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# Novel local CFT and exact results on perturbations of N=4 super Yang Mills from AdS dynamics

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Abstract: We find new, local, non-supersymmetric conformal field theories obtained by relevant deformations of the N=4 super Yang Mills theory in the large $N$ limit. We contruct interpolating supergravity solutions that naturally represent the flow from the N=4 super Yang Mills UV theory to these non-supersymmetric IR fixed points. We also study the linearization around the N=4 superconformal point of N=1 supersymmetric, marginal deformations. We show that they give rise to N=1 superconformal fixed points, as expected from field-theoretical arguments.

Keywords: 1/N Expansion, String Duality, Supersymmetry and Duality.

## Contents





## 1. Introduction

Maldacena's conjecture [1] is a new powerful tool to study conformal field theories in the large $N$, large 't Hooft parameter [2] regime [3, 4]. According to this conjecture, one can describe the field theory encoding the low-energy dynamics of $N$ branes, present in a closed superstring theory, in terms of the classical dynamics of the closed superstrings in the near-horizon geometry generated by the branes. In the large $N$ limit, one can find dynamical regimes in which the closed superstring theory is accurately approximated by its low-energy effective supergravity. The example we will be concerned with is $N$ D3 branes in Type IIB superstring theory. The open-string sector of this theory is described by an interacting N=4 super Yang Mills (SYM) theory with gauge group $SU(N)$ plus a free, decoupled theory with gauge group $U(1)$. The coupling constant of the SYM theory is $g_{YM}^2 = g_S$, and the supergravity approximation holds whenever $g_S N \gg 1$, $N \to \infty$.

One natural question one may ask of N=4 SYM is whether it allows for deformations that either are (super)conformal or flow to an interacting conformal fixed point. This question cannot be answered using perturbative field theory, since the fixed point may occur at large value of the coupling constant. Supersymmetry gives a handle on the non-perturbative domain; in ref. [5] it was shown that N=4 SYM theories with gauge group $SU(N)$, $N > 2$ possess a three-parameter family of N=1 supersymmetric *exact* marginal deformations. No non-perturbative results exist about relevant deformations.[1]

[1]A perturbative analysis of mass deformations was carried out in [6].

In this paper, we use the Maldacena conjecture to study marginal and relevant deformations of N=4 SYM. In this approach the question to ask is whether Type IIB supergravity compactified on $AdS_5 \times S^5$ can be deformed to some other background with isometry $SO(2,4)$. The very existence of such an isometry guarantees that the open-string sector of Type IIB on such background is a four-dimensional conformal theory; conformal invariance is the easy part of our job. The difficult part is to connect the deformation of the background with an operator of the N=4 SYM theory, and to construct an $AdS$ analog of the renormalization group flow from the $AdS_5 \times S^5$ to the new one.

The first problem is addressed in Sections 2 and 3. In Section 2, we recall the dictionary linking composite operators in SYM to supergravity modes in Type IIB on $AdS_5 \times S^5$. This dictionary allows us to associate supergravity modes to marginal and relevant perturbations of SYM. Some relevant perturbations, namely mass deformations for scalars and fermions, can also be described in the five-dimensional N=8 gauged supergravity obtained by dimensional reduction of Type IIB on $S^5$. The second part of this Section is rather technical, and may be skipped on a first reading. The dimensionally reduced supergravity is described in Section 3. In the five-dimensional theory, one finds two new solutions with isometry $SO(2,4)$, besides the maximally supersymmetric one. They are stationary points of the 5-d scalar potential in which some 5-d scalar fields get a nonzero VEV. In the SYM/$AdS$ dictionary, these fields correspond precisely to mass perturbations of the SYM theory. In the 10-d Type IIB theory, the new stationary points of the 5-d theory are non-supersymmetric compactifications on manifolds with isometry $SO(5) \times SO(2,4)$ and $SU(3) \times U(1) \times SO(2,4)$, respectively.

The second problem is answered in Section 4. There, we show that the new stationary points of the 5-d theory are local conformal field theories, and we construct interpolating supergravity solutions joining the new minima with the old one. These interpolating solutions are most naturally interpreted as describing a renormalization group flow, with the new theory in the infrared and the N=4 SYM in the ultraviolet.

In Section 5 we turn to the description of marginal deformations in the supergravity approximation. Nonperturbative results in field theory [5] ensure the existence of exact marginal deformations of N=4 SYM. By studying the Killing spinor equations of Type IIB supergravity, we prove that such marginal deformations can be described in the supergravity approximation to linear order in the deformation, and we briefly comment on this result.

## 2. The spectrum of type IIB on $AdS_5 \times S^5$

According to the celebrated Maldacena conjecture [1], N=4 Yang-Mills theory has a dual description as the Type IIB string on $AdS_5 \times S^5$. This dual description can be used when the superstring is weakly coupled: this corresponds to the large $N$,

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large t'Hooft coupling $g = \sqrt{g_{YM}^2 N}$ regime of the N=4 Yang-Mills theory.[2] The perturbative expansions in $\alpha'$ and in the string coupling constant correspond to the double $1/g$ and $1/N$ expansions of the Yang-Mills theory [2], respectively.

In the holographic prescription [3, 4], supergravity and stringy modes are associated via boundary values in $AdS_5$ to the set of gauge invariant composite operators of the conformal field theory. The mass of an $AdS_5$ state is related to the conformal dimension $E_0$ of the corresponding operator via a formula involving the Casimirs of the conformal group $SO(2,4)$ [4, 7]. In the case of a scalar, we have

$$m^2 = E_0(E_0 - 4)\,. \tag{2.1}$$

In the weak coupling regime of string theory, the stringy states in $AdS_5 \times S^5$, or, equivalently, the CFT operators, split into two disjoint sets. The KK modes coming from the reduction on $S^5$ fall into short multiplets of N=8 supersymmetry, containing states with maximum spin 2. They correspond to the algebra of *chiral* operators of the N=4 Yang-Mills theory, with dimensions protected under renormalization. The stringy states, on the other hand, fall into long representations, containing up to spin 4. They have a mass squared of order $1/\alpha'$ which corresponds, by eq. (2.1), to an anomalous dimension $\sqrt{g}$. We see that, generically, in the large $N$ limit with $g \to \infty$, the stringy states have infinite anomalous dimension. They decouple from the OPE's, and the algebra of *chiral* operators closes.

In the large $N$ limit with $g \to \infty$, the supergravity on $AdS_5 \times S^5$ contains all the information about the N=4 chiral operators. It is the purpose of this paper to investigate, using the supergravity description, the relevant and marginal deformations of N=4 Yang-Mills theory that can be associated with chiral operators.

The spectrum of KK modes on $AdS_5 \times S^5$ was computed in [8]. In fig. 1, all the KK scalar states with zero or negative mass square are shown. According to eq. (2.1), they correspond to marginal or relevant operators in the CFT. They appear in the harmonic expansion around $S^5$ of the following fields: the Type IIB dilaton $B$, the complex antisymmetric two-form, $A_{\alpha\beta}$,[3] with indices $\alpha, \beta$ along the five-sphere, and a combination of the dilation mode of the internal metric, $h^\alpha_\alpha$, and the Type IIB five-form with indices along $S^5$. The $SU(4)$ representation is indicated in fig. 1 near each state.

The KK spectrum was decomposed in representations of the superconformal algebra in [9]. The multiplets are specified by an integer $p$ which corresponds to the conformal dimension of the lowest state. In fig. 1, the states in the same multiplet lie on a vertical. Notice that, for reasons that will soon become obvious, there is no

[2] In this paper we indicate the Yang-Mills coupling constant with $g_{YM}$ and we use the notation $g$ for the t'Hooft coupling which is relevant in the large $N$ limit. In the same way, in many formulae, even when non explicitly noticed, the various coupling constants multiplying composite operators must be understood as the rescaled coupling constants that remain finite in the large $N$ limit.

