NOTES AND CORRESPONDENCE

An Analysis of Errors in Drop Size Distribution Retrievals and Rain Bulk Parameters with a UHF Wind Profiling Radar and a Two-Dimensional Video Disdrometer

LAURA KANOFSKY AND PHILLIP CHILSON

School of Meteorology, University of Oklahoma, Norman, Oklahoma

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ABSTRACT

Vertically pointed wind profiling radars can be used to obtain measurements of the underlying drop size distribution (DSD) for a rain event by means of the Doppler velocity spectrum. Precipitation parameters such as rainfall rate, radar reflectivity factor, liquid water content, mass-weighted mean drop diameter, and median volume drop diameter can then be calculated from the retrieved DSD. The DSD retrieval process is complicated by the presence of atmospheric turbulence, vertical ambient air motion, selection of fall speed relationships, and velocity thresholding. In this note, error analysis is presented to quantify the effect of each of those factors on rainfall rate. The error analysis results are then applied to two precipitation events to better interpret the rainfall-rate retrievals.

It was found that a large source of error in rain rate is due to unaccounted-for vertical air motion. For example, in stratiform rain with a rainfall rate of $R = 10$ mm h^{-1}, a mesoscale downdraft of 0.6 m s^{-1} can result in a 34% underestimation of the estimated value of R. The fall speed relationship selection and source of air density information both caused negligible errors. Errors due to velocity thresholding become more important in the presence of significant contamination near 0 m s^{-1}, such as ground clutter. If particles having an equivalent volume diameter of 0.8 mm and smaller are rejected, rainfall rate errors from -4% to -10% are possible, although these estimates depend on DSD and rainfall rate.

1. Introduction

During periods of precipitation, vertically pointed wind profiling radars can be used to measure the drop size distribution (DSD) through the Doppler velocity spectrum. This is achieved by directly mapping the Doppler spectrum from velocity space into diameter space. This procedure assumes that the Doppler velocities detected by the profiler are primarily due to falling hydrometeors. That is, effects of vertical ambient air motion and atmospheric turbulence are either not present or have been removed. Furthermore, it is typical to assume that Rayleigh scatter from discrete particles is the dominant contribution to the radar signal for a 915-MHz profiler operating during periods of precipitation. Under these conditions, the DSD can be directly retrieved from the Doppler spectrum by applying an appropriate expression relating drop diameter to terminal fall speed. Precipitation parameters such as rainfall rate, radar reflectivity factor, liquid water content, mass-weighted mean drop diameter, and median volume drop diameter can in turn be calculated from the retrieved DSD. Unlike in situ instruments located at the surface, measurements from a UHF profiler can be used to investigate the evolution of these parameters with height.

Several factors complicate the DSD retrieval process. First, the retrieval method relies on the assumption that there is no significant vertical ambient air motion. The presence of undetected updrafts and downdrafts will bias the retrieved number concentration. Second, several different fall speed relationships have been published, so it is not immediately clear which of those expressions should be used to infer drop diameters from radar-observed terminal velocities. Since the retrieval method assumes a single valid fall speed relationship, different expressions will result in different

Corresponding author address: Laura Kanofsky, School of Meteorology, University of Oklahoma, 120 David L. Boren Blvd., Suite 5900, Norman, OK 73072-7307.
E-mail: laura.kanofsky@noaa.gov

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retrievals. Third, an inherent artifact of the DSD retrieval process prohibits accurate retrievals of the number concentration of very small drops. This mathematical artifact is caused by near-zero velocities and is described in detail in section 5a. Therefore, the DSD retrievals at very small drop sizes must be excluded from precipitation parameter calculations; however, the diameter threshold (minimum diameter at which retrieved DSD information should be included) becomes arbitrary. Ground clutter at the lowest sampling heights further exacerbates this issue. Each of these considerations introduces errors into DSD retrievals that propagate through the precipitation estimations. These factors are examined and error analyses are presented with supporting data. The effects of atmospheric turbulence and broadening of the Doppler spectrum are not considered here.

In this note, the effects of error analyses are examined with datasets from central Oklahoma precipitation systems. The principal measurements were made with a 915-MHz boundary layer radar (BLR) and a collocated two-dimensional video disdrometer (2DVD). Emphasis is placed on nonconvective systems because this retrieval method assumes that no significant updrafts or downdrafts are present. Rainfall parameters including rainfall rate, radar reflectivity factor, mass-weighted mean diameter, and median volume diameter are compared between the lowest sampled height from the BLR and the 2DVD. This study is motivated by ongoing comparisons with the National Oceanic and Atmospheric Administration/National Severe Storms Laboratory (NOAA/NSSL) polarimetric S-band weather radar KOUN (Teshiba et al. 2007, manuscript submitted to *J. Atmos. Oceanic Technol.*).

2. Background

Atmospheric studies with wind profilers span several decades. This section briefly summarizes several investigations of DSD retrievals and some of the ways that profilers have been used to investigate issues that may affect those retrievals. Most of the DSD retrieval studies mentioned here used a vertically pointed profiler operating at a single frequency, matching the instrument and operating mode used for data collection in this study.

