Letters to the Editor

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Notes on Magneto-Hydrodynamic Equilibrium

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A proper coordinate system and a suitable form of equilibrium equations for a successive approximation method are reported in this letter. By these means we can solve equilibrium equations in the case of a rotational transformed field.[1)2)] But we could not apply these methods to the case in which lines of force are closed and have no symmetric property.

As is well known, lines of electric current and lines of magnetic force lie on equi-pressure surfaces in the M. H. equilibria. We have proved[3)] that it is possible to construct a global coordinate system on each equi-pressure surface by taking these two kinds of lines as coordinate lines and defining coordinate x_i on these lines according to

$$dx_i = ds_i/|\boldsymbol{X}_i| \quad (i=1,2) \qquad (1)$$

In Eq. (1), $\boldsymbol{X}_1$ and ds_1 are the magnetic field strength and its line-element, $\boldsymbol{X}_2$ and ds_2, the current density and its line-element, respectively.

We have studied the structure of these coordinate systems and got three main results.[3)] The first is that any closed equi-pressure surface without singular points is topologically torus, as was said by many other authors.[2)] The second is that there must exist simple closed loops which lie on each equi-pressure surface with non-vanishing pressure-gradient and have the following property, that is,

$$\int_A^B \frac{ds_1}{|\boldsymbol{X}_1|} = \text{const. on the surface} \qquad (2)$$

where the integrals are carried out along lines of force from an arbitrary starting point A on the loop to the final point B on it. Generally, there are one or more loops passing through any given point on the surface, turning arbitrary given times along and around the magnetic axis in an equilibrium configuration.

The third result is that by taking these loops belonging to two different homology classes as coordinate lines and defining coordinates ξ_i on these lines in proportion to magnetic flux through an infinitesimally narrow ribbon between the surface and its neighbouring one, limited by $\xi_i = 0$ and ξ_i itself, we can construct a many valued periodic coordinate system on each equi-pressure surface, and in this system, lines of current and lines of force are represented as parallel straight lines, respectively.

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But when the rotational transform ratio is rational, such a surface may be called a rational one and in the other case the surface may be called an irrational one; this result will fail unless the loops are carefully selected. The reason is the following. In almost all cases there exists one and only one loop through a given point, belonging to a given homology class for an irrational surface. But there exists an infinite number of such loops for a rational surface when the loops exist. Therefore, a selection rule is required for rational cases, but, at present, we cannot get it except in the case with symmetric property.

Equation (2) can be used as a necessary condition for an external field to be a magnetic trap, because Eq. (2) must hold as the fluid pressure tends to 0. Equation (2) is the condition for no charge separation. By means of the second and the third results we can get useful knowledge of current density in the first order of pressure when a rotational transformed external field is given.

We have transformed ordinary equilibrium equations into suitable forms for the successive approximation method. That is,

$$\nabla\chi=\boldsymbol{H}+4\pi p\nabla x_1,\ \nabla\cdot\boldsymbol{H}=0,\ \boldsymbol{H}\nabla p=0 \tag{3}$$

where p and $\boldsymbol{H}$ are the fluid pressure and the magnetic field strength, x_1 is defined in Eq. (1), and χ is a many valued scalar function which is obtained by solving the Poisson equation successively together with suitable boundary conditions and periods.

When p is sufficiently small and the external field is rotationally transformed, p and x_1 are given by the data in lower order and we can make the iteration to higher order of p.

Equation (2) suggests the possibility of a magnetic trap without rotational transform. In this case Eq. (2) gives magnetic surfaces. B. B. Kadomcev has already discussed this possibility in detail.

Finally, we have studied an effect of rational surfaces in a stellarator with varying rotational transform ratio. Generally speaking, almost all rational surfaces are incompatible with Eq. (2). Thus $|\nabla p|$ is zero on the surfaces densely distributed in the device, or else charge separation should occur. By means of the second and the third results, it can be shown that lines of current describe rapidly oscillating curves in the neibourhood of such surfaces. Therefore, the diffusion velocity of fluid across magnetic lines increases due to the finite conductivity of fluid and the long paths of current lines in that region. Probably, a vortex flow may also occur under some conditions. But according to our semi-quantitative treatments, the diffusion effect will be small in practical cases.

Detailed discussions will be published elsewhere.

1) L. Spitzer, 2nd conf. at Geneva, A/CONF. 15/p/2170
2) M. Kruskal ane R. Kulsrud, 2nd conf. at Geneva, A/CONF. 15/p/1876
3) S. Hamada, Kakuyugo-Kenkyu (Nuclear Fission Research in Japan), 1 (1958), 542

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