Novel Centroid Localization Algorithm for Three-Dimensional Wireless Sensor Networks

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***Abstract*—In this paper, we propose a new localization algorithm which can be effectively used in three-dimensional (3D) wireless sensor networks. This scheme needs no additional hardware support and can be implemented in a distributed way. The proposed method can improve the location accuracy with relatively low communication traffic and computing complexity. Simulation results show that the performance of the proposed algorithm is superior to that of the conventional centroid algorithm.**

***Index Terms*—WSNs, Centroid Localization algorithm, localization scheme, Volume-coordinate system.**

I. Introduction

Energy-efficient wireless sensor networks (WSNs) can be helpful in many different areas such as military surveillance, medical care, environmental monitoring, public service and home automation applications. Design and analysis of WSNs has been a major research topic in the computer and communication fields [1]. In a significant amount of applications, for example humidity and temperature monitoring, data collected by sensor nodes should include the information of their physical locations. Otherwise, the data will be irrelevant. Because of the constraints in size, power, and computing capabilities in WSNs, it is challenging to provide satisfactory localization algorithms to meet high accuracy requirement [2].

Localization algorithms for WSNs can be divided into two categories: (i) range-based and (ii) range-free protocols. Range-based protocols [3,6,7] employ absolute point-to-point distance or angular information to identify the locations among neighboring nodes. The measurements used in range-based localization including angle-of-arrival (AOA), received signal strength indicator (RSSI) [7], time-of-arrival (TOA) [3] and time-difference-of-arrival (TDOA) [6] schemes. On the other hand, we may adopt the range-free approach in which one can find the positions of non-anchor nodes by calculating their distances from the designated but sparse anchor nodes with known positions, also known as landmarks [4]. Although various localization algorithms have been proposed for two-dimensional (2D) sensor networks, there are relatively few localization schemes for three-dimensional WSNs. Tian *et al.*[8] developed a multilateration algorithm which could reliably localize and synchronize underwater sensor networks by acoustic ranging. Cheng *et al.* [10] proposed a novel underwater localization algorithm for sparse 3D acoustic sensor networks, which transformed 3D underwater localization to 2D. In this paper, a novel localization algorithm for three-dimensional WSNs, which can obtain high location performance, is developed based on the centroid theorem of coordinate-tetrahedron [9][11].

This paper makes four major contributions to the localization problem in WSNs. First, we present a practical, fast and easy-to-use localization scheme with relatively high accuracy and low cost for WSNs. Second, we propose the centroid theorem of coordinate-tetrahedron in the volume-coordinate system, which acts as a key component of our estimation approach. Third, the proposed localization algorithm improves location accuracy than the centroid localization algorithm. Fourth, the proposed algorithm can be effectively applied in 3D WSNs, which is proving to be a promising technique for several application scenarios such as in space, under water, or on hilly terrains. The rest of the paper is organized as follows. Section II describes the details of the new algorithm and its derivation. In Section III, simulation results are reported and a comparative study of the localization performance is conducted. Finally, Section IV gives the concluding remarks.

II. Algorithm Development

In this section, we are going to derive a novel node localization method for 3D WSNs. Before proceeding, we review the centroid algorithm.

2.1 Review of Centroid Algorithm

Bulusu and Heidemann [2] have proposed the centroid localization algorithm, which is a range-free, proximity-based, coarse-grained localization algorithm. The algorithm implementation contains three core steps. First, all anchors send their positions to all sensor nodes within their transmission range. Each unknown node listens for a fixed time period t and collects all the beacon signals it receives from various reference points. Second, all unknown sensor nodes calculate their own positions by a centroid determination from all n positions of the anchors in range.