The main issues of DSD retrieval are described by Rogers (1967). Atlas et al. (1973) present a review and extension of earlier work on DSD retrievals from profilers. As in many early papers, Atlas et al. (1973) assumed a functional form of the DSD to derive analytical expressions for the Doppler spectrum in terms of the DSD and expressions for DSD moments. Other researchers have made assumptions concerning the form of the DSD to facilitate comparisons between instruments. For example, Ellis et al. (2003) assumed a modified gamma drop size distribution to compare retrieved DSD parameters between a polarimetric radar, a wind profiler, and a 2DVD, finding that the instruments agreed well except when large raindrops were present. Although an assumed functional form of the DSD has many advantages for researchers, it imposes a specific solution on the retrievals regardless of whether or not that solution is appropriate for the rain event. The more general approach is to retrieve the DSD directly from the Doppler spectrum without any assumptions on the form of the DSD, as demonstrated by Gossard (1988).

A perennial issue affecting DSD retrievals from wind profilers is that the Doppler spectrum includes a contribution from air motion as well as from hydrometeors. If the clear-air contribution is known, it can be removed from the Doppler spectrum prior to retrieving DSDs. Several methods for isolating and removing the clear-air contribution have been explored, including the use of multiple radar frequencies such as UHF and VHF (Schafer et al. 2002; Rajopadhyaya et al. 1999), cluster analysis (Williams et al. 2000), deconvolution (Rajopadhyaya et al. 1993; Lucas et al. 2004), fitting model spectra to observed spectra (Wakasugi et al. 1986; Kobayashi and Adachi 2005), and using measurements from a single VHF wind profiler to quantify both precipitation signals and the ambient air velocity (Campos et al. 2007).

In addition to DSD retrievals, vertically pointed wind profilers have also been used to calibrate traditional scanning radars. Campos and Zawadzki (2000) describe how a well-calibrated profiler was used to validate a disdrometer calibration. Williams et al. (2005) describe how a disdrometer was used to calibrate a profiler, which in turn was used to calibrate a scanning radar. The profiler observations were averaged in time and space to match the temporal and spatial resolution of the scanning radar. Similar work was done by Gage et al. (2000) and includes recommendations for optimal calibration conditions. Both Williams et al. (2005) and Gage et al. (2000) conclude that a well-calibrated profiler can be used to calibrate or validate observations from a scanning radar.

Wind profilers have also been used to study vertical reflectivity gradients in precipitation. One such study found that Z–R relationships (where Z is reflectivity and R is rainfall rate) in weak and medium rainfall rates are nearly constant from the ground up to below the melting layer, but that Z–R relationships for heavy rainfall rates exhibit strong height dependence (Peters et al. 2005). Another study found reflectivity gradients in the lowest few hundred meters above ground before raindrops reach the surface and noted that this causes uncertainty in calibrating a profiler with a disdrometer (Clark et al. 2005). Johnston et al. (2002) found that using a standard radar equation under conditions of sharp vertical gradients of reflectivity can cause range location errors (due to a signal processing artifact) and developed a technique to correct for these errors.

It should be noted that a treatment of error analysis can be found in Atlas et al. (1973). The work presented by Atlas et al. (1973), however, focuses on the error in the DSD due to a given error in vertical air motion w. The work presented here examines the error in rainfall rate due to vertical air motion w, different fall speed relationships, different sources of air density information, and velocity thresholding.

3. Retrieval procedure

For a vertically pointed wind profiler, a DSD can be retrieved from each Doppler spectrum at each sampled height. Two main steps compose the retrieval process. The first step is to map the Doppler spectrum (a distribution of power-weighted radial velocities; Doviak and Zrnić 1993) to a distribution of drop diameters. This is accomplished by assuming a fall speed relationship such that a value of velocity may be calculated for any value of diameter. The distribution of velocities can then be recast as a distribution of diameters by applying the inverse of the equation. One such relationship is given by Atlas et al. (1973):

$$v(D) = 3.78D^{0.67}, \quad D < 3\text{ mm},$$
$$v(D) = 9.65 - 10.3e^{-0.6D}, \quad D > 3\text{ mm}, \tag{1}$$

where D is the drop diameter in millimeters and v is the terminal velocity in meters per second. This hybrid relationship spans two diameter regimes. Another relationship is given by Brandes et al. (2002):

$$v(D) = -0.1021 + 4.932D - 0.9551D^2 + 0.079\,34D^3 - 0.002\,362D^4. \tag{2}$$

Unless otherwise stated, the fall speed relationship used in this study is Eq. (2).

After transforming the set of velocity measurements into a set of assumed diameters, the second step is to use the Doppler spectrum to obtain information about the number of drops $N(D_e)$ for each diameter bin (interval). This is done by using the Doppler spectrum to find the power, which in turn is related to the equivalent radar reflectivity factor Z_e; Z_e is measured by the radar while Z is a theoretical value calculated for a given DSD. The radar reflectivity factor is known in terms of drop diameters:

$$Z = \int_0^\infty N(D_e)D_e^6\,dD_e, \tag{3}$$

where D_e is the equivalent drop diameter in millimeters, $N(D_e)$ is the number density of hydrometeors per unit diameter bin per unit volume of air (units of m^{-3} mm^{-1}), and Rayleigh scattering is assumed. Then Z has units of mm^6 m^{-3}. When a calibrated profiler observes Rayleigh scattering from spherical raindrops in the absence of noise, vertical motion, and turbulence, Z_e calculated from a Doppler spectrum should equal Z calculated from the DSD.

