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Novel hyperbolic and exponential ansatz methods to the fractional fifth-order Korteweg–de Vries equations

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Abstract

This paper aims to investigate the class of fifth-order Korteweg–de Vries equations by devising suitable novel hyperbolic and exponential ansätze. The class under consideration is endowed with a time-fractional order derivative defined in the conformable fractional derivative sense. We realize various solitons and solutions of these equations. The fractional behavior of the solutions is studied comprehensively by using 2D and 3D graphs. The results demonstrate that the methods mentioned here are more effective in solving problems in mathematical physics and other branches of science.

Keywords: Fractional derivative; Fifth-order KdV equations; Hyperbolic wave solutions; Exponential wave solutions; Solitary wave solutions

1 Introduction

Nonlinear partial differential equations (PDEs) play a significant role in several scientific and engineering fields [1–5]. Since the discovery of the soliton in 1965 by Zabusky and Kruskal [6], many nonlinear PDEs have been derived and extensively applied in different branches of physics and applied mathematics [7–17]. Nonlinear PDEs appear in condensed matter, solid state physics, fluid mechanics, chemical kinetics, plasma physics, nonlinear optics, propagation of fluxion in Josephson junctions, ocean dynamics and many others [18–27]. In order to understand the different nonlinear phenomena, various methods for obtaining exact solutions to nonlinear PDEs have been proposed [28–31].

One of the most interesting evolution equations with a lot of applications in describing different phenomena is the Korteweg–de Vries equation [32, 33]. This equation occurs in different types, orders and lots of modifications [34–46]. Certain applications of the equation are found in many fields including fluids dynamics, plasma physics and shallow water and nonlinear waves processes, respectively. The main motivation of this work is to study the fifth-order Korteweg–de Vries equation endowed with a time-fractional order derivative in time that reads [47]

$$u_t^\alpha + au^2u_x + bu_xu_{xx} + cuu_{xxx} + du_{xxxxx} = 0, \quad (1)$$

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where $\alpha \in (0, 1]$ is the fractional order derivative and a, b, c and d are non-zero real constants. The fractional order derivative α in the above equation is considered to be taken in the recent conformable fractional derivative sense [48–50]. It is worth to be noticed that the field of fractional calculus is an old area of research that has gained much interest in the last few decades [51–56].

Many researchers have proposed various forms of Eq. (1) by suitably introducing different values of the non-zero real constants a, b, c and d . Some of the famous examples with fractional order derivatives in time include [47]:

1) *The fractional Sawada–Kotera equation*

$$u_t^\alpha + 45u^2u_x + 15u_xu_{xx} + 15uu_{xxx} + u_{xxxxx} = 0, \tag{2}$$

2) *the fractional Caudrey–Dodd–Gibbon equation*

$$u_t^\alpha + 180u^2u_x + 30u_xu_{xx} + 30uu_{xxx} + u_{xxxxx} = 0, \tag{3}$$

3) *the fractional Lax equation*

$$u_t^\alpha + 30u^2u_x + 30u_xu_{xx} + 10uu_{xxx} + u_{xxxxx} = 0, \tag{4}$$

4) *the fractional Kaup–Kuperschmidt equation*

$$u_t^\alpha + 20u^2u_x + 25u_xu_{xx} + 10uu_{xxx} + u_{xxxxx} = 0, \tag{5}$$

and

5) *the fractional Ito equation*

$$u_t^\alpha + 2u^2u_x + 6u_xu_{xx} + 3uu_{xxx} + u_{xxxxx} = 0. \tag{6}$$

However, we tackle in this paper the class of time-conformable fractional fifth-order Korteweg–de Vries equations given in Eqs. (2)–(6) by devising suitable novel hyperbolic and exponential ansatze. The essential advantage of these techniques over the other methods in the literature is that they present novel explicit analytical wave solutions including many real free parameters. The closed-form wave answers of nonlinear PDEs have a significant meaning revealing the interior working of the physical phenomena. Furthermore, the calculations in these methods are very simple and vital in presenting new solutions compared with the steps in other approaches.

