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NOTES ON THE BERGMAN PROJECTION TYPE OPERATOR IN $\mathbb{C}^n$

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ABSTRACT. In this paper, we will define the Bergman projection type operator P_r and find conditions on which the operator P_r is bound-ed on $L^p(B, d\nu)$. By using the properties of the Bergman projection type operator P_r, we will show that if $f \in L^p_a(B, d\nu)$, then $(1- \| w \|^2)\nabla f(w) \cdot z \in L^p(B, d\nu)$. We will also show that if $(1- \| w \|^2) \frac{\nabla f(w) \cdot z}{\langle z, w \rangle} \in L^p(B, d\nu)$, then $f \in L^p_a(B, d\nu)$.

1. Introduction

Throughout this paper, $\mathbb{C}^n$ will be the Cartesian product of n copies of $\mathbb{C}$. For $z = (z_1, z_2, \ldots, z_n)$ and $w = (w_1, w_2, \ldots, w_n)$ in $\mathbb{C}^n$, the inner product is defined by $\langle z, w \rangle = \sum_{j=1}^n z_j \overline{w_j}$ and the norm by $\| z \|^2 = \langle z, z \rangle$.

Let B be the open unit ball in the complex space $\mathbb{C}^n$. Let ν be the Lebesgue measure in $\mathbb{C}^n$ normalized by $\nu(B) = 1$. We let $L^2(B, d\nu)$ be the usual space of Lebesgue square-integrable complex valued functions on B. The Bergman space $L^2_a(B, d\nu)$ is defined to be the subspace of $L^2(B, d\nu)$ consisting of analytic functions.

The measure μ_r is the weighted Lebesgue measure:

$$d\mu_r(z) = c_r(1 - \| z \|^2)^r d\nu(z),$$

where $r > -1$ is fixed, and c_r is a normalization constant such that $\mu_r(B) = 1$.

If we equip $L^2_{a,r} = L^2_a(B, d\mu_r)$ with the norm $\| f \|_{2,r} = \sqrt{\int_B |f|^2 d\mu_r}$, then $L^2_{a,r}$ is a Banach space for each $r > -1$. The orthogonal projection

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operator from $L^2(B, d\mu_r)$ to $L^2_{a,r}$ is denoted by P and P is called the Bergman projection. The Bergman projection is used in many areas related with Hankel operators and Toeplitz operators (See [2, 6, 9, 14, 15]).

The space $L^2_{a,r}$ is a Hilbert space with the inner product

$$\langle f, g\rangle = \int_B f(z)\overline{g(z)}d\mu_r(z).$$

Fix a point $z \in B$. Since the functional e_z given by $e_z(f) = f(z), f \in L^2_{a,r}$, is continuous, there exists a function $k_{r,z} \in L^2_{a,r}$ such that

$$f(z) = \int_B f(w)\overline{k_{r,z}(w)}d\mu_r(w)$$

by the Riesz representation theorem. The function $K_r(z,w) = \overline{k_{r,z}(w)}$ is called the weighted Bergman kernel. Also it is well known that

$$K_r(z,w) = \frac{1}{(1-\langle z,w\rangle)^{r+n+1}}$$

(See [12]). It was shown in [10] that if $f \in L^1_{a,r}, r > -1$, then

$$f(z) = c_r \int_B \frac{(1-\| w \|^2)^r}{(1-\langle z,w\rangle)^{n+r+1}} f(w)d\nu(w).$$

Suppose $1 \leq p < +\infty$ and $r > 0$. Let $L^p_{a,r}$ be the subspace of $L^p(B, d\mu_r)$ consisting of analytic functions. Define the Bergman projection type operator P_r by

$$\begin{aligned} P_r f(z) &= \int_B f(w)\overline{k_{r,z}(w)}d\mu_r(w) \\ &= c_r \int_B \frac{(1-\| w \|^2)^r}{(1-\langle z,w\rangle)^{n+r+1}} f(w)d\nu(w). \end{aligned}$$

Since $P_r f = f$ for all analytic f in $L^1(B, d\mu_r)$, P_r is a projection from $L^1(B, d\mu_r)$ onto $L^1_a(B, d\mu_r)$. In general, P_r is not a bounded operator (See Corollary 6).

In section 2, we will find conditions on which the operator P_r is a bounded operator from $L^p(B, d\mu_r)$ onto $L^p_{a,r}$. In particular, we will show that if $1 \le p < +\infty$ and $r > 0$, then the operator P_r is a bounded projection from $L^p(B, d\nu)$ onto $L^p_a(B, d\nu)$.