Under the assumption that the Doppler spectrum due to falling hydrometeors is associated only with velocities greater than zero and using the convention that $v > 0$ indicates downward motion, the contribution of N drops of size D_e to the Doppler spectrum S at speed v can be readily calculated. This is achieved through the relationship

$$\frac{S_{rc}(v)dv}{P} = \frac{N(D_e)D_e^6 dD_e}{Z}, \tag{4}$$

where $P = \int S_{rc}(v)dv$, $S_{rc}(v)$ is the range-corrected and calibrated Doppler spectrum, D_e is the drop diameter, $N(D_e)$ is the number of drops of diameter D_e per unit diameter bin per unit volume of air, and dD_e/dv comes from the relationship between drop diameter and terminal velocity (Atlas et al. 1973; Gossard 1988). Noise estimation and removal is accomplished with the method described in Hildebrand and Sekhon (1974). Since all of the quantities except $N(D)$ are known, rearranging this equation yields

$$N(D_e) = \frac{S_{rc}(v)}{D_e^6}\frac{Z_e}{P}\frac{dv}{dD_e}, \tag{5}$$

as given in Rogers et al. (1993). In this manner, $N(D_e)$ can be retrieved from a Doppler spectrum.

To compare retrieved DSDs to those measured by the 2DVD, this study focuses on time averaging and integral parameters such as reflectivity factor Z, rain rate R, mass-weighted mean diameter D_m, and median volume diameter D_0. The reflectivity factor Z is given in Eq. (3). The rain rate R is given by

$$R = (6\pi \times 10^{-4})\int_{D_{min}}^{D_{max}} N(D_e)D_e^3 v(D_e)\,dD_e, \tag{6}$$

where D_{min} and D_{max} are the minimum and maximum diameters, respectively; $N(D_e)$ has units of m^{-3} mm^{-1};

$v(D_e)$ is the terminal velocity of the drop in meters per second; D_e is in millimeters; and R has units of millimeters per hour. In the presence of ambient air motion, $v(D_e)$ is replaced by $v(D_e) + w$, where w is the vertical velocity of the ambient air ($w > 0$ for downward motion). The mass-weighted mean diameter D_m is given by

$$D_m = \frac{\int_0^\infty D_e^4 N(D_e)\, dD_e}{\int_0^\infty D_e^3 N(D_e)\, dD_e}, \tag{7}$$

where $N(D_e)$ is the number concentration with units of $\mathrm{m^{-3}\ mm^{-1}}$, D_e is the diameter in millimeters, and D_m is in millimeters. The median volume diameter is the diameter that divides the liquid water content of the distribution in half (Blanchard 1953). The median volume diameter D_0 is given by

$$\int_0^{D_0} D_e^3 N(D_e)\, dD_e = \int_{D_0}^\infty D_e^3 N(D_e)\, dD_e, \tag{8}$$

where $N(D_e)$ is the number concentration with units of $\mathrm{m^{-3}\ mm^{-1}}$, D_e is the diameter in millimeters, and D_0 is the median volume diameter in millimeters. For measured DSDs (as opposed to an analytical model), the integration limits of zero and infinity are replaced by $D_{\min}$ and $D_{\max}$, respectively.

4. Instrumentation

a. 2DVD

The two-dimensional video disdrometer directly measures DSDs by creating a virtual measuring area with two orthogonal light sheets. Each light sheet is monitored by a line-scan camera, and drops that pass through the virtual measuring area create shadows that are detected by the cameras. This information is then processed to determine raindrop properties such as diameter, oblateness, and the number of drops of each size that fell through the measuring area. The 2DVD collects data continuously and reports these properties for consecutive 1-min periods. The 2DVD used to collect data for this study can measure drop diameters from 0.1 to 8.1 mm at intervals of 0.2 mm.

The use of parallel planes of light separated by a known vertical distance means that drop fall speed can be directly calculated by the 2DVD software. The software determines the time it takes for a drop to fall from the first light sheet to the second light sheet to calculate the velocity (Kruger and Krajewski 2002). For a more complete description of 2DVD operation, see Kruger and Krajewski (2002).

TABLE 1. Typical operating parameters for the UHF profiler used in this study.

Frequency	915 MHz
Beamwidth	9°
Average transmit power	500 W
Transmit pulse width	700 ns
Range resolution	105 m
No. of coherent integrations	100
No. of FFT points	128
Interpulse period	60 μs
Nyquist velocity	13.6 $\mathrm{m\ s^{-1}}$
Effective dwell time	23 s
No. of height gates	60
Lowest sampled height	142.5 m
Highest sampled height	6.3 km

b. Wind profiler

The wind profiler used in this investigation is a UHF boundary layer radar that operates at 915 MHz. The radar is of the design developed by the National Oceanic and Atmospheric Administration and described in Carter et al. (1995). It has a beamwidth of 9° and typically runs with an average transmit power of 500 W. Table 1 lists typical operating parameters of the vertical beam mode used during this study. For more information about wind profilers, see Carter et al. (1995). It should be noted that the wind profiler was operated without a clutter screen during the present study. The clutter screen is typically erected around the profiler and is used to minimize signals from targets near the horizon, which are illuminated by the antenna sidelobes. The emitting panel itself is covered by a radome. During this study, we did not have the clutter screen in place but the emitting panel was covered.