In doing so, various solitons and solutions of the equations will be realized and further depicted graphically to confirm their shapes. One can well see various analytical and numerical methods used in tackling different forms of the Korteweg–de Vries equations and evolution equations in the papers cited above and also in [57–61]. The current paper is organized as follows: Sect. 2 presents some preliminaries about the conformable fractional derivative. Section 3 gives the concept of the methodology. Section 4 is reserved for the application. Section 5 gives some graphical results and a discussion. Finally, Sect. 6 gives the conclusion.

2 Fractional conformable derivative

Definition 1 ([48]) Let $u : [0, \infty) \rightarrow \mathbb{R}$ be a real-valued function. The fractional conformable derivative of order α for $u(t)$ is defined by

$$D_t^\alpha(u(t)) = \lim_{\delta \rightarrow 0} \left(\frac{u(t + \delta t^{1-\alpha}) - u(t)}{\delta} \right), \quad t > 0, \alpha \in (0, 1]. \tag{7}$$

Theorem 1 ([48]) Suppose $v(t)$ and $w(t)$ are α -differentiable for $\alpha \in (0, 1]$ and $t > 0$, then

- (a) $D_t^\alpha(t^n) = nt^{n-\alpha}, \forall n \in \mathbb{R}$,
- (b) $D_t^\alpha(C) = 0, \forall C \in \mathbb{R}$,
- (c) $D_t^\alpha(xv(t) + yw(t)) = xD_t^\alpha v(t) + yD_t^\alpha w(t), \forall x, y \in \mathbb{R}$,
- (d) $D_t^\alpha(v(t)w(t)) = v(t)D_t^\alpha(w(t)) + w(t)D_t^\alpha(v(t))$,
- (e) $D_t^\alpha\left(\frac{v(t)}{w(t)}\right) = \frac{1}{w^2(t)}(w(t)D_t^\alpha v(t) - v(t)D_t^\alpha w(t)), w(t) \neq 0$,
- (f) *Importantly, if the first derivative of $v(t)$ exists, then*

$$D_t^\alpha v(t) = t^{1-\alpha} \frac{dv}{dt}. \tag{8}$$

Theorem 2 ([48]) Let it be given that $v(t)$ is α -differentiable for $\alpha \in (0, 1]$. Let $w(t)$ be a differentiable function defined in the range of $v(t)$, then

$$D_t^\alpha(v(t) \circ w(t)) = t^{1-\alpha} w'(t)v'(w(t)). \tag{9}$$

The proof of Theorem 1 and Theorem 2 can be found in [48].

3 The hyperbolic and exponential ansatz methods

We consider the nonlinear time-fractional differential equation of the form

$$P(u, D_t^\alpha u, D_x u, D_{tt}^{2\alpha} u, D_{xx} u, D_t^\alpha D_x u, \dots) = 0, \tag{10}$$

where $\alpha \in (0, 1]$ is the fractional order derivative. Further, we make use of the following transformations:

$$u(x, t) = v(\xi), \quad \xi = f\left(x, \frac{t^\alpha}{\alpha}\right), \tag{11}$$

where f is a function of x and t , depending on the application. Moreover, we derive the following hyperbolic (see periodic ansätze in [35]) and exponential [41] ansätze for transformations for the fractional fifth-order Korteweg–de Vries equation under consideration.

1) *Hyperbolic ansatz method*

$$v(\xi) = \begin{cases} A_0 + A_1 \operatorname{sech}^2(\xi), & \text{bright soliton solution,} \\ A_0 + A_1 \operatorname{tanh}^2(\xi), & \text{dark soliton solution,} \\ A_0 + A_1 \operatorname{csch}^2(\xi), & \text{singular soliton solution,} \\ A_0 + A_1 \operatorname{coth}^2(\xi), & \text{singular soliton solution.} \end{cases} \tag{12}$$

2) *Exponential ansatz method*

$$v(\xi) = A_1 + A_2 \frac{e^\xi}{(1 + e^\xi)^2}, \tag{13}$$

where $A_0, A_1 \neq 0$ and $A_2 \neq 0$ are non-zero constants to be determined. Substituting either Eq. (12) or (13) as the case maybe into Eq. (10) and retaining the relevant coefficients in ξ , we get a system of algebraic equations. This system of equations will then be solved simultaneously to determine the unknowns with the help of computer software to obtain the solutions of Eq. (10).