Let N denote the set of natural numbers. A multi-index α is an ordered n-tuple $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ with $\alpha_j \in N, j = 1, 2, \dots, n$. For a multi-index α and $z \in \mathbb{C}^n$, set

$$\begin{aligned} |\alpha| &= \alpha_1 + \alpha_2 + \cdots + \alpha_n, \\ \alpha! &= \alpha_1! \alpha_2! \cdots \alpha_n!, \\ z^\alpha &= z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_n^{\alpha_n}. \end{aligned}$$

For all multi-indices α, we will write

$$\partial^\alpha f(w) = \frac{\partial^{|\alpha|} f(w)}{\partial {w_1}^{\alpha_1} \cdots \partial {w_n}^{\alpha_n}}.$$

For a holomorphic function f, we will also write

$$\nabla f(w) \cdot z = \frac{\partial f(w)}{\partial w_1} z_1 + \cdots + \frac{\partial f(w)}{\partial w_n} z_n.$$

Suppose $p \ge 1, |\alpha| \ge 1$ and $f \in L^p_a(B, d\nu)$. In section 3, we will show that

$$(1 - \| w \|^2)^{|\alpha|} \partial^\alpha f(w)$$

is in $L^p(B, d\nu)$. By using this result and the properties of the Bergman type projection in section 2, we will show that if $f \in L^p_a(B, d\nu)$, then $(1 - \| w \|^2) \nabla f(w) \cdot z \in L^p(B, d\nu)$. We will also show that if $(1 - \| w \|^2)$ $\frac{\nabla f(w) \cdot z}{\langle z, w \rangle} \in L^p(B, d\nu)$, then $f \in L^p_a(B, d\nu)$.

2. Bergman projection type operator P_r

THEOREM 1. *For $z \in B$, c is real, $t > -1$, define*

$$I_{c,t}(z) = \int_B \frac{(1 - \| w \|^2)^t}{|1 - \langle z, w \rangle|^{n+1+c+t}} d\nu(w).$$

Then,

(i) *$I_{c,t}(z)$ is bounded in B if $c < 0$;*

(ii) *$I_{0,t}(z) \sim -\log(1 - \| z \|^2)$ as $\| z \| \to 1^-$;*

(iii) *$I_{c,t}(z) \sim (1 - \| z \|^2)^{-c}$ as $\| z \| \to 1^-$ if $c > 0$.*

PROOF. See [12, Proposition 1.4.10]. □

THEOREM 2. *Suppose (X, μ) is a measure space and K is a measurable function on $X \times X$. Let T be the integral operator induced by K, that is,*

$$Tf(x) = \int_X K(x,y)f(y)d\mu(y).$$

Suppose $1 < p < +\infty$ with $\frac{1}{p} + \frac{1}{q} = 1$. If there is a constant $c > 0$ and a positive measurable function h on X such that

$$\int_X |K(x,y)|h(y)^q d\mu(y) \leqq ch(x)^q$$

for $\mu-$almost every x in X and

$$\int_X |K(x,y)|h(x)^p d\mu(x) \leqq ch(y)^p$$

for $\mu-$almost every y in X, then T is bounded on $L^p(X, d\mu)$ with norm less than or equal to c.

PROOF. See [16, Theorem 3.2.2]. □

THEOREM 3. *If $f \in L^1_{a,r}, r > -1$, then*

$$f(z) = c_r \int_B \frac{(1- \| w \|^2)^r}{(1 - \langle z, w\rangle)^{n+r+1}} f(w)d\nu(w).$$

PROOF. See [10, Theorem 2]. □

THEOREM 4. *X and Y are Banach spaces. $\mathcal{L}(X, Y)$ is the set of bounded linear transformations from X to Y. If A^* is the adjoint of $A \in \mathcal{L}(X, Y)$, then:*
(1) $\| A^* \| = \| A \|$;
(2) if A is invertible, then A^* is invertible and $(A^*)^{-1} = (A^{-1})^*$.