[3] $A_{\alpha\beta}$ is a linear combination of the NS-NS and R-R two-forms.



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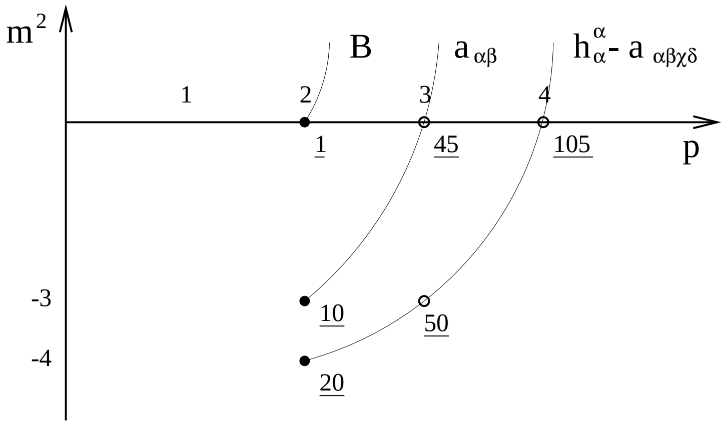


**Figure 1:** KK scalar states with zero or negative mass square. Supersymmetry multiplets correspond to vertical lines. Filled circles are associated with the states in the graviton multiplet. The $SU(4)$ representation of each scalar is indicated, together with the space-time field whose harmonic expansion gives rise to the particular state.

multiplet for $p = 1$. The states with $p = 2$ belong to the graviton multiplet in $AdS_5$ and are indicated by filled circles in fig. 1. The structure of the generic multiplet, with the $SU(4)$ quantum numbers and the conformal dimension of the states, can be found in table 1 of ref. [9].

Here we follow an approach based on N=4 on-shell superfields [7], which has the advantage of giving explicitly both the structure of the multiplets and the CFT operators corresponding to the supergravity modes.

The N=4 chiral multiplets can be obtained by products of the N=4 on-shell superfield [10]. This superfield, defined in N=4 superspace, is a Lorentz scalar, transforming in the $\underline{6}$ of $SU(4)$ and satisfying some constraints. These constraints can be found in [10]; they remove all states with spin greater than 1. For our purpose, we simply need to know that the physical components lie in the first few terms of the $\theta$ expansion,

$$W_{[AB]} = \phi_{[AB]} + \theta_A \lambda_B + \theta_A \theta_B \sigma^{\mu\nu}_{AB} F^{-}_{\mu\nu} + \cdots + c.c., \qquad A, B = 1, \ldots, 4\,. \tag{2.2}$$

This is not a chiral superfield, but it satisfies a generalised notion of chirality when defined in harmonic superspace [11, 12]. We will also use the label $i = 1, \ldots, 6$ instead of the pair of antisymmetric indices $[AB]$.

The spectrum of chiral operators of N=4 Yang-Mills, corresponding to the entire spectrum of KK states in $AdS_5 \times S^5$, is contained in the series of composite operators $A_p = \mathrm{Tr}\, W_{\{i_1 \ldots i_p\}}$ − traces, obtained by symmetrized traceless products of the $W_i$ superfield [4, 7]. Due to the trace on colour space, the first non-trivial multiplet corresponds to $p = 2$. It is the N=4 multiplet of supercurrents and it is associated with the graviton multiplet in $AdS_5$. Being conserved, it contains a number of states lower than the multiplets with $p \geq 3$; this corresponds in $AdS_5$ to the fact that the massless multiplet of the graviton sustains a gauge invariance. By multiplying superfields, it is easy to recover the structure and the quantum numbers of the states belonging to $p$-th multiplet, as quoted in table 1 of ref. [9].

The superfield $A_p$ has dimension $p$. We can assign dimension $-1/2$ to the $\theta$'s. It is then obvious that operators with dimension $\leq 4$ can be found only in the very first superfields, namely, $A_2, A_3$ and $A_4$. The lowest components of $A_p$ is the scalar $\mathrm{Tr}\,\phi_{\{i_1}...\phi_{i_p\}}$ $-$ traces, in the $(0,p,0)$ representation of $SU(4)$ with dimension $p$. For $p \leq 4$ we obtain the states in fig. 1, transforming in the $\underline{20}, \underline{50}$ and $\underline{105}$ of $SU(4)$, with dimensions 2,3 and 4, respectively. They all come from the harmonic expansion of the same Type IIB field, namely, a linear combination of the internal dilation mode and the five-form on $S^5$. In addition to these states, we can find Lorentz scalars in the $\theta^2$ and $\theta^4$ components of $\mathrm{Tr}\,W^2$: these are the states in the $\underline{10}$ and $\underline{1}$ of $SU(4)$ in fig. 1, with dimension 3 and 4, respectively. Finally, there is a scalar in the $\theta^2$ component of $\mathrm{Tr}\,W^3$, in the $\underline{45}$ of $SU(4)$ with dimension 4: it is the last scalar in fig. 1. By multiplying superfields, we can write an explicit expression for these operators,

$$\begin{array}{ll}
(E_0 = 2,\ \underline{20}) & \mathrm{Tr}\,\phi_{\{i}\phi_{j\}} - \mathrm{traces}\,, \\
(E_0 = 3,\ \underline{50}) & \mathrm{Tr}\,\phi_{\{i}\phi_j\phi_{k\}} - \mathrm{traces}\,, \\
(E_0 = 3,\ \underline{10}) & \mathrm{Tr}\,\lambda_A\lambda_B + \mathrm{cubic\ terms\ in}\,\phi\,, \\
(E_0 = 4,\ \underline{105}) & \mathrm{Tr}\,\phi_{\{i}\phi_j\phi_k\phi_{t\}} - \mathrm{traces}\,, \\
(E_0 = 4,\ \underline{45}) & \mathrm{Tr}\,\lambda_A\lambda_B\phi_i + \mathrm{quartic\ terms\ in}\,\phi\,, \\
(E_0 = 4,\ \underline{1}) & \mathrm{The\ N} = 4\,\mathrm{Lagrangian}\,.
\end{array} \tag{2.3}$$

The cubic and quartic terms in the $\underline{45}$ and $\underline{105}$ come from the interactions of the N=4 theory. Their explicit expression can be found by applying the supersymmetry generators to the lowest states of each multiplet, or by writing the N=4 theory in N=1 superspace language and solving the equations of motion for the F and D terms.

The last scalar in eq. (2.3) has a special status. Being the highest component of its multiplet, it preserves N=4 supersymmetry. The corresponding perturbation of the N=4 theory is simply a change in the (complexified) coupling constant. It is an exactly marginal deformation, because the N=4 Yang-Mills theory is conformal for each value of the coupling. This is consistent with the supergravity description, where this operator is the zero mode of the (complexified) dilaton, which decouples from the Type IIB equations of motion.

We can study two other marginal deformations of the N=4 theory by looking at the $\underline{45}$ and $\underline{105}$ operators, and two relevant deformations corresponding to masses for scalars or fermions (plus additional cubic terms) by looking at the $\underline{20}$ and $\underline{10}$ operators.

We can check if the deformations corresponding to the operators in fig. 1 preserve some supersymmetry. This can be easily done by writing the N=4 superfields in N=1 language. The superfield $W_i$ decomposes into three chiral multiplets $\Sigma_i$, transforming as a triplet of $SU(3)$ in the decomposition $SU(4) \to SU(3) \times U(1)$, and an $SU(3)$-singlet vector-multiplet field strength $W_\alpha = \bar{D}^2 e^{-V} D_\alpha e^V$,

$$W_i = \{\Sigma_i, W_\alpha\} \,. \tag{2.4}$$

The superfield $A_p$, being the product of $p$ fundamental superfields, decomposes in all possible products of $p$ N=1 superfields $\Sigma_i$ and $W_\alpha$, subject to some restrictions coming from total symmetrization and removal of traces in $A_p$. The complete decomposition for $A_2, A_3$ and $A_4$ can be found in [13].