4 Application

In this section, we employ the presented hyperbolic and exponential ansatz function methods to construct bright soliton solutions, dark soliton solutions, singular soliton solutions and exponential solutions for the class of fractional fifth-order Korteweg–de Vries equations under consideration.

4.1 The fractional Sawada–Kotera equation

4.1.1 Bright soliton solution

Let $A_0, A_1 \neq 0, k, s$ and w be arbitrary constants. Then we assume a bright soliton solution of the form

$$u(x, t) = A_0 + A_1 \operatorname{sech}^2(\xi), \quad \xi = sx - k \frac{t^\alpha}{\alpha} + w. \tag{14}$$

Substituting Eq. (14) into (2) and simplify as explained in Sect. 3, we get the following system of algebraic equations:

$$\begin{aligned} -2A_1k + 32A_1s^5 + 120A_0A_1s^3 + 90A_0^2A_1s &= 0, \\ -480A_1s^5 + 240A_1^2s^3 - 360A_0A_1s^3 + 180A_0A_1^2s &= 0, \\ 720A_1s^5 - 540A_1^2s^3 + 90A_1^3s &= 0. \end{aligned} \tag{15}$$

Solving the above system gives

$$A_0 = \frac{\pm\sqrt{5}\sqrt{ks + 4s^6} - 10s^3}{15s}, \quad A_1 = 2s^2, \tag{16}$$

which yields the following bright soliton solution:

$$u_1(x, t) = \frac{\pm\sqrt{5}\sqrt{ks + 4s^6} - 10s^3}{15s} + 2s^2 \operatorname{sech}^2\left(sx - k \frac{t^\alpha}{\alpha} + w\right). \tag{17}$$

4.1.2 Dark soliton solution

Let $A_0, A_1 \neq 0, k, s$ and w be arbitrary constants. Then we assume a bright soliton solution of the form

$$u(x, t) = A_0 + A_1 \tanh^2(\xi), \quad \xi = sx - k \frac{t^\alpha}{\alpha} + w. \tag{18}$$

Inserting Eq. (18) into (2), we get the following system of algebraic equations:

$$\begin{aligned}
 2A_1k - 272A_1s^5 - 60A_1^2s^3 + 240A_0A_1s^3 - 90A_0^2A_1s &= 0, \\
 -2A_1k + 1232A_1s^5 + 540A_1^2s^3 - 600A_0A_1s^3 - 180A_0A_1^2s + 90A_0^2A_1s &= 0, \\
 -1680A_1s^5 - 1020A_1^2s^3 + 360A_0A_1s^3 - 90A_1^3s + 180A_0A_1^2s &= 0, \\
 720A_1s^5 + 540A_1^2s^3 + 90A_1^3s &= 0.
 \end{aligned}
 \tag{19}$$

Solving the above system gives

$$A_0 = \frac{20s^3 \pm \sqrt{5}\sqrt{ks + 4s^6}}{15s}, \quad A_1 = -2s^2,
 \tag{20}$$

which yields the following dark soliton solution:

$$u_2(x, t) = \frac{20s^3 \pm \sqrt{5}\sqrt{ks + 4s^6}}{15s} - 2s^2 \tanh^2\left(sx - k\frac{t^\alpha}{\alpha} + w\right).
 \tag{21}$$

4.2 The fractional Caudrey–Dodd–Gibbon equation

4.2.1 Singular soliton solutions

Let $A_0, A_1 \neq 0, s, k$ and w be arbitrary constants. We have the following two cases:

Case (I). Assume a singular soliton solution of the form

$$u(x, t) = A_0 + A_1 \operatorname{csch}^2(\xi), \quad \xi = sx - k\frac{t^\alpha}{\alpha} + w.
 \tag{22}$$