Typically, wind profilers and scanning Doppler radars use the convention that $v_r < 0$ indicates motion toward the radar. This study employs the opposite convention, namely, that $v_r > 0$ indicates motion toward the radar. Since the profiler antenna points vertically, motion toward the antenna indicates falling (downward motion of) hydrometeors. During periods of precipitation and in the absence of significant vertical ambient air motion and turbulence, the range of velocities composing the Doppler spectrum is due to a range of sizes of hydrometeors within that resolution volume.

It is necessary to calibrate the profiler so that the Doppler spectra will be properly scaled; otherwise, the retrieved values of $N(D)$ will be incorrect. The profiler was calibrated in a statistical manner following Williams et al. (2005). This method involves comparing uncalibrated values of Z from the profiler (lowest sampled height) to values of Z calculated from data collected by a collocated 2DVD. The calibration constant for the profiler is the mean of the differences between profiler and 2DVD measurements.

Both instruments used in this study were located at the National Severe Storms Laboratory for the 2 May 2005 event and at the Kessler Farm Field Laboratory (KFFL) (Chilson et al. 2007) for the 17 September 2006 event.

5. Error analysis

a. Vertical air motion

This DSD retrieval method assumes that all contributions to the Doppler spectrum are due to hydrometeors falling through quiescent air. If the air itself is also moving, the fall speeds measured by the profiler will represent the combined effects of the hydrometeors and the air rather than the hydrometeors alone. In stratiform rain, typical vertical velocities for ambient air are about 20–60 cm s^{-1} (Rutledge et al. 1988). Ideally, the clear-air component of the Doppler spectrum could be used to estimate the ambient air motion and correct for the bias (Wakasugi et al. 1986). It was found during this study that the precipitation signal overwhelmed the clear-air signal. Additionally, significant ground clutter also tended to mask the clear-air signal. In the absence of external information regarding the vertical air motion, this retrieval method is restricted to cases with limited vertical air motion (e.g., stratiform rain).

To gain insight into how a hidden bias in the Doppler spectrum will affect integral parameters for retrieved DSDs, it is useful to first examine the effect of vertical air motion on integral parameters for analytical DSDs and then extend the analysis to consider the effect on DSD retrievals. Table 2 lists the true rain rate for two analytical DSDs along with the rain rates that result from applying an updraft ($w < 0$, upward motion) and a downdraft ($w > 0$, downward motion). The two DSDs are Marshall–Palmer (Marshall and Palmer 1948) and Laws and Parsons (Laws and Parsons 1943). The Marshall–Palmer distribution is described by

$$N(D_e) = N_0 e^{-\lambda D_e}, \lambda = 4.1R^{-0.21}, \quad (9)$$

where $N(D_e)$ is the number concentration in m^{-3} mm^{-1}, D_e is the diameter in millimeters, R is the rainfall rate in millimeters per hour, λ has units of inverse millimeters, and $N_0 = 8000$ m^{-3} mm^{-1} is a constant. The Laws and Parsons distribution is described by

$$N(D_e) = 198\,00R^{-0.34} D_e^{2.93} e^{-5.38 D_e R^{-0.186}}, \quad (10)$$

TABLE 2. Effect of ambient air velocity on rain rate. For both cases, $|w| = 0.6$ m s^{-1}, representing a worst-case scenario for stratiform rain. The distributions are given by Marshall and Palmer (1948) and Laws and Parsons (1943).

DSD	True r (mm h^{-1})	Updraft R (mm h^{-1})	Downdraft R (mm h^{-1})
Marshall–Palmer	5	4.37	5.63
Marshall–Palmer	10	8.86	11.14
Marshall–Palmer	20	17.91	22.09
Marshall–Palmer	50	45.31	54.69
Laws and Parsons	5	4.48	5.52
Laws and Parsons	10	9.03	10.97
Laws and Parsons	20	18.20	21.80
Laws and Parsons	50	45.86	54.14

where $N(D_e)$ is the number concentration in m^{-3} mm^{-1}, D_e is the diameter in millimeters, and R is the rainfall rate in millimeters per hour. Inspection of Eq. (6) shows that the presence of an updraft (downdraft), when not taken into consideration, will result in a decrease (increase) in the radar retrieved estimate of R. Table 2 shows this expected result. The thought experiment for Table 2 is as follows: suppose we have particles for a known analytical DSD sitting in a box. If we release those particles in calm air [particle velocity is $v(D_e)$ in Eq. (6)] and calculate the rainfall rate, we would get the same rainfall rate that was used to define the analytical DSD in the first place. Now, suppose we release the particles in an updraft/downdraft instead of calm air [where the particle velocity is now $v(D_e) + w$ instead of $v(D_e)$]. The value of R given in Table 2 reflects the rainfall rates that would be measured under these conditions.