The $\underline{20}, \underline{50}$ and $\underline{105}$, which are lowest components of an $A_p$ superfield, can be written as lowest components of products of N=1 superfields, $\Sigma_i$ and $\bar{\Sigma}_i$, and therefore break supersymmetry completely.

The $\underline{10}$ in $A_2$, on the other hand, decomposes under $SU(3)$ as $\underline{1} + \underline{3} + \underline{6}$ and corresponds to the components with dimension 3 in the N=1 superfields

$$\underline{1} + \underline{3} + \underline{6} \qquad\qquad \{\mathrm{Tr}\, W_\alpha W^\alpha,\ \mathrm{Tr}\, \Sigma_i W_\alpha,\ \mathrm{Tr}\, \Sigma_i \Sigma_j\} \,. \tag{2.5}$$

We see that the $\underline{6}$ is the highest component of a chiral superfield, and therefore preserves N=1 supersymmetry. The corresponding deformation is a supersymmetric mass term: $\int d^2\theta m_{ij} \mathrm{Tr}\, \Sigma_i \Sigma_j$. The $AdS$/CFT correspondence has little to say about this deformation; it explicitly breaks conformal invariance and, for a generic $m_{ij}$ flows in the IR to pure N=1 Yang-Mills which is also not conformal. The other two terms $\underline{1}$ and $\underline{3}$, being not the highest components of their N=1 multiplets, break all supersymmetries. We will see however that, despite the lacking of powerful supersymmetric methods, we will be able to discuss one of these mass deformations using N=8 supergravity. We will show in Section 3 and 4 that the deformation in the $\underline{1}$ (as well as an analogous mode in the $\underline{20}$ of $SU(4)$) leads in the IR to a novel N=0 conformal local quantum field theory.

We can repeat the same argument for the $\underline{45}$ in $A_3$ and consider all the possible products of three N=1 superfields chosen among $\Sigma_i$ and $W_\alpha$. It is easy to show that in the decomposition $\underline{45} \to \underline{10} + \underline{15} + \underline{8} + \underline{6} + \underline{3} + \underline{\bar{3}}$, the only state that preserves N=1 supersymmetry is $\underline{10}$, being the highest ($\theta^2$) component of the chiral superfield $\int d\theta^2 \mathrm{Tr}\, \Sigma_i \Sigma_j \Sigma_k$. We can therefore investigate the following N=1 deformation of the N=4 Yang-Mills theory,

$$L_{N=4} + \int d\theta^2 Y_{ijk} \mathrm{Tr}\, \Sigma_i \Sigma_j \Sigma_k \,. \tag{2.6}$$

The remaining N=1 supersymmetry allows one to derive exact results in the CFT. It is indeed known [5] that there exists a three-dimensional complex manifold of superconformal N=1 fixed points, containing N=4 Yang-Mills, as well as its deformation corresponding to eq. (2.6), and a modification of the superpotential coupling. The supergravity analysis of this deformation is the subject of Section 5.

Let us conclude this analysis of the KK spectrum of $AdS_5 \times S^5$ in terms of N=4 Yang-Mills operators by noticing that the deformations corresponding to *chiral* operators by no means exhaust the class of interesting perturbations of the theory. In the case of relevant deformations, for example, the mass term for the scalar (the $\underline{20}$), being traceless, is not the most general one. The diagonal mass term $\sum_i \mathrm{Tr}\,\phi_i^2$, is not chiral. It is indeed the simplest example of a non-chiral operator, since it is the lowest component of the long multiplet $\mathrm{Tr}\,W_i W_i$, that contains the Konishi current. In the case of marginal deformations, the simplest deformation of N=4 to N=1 corresponds, in N=1 language, to changing the coefficient of the superpotential, and it is non-chiral:

$$L_{N=4} \to L_{N=1DEF} = \int d\theta^2 d\bar{\theta}^2 \mathrm{Tr}\,\bar{\Sigma}_i e^V \Sigma_i + \int d\theta^2 \frac{1}{g^2} \mathrm{Tr}\,W_\alpha W^\alpha + h\epsilon_{ijk} \mathrm{Tr}\,\Sigma_i \Sigma_j \Sigma_k + \cdots \tag{2.7}$$

Indeed, the superpotential of N=4 belongs to the long Konishi multiplet, as the N=1 superfield equation of motion implies

$$\mathcal{W} \equiv \epsilon_{ijk} \mathrm{Tr}\,\Sigma_i \Sigma_j \Sigma_k = \bar{D}^2 (\mathrm{Tr}\,\bar{\Sigma}_i \Sigma_i)\,. \tag{2.8}$$

It is clear that all these non-chiral deformations cannot be easily described in terms of low energy Type IIB supergravity, because the corresponding $AdS_5$ modes are stringy modes. This will be discussed further in Section 5.

## 3. Gauged N=8 supergravity in five dimensions and relevant perturbations

The fact that the low-energy Lagrangian for the states in the graviton multiplet exists in the form of N=8 gauged supergravity [14] enables us to study the deformations corresponding to the $\underline{1}, \underline{10}$ and $\underline{20}$ directly from a Lagrangian point of view. These 42 scalars have a non-trivial potential, which was studied in [15]. There is a stationary point of the potential when all scalar VEVS are zero, with unbroken $SU(4)$ gauge group. This corresponds to the N=4 Yang-Mills theory. We know almost everything about the deformation in the $\underline{1}$ . It corresponds to moving along the complex line of fixed points parametrised by the complex coupling constant. As it must be, on the supergravity side, the gauged N=8 Lagrangian has a potential which is invariant under $SU(1,1)$. In other terms, $\underline{1}$ is a flat direction.

The interesting point is that, beside the maximally $SO(6)$-symmetric case, there are other isolated stationary points of the N=8 gauged supergravity, corresponding to VEVs of the $\underline{20}$ and $\underline{10}$. More precisely, two other stationary points with metric $AdS_5$ were found in [15]. Both of them completely break supersymmetry. In the spirit of the Maldacena $AdS$/CFT correspondence, we are tempted to interpret these two new $AdS_5$ minima as corresponding to two N=0 conformal field theories. The fact that they can be obtained in the N=8 gauged supergravity by giving VEVs to some scalars can be interpreted as the fact that there is some relevant deformation of N=4 Yang-Mills, which flows in the IR to these novel conformal theories. The CFT operators associated with the $\underline{10}$ and $\underline{20}$, as discussed in the previous Section, are masses for the N=4 Yang-Mills fermions or scalars. It must be noticed that, since the new minima are not continuously connected to the maximally symmetric one, the linearization around the N=4 theory can not be completely trusted. Higher order corrections, giving rise to higher dimensional operators in the deformed N=4 theory, must be included. As a conclusion, we do not know an explicit Lagrangian realization for these theories, but, as we will show in the next Section, we can prove their existence as local quantum field theories, and use supergravity to predict their symmetries.

In [15] it was suggested that these two N=0 solution correspond to explicit known compactifications of the type IIB string.