Inserting Eq. (22) into (3), we get the following system of algebraic equations:

$$\begin{aligned}
 A_1k - 32A_1s^5 - 240A_0A_1s^3 - 360A_0^2A_1s &= 0, \\
 -480A_1s^5 - 480A_1^2s^3 - 720A_0A_1s^3 - 720A_0A_1^2s &= 0, \\
 -720A_1s^5 - 1080A_1^2s^3 - 360A_1^3s &= 0.
 \end{aligned}
 \tag{23}$$

Thus, solving the above system gives

$$A_0 = \frac{\pm\sqrt{5}\sqrt{ks + 4s^6} - 10s^3}{30s}, \quad A_1 = -s^2,
 \tag{24}$$

which yields the following singular soliton solution:

$$u_3(x, t) = \frac{\pm\sqrt{5}\sqrt{ks + 4s^6} - 10s^3}{30s} - s^2 \operatorname{Csch}^2\left(sx - k\frac{t^\alpha}{\alpha} + w\right).
 \tag{25}$$

Case (II). Assume a singular soliton solution of the form

$$u(x, t) = A_0 + A_1 \operatorname{coth}^2(\xi), \quad \xi = sx - k\frac{t^\alpha}{\alpha} + w.
 \tag{26}$$

Inserting Eq. (26) into (3), we get the following system of algebraic equations:

$$2A_1k - 272A_1s^5 - 120A_1^2s^3 + 480A_0A_1s^3 - 360A_0^2A_1s = 0,$$

$$\begin{aligned}
 & -2A_1k + 1232A_1s^5 + 1080A_1^2s^3 - 1200A_0A_1s^3 - 720A_0A_1^2s + 360A_0^2A_1s = 0, \\
 & -1680A_1s^5 - 2040A_1^2s^3 + 720A_0A_1s^3 - 360A_1^3s + 720A_0A_1^2s = 0, \\
 & 720A_1s^5 + 1080A_1^2s^3 + 360A_1^3s = 0.
 \end{aligned} \tag{27}$$

Thus, solving the above system gives

$$A_0 = \frac{20s^3 \pm \sqrt{5}\sqrt{ks + 4s^6}}{30s}, \quad A_1 = -s^2, \tag{28}$$

which yields the following singular soliton solution:

$$u_4(x, t) = \frac{20s^3 \pm \sqrt{5}\sqrt{ks + 4s^6}}{30s} - s^2 \coth^2\left(sx - k\frac{t^\alpha}{\alpha} + w\right). \tag{29}$$

4.3 The fractional Lax equation

4.3.1 Bright soliton solution

Let $A_0, A_1 \neq 0, s, k$ and w be arbitrary constants. Assume a bright soliton solution of the form

$$u(x, t) = A_0 + A_1 \operatorname{sech}^2(\xi), \quad \xi = sx - k\frac{t^\alpha}{\alpha} + w. \tag{30}$$

Substituting Eq. (30) into (4), we get the following system of algebraic equations:

$$\begin{aligned}
 & -2A_1k + 32A_1s^5 + 80A_0A_1s^3 + 60A_0^2A_1s = 0, \\
 & -480A_1s^5 + 320A_1^2s^3 - 240A_0A_1s^3 + 120A_0A_1^2s = 0, \\
 & 720A_1s^5 - 600A_1^2s^3 + 60A_1^3s = 0.
 \end{aligned} \tag{31}$$

Thus, solving the above system gives

$$A_0 = \frac{1}{3}(\pm\sqrt{13s^2 - 5s^2}), \quad A_1 = \mp\sqrt{13s^2 + 5s^2}, \quad k = 4(\mp5\sqrt{13s^5 + 19s^5}), \tag{32}$$

which yields the following bright soliton solution:

$$u_5(x, t) = \frac{1}{3}(\pm\sqrt{13s^2 - 5s^2}) + (\mp\sqrt{13s^2 + 5s^2}) \operatorname{sech}^2\left(sx - k\frac{t^\alpha}{\alpha} + w\right). \tag{33}$$

4.3.2 Dark soliton solution

Let $A_0, A_1 \neq 0, s, k$ and w be arbitrary constants. Let us assume a bright soliton solution in the form

$$u(x, t) = A_0 + A_1 \tanh^2(\xi), \quad \xi = sx - k\frac{t^\alpha}{\alpha} + w. \tag{34}$$

