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Novel equivalent function for deadzone nonlinearity: applied to analytical solution of beam vibration using He's Parameter Expanding Method

Abstract

This study intends to introduce the novel and efficient exact equivalent function (EF) for well-known deadzone nonlinearity. To indicate the effectiveness of this EF, the nonlinear vibration of cantilever beam in presence of deadzone nonlinear boundary condition is studied. The powerful analytical method, called He's Parameter Expanding Method (HPEM) is used to obtain the exact solution of dynamic behavior of mentioned system. It is shown that one term in series expansions is sufficient to obtain a highly accurate solution. Comparison of the obtained solutions using numerical method shows the soundness of this analytical EF.

Keywords

Deadzone nonlinearity, Equivalent function, He's Parameter Expanding Method, cantilever beam

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1 INTRODUCTION

The nonlinear free vibration of beams is of considerable interest to engineers and has been much studied. From the engineering point of view and to be more accurate, structures such as bridges, buildings, and space-craft arms should be considered as flexible beams. In some cases natural responses of these structures are essentially nonlinear and hence are described by nonlinear equations. Otherwise, the application of different numerical techniques is unavoidable.

The sources of nonlinearity of vibration systems are generally considered as due to the following aspects: (1) the physical nonlinearity, (2) the geometric nonlinearity and, (3) the nonlinearity of boundary conditions. As it is reported in many research papers, the deadzone nonlinearity is an on-differentiable function. This input characteristic is ubiquitous in a wide range of mechanical and electrical components such as valves, gear vibration, DC servo motors, and other devices. However, approximation of this nonlinear condition to obtain analytical solution of behavior of mentioned systems is always the major difficulty of engineer's computations. Marcio and Leandro [16] used the error function as an approximation of deadzone-type nonlinearity in deriving analytical models for the Least Mean Square (LMS) adaptive algorithm. Chengwu and Rajendra [7] used the arctangent function to approximate the non-analytical deadzone relationship in preloaded spring in a mechanical oscillator. To analyze the drillstring vibrations in a near vertical hole, Hakimi and Moradi [13] modeled contact between the drillstring and formation wall by series of springs with deadband gap using DQM. Recently, considerable attention has been directed towards analytical solutions for nonlinear equations without small parameters. Many new techniques have been appeared in the literature such as perturbation techniques [11, 20, 29], variational iteration method [6], iteration perturbation method [9], He's Improved Amplitude-Formulation (IAFF)[6], HAM [24], HPM [8], MHPM [18], Meshless analysis [14], Modified wave approach [2] and Min-Max method [5] are used to solve nonlinear problems. He's Parameter expanding method (HPEM) is the most effective and convenient method to analytically solve of nonlinear differential equations. HPEM has been shown to effectively, easily and accurately solve large nonlinear problems with components that converge rapidly to accurate solutions. Tao [28] suggested He's parameter expanding method for strongly nonlinear oscillators and propose frequency–amplitude relationship of nonlinear oscillators using He's Parameter expanding method. Furthermore, during the past decades, the nonlinear vibrations of Euler-Bernoulli beams have received considerable attention by many researchers [4, 10, 12, 15, 17, 19, 21–23, 25–27, 27, 29, 30]. But, heretofore, deadzone nonlinearity, as a nonlinear boundary condition, due to its inherent difficulty, hasn't been modeled exactly by researchers.

The main objective of this paper was to obtain analytical expressions for geometrically nonlinear vibration of Euler–Bernoulli beam using HPEM, with deadzone nonlinear boundary condition, by introducing novel and efficient EF. First the nonlinear partial differential equation of motion reduced by implementation of Bubnov-Galerkin method, and then mentioned EF has been used for deadzone nonlinear boundary condition. As we can see, the results presented in this paper reveal that the method is very effective and convenient for nonlinear oscillators for which the highly nonlinear boundary condition exists. To validate the EF, it's shown that one term in series expansions is sufficient to obtain a highly accurate solution of the problem.

2 EQUATION OF MOTION

Figure 1 shows a clamped-free flexible beam of length L, a cross-sectional area A, the mass per unit length of the beam m, a moment of inertia I, and a modulus of elasticity E. Linear spring with constant K is in contact at free end of cantilever beam with a deadzone clearance δ. Assume that the beam considered here is the Euler–Bernoulli beam. The symbol w denotes the displacement of a point in the middle plane of the flexible beam in y direction.