The first stationary point corresponds to a VEV for the $\underline{20}$ which preserves an $SO(5)$ subgroup of $SO(6)$. The $AdS$ gauge group is identified with the global symmetry of the conformal field theory. According to the general philosophy of the $AdS$/CFT, we expect that this CFT, with $SO(5)$ global symmetry, corresponds to some compactification of the Type IIB string on a manifold with isometry $SO(2,4)\times SO(5)$. Luckily enough, a manifold with the right properties was identified in [16], as noticed in [15]. Topologically, it is a direct product $AdS_5 \times H$, where $H$ is a compact manifold. Metrically, it cannot be written as a direct product; rather, the $AdS$ metric is multiplied by a "warp factor" depending only on the coordinates of $H$. We denote such a manifold by $AdS_5 \times_W H$. It can be continuously connected to $S^5$ in the following sense: there exists (at least) a one-parameter class of manifolds $AdS_5 \times_W H(\alpha)$ that solves the Type IIB equations of motion only for $\alpha = 0$ and, say, $\alpha = 1$. At $\alpha = 0$ the solution reduces to $AdS_5 \times S^5$; at $\alpha = 1$ it reduces to $AdS_5 \times_W H$ [16]. Moreover, the linearization around $S^5$ shows that we are deforming with a non-trivial dilation mode of the $S^5$ metric, whose harmonic expansion gives rise, among other things, to our deformation in the $\underline{20}$, as shown in fig. 1. This strongly suggests that the solution of [16] can be identified with the stationary point found by [15].

The second stationary point corresponds to a VEV in the $\underline{10}$, which preserves an $SU(3)$ subgroup of $SU(4)$. This is the mode $\underline{1}$, lowest component of the superfield $W_\alpha W^\alpha$ in eq. (2.5). As shown in [15], the minimum leaves an unbroken gauge group $SU(3) \times U(1)$, where the $U(1)$ is a combination of the $U(1)$ in the decomposition $SU(4) \to SU(3) \times U(1)$ and a $U(1)$ subgroup of $SU(1,1)$. We see that the $SL(2;Z)$ symmetry of the N=4 theory must play an important role in the definition of this N=0 theory. Luckily again, a candidate for the manifold $H$ exists also in this case. It was found in [17]. It has the same property as the previous solution, namely, it is connected to the maximal $S^5$ case by a one parameter series of manifolds $H(\alpha)$, with the same properties as before. By linearising around $S^5$, we identify the deformation with a non-zero value of the two-form antisymmetric tensor on $S^5$, which gives rise, after KK expansions, to our mode $\underline{10}$. The $U(1)$ factor among the symmetries of the solution, which was initially overlooked in [17], involves a combination of a geometrical $U(1)$ and the $U(1)$ subgroup of $SU(1,1)$, as noticed in [15], in complete analogy with the supergravity analysis. This again strongly suggests that the solution of [17] can be identified with the stationary point found in [15].

The explicit parametrisation of the potential and the value of the cosmological constant at the stationary points will be discussed in the following Section.

Both N=0 conformal theories should belong to a complex line of fixed points because of the $SU(1,1)$ invariance of the supergravity minima. In addition to that, supergravity on $AdS_5 \times_W H$ should be interpreted as a strong coupling limit, large $N$ expansion of these theories. We do not know much about these theories, but the existence of corresponding Type IIB compactifications allows us to predict, by analysing the KK excitations, the full spectrum of operators which have finite dimension in the large $N$, strong coupling expansion of the theories. In the same limit, the Type IIB equations of motion would reproduce the Green functions of the theories, according to the holographic prescription [3, 4]. The study of the spectrum and of the Green functions in this particular regime is reduced to the study of a classical supergravity theory.

## 4. The renormalization group flow

In the previous Section we have found three stationary points of the gauged N=8 supergravity in five dimensions. One, with N=8 supersymmetry, corresponds to the standard compactification of Type IIB supergravity on the round five-sphere and is $SO(6)$-symmetric. The other two are non-supersymmetric and preserve $SO(5)$ and $SU(3) \times U(1)$, respectively. Even though these stationary points were derived by minimisation of the potential of the dimensionally reduced theory, they are most probably true compactifications of Type IIB supergravity [16, 17].[4]

In the $SO(5)$-symmetric stationary point, some scalars in the $\underline{20}$ of $SU(4)$ get a nonzero expectation value; while in the $SU(3) \times U(1)$-symmetric vacuum scalars in the $\underline{10}$ of $SU(4)$ have nonzero expectation values.

In both stationary points the five-dimensional space-time is $AdS_5$, thus, the 4-d boundary theory associated with these stationary points is automatically conformally invariant. We are not guaranteed *a priori* that the boundary theory is a local quantum field theory. A counterexample is the linear-dilaton background describing the near-horizon geometry of $N$ NS fivebranes at $g_s \to 0$ [19].

[4] This conclusion is also supported by the fact that in the analogous case of 4-d gauged $SO(8)$ supergravity this is a *theorem* [18]: any stationary point of the 4-d superpotential can be lifted to a solution of 11-d supergravity.

By associating a 10-d type IIB background of the form $AdS_5 \times_W X$ to these new stationary points, locality follows because the construction of local operators in the boundary theory proceeds by looking at the asymptotic behavior of perturbations in the interior.[5]

In this Section, we will prove locality in a different way. Namely, we will use the UV/IR relations of $AdS$ dynamics to find an analog to the renormalization group flow. We will find a solution of N=8 5-d gauged supergravity with two asymptotic regions, in which the scalar fields depend on the radial $AdS$ coordinate $U$ as follows: in the region near the $AdS$ horizon ($U$ small) the scalars are close to the new stationary point, while as $U$ increases they roll towards the $SO(6)$-symmetric stationary point. Since $U$, the distance from the horizon, is always linearly proportional to the energy scale of the boundary theory [20] one can interpret this solution as describing the RG evolution from an instable IR fixed point (the new stationary point, non supersymmetric), towards a stable UV fixed point: N=4 Super Yang Mills. The UV theory is a local field theory; therefore, the IR fixed point is also local. Besides helping with locality, our construction also gives a function equal to the central charge at the critical points of the scalar potential, and always increasing along the IR→UV RG flow.

Both the new stationary points of 5-d $AdS$ supergravity can be obtained by giving a nonzero VEV to a single real scalar, that breaks the $SO(6)$ symmetry to $SO(5)$ or $SU(3)\times U(1)$. Also, in both cases, one can consistently put to zero all fields except the 5-d metric $g_{IJ}$ and the scalar $\lambda$. This is possible because all other scalar fields are nonsinglets of the residual symmetry and must by consequence appear at least quadratically in the action. Also, with the explicit parametrisation given in [15], it can be checked that these scalars have no current, so the coupling to vector fields is also at least quadratic.

The 4-d Poincaré-invariant ansatz for the metric is:

$$ds^2 = e^{2\phi(\rho)}(d\rho^2 + dx^\mu dx_\mu), \qquad \mu, \nu = 0, \ldots, 3\,. \tag{4.1}$$

Here $\rho = 1/U$, and the $AdS$ background is $e^\phi = U$. The Lagrangian density of the scalar $\lambda$ can be written as

$$L = \frac{1}{2} m g^{IJ} \partial_I \lambda \partial_J \lambda + V(\lambda)\,, \tag{4.2}$$

where $m$ is a nonzero constant. In our ansatz, the scalar too depends only on $\rho$. The Einstein tensor $G_{IJ} = R_{IJ} - 1/2 g_{IJ} R$ has only two independent nonzero components

$$G_{\rho\rho} = 6\frac{d\phi}{d\rho}\frac{d\phi}{d\rho}, \qquad G_{00} = -3\frac{d\phi}{d\rho}\frac{d\phi}{d\rho} - 3\frac{d^2\phi}{d\rho^2}\,. \tag{4.3}$$

[5]We thank E. Witten for this remark.

Einstein's equations reduce to:

$$G_{\rho\rho} = m\frac{d\lambda}{d\rho}\frac{d\lambda}{d\rho} - 2e^{2\phi}V(\lambda)\,, \tag{4.4}$$

$$G_{00} = m\frac{d\lambda}{d\rho}\frac{d\lambda}{d\rho} + 2e^{2\phi}V(\lambda)\,. \tag{4.5}$$

The scalar's equation of motion is instead:

$$m\frac{d}{d\rho}\left(e^{3\phi}\frac{d}{d\rho}\lambda\right) = e^{5\phi}V'(\lambda)\,, \tag{4.6}$$

(the prime denotes derivative with respect to $\lambda$).