The centroid localization algorithm, which uses anchor nodes (reference nodes), containing location information (x_i, y_i), to estimate node position. After receiving these beacons, a node estimates its location using the following centroid formula:

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$$(x_{est}, y_{est}) = \left(\frac{x_1 + \ldots + x_N}{N}, \frac{y_1 + \ldots + y_N}{N} \right) \quad (1)$$

The centroid localization algorithm is simple but the location error is high due to the centroid formula. Besides, the conventional centroid localization algorithm only focuses on node self-localization for 2D networks. Practical nodes of WSNs are generally arranged in 3D scenarios such as in space, under water, or on hilly terrains, rather than on pure 2D planes, thus 3D position information of nodes is commonly required. Node self-localization algorithm for 3D WSNs is studied in this paper.

2.2 Development of Novel Centroid Algorithm for 3D WSNs

This subsection describes our proposed 3D positioning algorithm for WSNs in detail. Wan *et al.* [11] proposed a localization algorithm for mobile system based on a linear relationship between the rectangular and the volume coordinates. Our work is inspired by [11]. Based on [11], we extend their work and propose an improved centroid algorithm for WSNs. Our proposed localization algorithm will not use 2D centroid theorem, but present the centroid theorem of coordinate-tetrahedron in the volume-coordinate system, which acts as a key component of our estimation approach. Using centroid theorem of coordinate-tetrahedron [9]-[11], the proposed positioning algorithm can be used in 3D WSNs and also can improve location accuracy than the conventional centroid localization algorithm.

The linear relationship between the rectangular and the volume coordinates will be reviewed first as shown in Eqs.(5)-(12), which have been presented in [11]. Then the detailed derivation procedure for the centroid theorem of coordinate-tetrahedron in the volume-coordinate system will be described.

Lemma 1: Assume that the rectangular coordinate of the vertex A_i of the tetrahedron is (x_i, y_i, z_i) $(i = 1,2,3,4)$. Then its signed volume can be expressed in the form of determinant as follows:

$$V = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{vmatrix} = \frac{1}{6}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\ x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \end{vmatrix} \quad (2)$$

Lemma 2: The volume of the general tetrahedron is also given by the determinant

$$V^2 = \frac{1}{288}\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & r_{12}^2 & r_{13}^2 & r_{14}^2 \\ 1 & r_{12}^2 & 0 & r_{23}^2 & r_{24}^2 \\ 1 & r_{13}^2 & r_{23}^2 & 0 & r_{34}^2 \\ 1 & r_{14}^2 & r_{24}^2 & r_{34}^2 & 0 \end{vmatrix} \quad (3)$$

where r_{ij} is the range between vertex A_i and A_j.

Lemma 3: In the volume-coordinate system, let P be a point on the line M_1M_2. If $\frac{M_1P}{PM_2} = \lambda$, then the volume coordinates of P can be calculated as follows:

$$v_i = \frac{v_i^1 + \lambda v_i^2}{1 + \lambda}, \; i = 1,2,3,4. \quad (4)$$

where v_i^1 and v_i^2 are the volume coordinates of points M_1 and M_2, respectively.

Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix}$. Let $K(x, y, z)$ be the unknown target node location and (x_i, y_i, z_i) be the known location of the *i*th anchor node. The random 4 anchor nodes A_1, A_2, A_3, A_4 will be used to calculate the position of target node. The anchor nodes A_1, A_2, A_3, A_4 can form a tetrahedron. Then using Lemma 1 we can get the volume coordinates of the unknown node $K(v_1, v_2, v_3, v_4)$ as follows

$$v_1 = V_{KA_2A_3A_4} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x & x_2 & x_3 & x_4 \\ y & y_2 & y_3 & y_4 \\ z & z_2 & z_3 & z_4 \end{vmatrix} = \frac{1}{6}(A_{11} + A_{21}x + A_{31}y + A_{41}z) \quad (5)$$

$$v_2 = V_{A_1KA_3A_4} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x & x_3 & x_4 \\ y_1 & y & y_3 & y_4 \\ z_1 & z & z_3 & z_4 \end{vmatrix} = \frac{1}{6}(A_{12} + A_{22}x + A_{32}y + A_{42}z) \quad (6)$$

$$v_3 = V_{A_1A_2KA_4} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x & x_4 \\ y_1 & y_2 & y & y_4 \\ z_1 & z_2 & z & z_4 \end{vmatrix} = \frac{1}{6}(A_{13} + A_{23}x + A_{33}y + A_{43}z) \quad (7)$$

$$v_4 = V_{A_1A_2A_3K} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x \\ y_1 & y_2 & y_3 & y \\ z_1 & z_2 & z_3 & z \end{vmatrix} \quad (8)$$

$$= \frac{1}{6}(A_{14} + A_{24}x + A_{34}y + A_{44}z)$$

The values of v_1, v_2, v_3 and v_4 can also be calculated by (3) using Lemma 2. From (5)-(8), we get

