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NOVEL CONTROLLER FOR INTEGRATING AND STABLE PROCESSES WITH LONG TIME DELAY

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Abstract: In this paper, a novel control scheme is proposed on the basis of the well-known Smith predictor for the control of integrating processes with long time delay. It can provide one and half degree-of-freedom for the closed loop system tuning. Systematic design procedure is developed by employing optimal control theory, and simple and efficient tuning rules are derived analytically. It is shown that the design procedure can be directly extended to the control of stable processes with time delay. Numerical examples are given to illustrate the proposed method. *Copyright © 2002 IFAC*

Keywords: Linear system, integrating process, time delay, Smith predictor, optimal control, robust control

1. INTRODUCTION

A frequently encountered problem in the process industries is that of controlling processes with a significant time delay. This feature results from the presence of heat flow, material transportation, hydraulic and pneumatic transmission, chemical reactors and distillation columns. Dealing with this problem serves then as a starting point for the design of almost any process control system, regardless of its configuration.

"Time delay compensator" were developed in an attempt to overcome the detrimental effects of the time delay. The Smith predictor (Smith,1957) is the first of the compensation techniques. This technique utilizes an inner feedback compensation loop based on a first order plus time delay model of the process. The main drawback with the method is that it is ineffective for controlling integrating processes (Watanabe and Ito,1981; Zhang and Xu, 1999; Astrom *et al.*, 1994).

To solve the problem, Astrom *et al.*(1994) presented a modified Smith predictor for integrator/time delay processes. The method provides superior performance to that of previous algorithms. One main merit of the new Smith predictor is that it decouples the setpoint response from the disturbance response. In other words, it is of two degree-of-freedom. Thus, the setpoint response and disturbance response can be optimized independently. The scheme was simplified and improved by Zhang and Sun(1996) and Tian and Gao (1998; 1999) in both structure and tuning rules and had been extended to the control of a first order plus time delay process (Zhang *et al.*, 1998).
The objective of this paper is to develop a novel scheme for the control of integrating processes with long time delay. The feature of the new scheme is that it can provide one and half degree-of-freedom for the closed loop system tuning. Only setpoint loop can be tuning independently. A systematic procedure for the design of controllers is formulated by employing optimal control techniques, and simple and efficient tuning rules are derived analytically. It is shown that the design procedure can be directly extended to the control of stable processes with time delay.

We preview the some of the contents. In section 2, the optimal performance criterion is defined for setpoint loop. Analytical method is developed for the controller design. A compensating structure is introduced in section 3, and the controller of disturbance loop is then derived analytically. In section 4 the proposed design method is extended to the controller design of stable processes with time delay. The conclusions are finally given in section 5.

2. CONTROLLER DESIGN FOR TRACKING

Optimal control is a well-established branch of control theory and is concerned with obtaining the best performance, in some sense, from a system. A statement of the optimal control problem usually consists of a definition of the system structure and a performance criterion. The control law is then obtained as the solution that minimizes the specified criterion within the admissible set of control signals.

In this paper, we consider a new modified Smith predictor of which the structure is shown in Fig. 1. In the figure, *R(s)* is the controller of setpoint loop, *D(s)* is the controller of disturbance loop, *G(s)* is the plant, and $G_{mo}(s)$ is the delay-free part of the plant model $G_m(s)$. It is seen that the control structure of setpoint loop is similar to that of internal model control (Zhang and Xu, 1999; Garcia *et al.*, 1982). Hence, instead of the controller *R(s)* we first design the controller *Q(s)*

$$Q(s) = R(s)/(1 + R(s)G_{mo}(s)) \tag{1}$$

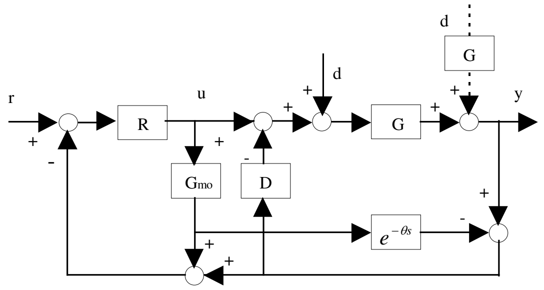

Fig. 1. Generalized structure of modified Smith predictor

The integrating process is given by

$$G(s) = A(s)e^{-\theta s}/(TsB(s)) \tag{2}$$

where T is the integral constant, $A(s)$ and $B(s)$ are polynomials in the Laplace transform variable s, all the roots of $A(s)$ and $B(s)$ are in the left half plane, $A(0)=B(0)=1$, and $\deg(B(s))\geq\deg(A(s))$. If the model is exact, i.e., $G_m(s)=G(s)$, then the transfer function of setpoint loop can be written as

$$H_r(s) = \frac{R(s)G(s)}{1 + R(s)G_{mo}(s)} = \frac{Q(s)A(s)}{TsB(s)}e^{-\theta s} \tag{3}$$