The last equation is not independent; rather, it is a linear combination of Einstein's equations. It is convenient to change variable from $\rho$ to $x$ such that $\exp(-\phi) = d\rho/dx$, and to choose as independent equations the equations of motion of the scalar and of $g_{\rho\rho}$

$$m\frac{d^2\lambda}{dx^2} + 4m\frac{d\phi}{dx}\frac{d\lambda}{dx} = V'(\lambda), \qquad 6\frac{d\phi}{dx}\frac{d\phi}{dx} = m\frac{d\lambda}{dx}\frac{d\lambda}{dx} - 2V(\lambda)\,. \tag{4.7}$$

By solving with respect to $d\phi/dx$ we find the single equation:

$$m\frac{d^2\lambda}{dx^2} \pm \frac{4}{\sqrt{6}}m\sqrt{m\frac{d\lambda}{dx}\frac{d\lambda}{dx} - 2V(\lambda)}\frac{d\lambda}{dx} = V'(\lambda)\,. \tag{4.8}$$

The sign in eq. (4.8) is fixed to + as follows.

1. The equations of motion (4.4, 4.5, 4.6) imply that $d^2\phi/d^2x \leq 0$.

2. We are looking for a solution that approaches the $SO(6)$ symmetric $AdS$ solution in the far "future" i.e. for $x \to \infty$. Moreover we want that increasing $x$ corresponds to increasing energy, i.e. distance from the $AdS$ horizon; this means $\phi(x) \to x/R_2$ when $x \to \infty$. $R_2$ is the $AdS$ radius. From 1 and 2 it follows that $d\phi/dx \geq 1/R_2 > 0$, always.

Eq. (4.8) has a simple interpretation: it describes the motion of a particle of mass $m$ in the potential $-V$, subject to a damping with never-vanishing coefficient $4d\phi/dx$.

In ref. [15], an explicit parametrisation was given for the scalar field configuration that breaks the $SU(4)$ symmetry of $AdS$ 5-d supergravity to $SO(5)$ or $SU(3)\times U(1)$. Let us use that parametrisation and analyse the $SO(5)$ case first.

By calling $\lambda$ the real scalar in the $\underline{20}$ of $SU(4)$ that gets a nonzero VEV at the $SO(5)$-symmetric minimum, we obtain a Lagrangian of the form given in eq. (4.2), with $m = 45/12$ and a potential [15]

$$V(\lambda) = -\frac{1}{32}g^2(15e^{2\lambda} + 10e^{-4\lambda} - e^{-10\lambda})\,. \tag{4.9}$$

It has two stationary points. At the first point, $SO(6)$ symmetric, $\lambda = 0$ and $V = -3g^2/4$. At the other, the symmetry is $SO(5)$, $\lambda = -(\log 3)/6$, and $V = -3^{5/3}g^2/8$.

We want to prove that there exists an interpolating solution leaving $\lambda = -(\log 3)/6$ at $x = -\infty$ and stopping at $\lambda = 0$ at $x = +\infty$. This is obvious since that solution describes a particle subject to a never-vanishing damping moving away from a local maximum of the upside-down potential ($\lambda = -(\log 3)/6$, $-V = 3^{5/3}g^2/8$) and rolling to rest at a local minimum ($\lambda = 0$, $-V = 3g^2/4$). The shape of the upside-down potential, shown in fig. 2, also shows that the interpolating solution is generic; namely, that by *increasing* $\lambda$ by an arbitrary small amount at $x = -\infty$, one always reaches $\lambda = 0$ at $x = +\infty$.

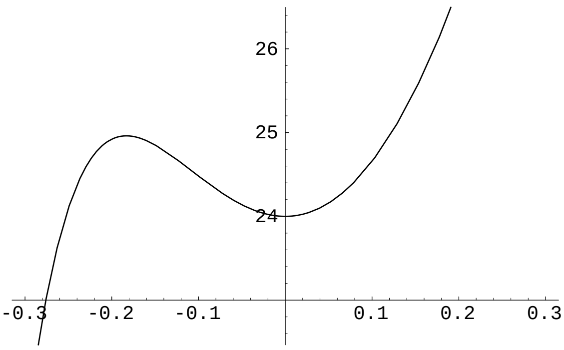


**Figure 2:** Shape of the $SO(5)$-symmetric upside-down potential $-V(\lambda)$; units on the coordinate axis are conventional.

The same argument can be applied verbatim to the case of the $SU(3) \times U(1)$ symmetric deformation. Denoting by $\lambda$ the scalar that breaks $SO(6)$ to $SU(3)\times U(1)$ one finds again a Lagrangian as in eq. (4.8), with a potential [15]

$$V(\lambda) = \frac{3}{32}g^2 \left[\cosh(4\lambda)^2 - 4\cosh(4\lambda) - 5\right] . \tag{4.10}$$

This potential is even in $\lambda$. It has three stationary points; that at $\lambda = 0$ is the old $SO(6)$ symmetric one, with $-V = 3g^2/4$, while those at $\cosh(4\lambda) = 2$ are $SU(3) \times U(1)$ symmetric, and there $-V = 27g^2/32$. Again, the existence of a solution interpolating between an $SU(3) \times U(1)$ symmetric vacuum and the $\lambda = 0$ one is obvious. Also, as in the previous case, it is generic: any small perturbation such that $\cosh(4\lambda) < 2$ at $x = -\infty$ gives rise to a $\lambda(x)$ that rolls towards $\lambda = 0$, and stops there at $x = +\infty$. The shape of the potential $-V(\lambda)$ is shown in fig. 3.

The existence of generic interpolating solutions means that, N=8 gauged, the 5-d supergravity admits solutions on space-times with two asymptotic $AdS$ regions, one near and the other far from the horizon. The holographic correspondence with 4-d boundary CFTs, and the link existing between the distance from the horizon and the energy scale of the boundary theory, tell us that this solution can be interpreted as a renormalization group flow. Namely, it corresponds to a flow from an IR non-supersymmetric theory to N=4 super Yang Mills. The

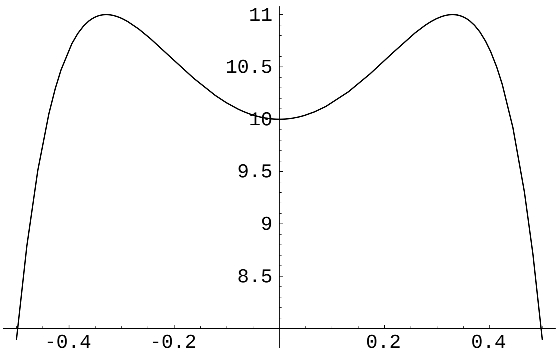


**Figure 3:** Shape of the $SU(3) \times U(1)$-symmetric upside-down potential $-V(\lambda)$

novel IR theories may not have a Lagrangian formulation, but they are local, as explained at the beginning of this Section. Also, our construction of an RG flow explicitly gives a function, $c(\rho)$, that equals the central charge at the critical points of the scalar potential, and that obeys a c-theorem, i.e. that increases along the RG trajectory:

$$c(\rho) = \text{const}\,(T_{xx})^{-3/2}\,. \tag{4.11}$$

At the critical points, the kinetic energy is zero and $T_{xx}$ equals minus the scalar potential, $V_{crit}$. In ref. [21] it was shown that precisely $-V_{crit}^{-3/2}$ is proportional to the central charge.

In both cases, we found that the new theories are UV instable: in the UV they flow back to N=4. Conversely, this means that an *appropriately chosen* small perturbation of N=4 will flow in the IR to one of our novel theories. To identify completely the perturbation goes beyond the possibilities of today's $AdS$/CFT techniques. This identification would require a complete control over all non-renormalizable operators present in N=4. We can nevertheless expand the perturbation in a power series of the perturbation parameter, $\epsilon$, and identify the first term in the series.

