicability of the model, but, if applicable, would also determine the level of influence the variable B has on the variable T [i.e., the functions $\tilde{\beta}(R_B)$ and $\tilde{G}(R_B)$]. If uncertainty in B has no effect on the identification of T, it should also have no effect on discrimination, and the two dimensions would be separable [i.e., $\tilde{\beta}(R_B) = \beta$, $\tilde{G}(R_B) = G$ for all R_B]. If uncertainty affects identification but not discrimination, the interaction is at the level of memory, and the dimensions are sensation separable but context integral [i.e., $\tilde{\beta}(R_B) = \beta$, $\tilde{G}(R_B) > G$ for nonzero R_B]. If uncertainty affects both tasks, and the effect on identification can be completely accounted for by the effect on discrimination, the interaction is at the level of sensation and the dimensions are sensation integral but context separable [i.e., $\tilde{G}(R_B) = G$, $\tilde{\beta}(R_B) > \beta$ for nonzero R_B]. The fourth possibility, that both sensation and memory interactions exist, follows if there is an identification uncertainty effect that cannot be accounted for by a nonzero discrimination effect [i.e., $\tilde{\beta}(R_B) > \beta$ and $\tilde{G}(R_B) > G$ for nonzero R_B].

To the extent that the model is correct, it can be used to assess sensation and context separability even without varying R_T and R_B, because the separability attributes depend only on ratios of model parameters. More explicitly, suppose that d' for a stimulus pair (or total d' for an entire stimulus range) has been measured in both discrimination and identification, each with fixed and roved B. Then

$$(\tilde{\beta}/\beta)^2 = (d'_{\text{FIXEDdisc.}}/d'_{\text{ROVEDdisc.}})^2, \quad (3)$$

and

$$(\tilde{G}/G)^2 = \frac{(d'_{\text{ROVEDident.}})^{-2} - d'_{\text{ROVEDdisc.}})^{-2}}{(d'_{\text{FIXEDident.}})^{-2} - (d'_{\text{FIXEDdisc.}})^{-2}}. \quad (4)$$

Two recent studies in our laboratory, one auditory and one tactile, illustrate the application of the model. In the auditory experiment (summarized in Macmillan, Braida, & Golderg, 1987), discrimination and identification were measured on a synthetic six-step vowel continuum. In conditions with uncertainty, the fundamental frequency F_0 varied randomly among four values; in conditions without uncertainty, it was constant. Discrimination performance was essentially unchanged by uncertainty [the estimate of $(\tilde{\beta}/\beta)^2$ from Equation 3 was 1.02], so the dimensions F_0 and vowel value were sensation separable. Identification was moderately reduced by uncertainty [$(\tilde{G}/G)^2$ was 2.24], indicating some degree of context integrality.

In the tactile experiment (Tan, Rabinowitz, & Durlach, in press), a four-dimensional display was used. Unidimensional identification and discrimination experiments were performed for each dimension while the other parameters were held fixed. The unidimensional experiments were then repeated with the background parameters (all of them) varying randomly over their entire ranges. Results for the four dimensions differed, but all showed a greater degree of sensation integrality than context integrality: $(\tilde{\beta}/\beta)^2$ averaged 10.5, whereas $(\tilde{G}/G)^2$ averaged only 2.9. For this multidimensional stimulus set, the primary effect of uncertainty in the background parameters is on sensation variance.

In general, these interaction effects need not be symmetric (e.g., see Wood, 1974). Thus, a complete evaluation would require measurements with the roles of the parameters T and B reversed. Similarly, the designations of *separable* and *integral* must be subscripted to indicate the direction of the influence ($B \rightarrow T$ or $T \rightarrow B$).

Finally, it should be noted that in the above discussion we have ignored cases in which resolution in the target parameter T depends on the value of the background parameter B in the FIXED experiments. In other words, in terms of the model, we have ignored cases in which one or both of the parameters β and G associated with the FIXED case changes significantly with a change in B. If the dependence of β and G on B in the FIXED paradigm is strong over the range of B used in the ROVED paradigm, then one must consider the ratios $\tilde{\beta}/\beta$ and $\tilde{G}/G$ for a variety of B in this range. The overall effect of B uncertainty can then be summarized by computing averages over B. Alternatively, this effect can be measured by comparing, for each value of B, the values of β and G obtained from FIXED experiments to the values of $\tilde{\beta}$ and $\tilde{G}$ obtained by segregating the responses according to the value of B in the ROVED experiment. Clearly, adequate modelling of these more complicated cases requires further study.