PROOF. See [7, Proposition 1.4]. □

THEOREM 5. *Suppose $1 \leq p < +\infty$ and $r > 0$. Then the operator P_r is a bounded projection from $L^p(B, d\nu)$ onto $L^p_a(B, d\nu)$.*

PROOF. We first prove the case $p = 1$. If P_r is bounded on $L^p(B, d\nu)$,

$$\begin{aligned}
&\langle P_r f, g\rangle \\
&= \int_B P_r f(z)\overline{g(z)}d\nu(z) \\
&= \int_B \int_B \frac{c_r(1-\| w \|^2)^r f(w)}{(1-\langle z,w\rangle)^{n+r+1}} d\nu(w)\overline{g(z)}d\nu(z) \\
&= \int_B c_r(1-\| w \|^2)^r f(w) \int_B \frac{\overline{g(z)}}{(1-\langle z,w\rangle)^{n+r+1}} d\nu(z)d\nu(w),
\end{aligned}$$

where $g \in L^\infty(B)$. Let P_r^* be the adjoint of P_r under the usual integral pairing. Then above result shows that

$$P_r^* g(w) = c_r(1-\| w \|^2)^r \int_B \frac{g(z)}{(1-\langle w,z\rangle)^{n+r+1}} d\nu(z).$$

By Theorem 1, if $r > 0$

$$\sup_{w\in B}(1-\| w \|^2)^r \int_B \frac{d\nu(z)}{(1-\langle w,z\rangle)^{n+r+1}} < \infty.$$

This shows that if $r > 0$, then P_r^* is bounded on $L^\infty(B, d\nu)$. By Theorem 4, P_r is bounded on $L^1(B, d\nu)$ if $r > 0$.

Next assume $1 < p < +\infty$, $\frac{1}{p} + \frac{1}{q} = 1$. Let $h(z) = (1-\| z \|^2)^{-\frac{1}{pq}}$.

$$\begin{aligned}
&\int_B \frac{c_r(1-\| w \|^2)^r}{|1-\langle z,w\rangle|^{n+r+1}}(1-\| w \|^2)^{-\frac{1}{p}} d\nu(w) \\
&= \int_B \frac{c_r(1-\| w \|^2)^{r-\frac{1}{p}}}{|1-\langle z,w\rangle|^{n+r+1}} d\nu(w) \\
&= c_r(1-\| z \|^2)^{-\frac{1}{p}}.
\end{aligned}$$

$$\begin{aligned}
&\int_B \frac{c_r(1-\| w \|^2)^r}{|1-\langle z,w\rangle|^{n+r+1}}(1-\| z \|^2)^{-\frac{1}{q}} d\nu(z) \\
&= c_r(1-\| w \|^2)^r \int_B \frac{(1-\| z \|^2)^{-\frac{1}{q}}}{|1-\langle z,w\rangle|^{n+r+1}} d\nu(z) \\
&= c_r(1-\| w \|^2)^r \int_B \frac{(1-\| z \|^2)^{-\frac{1}{q}}}{|1-\langle z,w\rangle|^{n+(r+\frac{1}{q})+1-\frac{1}{q}}} d\nu(z).
\end{aligned}$$

Since $r > 0$, then $r + \frac{1}{q} > 0$. This shows that

$$\int_B \frac{c_r(1-\| w \|^2)^r}{|1-\langle z, w\rangle|^{n+r+1}}(1-\| z \|^2)^{-\frac{1}{q}} d\nu(z) \le c_r(1-\| w \|^2)^{-\frac{1}{q}}.$$

By Theorem 2, P_r is bounded on $L^p(B, d\nu)$ if $r > 0$. If $f \in L^p_a(B, d\nu)$, then $f \in L^1(B, d\mu_r)$. Since $P_r f = f$, the operator P_r is a bounded projection from $L^p(B, d\nu)$ onto $L^p_a(B, d\nu)$. □

COROLLARY 6. *If $rq \le -1$ where $\frac{1}{p} + \frac{1}{q} = 1$, then P_r is not bounded.*

PROOF.

$$\begin{aligned}
&\int_B \{P_r^* 1(w)\}^q d\nu(w) \\
&= \int_B \{c_r(1-\| w \|^2)^r \int_B \frac{d\nu(z)}{(1-\langle w, z\rangle)^{n+r+1}}\}^q d\nu(w) \\
&= \int_B c_r^q(1-\| w \|^2)^{rq}\{\int_B \frac{d\nu(z)}{(1-\langle w, z\rangle)^{n+r+1}}\}^q d\nu(w).
\end{aligned}$$

$\int_B \frac{d\nu(z)}{(1-\langle w,z\rangle)^{n+r+1}} d\nu(w)$ is bounded by Theorem 1, but $\int_B (1-\| w \|^2)^{rq}$ $d\nu(w) = \infty$ since $rq \le -1$. This shows that if $rq \le -1$, then P_r is not bounded. □

COROLLARY 7. *If $1 < p < +\infty$, then P is bounded on $L^p(B, d\nu)$.*

PROOF. This follows from Theorem 5. □

3. Results related with P_r

Remember that, for all multi-indices $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$,

$$\partial^\alpha f(w) = \frac{\partial^{|\alpha|} f(w)}{\partial {w_1}^{\alpha_1} \cdots \partial {w_n}^{\alpha_n}}.$$