In the first example we examined, we gave a nonzero expectation value to a supergravity scalar in the $\underline{20}$ of $SU(4)$. This scalar has $AdS$ mass square equal to $-4$. Using the correspondence between supergravity fields and composite operators in N=4 established in the previous Sections, we identify this perturbation with a composite operator of N=4 super Yang Mills of dimension 2: a mass term for the scalars $\phi_i$, symmetric and traceless in the index $i$. Thus the perturbation breaking $SO(6)$ to $SO(5)$ is:

$$\mathcal{O} = \epsilon\left(\sum_{i=1}^{5}\text{Tr}\,\phi_i^2 - 5\text{Tr}\,\phi_6^2\right) + O(\epsilon^2)\,. \tag{4.12}$$

The trace is taken over the gauge-group indices; the $O(\epsilon^2)$ terms are higher-dimension operators that, among other things, stabilise the runaway direction $\phi_6$. In the second example, we gave an expectation value to a supergravity scalar in the $\underline{10}$ of $SU(4)$, with $AdS$ mass square equal to $-3$. In the N=4 CFT it roughly corresponds to a mass term for the fermion $\lambda^4$ (the N=1 gaugino). Thus, the perturbation breaking $SO(6)$ to $SU(3)\times U(1)$ is

$$\mathcal{O} = \epsilon\left(\text{Tr}\,\lambda^4\lambda^4 + \text{cubic terms in}\,\phi_i\right) + O(\epsilon^2) + h.c.\ldots \tag{4.13}$$

**Addendum.** A few days after this paper was posted on the web, a paper appeared [22] that closely parallels the results of this section. In that paper, the spectrum of our supergravity solutions was also computed, with the result that while the $SU(3)\times U(1)$ stationary point is stable, the $SO(5)$ is not. Namely, a scalar in the 14 of $SO(5)$ has a negative mass square exceeding the Breitenlohner-Freedman bound [23]. The corresponding field in the boundary conformal field theory would have then a complex conformal weight.

This result is puzzling because one may think that the infrared fixed point of a positive, local field theory (N=4 SYM) should not exhibit such pathology. We do not have a definite answer to this puzzle; we may just notice that the $SO(5)$ point was obtained in the first place by a somewhat peculiar perturbation, eq. (4.12), which is tachionic at zero scalar VEVs. This tachion signals that under this perturbation the SYM scalars must get a nonzero VEV, and the configuration with $N$ coincident 3-branes is unstable and must break apart. The instability of the $SO(5)$-symmetric point probably means that its vacuum state has a finite lifetime. This conjecture is supported by the following observation.

Let us call $\phi$ all supergravity fields on $AdS_5$, and $Z[\phi_0]$ the supergravity partition function on $AdS_5$, computed with the boundary condition that $\phi = \phi_0$ at infinity. If $\phi_0$ is a stationary point of the supergravity scalar potential, the $AdS$/CFT correspondence states that

$$Z[\phi_0] = \langle 0|0\rangle_{CFT}\,, \tag{4.14}$$

where $|0\rangle$ is the vacuum state of the CFT defined by the stationary point of the scalar potential. At leading order in the $1/N$ expansion, $Z[\phi_0] = \exp\{-S[\phi_0]\}$, where $S[\phi_0]$ is the classical supergravity action. This action is real, but to next order in the expansion one gets $Z[\phi_0] = \exp\{-S[\phi_0] - 1/2\mathrm{Str}\,\log S''[\phi_0]\}$, with $S''[\phi_0]$ the matrix of the quadratic fluctuations around $\phi_0$. The supertrace is complex when there exist fluctuations that do not satisfy the Breitenlohner-Freedman bound. Its imaginary part is *nonvanishing* in the large $N$ limit; therefore, the lifetime of the vacuum is finite. Thanks to eq. (4.14), the vacuum decay rate per unit volume is: $\Gamma = (2VT)^{-1}\mathrm{Im}\,\mathrm{Str}\,\log S''[\phi_0]$, where $VT$ is the 4-d space-time volume.[6]

## 5. Marginal deformations of N=4 super Yang Mills in the supergravity limit

It is known that there exists a manifold of N=1 fixed points that contains the N=4 Yang-Mills theory [5]. The corresponding theories can be described in N=1 language as containing the same fields as N=4 but with a superpotential

$$\mathcal{W} = h\epsilon_{ijk}\mathrm{Tr}\,\Sigma_i\Sigma_j\Sigma_k + Y_{ijk}\mathrm{Tr}\,\Sigma_i\Sigma_j\Sigma_k\,, \tag{5.1}$$

where $Y_{ijk}$ is a generic symmetric tensor of $SU(3)$, or, in other terms, an element of the $\underline{10}$ of $SU(3)$. The N=4 theory is recovered for $Y_{ijk} = 0$ and $h = g$. There is a particular relation between $g, h, Y_{ijk}$ for which the theory is superconformal. The reason is very simple [5]. The theory is conformal if the anomalous dimension matrix $\gamma_i^j$ for the matter fields $\Sigma_i$ vanishes. The reason is that non-perturbative exact

[6] A somewhat related analysis of conditions for the stability of the large $N$ supergravity approximation of non-supersymmetric theories was performed in [24].

formulae [25] relate the N=1 gauge beta function to the anomalous dimensions of the matter fields. When the N=1 non-renormalization theorems are used, the vanishing of $\gamma_i^j$ is enough to guarantee the vanishing of the beta functions for all the couplings of the theory. In our case, with 24 real parameters, the 9 conditions

$$\gamma_i^j(g, h, Y_{ijk}) = 0\,, \tag{5.2}$$

combined with the modding by $SU(3)$ and the $U(1)$ R-symmetry, yield a three-complex dimensional manifold of fixed points.

If the N=4 Yang-Mills theory can be embedded in this larger manifold of N=1 fixed points, we should be able to find a continuous family of solutions of the Type IIB string theory associated with backgrounds of the type $AdS_5 \times_W H(g, h, Y_{ijk})$, continuously connected to $AdS_5 \times S^5$.[7] We will take a perturbative point of view: if we can identify $h$ and $Y_{ijk}$ with KK modes in the spectrum of $AdS_5 \times S^5$, we can deform the symmetric solution by turning on the corresponding $S^5$ harmonic, and check order by order in a perturbative expansion in $h$ and $Y_{ijk}$ whether or not the new background is a solution of the Type IIB equations of motion.[8]

We discussed the $AdS_5$ interpretation of both terms in (5.1) in Section 1. The conclusion was that, while the deformation in the $\underline{10}$ can be identified with a particular KK mode (part of the $\underline{45}$ in fig. 1) and can be therefore studied in the supergravity approximation, the coupling $h$ of the N=4 superpotential must be associated with a string state, which we are not able to study within the supergravity approximation. At first order in the deformation this is not a problem, since we can consistently study the case $g = h$ in the supergravity limit, and $g - h$ is quadratic in the deformation. Next, we turn to the explicit supergravity calculation.

As already explained in Section 1, the marginal deformation $Y_{ijk}$ of N=4 Super Yang-Mills theory can be identified with part of the KK scalar mode in the $\underline{45}$ of $SO(6)$. More precisely, this scalar corresponds to the second two-form harmonic $Y^I_{[\alpha\beta]}$ ($k = 2$ in the language of ref. [8]) in the expansion of the antisymmetric two-form, $A_{\alpha\beta}$, with components along the five-sphere [8]

$$A_{\alpha\beta} = \sum a^I(x) Y^I_{[\alpha\beta]}(\theta_\alpha)\,. \tag{5.3}$$

Here $x$ is the $AdS$ coordinate and $\theta_\alpha$ is a set of angular coordinates on $S^5$. $I$ is an index labelling the $SU(4)$ representation of the harmonic $Y$; in this case, $I$ labels the $\underline{45}$ of $SU(4)$. We can then try to construct a new set of supergravity solutions by turning on this mode. The ansatz for our solution is, to linear order in the

[7] Notice that, unlike the discussion in Section 3, we are now looking for backgrounds that are Type IIB solution for each value of the parameters and not only for particular ones.