The general problem presented by this retrieval method in the presence of an unaccounted-for updraft/downdraft is that the estimate of the DSD itself will be affected (biased). This is due to the fact that the fall speed of the water drops is given by $v'(D_e) = w + v(D_e)$ rather than $v(D_e)$. One way to study this problem is to take a known DSD, assume the drops fall with velocity $v(D)$ in calm air, uniformly bias the fall speeds by an updraft/downdraft to obtain $v'(D)$ [$v'(D) = v(D) + w$], retrieve the DSD corresponding to $v(D_e) \approx v'(D_e)$, calculate integral parameters for the retrieved DSD, and compare the original integral parameters to those from the retrieved DSD. In other words, an analytical DSD is projected into a simulated Doppler velocity spectrum, the simulated spectrum is shifted by the air motion, and a new DSD is retrieved from the shifted Doppler spectrum using the fall speed relationship from Eq. (2). The difference between the two sets of integral parameters shows how the presence of an unaccounted-for updraft/downdraft affects the retrieved

TABLE 3. Effect of ambient air velocity on integral parameters for retrieved DSDs. The original DSD follows Marshall–Palmer with $R = 10$ mm h^{-1}. Fall speeds for the drops were calculated for terminal velocities in the range corresponding to the original drop diameters, biased with an updraft ($w < 0$) or downdraft ($w > 0$), and used to construct a Doppler spectrum. The DSD was then retrieved from the constructed spectrum, and all integral parameters were calculated from the retrieved DSD. Note that only values of not-a-number and infinity were excluded and no diameter thresholding was applied.

Marshall–Palmer $R = 10$ mm h^{-1}			
	Original	$w = -0.6$ m s^{-1}	$w = 0.6$ m s^{-1}
R	10 mm h^{-1}	15.17 mm h^{-1}	6.50 mm h^{-1}
Z	39.4 dB	43.9 dB	36.98 dB
D_0	1.40 mm	1.24 mm	1.58 mm
D_m	1.58 mm	1.21 mm	1.91 mm

DSD and carries through to affect the integral parameters. Table 3 lists integral parameters for a Marshall–Palmer DSD (Marshall and Palmer 1948) ($R = 10$ mm h^{-1}) as well as integral parameters from the DSD retrieved for nonzero vertical ambient air motions. Although only one DSD (Marshall–Palmer) at one rain rate ($R = 10$ mm h^{-1}) is shown in Table 3, other rain rates and analytical DSDs produce similar results.

The thought experiment for Table 3 is more subtle than for Table 2. Suppose we have the same DSD as before, but this time we want to know how updrafts/downdrafts (which affect the particles' velocities) change the retrieved DSD and to what extent the integral parameters calculated from this biased DSD are influenced. If we release the particles in an updraft/downdraft of unknown magnitude, the measured fall speeds will be $v(D_e) + w$ instead of $v(D_e)$. We can still retrieve a DSD and calculate integral parameters, but all quantities will be incorrect by some amount because the retrieval method treats $v(D_e) + w$ as if it were $v(D_e)$. In other words, if the fall speeds are biased by some unknown amount, and if those biased fall speeds are treated as if they were unbiased and used to retrieve a DSD, how much of an effect will that bias have on the integral parameters that are calculated from that retrieved DSD? These results are presented Table 3.

Table 3 shows an initially counterintuitive result: rain rates from retrieved DSDs decrease in downdrafts and increase in updrafts. To understand this result, consider the idealized Doppler spectrum shown in Fig. 1. In the presence of an updraft (or, equivalently, after removing a downdraft), all drops fall more slowly. Therefore, the entire spectrum shifts to the left. The contribution to the spectrum $S_{rc}(v)dv$ at low v is higher than it was before the transformation. Even though the values of S_{rc} themselves have not changed, they are now attributed to lower values of v. Since $D \propto v$, the spectral contribution to S_{rc} is now larger at smaller drop diameters. The contribution to the spectrum $S_{rc}(v)$ is directly proportional to $N(D)D^6$. If the values of S_{rc} remain constant and D decreases, then $N(D)$ must increase. In other words, it takes far more small drops than large drops to produce a given spectral contribution to S_{rc}. The rain rate R from retrieved DSDs increases in an updraft (or after removing a downdraft) because R is proportional to $N(D)D^3$. Similarly, in the presence of a downdraft (or after removing an updraft), R from retrieved DSDs will decrease.

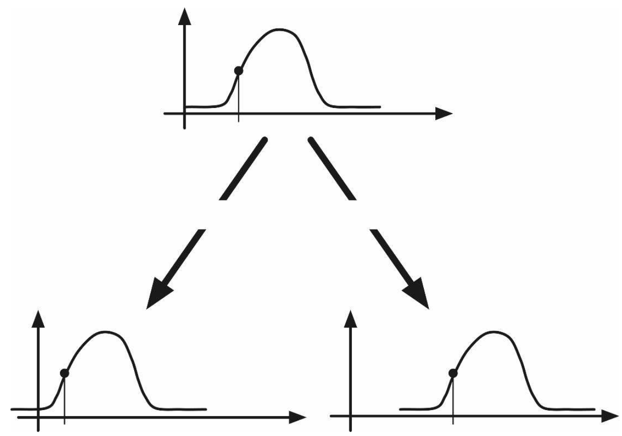

FIG. 1. Diagram of an idealized Doppler spectrum in (top) still air, (left) an updraft, and (right) a downdraft. A representative spectral contribution S is marked on all diagrams with a thin vertical line. The presence of vertical air motion affects the retrieved DSD with counterintuitive results. See text for further discussion.

b. Fall speed relationships

The DSD retrieval process relies on the existence of a relationship between a drop's diameter and its fall speed. Since a variety of fall speed relationships have been published to cover a wide range of drop diameters, one source of error in retrieved rainfall parameters may be due to the selection of a particular fall speed relationship. Time histories of rainfall rates from several precipitation events (not shown) based on retrieved DSDs using Eqs. (1)–(2) show that the choice of fall speed relationship has very little impact on rainfall rate provided that one of the conventionally used relationships is chosen.