2. *Generalization of additivity laws relating multidimensional and unidimensional resolution measures*

Consider now the case of two-dimensional identification in which statistically independent variation occurs in both T and B, and the task is to identify the pair (T,B).

The idea that performance on the two-dimensional task is equal to the sum of the performance measures on the component one-dimensional tasks when the individual dimensions are "independent" has been expressed in a variety of metrics. In detection-theory terms,

$$d'^2[(T_1,B_1),(T_2,B_2)] = d'^2(T_1,T_2) + d'^2(B_1,B_2), \quad (5)$$

where $d'[(T_1,B_1),(T_2,B_2)]$ is the two-dimensional sensitivity index, and $d'(T_1,T_2)$ and $d'(B_1,B_2)$ are the unidimensional sensitivity indices. In information-theory terms,

$$I(T,B) = I(T) + I(B), \quad (6)$$

where $I(T,B)$ is the information transfer for the two-dimensional case, and $I(T)$ and $I(B)$ are the information transfers for the corresponding unidimensional cases. Exact conditions for Equations 5 and 6 to hold have been investigated by Ashby and Townsend (1986). Since methods for estimating $d'[(T_1,B_1),(T_2,B_2)]$ from the multidimensional confusion matrix are only now being fully developed (e.g., see Braida, 1988; Nosofsky, 1986), in this note we restrict our attention to Equation 6.

In most cases examined empirically, multidimensional information transfer is less than the sum of the component unidimensional transfers (e.g., see Egeth & Pachella, 1969; Garner, 1962; Rabinowitz, Houtsma, Durlach, & Delhorne, 1987; Tan et al., in press). In these and other applications, however, multidimensional performance is compared with one-dimensional performance in experiments with FIXED background parameters. A general principle of our unidimensional model that might be extended to the multidimensional case is that the variability limiting performance is a function of the *stimulus set*. According to this principle, multidimensional identification performance is best predicted from unidimensional identification with ROVED background parameters. In fact, and as discussed recently in Tan et al. (in press), it is possible that Equation 6 *always* holds, independent of whether the variables are separable or integral, provided only that the background parameters are roved over the appropriate ranges in the unidimensional tests. Only in the case of complete sensation and context separability will the results on multidimensional identification be predictable from results on unidimensional identification with FIXED background.

Evaluation of these ideas requires that we conduct the following experiments, in addition to Experiments 1 and 2 of section 1:

3. All the experiments in series 1 and 2 but with the roles of parameters T and B interchanged (to evaluate the effect of uncertainty in T on resolution in B).

4. Two-dimensional identification experiments with statistically independent variation in T and B, using the same ranges of T and B employed in the above experiments.

This sequence of experiments, which we hope to perform in the future, clearly represents a lengthy undertaking. For cases of more than two dimensions (for which the theoretical generalizations are obvious), empirical testing will be even more onerous. Limited data bearing on Equation 6 have, however, been collected by Tan et al. (in press) using a tactile display. In addition to the one-dimensional tasks already described, Tan et al. measured four-dimensional identification, with each variable covering the same range as in the one-dimensional tasks. Transmitted information, averaged over subjects and corrected for bias (see Houtsma, 1983), was 3.3 bits for four-dimensional identification, 6.5 bits for the sum of the four one-dimensional FIXED tests, and 3.4 bits for the sum of the four one-dimensional ROVED tests. In other words, the deficit in the multidimensional information transfer (relative to the sum of the unidimensional FIXED information transfers) is adequately reflected in the difference between the sum of the FIXED information transfers and the sum of the ROVED information transfers. This implies, in particular, that the reduction in performance for multidimensional identification caused by the increased complexity of the response code in multidimensional identification is negligible. Note also that, together with the extension of our unidimensional model to the case of roved parameters (discussed in section 1), it greatly simplifies the task of extending our model to the cases of multidimensional identification.

In conclusion, we note that the above-mentioned ideas have significant practical as well as theoretical implications. For example, to the extent that further research confirms the general additivity law, the amount of experimental time required to estimate multichannel information transfer should be drastically reduced: one can estimate N one-dimensional information transfers much more quickly than one N-dimensional information transfer (to the same degree of accuracy). This saving, combined with the savings that can be achieved through application of the extended model described in section 1, can be substantial.

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