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} A_{11} & A_{21} & A_{31} & A_{41} \\ A_{12} & A_{22} & A_{32} & A_{42} \\ A_{13} & A_{23} & A_{33} & A_{43} \\ A_{14} & A_{24} & A_{34} & A_{44} \end{bmatrix}\begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix} \quad (9)$$

$$= \frac{1}{6}A^*\begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = VA^{-1}\begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix} \quad (10)$$

Let $A^{-1} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix}$

Then (10) can be converted into

$$h_c = G_c Z_c \quad (11)$$

Where $Z_c = [x, y, z]^T$

$$h_c = \frac{1}{V}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}, \quad G_c = \begin{bmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \\ b_4 & c_4 & d_4 \end{bmatrix}$$

Using least square (LS) algorithm, we get

$$Z_c = (G_c^T G_c)^{-1} G_c^T h_c \quad (12)$$

As shown in Fig. 1, D is the midpoint of A_2A_4, Q is the barycenter of triangle $A_2A_3A_4$, and M is the centroid of tetrahedron $A_1A_2A_3A_4$. The volume coordinates for A_1, A_2, A_3, A_4 will be (V,0,0,0), (0,V,0,0), (0,0,V,0), (0,0,0,V), respectively.

Since $\frac{A_2D}{DA_4} = 1$, then D's volume coordinate will be (0, V/2, V/2,0) using Lemma 3. Because $\frac{A_3Q}{QD} = 2$, Q's volume coordinate will be (0, V/3, V/3, V/3) using Lemma 3. Since $\frac{A_1M}{MQ} = 3$, thus, the M's volume coordinate will be (V/4, V/4, V/4, V/4) using Lemma 3.

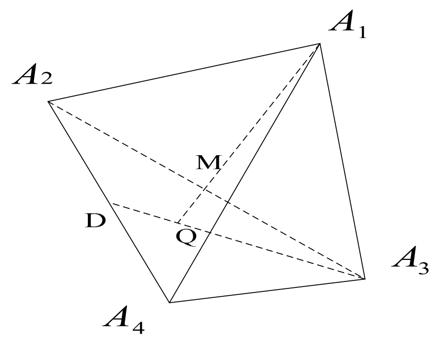

Fig.1. Tetrahedron $A_1A_2A_3A_4$

Then using the M's (the barycenter of tetrahedron $A_1A_2A_3A_4$) volume coordinate we get

$$h_{c1} = \frac{1}{V}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}. \quad (13)$$

From (11) and (13), the estimated position for unknown target nodes can be calculated using

$$Z_{c1} = (G_c^T G_c)^{-1} G_c^T h_{c1} \quad (14)$$

To summarize, our proposed localization algorithm is summarized as follows:

1. All anchors send their positions to all sensor nodes within their transmission range. Each unknown node collects all the beacon signals it receives from various reference points.
2. Each unknown node randomly selects four anchor nodes in range to form a series of tetrahedrons.
3. Apply the proposed centroid theorem of coordinate-tetrahedron in the volume-coordinate system described in this subsection to calculate the barycenter of each tetrahedron, then we use the average coordinates of these barycenters as the final estimated position of *i*th unknown node, $i = 1, 2, \cdots M$.

III. SIMULATION RESULTS

In this section, simulation results are presented and analyzed. The performance evaluation focuses on the position estimation accuracy of the proposed algorithm. Since the conventional centroid localization algorithm can only obtain 2D localization performance on 2D scenario, we simply extend the conventional Centroid localization algorithm to the three-dimensional space via adding z-coordinate.