[8] We thank O. Aharony for pointing out a mistake in an earlier version of this manuscript, that changed the conclusions of this Section.

deformation,

$$
\begin{aligned}
g_{MN} &= \dot{g}_{MN} \\
A_{MN} &= \begin{cases} a^I Y^I_{[\alpha\beta]} & \text{for } M,N = \alpha,\beta = 5,\ldots,9\,; \\ 0 & \text{otherwise.} \end{cases} \\
F_{MNRST} &= \begin{cases} \epsilon_{\mu\nu\rho\sigma\tau}\,, & \mu,\nu = 0,\ldots,4\,, \\ \epsilon_{\alpha\beta\gamma\delta\epsilon}, & \alpha,\beta = 5,\ldots,9\,, \end{cases} \\
B &= \text{cost}\,.
\end{aligned} \tag{5.4}
$$

Here $\dot{g}_{MN}$ is the metric on $AdS_5 \times S^5$ and all fermion fields are set to zero.

We know from [8] that, at the linearised level, the Type IIB equations of motion reduce to:

$$
D_\mu D^\mu a^I(x) = 0\,. \tag{5.5}
$$

A field with zero mass in $AdS_5$ corresponds to a marginal operator in the CFT, according to eq. (2.1). Since we are looking for supergravity solutions corresponding to conformal quantum field theory, we do not want the $AdS$ geometry of the five dimensional space-time to be modified. Therefore we take the coefficient $a^I$ independent of the $AdS$ coordinates $x$. This is obviously a solution of the Type IIB equations of motion.

To check that this deformation is indeed a solution of Type IIB compactified on $AdS_5$ times a continuous deformation of $S^5$, we should in principle verify that it satisfies, order by order in the perturbative parameter $a^I$, the type IIB equations of motion. Moreover, this new background must be supersymmetric. On the field theory side of the $AdS$/CFT correspondence, indeed, we want a family of N=1 superconformal theories; thus, the supergravity solution must have N=2 supersymmetry. In such a background, the supersymmetry shift of the fermionic fields must be zero. The general supersymmetry transformations for the fermions are given in [26]; to linear order in the deformation they read:

$$
0 = \delta\lambda = -\frac{i}{24}\Gamma^{MNP} G_{MNP}\dot{\epsilon}\,, \tag{5.6}
$$

$$
0 = \delta\psi_M = \hat{D}_M \epsilon^{(1)} + \frac{1}{96}\left(\Gamma_M{}^{NPQ} G_{NPQ} - 9\Gamma^{NP} G_{MNP}\right)\dot{\epsilon}^c\,, \tag{5.7}
$$

where $G_{MNP} = 3\partial_{[M}A_{NP]}$, $\hat{D}_M$ is the $AdS_5 \times S^5$ covariant derivative and $\epsilon$ is a ten dimensional left-handed spinor. We also denoted by $\epsilon^c$ the charge-conjugated spinor. $\dot{\epsilon}$ is the zero-order spinor that satisfies the above equation for the maximally symmetric background of ref. [8]: $\hat{D}_M\dot{\epsilon} = 0$, while $\epsilon^{(1)}$ denotes the first order correction in the deformation.

We will use the following decomposition for the ten-dimensional $\Gamma$ matrices

$$
\Gamma^\mu = \gamma^\mu \otimes \mathbf{1}_4 \otimes \sigma^1, \qquad \Gamma^\alpha = \mathbf{1}_4 \otimes \tau^\alpha \otimes (-\sigma^2)\,, \tag{5.8}
$$



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where $\gamma^\mu$ and $\tau^\alpha$ are two sets of five-dimensional gamma matrices for the space-time and internal dimensions, respectively. The reduction to five dimensions is performed by expanding in harmonics also the supersymmetry spinor, $\epsilon$,

$$\epsilon(x,\theta) = \sum \epsilon^I(x) Y_a^I(\theta)\,, \tag{5.9}$$

where $a = 1, ..., 4$ denotes a spinorial index on $S^5$. $\dot\epsilon$ simply coincides with the first harmonic of the expansion (5.9), which corresponds to the representation $\underline{\bar 4}$ of $SU(4)$: these are the four complex spinor parameters of N=8 supersymmetry.

We must satisfy the equations:

$$0 = \delta\lambda = -\frac{i}{24}\Gamma^{\alpha\beta\gamma}\dot\epsilon G_{\alpha\beta\gamma} \tag{5.10}$$

$$0 = \delta\psi_M = \begin{cases} \hat D_\mu \epsilon^{(1)} + \frac{1}{96}\Gamma_\mu{}^{\alpha\beta\gamma}\,, G_{\alpha\beta\gamma}\dot\epsilon^c\,, \\ \hat D_\alpha \epsilon^{(1)} + \frac{1}{96}\left(\Gamma_\alpha{}^{\beta\gamma\delta} G_{\beta\gamma\delta} - 9\Gamma^{\beta\gamma} G_{\alpha\beta\gamma}\right)\dot\epsilon^c\,. \end{cases} \tag{5.11}$$

Both $G_{\alpha\beta\gamma}$ and $\dot\epsilon$ are known functions on $S^5$ transforming in the $\underline{45}$ and $\underline{\bar 4}$ of $SU(4)$, respectively. The various products of these functions that appear in eq. (5.10) can be decomposed into sums of harmonics:

$$\begin{array}{lll} 0 = \delta\lambda_a = \sum Y_a^I, & I \in \underline{45}\times\underline{\bar 4} \to \underline{140}+\underline{20}+\underline{20'} \\ 0 = \delta\psi_{a\mu} = \hat D_\mu\epsilon^{(1)} + \Gamma_\mu \sum Y_a^I, & I \in \underline{45}\times\underline{4} \to \underline{84}+\underline{60}+\underline{36} \\ 0 = \delta\psi_{a\alpha} = \hat D_\alpha\epsilon^{(1)} + \sum Y_{a\alpha}^I, & I \in \underline{45}\times\underline{4} \to \underline{84}+\underline{60}+\underline{36}. \end{array} \tag{5.12}$$

Not all the representations indicated in eq. (5.12) actually appear. A generic harmonic $Y_i^I$ of $S^5 = SO(6)/SO(5)$ is specified by giving the representation $I$ under the isometry group of the five-sphere, $SO(6)$, and the representation $i$ under the local Lorentz group of the sphere, $SO(5)$. In our case, the gaugino transforms in the 4 of $SO(5)$ and the gravitino in the $4+16$ of $SO(5)$. As a general rule [27], the representation $i$ must appear in the decomposition of the representation $I$ of $SO(6)$ under $SO(5)$. In fig. 4, where the product of the relevant representations is expressed in terms of $SU(4)$ Young tableaux, the representations containing the 4 or 16 of $SO(5)$ are explicitly indicated. We see that, for example, $\underline{84}$ and $\underline{20}$ do not contribute to the right hand side of eq. (5.12).