c. Minimum included diameter

Since this retrieval method assumes that only hydrometeors contribute to the Doppler spectrum, clutter contamination and clear-air signals near 0 m s^{-1} are

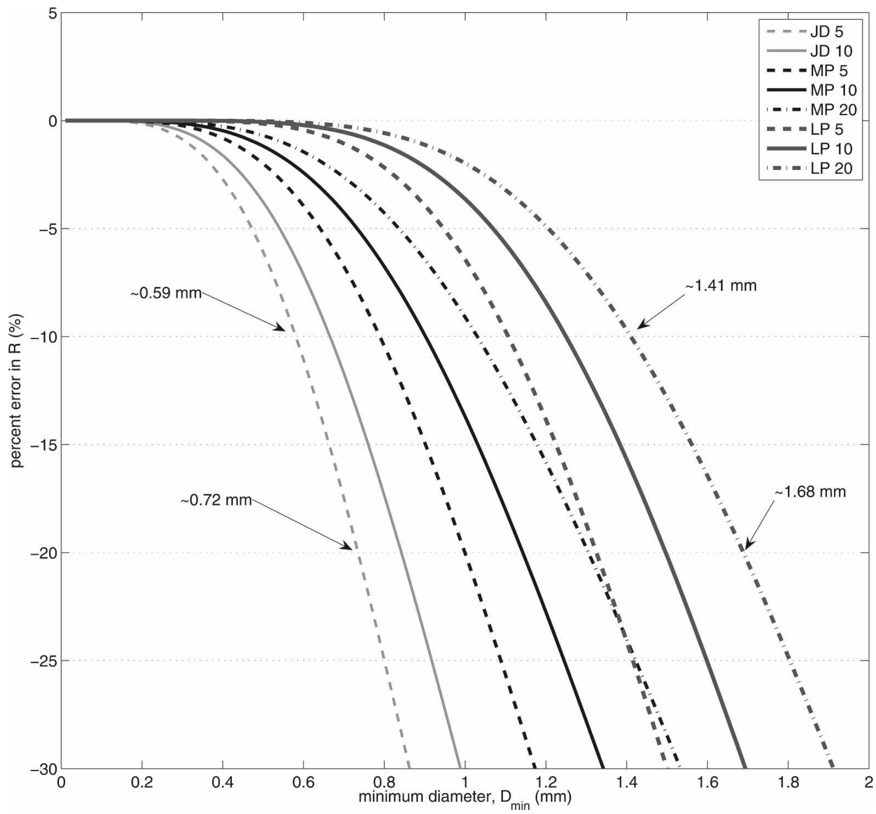

FIG. 2. Effect of velocity thresholding on rain rate. Here D_{min} refers to the minimum diameter that is included in the rain-rate calculation. Higher values of D_{min} correspond to higher velocity thresholds and more truncation. The arrows point to the intersection of the −10% and −20% error lines with the rain-rate curves for the three DSDs (Marshall–Palmer, Laws and Parsons, and Joss–Drizzle). The four numbers refer to the diameters at the intersections. For example, for a given acceptable error of −10%, the cutoff diameter must be no larger than ≈0.59 mm for a Joss–Drizzle DSD with $R = 5$ mm h^{-1} and no larger than ≈ 1.41 mm for a Laws and Parsons DSD with $R = 20$ mm h^{-1}. Note that a 20 mm h^{-1} rainfall rate for the Joss–Drizzle DSD does not seem realistic and has not been included in the figure.

erroneously interpreted as the result of very large numbers of very small drops. Applying velocity thresholding to a Doppler spectrum prior to retrieving the DSD ensures that clutter contamination is excluded, but it also imposes a lower bound on the retrieved drop diameters. Applying a minimum threshold in velocity is equivalent to truncating the DSD, which introduces errors in the integral parameters. To estimate this error, it is useful to examine analytical DSDs for which the rain rate is known. Figure 2 shows the effect of velocity thresholding on rainfall-rate calculation for three DSDs: Marshall–Palmer (Marshall and Palmer 1948), Laws and Parsons (Laws and Parsons 1943), and Joss–Drizzle (Joss and Gori 1978). The Marshall–Palmer and Laws and Parsons DSDs were defined above in Eqs. (9)–(10). The Joss–Drizzle DSD is described by

$$N(D_e) = 30\,000 e^{-5.7 D_e R^{-0.21}}, \tag{11}$$

where $N(D_e)$ is the number concentration in m^{-3} mm^{-1}, D_e is the diameter in millimeters, and R is the rainfall rate in millimeters per hour. As expected, as the velocity threshold is raised, more of the DSD is truncated and the rain rate decreases. It was found in this study that a threshold of 2.5–2.7 m s^{-1} was appropriate for most of the precipitation events studied. A threshold of 2.5–2.7 m s^{-1} corresponds to drop diameters of 0.5–0.7 mm using Eq. (2). Although the percent error is rain rate and DSD dependent, Fig. 2 suggests that the percent error in rain rate due to thresholding is a few percent or less at best and around 15% at worst.

d. Air density

Since air density decreases with height, it is necessary to correct for this effect prior to retrieving DSDs. This is accomplished by applying a correction factor to the velocity (Foote and duToit 1969). A drop of size D