The governing equation of motion for the uniform beam shown in Fig. 1 is given by [29]:

$$m\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} - \frac{EA}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx = 0 \quad (1)$$

which is subjected to the following boundary conditions

$$w(0,t) = \frac{\partial w}{\partial x}(0,t) = 0, \ \ \frac{\partial^2 w}{\partial x^2}(L,t) = 0, \ \ EI\frac{\partial^3 w}{\partial x^3}(L,t) = F_{dz}(L,t) \quad (2)$$

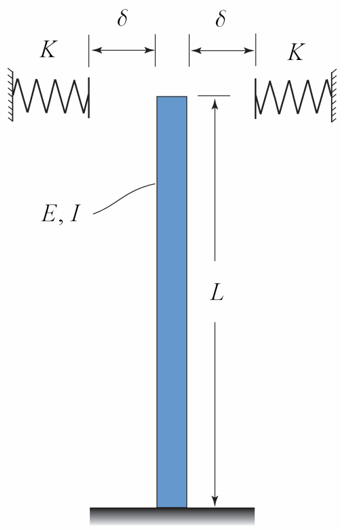


Figure 1 Cantilever beam with deadzone nonlinear boundary condition.

where $F_{dz}(L,t)$ is described by the following nonlinear deadzone formula

$$F_{dz}(L,t) = \begin{cases} K(w(L,t)-\delta) & w(L,t) > \delta \\ 0 & -\delta \le w(L,t) \le \delta \\ K(w(L,t)+\delta) & w(L,t) < -\delta \end{cases} \tag{3}$$

Assuming $w(x,t) = q(t)\,\varphi(x)$, where $\varphi(x)$ is the first eigenmode of the clamped-free beam and can be expressed as:

$$\varphi(x) = \cosh(\lambda x) - \cos(\lambda x) - \alpha(\sinh(\lambda x) - \sin(\lambda x)) \tag{4a}$$

and

$$\alpha = \frac{\cosh(\lambda L) + \cos(\lambda L)}{\sinh(\lambda L) + \sin(\lambda L)} \tag{4b}$$

where $\lambda = 1.875$ is the root of characteristic equation for first eigenmode. Applying the Bubnov-Galerkin method yields:

$$\int_0^L \left(m\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} - \frac{EA}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \right)\varphi(x)\,dx = 0 \tag{5}$$

to implement the end nonlinear boundary condition, applying integration by part on equation (5), it is converted to the following

$$\int_0^L \left(m\frac{\partial^2 w}{\partial t^2} - \frac{EA}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \right)\varphi(x)\,dx + \int_0^L EI\frac{\partial^4 w}{\partial x^4}\varphi(x)\,dx = 0 \tag{6}$$

$$\int_0^L \left(m\frac{\partial^2 w}{\partial t^2} - \frac{EA}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \right)\varphi(x)\,dx + EI\frac{\partial^3 w}{\partial x^3}\varphi(x)\Big|_0^L \int_0^L EI\frac{\partial^3 w}{\partial x^3} d(\varphi(x)) = 0 \quad (7)$$

and the nonlinear equation of motion can be written as

$$\frac{d^2 q}{dt^2} + \beta_1 q(t) + \beta_2 (q(t))^3 + F_{dz} = 0 \quad (8)$$

where

$$\beta_1 = 12.362EI/mL^4,\ \beta_2 = -1.994EA/mL^4 \quad (9)$$

To solve nonlinear ordinary equation (8) analytically, the deadzone conditionF_{dz}, must be formulated, properly. We introduce suitable and novel exact equivalent function for this nonlinearity as:

$$F_{dz} = {}^{K}\!/_{2}\left(2w(L,t) + |w(L,t) - \delta| - |w(L,t) + \delta|\right) \quad (10)$$

Figure 2 shows the equivalent function for F_{dz} with deadzone clearance δ, graphically.

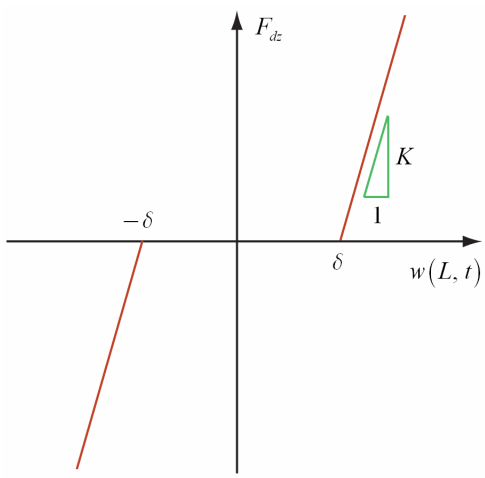


Figure 2 Plot of EF deadzone nonlinearity.