In order to track the setpoint asymptotically, $H_r(s)$ should satisfy the following constraint $\lim_{s\to 0} H_r(s) = 1$. This implies that the *Q(s)* should be of the form $Q(s)=TsQ_1(s)$ with $Q_1(0)=1$, or equivalently, $Q(s) = Ts(1 + sQ_2(s))$, where $Q_1(s)=1+sQ_2(s)$ and $Q_2(s)$ is stable.

A central concept in automatic control is the idea that the system output *y* can track the reference value *r* as well as possible in a suitable sense. The idea gives rise to the following performance criterion $\min\|W(s)(1 - H_r(s))\|_2$, where *W(s)* is the weighting function and $\|\cdot\|_2$ denotes the 2 norm of the transfer function.

In process control, the controller is always designed for step setpoint. Thus, *W(s)* can be selected as $1/s$. By utilizing the *n/n* order all-pass Pade approximation (Saff and Varga, 1977) we have

$$G(s) = \frac{A(s)}{TsB(s)}\frac{Q_{nn}(-\theta s)}{Q_{nn}(\theta s)} \tag{4}$$

where $Q_{nn}(\theta s) = 1 + q_1 s + q_2 s^2 + \cdots + q_n s^n$, $q_j = \frac{(2n-j)!n!}{(2n)!j!(n-j)!}\theta^j$, $j = 1,2,\ldots,n$, and n is an integer large enough. It follows that

$$\begin{aligned}&\|W(s)(1 - H_r(s))\|_2^2\\ &= \left\|\frac{1}{s}\left(1 - \frac{Q(s)A(s)}{TsB(s)}\frac{Q_{nn}(-\theta s)}{Q_{nn}(\theta s)}\right)\right\|_2^2\\ &= \left\|\begin{matrix}\frac{Q_{nn}(\theta s) - Q_{nn}(-\theta s)}{sQ_{nn}(-\theta s)}\\ + \frac{(B(s) - A(s)) - sA(s)Q_2(s)}{sB(s)}\end{matrix}\right\|_2^2\end{aligned} \tag{5}$$

Notice that $Q_{nn}(\theta s)$, $Q_{nn}(-\theta s)$, $A(s)$, and $B(s)$ are polynomials with $Q_{nn}(0)=Q_{nn}(0)=A(0)=B(0)=1$. Hence, s must be the factor of $Q_{nn}(\theta s)- Q_{nn}(-\theta s)$ and $B(s)-A(s)$. Furthermore, it is known that all of the roots of $Q_{nn}(-\theta s)$ are on the right half plane. With the orthogonality property of the 2 norm we have

$$\|W(s)(1-H_r(s))\|_2^2$$
$$=\left\|\frac{(Q_{nn}(\theta s)-Q_{nn}(-\theta s))/s}{Q_{nn}(-\theta s)}\right\|_2^2+$$
$$\left\|\frac{(B(s)-A(s))/s-A(s)Q_2(s)}{A(s)}\right\|_2^2$$

Notice that the right part of the above equation may not be proper. This does not appear to be a severe handicap, because poles can always be added to the denominator such that we can approximate the part arbitrarily closely without being improper. Minimizing the right side yields the optimal *Q(s)*

$$Q_{opt}(s)=TsB(s)/A(s)$$

Hence, we derive the controller analytically. To implement the controller physically we introduce a low-pass filter to roll *Q(s)* off at high frequency. Let

$$J_r(s)=1/(\lambda_1 s+1)^i \tag{7}$$

where λ_1 >0, *i*=deg(*B(s)*)-deg(*A(s)*)+1. The filter satisfies the asymptotic trajectory constraint. Then

$$Q(s)=Q_{opt}(s)J_r(s)=\frac{TsB(s)}{A(s)(\lambda_1 s+1)^i} \tag{8}$$

After routine algebra we obtain

$$R(s)=\frac{TsB(s)}{A(s)((\lambda_1 s+1)^i-1)} \tag{9}$$

It is found that *D(s)* has no effect on *R(s)*. Thus, the setpoint response can be adjusted independently. When λ_1 tends to be zero, the system recovers optimality.