We consider an experiment region in a 3D cubic region with a size of 100 m x 100 m x 100 m. As can be seen from [4], centroid location is robust under the effect of the irregularity of the radio pattern. The reason is that the centroid algorithm does not depend on hop-count and hopsize that the effect of degree of irregularity（DOI） is abated by the aggregation of beaconed information. So similar to [2][5], we also assume the sensor nodes have the same maximum radio range *R,* which is used for normalization only. Firstly, we deploy 100 sensor nodes randomly on the three-dimensional plane for the centroid localization algorithm. The number of sensor nodes and the radio range of sensor nodes will be varied then. The simulation results are averaged over 100 network instances. The simulation results are shown in Figs.2-3.

To begin with, we investigate the location errors resulted from the conventional centroid localization algorithm and the proposed algorithm. As observed from Fig. 2, the proposed algorithm has smaller location error in general with respect to different number of anchor nodes in the WSNs. Herein, the number of unknown nodes is fixed to 100. Generally, the location error decreases as the number of anchor nodes increases. For the same number of anchor nodes and in the same WSN environment, the position error in the proposed scheme is smaller than that in the conventional centroid localization algorithm. For example, with 20 anchor nodes, our proposed algorithm has an average error of approximately 0.54*R*, while the conventional centroid localization algorithm has an average error about 0.7*R*.

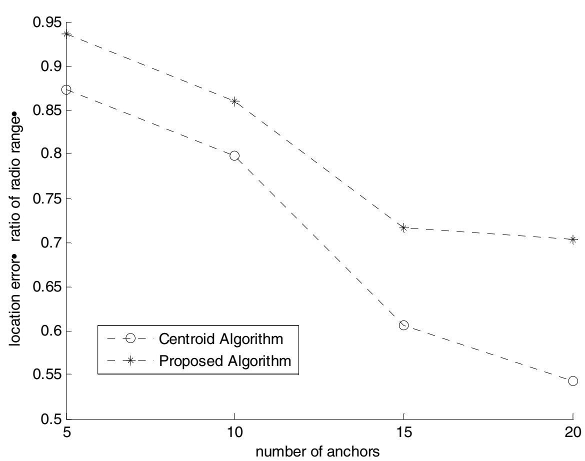

Fig. 2. Location error with respect to different number of anchor nodes (R=40m)

Figure 3 shows the location errors of both schemes in different node radio transmission range from 30 to 70 meters. Herein, the number of anchor nodes and unknown nodes are fixed to 20 and 100, respectively. As observed in Fig. 3, the location accuracy increases as the transmission range is increased. The increase in the location accuracy is due to the fact that an increase in the transmission range results in a range estimate closer to the Euclidean distance between two sensor nodes.

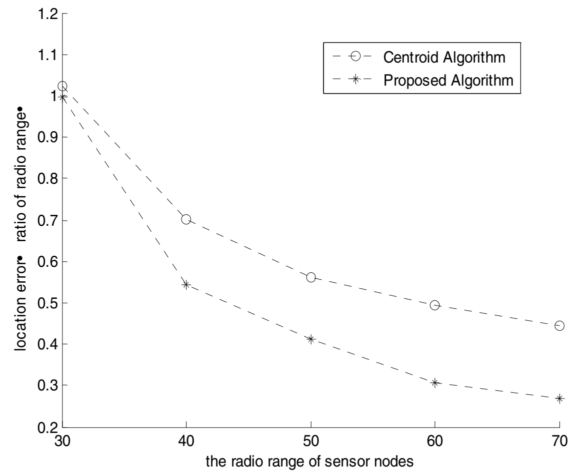

Fig.3. Location error with 20 anchor nodes

IV. CONCLUSION

We present a new localization method that improves the basic centroid localization algorithm significantly. It is shown in the simulation results that the proposed algorithm can improve location accuracy than the conventional centroid localization algorithm. The performance of our proposed scheme has identified that it has potential application advantage for 3D WSNs..

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