Using this new definition of F_{dz}, equation (9) is written as follows:

$$\frac{d^2 q}{dt^2} + \beta_1' q(t) + \left[\beta_2 (q(t))^3 + \beta_3 \{|2q(t) - \delta| - |2q(t) + \delta|\}\right] = 0 \quad (11)$$

where

$$\beta_1' = \beta_1 + 4K/mL\ ,\ \ \beta_3 = K/mL \quad (12)$$

3 SOLUTION PROCEDURE

Consider the equation (11) for the vibration of a cantilever Euler-Bernoulli beam with the following general initial conditions

$$q(0) = A\ ,\quad \dot{q}(0) = 0 \tag{13}$$

Free oscillation of a system without damping is a periodic motion and can be expressed by the following base functions

$$\cos(m\omega t)\ ,\quad m = 1, 2, 3, ... \tag{14}$$

We denote the angular frequency of oscillation by ω and note that one of our major tasks is to determine $\omega(A)$,i.e., the functional behavior of ω as a function of the initial amplitude A. In the HPEM, an artificial perturbation equation is constructed by embedding an artificial parameter $p \in [0, 1]$ which is used as an expanding parameter.

According to HPEM the solution of equation (11) is expanded into a series of p in the form

$$q(t) = q_0(t) + pq_1(t) + p^2 q_2(t) + ... \tag{15}$$

The coefficients 1 and β_1' in the equation (11) are expanded in a similar way

$$\begin{aligned} 1 &= 1 + pa_1 + p^2 a_2 + ... \\ \beta_1' &= \omega^2 - pb_1 - p^2 b_2 + ... \\ 1 &= pc_1 + p^2 c_2 + ... \end{aligned} \tag{16}$$

where a_i, b_i, c_i $(i = 1, 2, 3, ...)$are to be determined. When$p = 0$, equation (11) becomes a linear differential equation for which an exact solution can be calculated for$p = 1$. Substituting equations (15) and (16) into equation (11)

$$\begin{aligned} &\left(1 + pa_1 + p^2 a_2\right)\left(\ddot{q}_0(t) + p\ddot{q}_1(t) + p^2\ddot{q}_2(t)\right) + \left(\omega^2 - pb_1 - p^2 b_2\right)\left(q_0(t) + pq_1(t) + p^2 q_2(t)\right) \\ &+ \left(pc_1 + p^2 c_2\right)\left[\beta_2\left(q_0(t) + pq_1(t) + p^2 q_2(t)\right)^3 + \beta_3 f_{dz}\left(q_0(t) + pq_1(t) + p^2 q_2(t)\right)\right] = 0 \end{aligned} \tag{17}$$

where

$$f_{dz}(q(t)) = |2q(t) - \delta| - |2q(t) + \delta| \tag{18}$$

in equation (18) we have taken into account the following expression

$$\begin{aligned} &f_{dz}(q) = f_{dz}\left(q_0 + pq_1 + p^2 q_2 + ...\right) = ... \\ &... f_{dz}(q_0) + pq_1 f_{dz}'(q_0) + p^2\left[q_2 f_{dz}'(q_0) + \tfrac{1}{2} q_1^2 f_{dz}''(q_0)\right] + O\left(p^3\right) \end{aligned} \tag{19}$$

where

$$f_{dz}'(q) = \frac{df_{dz}}{dq} = 2\frac{|2q(t) - \delta|}{2q(t) - \delta} - 2\frac{|2q(t) + \delta|}{2q(t) + \delta}, f_{dz}''(q) = f_{dz}'''(q) = ... = 0 \tag{20}$$

collecting the terms of the same power of p in equation (17), we obtain a series of linear equations which the first equation is

$$\ddot{q}_0(t) + \omega^2 q_0(t) = 0, \quad q_0(0) = A, \quad \dot{q}_0(0) = 0 \tag{21}$$

with the solution

$$q_0(t) = A\cos(\omega t), \tag{22}$$

substitution of this result into the right-hand side of second equation gives

$$\begin{aligned}&\ddot{q}_1(t) + \omega^2 q_1(t) = \left(b_1 A - \tfrac{3}{4}c_1\beta_2 A^3 + 4c_1\beta_3 A + a_1 A\omega^2\right)\cos(\omega t)\\&+\tfrac{16}{3\pi}c_1\beta_3 A\cos(2\omega t) - \tfrac{1}{4}c_1\beta_3 A\cos(3\omega t),\end{aligned} \tag{23}$$