If the model is exact, the transfer function of the setpoint loop is

$$H_r(s)=e^{-\theta s}/(\lambda_1 s+1)^i \tag{10}$$

The characteristic equation is given as follows: $\lambda_1 s+1=0$. It is seen that the closed loop system is always stable. When there exists model-plant mismatch, the stability margin and the robustness of the closed loop system can be monotonously adjusted by the parameter λ_1. The property has been studied by many researchers (Zhang and Xu, 1999; Zhang and Sun, 1996; Tian and Gao, 1998;1999).

3. REJECTION OF DISTURBANCE

To simplify the analysis and design, we consider the effect caused by *dG(s)* without losing generality (Fig. 1). Suppose that the model is exact. It is easy to verify that the transfer function from the disturbance *dG(s)* to the system output *y* is

$$H_{dG}(s)=(1+G(s)Q(s))/(1+G(s)D(s)) \tag{11}$$

We find that the disturbance response is determined by not only *Q(s)* but also *D(s)*. This implies that the new scheme can only provide one and half degree-of-freedom for the closed loop system tuning. The characteristic equation of the closed loop system consists of the denominator of 1+*G(s)Q(s)* and the nominator of 1+*G(s)D(s)*. It is seen that there exists a time delay in the term 1+*G(s)D(s)*. This greatly complicates the analysis and design of the control system. It is well known that the attractiveness of Smith predictor comes from the fact that it removes the time delay from the closed loop characteristic equation, thus seemingly converting the design problem for a system with time delay to that for a system without time delay. Similarly, we will construct the structure of *D(s)* to eliminate the time delay. From the discussion of Zhang and Sun(1996) and Tian and Gao(1998; 1999) we know that many time delays in the characteristic equation can be eliminated by introducing an internal model. It might as well let

$$D(s)=\frac{TsB(s)D_0(s)}{1-G(s)TsB(s)D_0(s)} \tag{12}$$

where $D_0(s)$ is a stable rational function. This leads to

$$H_{dG}(s)=\frac{((\lambda_1 s+1)^i+e^{-\theta s})(1-A(s)D_0(s)e^{-\theta s})}{(\lambda_1 s+1)^i} \tag{13}$$

Such a transfer function allows the use of classical design techniques developed for rational transfer functions.

To reject the disturbance *dG(s)* asymptotically, $H_{dG}(s)$ should satisfy the following constraint

$$\lim_{s\to 0}H_{dG}(s)=\lim_{s\to 0}\frac{((\lambda_1 s+1)^i+e^{-\theta s})(1-A(s)D_0(s)e^{-\theta s})}{(\lambda_1 s+1)^i}=0 \tag{14}$$

Then we get $D_0(s)=1+sD_1(s)$, where $D_1(s)$ is stable. The controller $D_0(s)$ should be designed such that the system output *y* caused by the disturbance *dG(s)* is minimized, i.e. $\min\|W(s)H_{dG}(s)\|_2\big|_{\lambda_1\to 0}$.

In the context of process control, the disturbance is usually assumed to be a step. Therefore, we select the same weighting function *W(s)*. It follows that

$$\|W(s)H_{dG}(s)\|_2^2 = \left\|\frac{1}{s}(1+e^{-\theta s})(1-A(s)D_0(s)e^{-\theta s})\right\|_2^2 \quad (15)$$

Employing the all-pass Pade approximation gives

$$\|W(s)H_{dG}(s)\|_2^2 = \left\|\begin{array}{l}\frac{Q_{nn}(2\theta s)-Q_{nn}(-2\theta s)}{sQ_{nn}(-2\theta s)}-\\ \frac{A(s)D_0(s)Q_{nn}(\theta s)-Q_{nn}(-\theta s)}{sQ_{nn}(-\theta s)}+\\ \frac{Q_{nn}(\theta s)-A(s)Q_{nn}(-\theta s)}{sQ_{nn}(-\theta s)}\end{array}\right\|_2^2 + \|A(s)D_1(s)\|_2^2 \quad (16)$$

Minimize the right side we get the optimal $D_{1opt}(s)$=0. The optimal $D_0(s)$ can be consequently expressed as follows: $D_{0opt}(s)=1$. Since *G(s)* is an integrating process, the system is of type II. When we consider the asymptotic rejection of disturbance *d*, the following constraint is required

$$\lim_{s\to 0}\frac{d}{ds}H_{dG}(s) = \lim_{s\to 0}\frac{d}{ds}\frac{((\lambda_1 s+1)^i+e^{-\theta s})(1-A(s)D_0(s)e^{-\theta s})}{(\lambda_1 s+1)^i} = 0 \quad (17)$$