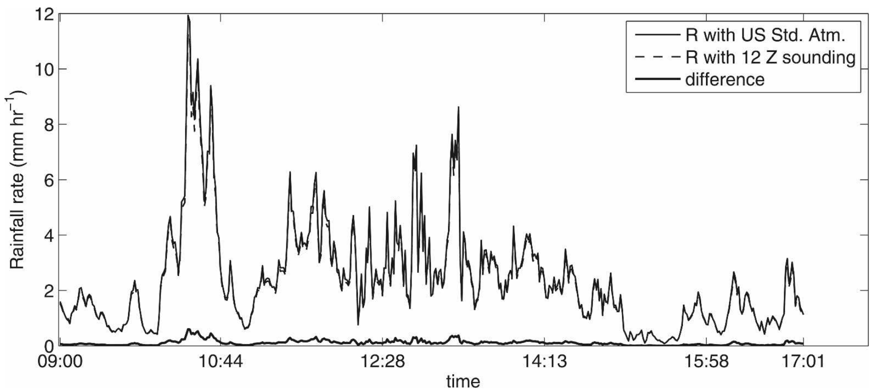

FIG. 3. DSDs were retrieved from the lowest usable range gate (200 m above ground level) for the same rain event using two different air density sources. One retrieval used density information from the U.S. Standard Atmosphere while the other retrieval used density information from a sounding that was released near the profiler site.

falling at some height above ground will fall at speed $v(D)$ given by

$$v(D) = v_o(D)\left(\frac{\rho_o}{\rho}\right)^{0.4}, \quad (12)$$

where $v_o(D)$ is the velocity given by any fall speed relationship at a given reference level, ρ_o is the density of air at the reference level, and ρ is the density of air at the height of interest. Similarly, a velocity measurement $v(D)$ for a drop located some distance above the reference level corresponds to a velocity of $v(D)(\rho/\rho_o)^{0.4}$ at the surface.

Some measure of air density information is required to calculate the correction factor. Two options for air density information are the U.S. Standard Atmosphere and environmental soundings. Although the differences between them are typically small, it is prudent to investigate whether or not those small differences will greatly affect DSD retrievals and subsequent calculations. Figure 3 shows rainfall rates from two DSD retrievals for the same rain event, one of which incorporated air density information from a sounding (launched less than a kilometer from the profiler site) while the other used only the U.S. Standard Atmosphere. The difference between rainfall rates is also shown. Differences are approximately 0.1–0.3 mm h^{-1} for this stratiform rain case. Based on these results, density correction with soundings is not significantly different from density correction with the U.S. Standard Atmosphere and there is no advantage to using soundings for this purpose. Unless otherwise stated, this study used air density information from the U.S. Standard Atmosphere.

6. Results

Here we present results from two precipitation events in support of the earlier error analysis. These events both occurred in central Oklahoma and were observed by a wind profiler collocated with a 2DVD. The 2 May 2005 event consisted entirely of stratiform rain while the 17 September 2006 event included stratiform rain and scattered embedded convection.

On 2 May 2005, measurements from the lowest usable range gate (200 m above ground level) of the profiler indicated that 18.65 mm of rain fell during the 8-h event. A collocated 2DVD reported 17.73 mm of rain and the Oklahoma Mesonet station (Brock et al. 1995) reported 16.76 mm of rain. The very good agreement between instruments illustrates the utility of this retrieval technique in stratiform rain. The calculation of rain based on profiler measurements is based on Eq. (2); using (1) yields a total of 19.02 mm. The difference between the two is quite small.

Figure 4 shows a comparison of rain rate between DSDs retrieved from the profiler, DSDs measured by the 2DVD, and rainfall measured over 5-min periods by the Oklahoma Mesonet station in Washington (collocated with the profiler and 2DVD). Since Mesonet stations record the accumulated rainfall over each 5-min period of the day (Brock et al. 1995), a conversion is required to obtain a rainfall rate representing the average rainfall rate over the 5-min period. Figure 4 shows that the profiler and 2DVD are generally in good agreement with each other and often in good agreement with the Mesonet station. Disagreements with the Mesonet station tend to occur during periods of low rain rates. During low rain-rate periods, the Mesonet tipping-bucket rain gauge may require several 5-min

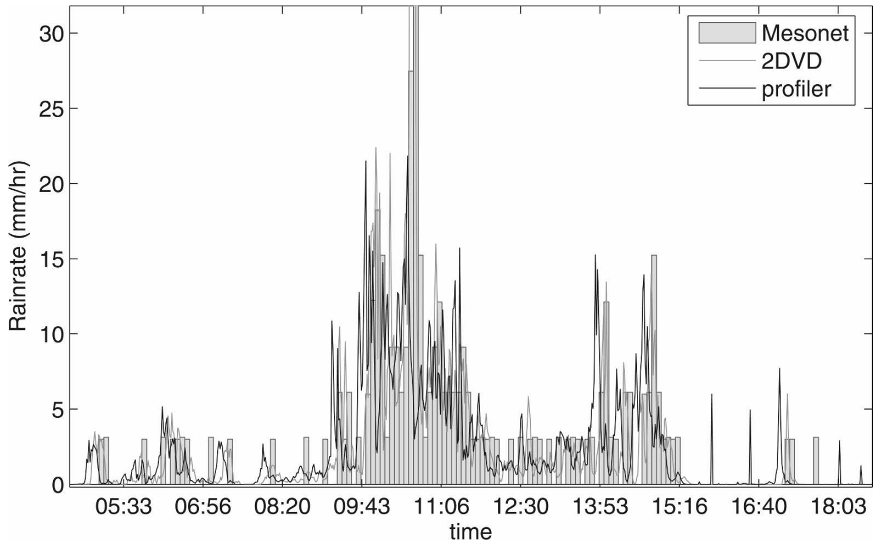

FIG. 4. Comparison of rain rate between three instruments for 17 Sep 2006.

periods to register a single tip (each tip is 0.254 mm, or 0.01 in.). The total accumulated rain measured by the different instruments is as follows: profiler, 27.61 mm; 2DVD, 38.19 mm; and Mesonet, 38.61 mm.