In the above equation, the possible following Fourier series expansion have been accomplished

$$\begin{aligned}f_{dz}(q_0) &= f_{dz}(A\cos(\omega t)) = \textstyle\sum_{n=1}^{\infty} h_n\cos(n\omega t) = h_1\cos(\omega t) + h_2\cos(2\omega t) + ...\\f'_{dz}(q_0) &= f'_{dz}(A\cos(\omega t)) = \textstyle\sum_{n=1}^{\infty} v_n\cos(n\omega t) = v_1\cos(\omega t) + v_2\cos(2\omega t) + ...\end{aligned} \tag{24}$$

where

$$\begin{aligned}h_n &= \tfrac{2}{\pi}\textstyle\int_{-\pi/2}^{\pi/2} f_{dz}(A\cos\theta)\cos(n\theta)\,d\theta,\\v_n &= \tfrac{2}{\pi}\textstyle\int_{-\pi/2}^{\pi/2} f'_{dz}(A\cos\theta)\cos(n\theta)\,d\theta,\end{aligned} \tag{25}$$

and the functions f_{dz}, f'_{dz} are substituted from equations (18) and (20). The first terms of the expansion in equations (25) are given by

$$\begin{aligned}h_1 &= \tfrac{2}{\pi}\textstyle\int_{-\pi/2}^{\pi/2} f_{dz}(A\cos\theta)\cos(\theta)\,d\theta = -4A\\v_1 &= \tfrac{2}{\pi}\textstyle\int_{-\pi/2}^{\pi/2} f'_{dz}(A\cos\theta)\cos(\theta)\,d\theta = -\tfrac{16}{\pi}\end{aligned} \tag{26}$$

No secular terms in $q_1(t)$ require eliminating contributions proportional to $\cos(\omega t)$on the right-hand side of equation (23)

$$b_1 A - \frac{3}{4}c_1\beta_2 A^3 + 4c_1\beta_3 A + a_1 A\omega^2 = 0 \tag{27}$$

But equation (16) for one term approximation of series respect to $p$and for $p = 1$ yields

$$a_1 = 0,\ b_1 = \omega^2 - \beta_1',\ c_1 = 1 \tag{28}$$

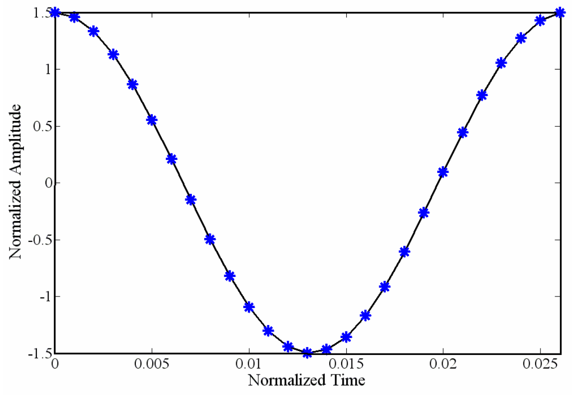
From equations (27) and (28) we can easily find that the solution $\omega$is

$$\omega(A) = \pm\sqrt{\beta_1' + \frac{3}{4}\beta_2 A^2 - 4\beta_3} \tag{29}$$

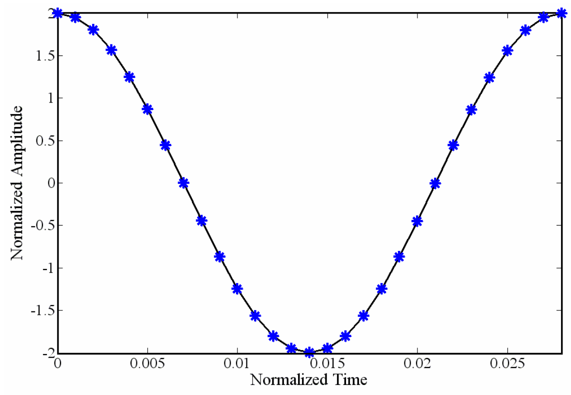
Replacing ω from equation (29) into equation (22) yields:

$$q(t) \approx q_0(t) = A\cos\left(\sqrt{\beta_1' + \frac{3}{4}\beta_2 A^2 - 4\beta_3}\, t\right) \tag{30}$$

To demonstrate the soundness of the obtained analytical results, the authors also calculate the variation of non-dimensional amplitude A/δ vs. $\tau = \omega t$, numerically. As can be seen in the figures 3a and 3b the first order approximation of $q(t)$ obtained using the HPEM with EF for deadzone nonlinearity has an excellent agreement with numerical results using fourth-order Runge–Kutta method.



(a) Comparison of the approximate first order periodic solution (continuous line) with the numerical solution (circles) with $A/\delta = 1.5$.



(b) Comparison of the approximate first order periodic solution (continuous line) with the numerical solution (circles) with $A/\delta = 2$.

Figure 3

4 CONCLUSION

In this study deadzone discontinuous nonlinearity has been considered as a boundary condition of a cantilever beam and redefined exactly using the basic continuous functions. Using the novel and efficient EF for the deadzone nonlinearity, an excellent first-order analytical approximate solution by HPEM was obtained which can predict the nonlinear frequency of mentioned system as a function of amplitude. It was demonstrated that the introduced EF can significantly make the analytical study of dynamic behavior of the nonlinear problems to be easier. We can see that the introduced method has special potential to be applied to the other strongly nonlinear oscillators with deadzone nonlinearity.

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