Therefore, the following filter should be selected

$$J_d(s)=\frac{\beta_1 s+\beta_0}{(\lambda_2 s+1)^j} \quad (18)$$

where λ_2 >0, *j*=deg(*B(s)*)+2. From the constraint (14) we obtain β_0 =1, and the constraint (17) gives

$$\beta_1=\theta+j\lambda_2-\frac{d}{ds}A(0)$$

Hence, the controller that rejects the disturbance at the plant input can be written as

$$D_0(s)=D_{0opt}J_d(s)=\frac{\beta_1 s+\beta_0}{(\lambda_2 s+1)^i} \quad (19$$

If the model is exact, the transfer function of the disturbance loop is

$$H_{dG}(s)=\frac{((\lambda_1 s+1)^i+e^{-\theta s})((\lambda_2 s+1)^j-A(s)(\beta_1 s+\beta_0)e^{-\theta s})}{(\lambda_1 s+1)^i(\lambda_2 s+1)^j} \quad (20)$$

We see that the loop is also stable. Compared with the setpoint loop, the stability margin and the robustness of the closed loop system is determined by not only the parameters λ_2 but also the parameters λ_1.

From the above results we obtain

$$D(s)=\frac{TsB(s)(\beta_1 s+\beta_0)}{(\lambda_2 s+1)^i-A(s)(\beta_1 s+\beta_0)e^{-\theta s}} \quad (21)$$

As there is a time delay in the denominator the implementation of the structure needs an inner loop. Sometimes, we hope to use a simpler controller. In this case, we can substitute the 1/1 order Pade approximation for the time delay (Saff and Varga, 1977). It follows that

$$D(s)=\frac{TsB(s)(\beta_1 s+\beta_0)(1+\frac{\theta}{2}s)}{(\lambda_2 s+1)^i(1+\frac{\theta}{2}s)-A(s)(\beta_1 s+\beta_0)(1-\frac{\theta}{2}s)} \quad (22)$$

For an integrator/time delay process one gets

$$D(s)=\frac{T((\theta+2\lambda_2)s+1)(1+\frac{\theta}{2}s)}{s(\frac{\theta\lambda_2^2}{2}s+\frac{\theta^2}{2}+2\lambda_2\theta+\lambda_2^2)}$$

It is found that the controller is in fact a PID controller.

Example 1 Consider the process described by the following transfer function

$$G(s)=e^{-5s}/s$$

For both the optimal controller and simplified controller we take λ_1=1/0.6 and λ_2=8. A unit step setpoint change is introduced at time t=0, and a step disturbance with magnitude 0.1 is introduced at time t=100. The responses are shown in Fig. 2 with a good process model. Fig. 3 shows the effect of 10% error in estimating the time delay. The practical time delay is 4.5. The controller is found to be robust against this degree of uncertainty.

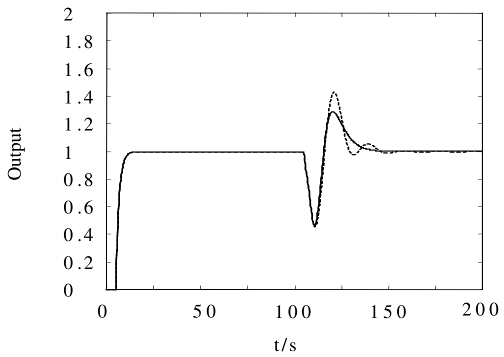

Fig. 2. Responses of nominal systems
(Solid line: optimal; dashed line: simplified)

Example 2 Consider a high order process given by the following transfer function

$$G(s) = \frac{e^{-5s}}{s(s+1)(0.5s+1)(0.2s+1)(0.1s+1)}$$

For the optimal controller we take λ_1=1/1.7 and λ_2=2. The responses of closed loop system are shown in Fig. 4.

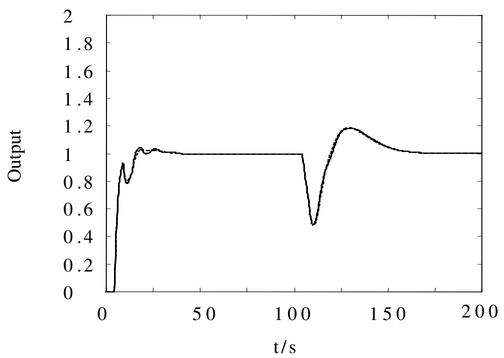

Fig. 3 Responses of uncertain systems
(Solid line: optimal; dashed line: simplified)

4. EXTENSION TO STABLE PROCESSES

Though a lot of methods have been developed for controlling stable processes with time delay, we extend the method proposed in this paper to these processes. The extension can provide some insight into the performance improvement resulted from modifying the control structure.