Substituting Eq. (34) into (4), we get the following system of algebraic equations:

$$\begin{aligned}
 & 2A_1k - 272A_1s^5 - 120A_1^2s^3 + 160A_0A_1s^3 - 60A_0^2A_1s = 0, \\
 & -2A_1k + 1232A_1s^5 + 760A_1^2s^3 - 400A_0A_1s^3 - 120A_0A_1^2s + 60A_0^2A_1s = 0,
 \end{aligned}$$

$$\begin{aligned}
 & -1680A_1s^5 - 1240A_1^2s^3 + 240A_0A_1s^3 - 60A_1^3s + 120A_0A_1^2s = 0, \\
 & 720A_1s^5 + 600A_1^2s^3 + 60A_1^3s = 0.
 \end{aligned}
 \tag{35}$$

Thus, solving the above system gives

$$A_0 = \frac{2}{3}(\pm\sqrt{13}s^2 + 5s^2), \quad A_1 = -\sqrt{13}s^2 \mp 5s^2, \quad k = 4(\mp 5\sqrt{13}s^5 + 19s^5),
 \tag{36}$$

which yields the following dark soliton solution:

$$u_6(x, t) = \frac{2}{3}(\pm\sqrt{13}s^2 + 5s^2) + (\mp 5\sqrt{13}s^5 + 19s^5) \tanh^2\left(sx - k\frac{t^\alpha}{\alpha} + w\right).
 \tag{37}$$

4.4 The fractional Kaup–Kuperschmidt equation

4.4.1 Exponential solution

Let $A_1, A_2 \neq 0, k$ and c be arbitrary constants. We assume an exponential solution in the form

$$u(x, t) = A_1 + A_2 \frac{e^\xi}{(1 + e^\xi)^2}, \quad \xi = k\left(x - c\frac{t^\alpha}{\alpha}\right).
 \tag{38}$$

Substituting Eq. (38) into (5), we get the following system of algebraic equations:

$$\begin{aligned}
 & 20A_1^2A_2k + 10A_1A_2k^3 - A_2ck + A_2k^5 = 0, \\
 & 60A_1^2A_2k + 40A_1A_2^2k - 90A_1A_2k^3 + 35A_2^2k^3 - 3A_2ck - 57A_2k^5 = 0, \\
 & 40A_1^2A_2k + 40A_1A_2^2k - 100A_1A_2k^3 + 20A_2^3k - 235A_2^2k^3 - 2A_2ck + 302A_2k^5 = 0, \\
 & -40A_1^2A_2k - 40A_1A_2^2k + 100A_1A_2k^3 - 20A_2^3k + 235A_2^2k^3 + 2A_2ck - 302A_2k^5 = 0, \\
 & -60A_1^2A_2k - 40A_1A_2^2k + 90A_1A_2k^3 - 35A_2^2k^3 + 3A_2ck + 57A_2k^5 = 0, \\
 & -20A_1^2A_2k - 10A_1A_2k^3 + A_2ck - A_2k^5 = 0.
 \end{aligned}
 \tag{39}$$

Thus, solving the above system gives

$$\begin{cases} \text{Case one: } A_1 = -k^2, & A_2 = 12k^2, & c = 11k^4, \\ \text{Case two: } A_1 = -\frac{k^2}{8}, & A_2 = \frac{3k^2}{2}, & c = \frac{k^4}{16}, \end{cases}
 \tag{40}$$

which yield the following solutions:

$$u_7(x, t) = -k^2 + \frac{12k^2 e^{k(x-11k^4\frac{t^\alpha}{\alpha})}}{(1 + e^{k(x-11k^4\frac{t^\alpha}{\alpha})})^2},
 \tag{41}$$

or

$$u_8(x, t) = -\frac{k^2}{8} + \frac{3k^2 e^{k(x-\frac{k^4}{16}\frac{t^\alpha}{\alpha})}}{2(1 + e^{k(x-\frac{k^4}{16}\frac{t^\alpha}{\alpha})})^2}.
 \tag{42}$$