For a multi-index $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ and $z = (z_1, z_2, \cdots, z_n) \in \mathbb{C}^n$, set

$$\overline{z}^\alpha = \overline{z_1}^{\alpha_1}\overline{z_2}^{\alpha_2} \cdots \overline{z_n}^{\alpha_n}.$$

LEMMA 8. *Suppose* $p \geq 1, |\alpha| \geq 1$ *and* $f \in L_a^p(B, d\nu)$. *Then*

$$(1 - \| z \|^2)^{|\alpha|} \partial^\alpha f(z)$$

is in $L^p(B, d\nu)$.

PROOF. We first prove the case $p = 1$. By Theorem 3,

$$f(z) = c_1 \int_B \frac{(1 - \| w \|^2)}{(1 - \langle z, w \rangle)^{n+2}} f(w) d\nu(w).$$

$$\begin{aligned} \partial^\alpha f(z) &= \frac{\partial^{|\alpha|} f(w)}{\partial {z_1}^{\alpha_1} \cdots \partial {z_n}^{\alpha_n}} \\ &= c_1 \int_B \frac{(1 - \| w \|^2)(n+2) \cdots (n + |\alpha| + 1) f(w) \overline{w}^\alpha}{(1 - \langle z, w \rangle)^{n+2+|\alpha|}} f(w) d\nu(w). \end{aligned}$$

$$\begin{aligned} &\left| \int_B (1 - \| z \|^2)^{|\alpha|} \partial^\alpha f(z) d\nu(z) \right| \\ &\leq |c_1| \frac{(n + |\alpha| + 1)!}{(n+1)!} \int_B (1 - \| z \|^2)^{|\alpha|} \\ &\quad \times \left| \int_B \frac{(1 - \| w \|^2) \overline{w}^\alpha}{(1 - \langle z, w \rangle)^{n+2+|\alpha|}} f(w) d\nu(w) \right| d\nu(z) \\ &\leq |c_1| \frac{(n + |\alpha| + 1)!}{(n+1)!} \int_B (1 - \| w \|^2) |f(w)| \\ &\quad \times \int_B \frac{(1 - \| z \|^2)^{|\alpha|}}{|1 - \langle z, w \rangle|^{n+2+|\alpha|}} d\nu(z) d\nu(w) \\ &\leq |c_1| \frac{(n + |\alpha| + 1)!}{(n+1)!} \int_B (1 - \| w \|^2)(1 - \| w \|^2)^{-1} |f(w)| d\nu(w) \\ &\leq |c| \int_B |f(w)| d\nu(w). \end{aligned}$$

This shows that if f is in $L_a^1(B, d\nu)$, then $(1 - \| z \|)^{|\alpha|} \partial^\alpha f(z)$ is in $L^1(B, d\nu)$.

Next assume $1 < p < +\infty$. By Theorem 3,

$$f(z) = \int_B \frac{f(w)}{(1 - \langle z, w \rangle)^{n+1}} d\nu(w).$$

$$
\begin{aligned}
&(1-\| z \|^2)^{|\alpha|}\partial^\alpha f(z) \\
&= (1-\| z \|^2)^{|\alpha|}\frac{(n+|\alpha|)!}{n!}\int_B \frac{\overline{w}^\alpha f(w)}{(1-\langle z,w\rangle)^{n+1+|\alpha|}}d\nu(w) \\
&= (1-\| z \|^2)^{|\alpha|}\frac{(n+|\alpha|)!}{n!}\int_B \frac{f_1(w)}{(1-\langle z,w\rangle)^{n+1+|\alpha|}}d\nu(w) \\
&= \frac{1}{c_{|\alpha|}}\frac{(n+|\alpha|)!}{n!}c_{|\alpha|}(1-\| z \|^2)^{|\alpha|}\int_B \frac{f_1(w)}{(1-\langle z,w\rangle)^{n+1+|\alpha|}}d\nu(w) \\
&= \frac{(n+|\alpha|)!}{n!}\frac{1}{c_{|\alpha|}}P^*_{|\alpha|}f_1(z),
\end{aligned}
$$

where $f_1(w) = \overline{w}^\alpha f(w)$. Since $P_{|\alpha|}$ is bounded on $L^q(B,d\nu)$(where $\frac{1}{p}+\frac{1}{q}=1$) by Theorem 4, $P^*_{|\alpha|}$ is bounded on $L^p(B,d\nu)$ and

$$(1-\| w \|^2)^{|\alpha|}\partial^\alpha f(w)$$

is in $L^p(B,d\nu)$. □

THEOREM 9. *Suppose that* $p \geq 1$. *If* $f \in L^p_a(B,d\nu)$, *then* $(1-\| w \|^2)\nabla f(w)\cdot z \in L^p(B,d\nu)$.