As shown in Table 3, an unaccounted-for mesoscale downdraft of 0.6 m s^{-1} may produce a 34% underestimate of R for a rain rate of 10 mm h^{-1}. If such a downdraft was present throughout this rain event, the 27.61-mm total listed above is an underestimate of 41.83 mm, which agrees favorably with the 2DVD and Mesonet. It should be noted that 41.83 mm is a first guess based solely on a 34% underrepresentation. Actually removing an assumed downdraft of 0.6 m s^{-1} from the data prior to retrieving DSDs results in total rainfall of 46.45 mm.

7. Discussion

Both precipitation cases presented above featured significant periods of stratiform rain spanning several hours. Although the 17 September 2006 event included scattered embedded convection, efforts were made to remove those portions of the dataset prior to retrieving DSDs. This was accomplished by comparing time–height cross sections from the profiler with Weather Surveillance Radar-1988 Doppler (WSR-88D) plan position indicator (PPI) plots. The rather poor agreement in total rainfall rate between the profiler and any other instrument for 17 September 2006 contrasts sharply with the very good agreement between instruments for 2 May 2005, a known stratiform case. It is believed that the poor agreement for 17 September 2006 is largely due to the nonuniform nature of the precipitation.

Since this DSD retrieval method relies on the presence of liquid water drops and the assumption of nearly quiescent ambient air, DSD retrievals are restricted to regions below the melting layer in stratiform rain. It is necessary to exclude periods of convective rain that may be present in the datasets from any DSD retrievals. It should be noted that this limitation is not present when using VHF wind profilers (Campos et al. 2007).

It was found that the largest errors in rain-rate estimates are due to unaccounted-for vertical ambient air motion. In stratiform rain with $R = 10$ mm h^{-1}, a mesoscale downdraft of 0.6 m s^{-1} may produce a 34% underestimate of R if the downdraft contribution is not removed prior to retrieving the DSD (as shown in Table 3), and the error is rain-rate dependent. This should be considered as a worst-case scenario for stratiform precipitation since 0.6 m s^{-1} is typically at the upper end of the range of ambient air velocities for these conditions.

Using two different fall speed relationships to retrieve DSDs and calculate rainfall rate resulted in a difference of approximately 1 mm over an 8-h dataset. Based on these results, we conclude that varying the choice of fall speed relationship produces only minimal errors in rain-rate estimates, provided that the selected relationship is valid over an appropriate range of diameters.

Velocity thresholding must be applied prior to retrieving the DSD and calculating precipitation parameters. It was found that an appropriate velocity threshold for these radar parameters was approximately 2.6–2.7 m s^{-1}, corresponding to a diameter threshold of

0.6–0.8 mm with Eq. (2). Simulations with analytical DSDs suggest a −4% to −10% error in rain rate due to truncating DSDs at 0.8 mm, although it varies with DSD and rain rate and may be as large as −25% or as small as −1%, as shown in Fig. 2. Attempts to fit these analytical models to retrieved DSDs resulted in very poor fits, so the applicability of this analysis to DSD retrievals remains unknown. It is thought that the poor fits simply reflect a degree-of-freedom issue in attempting to fit DSD models based on 2–3 moments to a retrieved DSD, which is based on only one moment (Z, with an additional parameter υ).

It was found that the error in rain rate due to the difference between actual air density (calculated from radiosondes) and air density given by the U.S. Standard Atmosphere was less than 0.3 mm h^{-1} for stratiform rain near the ground. In regions where this retrieval method is valid, the U.S. Standard Atmosphere is an appropriate source of air density information.

A promising area for future work (and one that was a motivation for this study) involves the combined use of data from the UHF profiler and KOUN, the polarimetric weather surveillance research radar at NSSL. Datasets from KOUN can be used for DSD retrievals by assuming that the DSD follows a modified gamma distribution and then calculating the parameters for the distribution (Zhang et al. 2001; Ryzhkov et al. 2005; Cao et al. 2008). DSD retrievals from range–height indicator (RHI) or PPI scans above the instrumentation site would provide additional observations of DSDs aloft, which could be compared to those obtained with the profiler. A method is currently being explored to retrieve vertical air motions from combined polarimetric weather radar and profiler measurements during precipitation periods (Teshiba et al. 2005; Teshiba et al. 2007, manuscript submitted to *J. Atmos. Oceanic Technol.*). These ambient air velocity estimates could be used to correct for vertical air velocities. Additional avenues for exploration include assessing the degree of agreement between instruments, determining whether KOUN DSD retrievals produce accurate estimates of rainfall rate at the surface, and comparing rainfall rates from polarimetric Z–R relationships to rainfall rates from retrieved DSDs.

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