Eqs. (5.12) can be satisfied to linear order in the deformation only if

$$G_{\alpha\beta\gamma}\Gamma^{\alpha\beta\gamma}\dot\epsilon = 0\,. \tag{5.13}$$

This equation is simply the gaugino variation. It is satisfied for the following reason. Only the harmonics transforming in the 4 of $SO(5)$ contribute to the equation, thus we see from fig. 4 that only $\underline{20'}$ may contribute. Also, we are not interested in satisfying these equations for all the 45 complex deformations in eq. (5.3), but only for those that transform as $\underline{10}$ under $SU(3)\subset SU(4)$. Moreover, we are interested

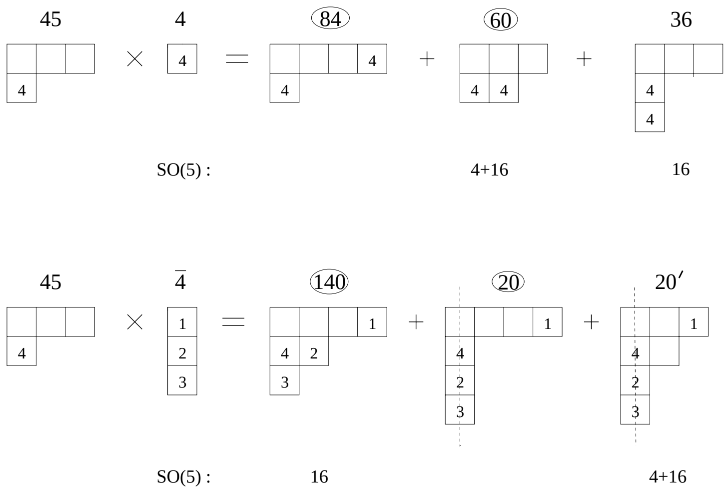


**Figure 4:** Products of harmonics in the fermion variations are decomposed into $SU(4)$ representations. Representations that contain the 4 or 16 of $SO(5)$ and can, therefore, contribute to the expansion into harmonics of the fermion shifts are indicated. The reduction to $SU(3)$ is obtained by splitting the indices of the $\underline{4}$ into $i = 1, 2, 3$ and 4, and by putting the index 4 in all the relevant boxes. The representations that contains a $\underline{10}$ of $SU(3)$ are encircled.

in preserving only one supersymmetry out of the initial 4: the $SU(3)$ singlet in the decomposition $\underline{4} \to \underline{1} + \underline{3}$ under $SU(3) \subset SU(4)$. Therefore, we have to consider only the $SU(3)$-terms $\underline{10} \times \underline{1}$ in the products $\underline{45} \times \underline{\bar{4}}$. The representations that contain a $\underline{10}$ are encircled in fig. 4. We see that eq. (5.13) can be satisfied because $\underline{20}'$ does not contain the $\underline{10}$ of $SU(3)$. Notice that we must choose $\dot{\epsilon}$ in the $\underline{\bar{4}}$; if we choose it in the $\underline{4}$ eq. (5.10) has no solution.

To cancel the gravitino shifts in eq. (5.11) we first recall that by definition:

$$\left( D_M + \frac{1}{2} ie\Gamma\Gamma_M \right) \dot{\epsilon} \equiv \hat{D}_M \dot{\epsilon} = 0\,. \tag{5.14}$$

Here $\Gamma = \Gamma_5 \ldots \Gamma_9$, and $e$ determines the curvature of $S^5$. From the first of eqs. (5.11) we find

$$\epsilon^{(1)} = -\frac{1}{96} ie^{-1} \Gamma \Gamma^{\alpha\beta\gamma} G_{\alpha\beta\gamma} \dot{\epsilon}^c\,. \tag{5.15}$$

We must check that this expression cancels the gravitino shifts with indices in $S^5$, given by the second of eqs. (5.11). Setting to zero the gamma trace of $\delta\psi_\alpha$ we find:

$$\Gamma^\alpha \delta\psi_\alpha = \left( \Gamma^\alpha D_\alpha - \frac{9}{2} ie\Gamma \right) \epsilon^{(1)} = 0\,. \tag{5.16}$$

This equation holds because the only harmonic contributing to $\delta\psi_\alpha$ is the $\underline{60}$, as shown in fig. 4. In ref. [8], it was shown that the equation satisfied by this spinor harmonic ($\Xi^{2,-}$ in the notations of [8]) is, precisely, eq. (5.16). The divergence $D^\alpha \delta\psi_\alpha$

gives another constraint on $\epsilon^{(1)}$. Using the identity $D^\alpha D_\alpha = (\Gamma^\alpha D_\alpha)^2 - 5e^2$, and the equation of motion of $G_{\alpha\beta\gamma}$ [26]

$$D^\delta G_{\delta\alpha\beta} = -\frac{2}{3} i \epsilon_{\alpha\beta\gamma\delta\epsilon} G^{\gamma\delta\epsilon} , \tag{5.17}$$

we find, after some elementary gamma-matrix algebra:

$$D^\alpha \psi_\alpha = \left[ (\Gamma^\alpha D_\alpha)^2 + \frac{3}{2} i e \Gamma \Gamma^\alpha D_\alpha + 27 e^2 \right] \epsilon^{(1)} = 0 . \tag{5.18}$$

This equation is satisfied whenever eq. (5.16) is.

To prove that $\epsilon^{(1)}$ cancels all gravitino shifts we must show that $\delta\psi_\alpha$ contains no transverse (i.e. divergenceless and gamma-transverse) term. This follows again from a result of ref. [8]: no transverse harmonic belongs to the $\underline{60}$ of $SU(4)$.

A second, direct proof is obtained by projecting $D_\alpha \epsilon^{(1)}$ on a complete basis of transverse vector-spinor harmonics, $\Xi^{I_T}_\alpha$ in the notations of [8]. By definition $\Xi^{I_T}_\alpha$ obeys $D^\alpha \Xi^{I_T}_\alpha = \Gamma^\alpha \Xi^{I_T}_\alpha = 0$. Using the Bianchi identity of $G_{\alpha\beta\gamma}$, its equation of motion, and elementary gamma-matrix manipulations we find, thanks to the transversality of $\Xi^{I_T}_\alpha$:

$$\begin{aligned} 0 &= \int_{S^5} d^5 x \bar{\Xi}^{\alpha\, I_T}(x) D_\alpha \epsilon^{(1)} \\ &= -\frac{1}{32} i e^{-1} \int_{S^5} d^5 x \bar{\Xi}^{\alpha\, I_T}(x) \left( \Gamma^\delta D_\delta - \frac{7}{2} i e \Gamma \right) G_{\alpha\beta\gamma} \Gamma^{\beta\gamma} \dot{\epsilon}^c . \end{aligned} \tag{5.19}$$

Integrating by parts, we see that this equation means that the only term in the gravitino shift that can contain a transverse part, $G_{\alpha\beta\gamma}\Gamma^{\beta\gamma}\dot{\epsilon}^c$, is at most proportional to the harmonic $\Xi^{I_T}_\alpha$ obeying $(\Gamma^\delta D_\delta - \frac{7}{2} i e \Gamma) \Xi^{I_T}_\alpha = 0$. Since this harmonic is the product of a $\underline{15}$ with a $\underline{4}$, it has no component in the $\underline{60}$ of $SU(4)$ [8]; therefore, $\delta\psi_\alpha$ has no transverse component at all.

We have now completed the proof that there exist a linearized deformation of the $AdS_5 \times S^5$ background with $SO(2,4)$ isometry and preserving N=2 supersymmetry.

To linear order, all deformations in the $\underline{10}$ of $SU(3)$, $Y_{ijk}$, preserve N=1 superconformal invariance. We do expect to see the equivalent of eq. (5.2) starting to second order in $Y_{ijk}$, similarly to field theory. In our case, the role of eq. (5.2) is played by the type IIB equations of motion, and, in particular, by the Einstein equations.

As a concluding remark, let us point out that the existence of superconformal deformations of type IIB on $AdS_5 \times S^5$ is far from trivial, even to linear order. When seen from the viewpoint of the type IIB theory, this result implies that on the $AdS$ background an N=8 supersymmetric solution can be continuously deformed to solutions with lower supersymmetry. This is not possible on a Minkowsky background [28]. The use of marginal deformations of type IIB as a means to obtain supersymmetry-changing transitions was also put forward in [29].

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