PROOF. By Lemma 8, $(1-\| w \|^2)\frac{\partial f(w)}{\partial w_i} \in L^p(B,d\nu)$ for $i=1,\dots,n$.

$$
\begin{aligned}
&(1-\| w \|^2)|\nabla f(w)\cdot z| \\
&\leq (1-\| w \|^2)\sqrt{(\frac{\partial f(w)}{\partial w_1})^2+\cdots+(\frac{\partial f(w)}{\partial w_n})^2} \\
&\leq (1-\| w \|^2)(|\frac{\partial f(w)}{\partial w_1}|+\cdots+|\frac{\partial f(w)}{\partial w_n}|).
\end{aligned}
$$

This shows that $(1-\| w \|^2)\nabla f(w)\cdot z \in L^p(B,d\nu)$. □

THEOREM 10. *Suppose* $z \in B$ *and* $\nabla f(w)\cdot z \in L^1(B,d\mu_r)$. *Then*

$$f(z) = f(0) + \frac{c_r}{n+r}\int_B \frac{(1-\| w \|^2)^r\nabla f(w)\cdot z}{\langle z,w\rangle(1-\langle z,w\rangle)^{n+r}}d\nu(w).$$

PROOF. See [10, Theorem 3]. □

THEOREM 11. *Suppose* $1 \le p < +\infty$. *If* $(1-\| w \|^2)\frac{\nabla f(w)\cdot z}{\langle z,w\rangle} \in L^p(B,d\nu)$, *then* $f \in L^p_a(B,d\nu)$.

PROOF. Note that $(1-\| w \|^2)\frac{\nabla f(w)\cdot z}{\langle z,w\rangle} \in L^p(B,d\nu)$ implies $(1-\| w \|^2)\nabla f(w)\cdot z \in L^p(B,d\nu)$. We first consider the case $p=1$. Since $\nabla f(w)\cdot z \in L^1(B,d\mu_2)$, by Theorem 10,

$$
\begin{aligned}
&f(z)-f(0)\\
&= \frac{c_2}{n+2}\int_B \frac{(1-\| w \|^2)^2\nabla f(w)\cdot z}{\langle z,w\rangle(1-\langle z,w\rangle)^{n+2}}d\nu(w)\\
&= \frac{c_2}{n+2}\frac{1}{c_1}\int_B \frac{c_1(1-\| w \|^2)}{(1-\langle z,w\rangle)^{n+2}}\frac{(1-\| w \|^2)\nabla f(w)\cdot z}{\langle z,w\rangle}d\nu(w)\\
&= \frac{c_2}{n+2}\frac{1}{c_1}(P_1F_z)(z),
\end{aligned}
$$

where $F_z(w)=\frac{(1-\|w\|^2)\nabla f(w)\cdot z}{\langle z,w\rangle}$. Since $F_z \in L^1(B,d\nu)$, $P_1F_z \in L^1_a(B,d\nu)$ by Theorem 5. This shows that $f \in L^1_a(B,d\nu)$.

Now suppose that $1 < p < +\infty$. Since $(1-\| w \|^2)\nabla f(w)\cdot z \in L^p(B,d\nu)$ and $p>1$, $(1-\| w \|^2)\nabla f(w)\cdot z \in L^1(B,d\nu)$. This implies that $\nabla f(w)\cdot z \in L^1(B,d\mu_1)$. By Theorem 10,

$$
\begin{aligned}
f(z)-f(0) &= \frac{c_1}{n+1}\int_B \frac{(1-\| w \|^2)\nabla f(w)\cdot z}{\langle z,w\rangle(1-\langle z,w\rangle)^{n+1}}d\nu(w)\\
&= \frac{c_1}{n+1}\int_B \frac{1}{(1-\langle z,w\rangle)^{n+1}}\frac{(1-\| w \|^2)\nabla f(w)\cdot z}{\langle z,w\rangle}d\nu(w)\\
&= \frac{c_1}{n+1}(PF_z)(z),
\end{aligned}
$$

where $F_z(w)=\frac{(1-\|w\|^2)\nabla f(w)\cdot z}{\langle z,w\rangle}$. Since $F_z \in L^p(B,d\nu)$, $PF_z \in L^p_a(B,d\nu)$ by Corollary 7. This shows that $f \in L^p_a(B,d\nu)$. □

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