4.5 The fractional Ito equation

4.5.1 Exponential solution

Let $A_1, A_2 \neq 0, k$ and c be arbitrary constants. Assume we have an exponential solution in the form

$$u(x, t) = A_1 + A_2 \frac{e^\xi}{(1 + e^\xi)^2}, \quad \xi = k \left(x - c \frac{t^\alpha}{\alpha} \right). \tag{43}$$

Substituting Eq. (43) into (6), we get the following system of algebraic equations:

$$\begin{aligned} 2A_1^2 A_2 k - A_2 c k + 3A_1 A_2 k^3 + A_2 k^5 &= 0, \\ 6A_1^2 A_2 k + 4A_1 A_2^2 k - 3A_2 c k - 27A_1 A_2 k^3 + 9A_2^2 k^3 - 57A_2 k^5 &= 0, \\ 4A_1^2 A_2 k + 4A_1 A_2^2 k + 2A_2^3 k - 2A_2 c k - 30A_1 A_2 k^3 - 63A_2^2 k^3 + 302A_2 k^5 &= 0, \\ -4A_1^2 A_2 k - 4A_1 A_2^2 k - 2A_2^3 k + 2A_2 c k + 30A_1 A_2 k^3 + 63A_2^2 k^3 - 302A_2 k^5 &= 0, \\ -6A_1^2 A_2 k - 4A_1 A_2^2 k + 3A_2 c k + 27A_1 A_2 k^3 - 9A_2^2 k^3 + 57A_2 k^5 &= 0, \\ -2A_1^2 A_2 k + A_2 c k - 3A_1 A_2 k^3 - A_2 k^5 &= 0. \end{aligned} \tag{44}$$

Thus, solving the above system gives

$$A_1 = -\frac{5k^2}{2}, \quad A_2 = 30k^2, \quad c = 6k^4, \tag{45}$$

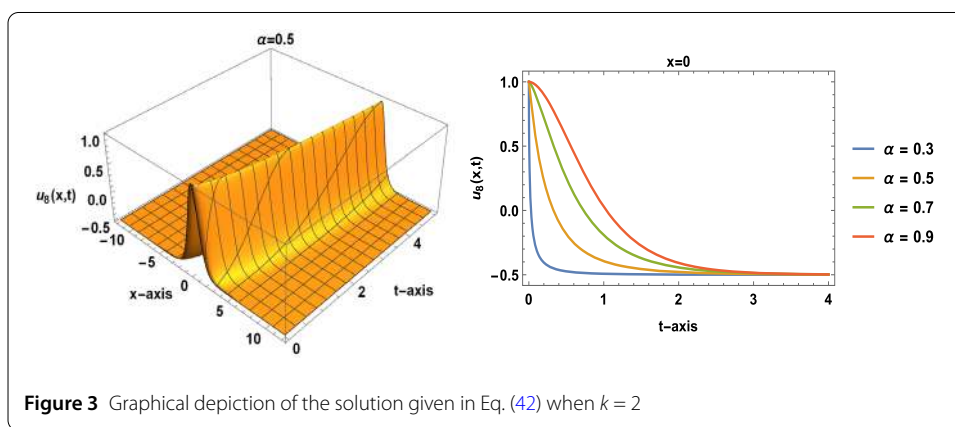
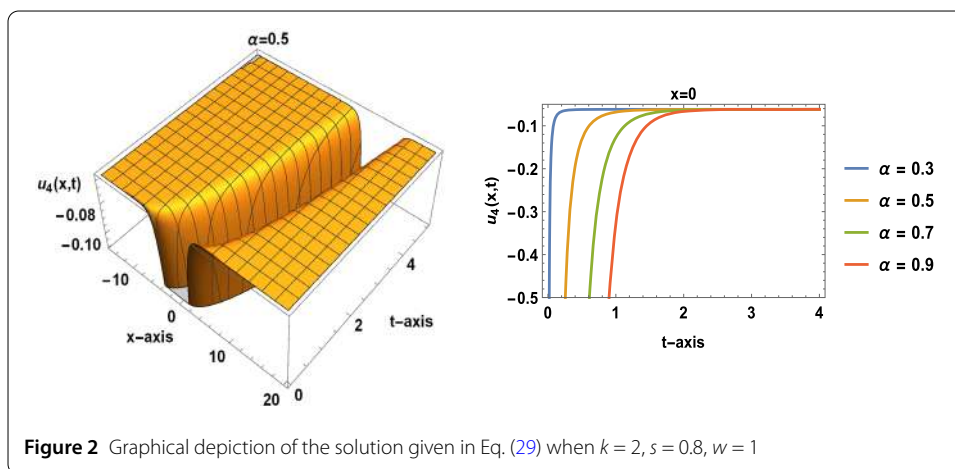
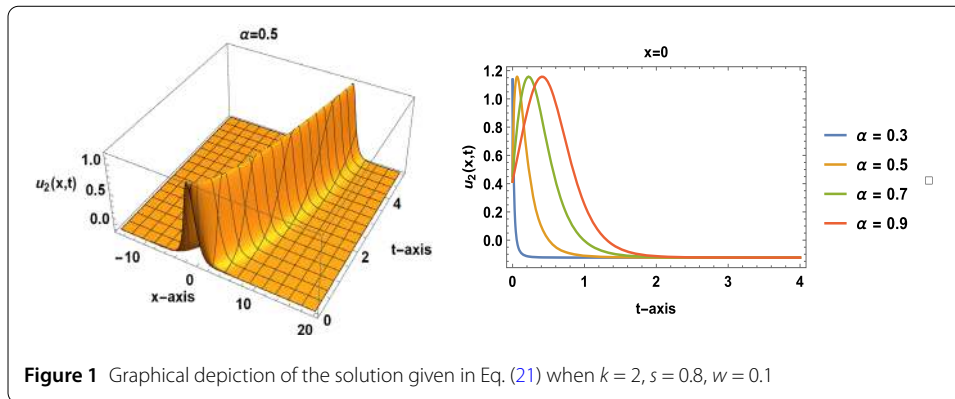
which yield the following solutions:

$$u_9(x, t) = -\frac{5k^2}{2} + \frac{30k^2 e^{k(x-6k^4 \frac{t^\alpha}{\alpha})}}{(1 + e^{k(x-6k^4 \frac{t^\alpha}{\alpha})})^2}. \tag{46}$$

5 Graphical results and discussion

In this section, we give some graphical depictions to some of the acquired solutions using the devised hyperbolic and exponential ansatz methods for the class of fifth-order Korteweg–de Vries equations under consideration. Both the 2-dimensional and the 3-dimensional plots are presented. Based on the ansatz methods devised, we have constructed bright and dark soliton solutions for the fractional Sawada–Kotera equation in Eqs. (17) and (21); singular soliton solutions for the fractional Caudrey–Dodd–Gibbon in equations (25) and (29); bright and dark soliton solutions for the fractional Lax equation in Eqs. (33) and (37); exponential solutions for the fractional Kaup–Kuperschmidt in Eqs. (41) and (42); and finally the exponential solution for the fractional Ito in Eq. (46). In Fig. 1, we plot the dark soliton solution of the fractional Sawada–Kotera equation given in equation (19). In Fig. 2, we plot the singular solution of the fractional Caudrey–Dodd–Gibbon given in Eq. (25). In Fig. 3, we plot the exponential solution of the fractional Kaup–Kuperschmidt equation given in Eqs. (36). In these figures, we study the effect of the fraction order on the variation of the wave displacement. Figures 1 and 3 clearly show bell-shaped solution, while Fig. 2 gives a singular solution representation.

In Figs. 1–3, the fractional order α clearly affected on the propagating of wave solution in which the amplitude is increased with the increase of α .



6 Conclusion

In summary, the present paper investigates a well-known class of fifth-order Korteweg–de Vries equations by devising novel hyperbolic and exponential ansätze in the presence of a time-fractional order derivative. The fractional derivative is considered to be taken in the sense of the conformable fractional derivative. Various solitons and solutions of the equations including bright solitons, dark solitons, singular solitons and certain exponential solutions have been realized in the study. We finally depict some of the obtained solutions

graphically and conclude that similar considerations of various evolution equations can be done using the devised ansatze.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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