Suppose that the process is described by the following transfer function

$$G(s) = KA(s)e^{-\theta s} / B(s) \tag{23}$$

where K is the gain, $A(s)$ and $B(s)$ are monic polynomial in s, all the roots of $A(s)$ and $B(s)$ are in the left half plane, and deg($B(s)$)>deg($A(s)$). If the model is exact, then the transfer function of setpoint loop can be written as

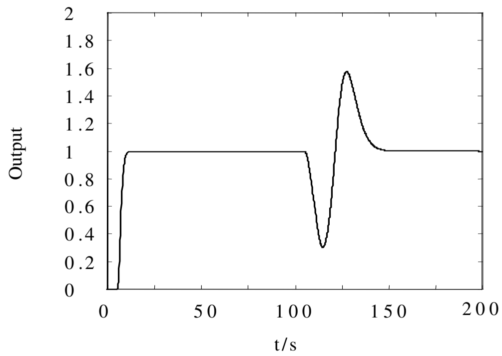

Fig. 4. Responses of high order systems
(Solid line: optimal)

$$\begin{aligned} H_r(s) &= \frac{R(s)G(s)}{1+R(s)G_{mo}(s)} \\ &= \frac{KQ(s)A(s)}{B(s)}e^{-\theta s} \end{aligned} \tag{24}$$

By similar design procedure one obtains

$$Q(s) = \frac{B(s)}{KA(s)(\lambda_1 s+1)^i} \tag{25}$$

and

$$R(s) = \frac{B(s)}{KA(s)((\lambda_1 s+1)^i - 1)} \tag{26}$$

where i=deg($B(s)$)-deg($A(s)$).
In the disturbance loop, $D(s)$ is selected as follows

$$D(s) = \frac{B(s)D_0(s)}{K - G(s)B(s)D_0(s)} \tag{27}$$

where $D_0(s)$ is a stable rational function. It follows that

$$H_{dG}(s) = \frac{((\lambda_1 s+1)^i + e^{-\theta s})(1-A(s)D_0(s)e^{-\theta s})}{(\lambda_1 s+1)^i} \quad (28)$$

The optimal $D_0(s)$ is then obtained

$$D_{0opt}(s) = 1 \quad (29)$$

It should be notice that the system is of type I. The following filter is selected

$$J_d(s) = \frac{1}{(\lambda_2 s+1)^j} \quad (30)$$

where j=deg($B(s)$). Hence, the controller can be written as

$$D_0(s) = \frac{1}{(\lambda_2 s+1)^i} \quad (31)$$

Example 3 Consider the following process (Huang *et al.*, 1990)

$$G(s) = \frac{e^{-3s}}{10s+1}$$

Take λ_1=2 and λ_2=2 for the optimal controller. A unit step setpoint change is introduced at time t=0, and a unit step disturbance is introduced at time t=100. It is found that the proposed controller provides better performance than that of the modified Smith predictor with improved disturbance rejection capability (MSP) (Huang *et al.*, 1990) (Fig. 5).

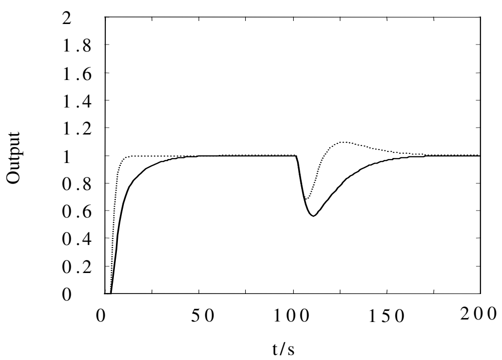

Fig. 5. Control of stable processes
(Solid line: MSP; dotted line: optimal)

5. CONCLUSIONS

The major objective of this paper is to focus on the design of a novel scheme for integrating and stable processes with long time delay. The main contribution of the paper is the analytical derivation of the optimal controller. It should be point out that such a procedure can be directly applied for nonminimum phase processes. The tuning of the resulted closed loop system is very simple, since the response is determined by the adjustable parameter monotonously. All of the traditional one degree-of-freedom controller, the one and half degree-of-freedom controller and the two degree-of-freedom controller have their own merits. The superiority of the proposed controller is that it provides more freedom than one degree-of-freedom controllers and less complexity than two degree-of-freedom controllers and this gives another selection for control system designers.

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