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NOVEL QUARK FRAGMENTATION FUNCTIONS AND THE NUCLEON'S TRANSVERSITY DISTRIBUTION

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ABSTRACT

We define twist-two and twist-three quark fragmentation functions in Quantum Chromodynamics (QCD) and study their physical implications. Using this formalism we show how the nucleon's transversity distribution can be measured in single pion inclusive electroproduction.

Recent measurements of the nucleon's dominant spin–dependent quark distribution have sparked renewed interest in deep–inelastic spin physics. [1, 2, 3] In addition to the debate over the unexpected result reported in Ref. [1], the classification and interpretation of spin dependent effects in deep inelastic scattering has been re–examined and extended. [4, 5] One of the most interesting consequences has been the discovery of a class of *chirally odd* quark distribution functions including one, $h_1(x, Q^2)$,[5, 6] which scales in the deep inelastic limit and provides the long-sought parton description of the quark distribution in a transversely polarized nucleon.[5] For reasons discussed in Ref. [5] we call $h_1(x, Q^2)$ the nucleon's *transversity* distribution. Chirally odd quark distributions are difficult to measure because they are suppressed in totally–inclusive deep inelastic scattering. Up to now, the only practical way to determine $h_1(x, Q^2)$ was muon pair production (the "Drell-Yan" process) with transversely polarized target and beam.[5,6]

In this Letter we show how to generalize the spin, twist and chirality analysis of deep inelastic processes to include quark *fragmentation* functions. Our analysis is complete at the leading order and, for special cases of interest, at orders $1/\sqrt{Q^2}$ and $1/Q^2$. As an application of this formalism – one of many – we show how a chirally odd fragmentation function can be exploited to enable a measurement of $h_1(x, Q^2)$ to be obtained in polarized electroproduction of pions from a transversely polarized nucleon. This is an experiment which could be performed at several existing facilities. Related suggestions involving semi–inclusive production of Λ–hyperons and of two pions have been discussed previously. [7, 8] Our proposal is simpler since it involves only one particle in the final state and does not require measurement of that particle's spin. The price we pay for this simplicity is suppression by a power of $\sqrt{Q^2}$.

The simplest quark fragmentation function is represented diagramatically in Fig. [1]. More complicated fragmentation processes, such as coherent fragmentation of several quarks and gluons, do contribute at order $1/\sqrt{Q^2}$ and beyond. For reasons discussed below, they will not concern us here. In Fig. [1], a quark of momentum k and helicity h fragments into a hadron of momentum P and helicity

H plus an unobserved final state X. The process then repeats in reverse as the unobserved system, X, plus the hadron of momentum P and helicity H' reconstitute the quark of momentum k and helicity h'. The scattering $k + P \to k + P$ is forward, *i.e.* collinear. For definiteness, we take the momentum of the quark–hadron system to be aligned along the $\hat{e}_3$–axis. Then helicity is conserved as a consequence of angular momentum conservation about this axis: $h - H = h' - H'$. The initial and final hadron helicities H and H' need not be equal because the hadron need not have been in a helicity eigenstate; likewise for the quark. This possibility arises when observed hadrons are polarized transversely to the direction of hard momentum flow in a deep inelastic process.[5]

Our first objective is to classify the spin and chirality dependence and twist (order in $1/Q^2$ as $Q^2 \to \infty$) of the possible quark fragmentation functions pictured in Fig. [1]. To do this it is necessary to decompose the Dirac spin space of the quark field component with momentum along the $\hat{e}_3$–axis. Consider the three mutually compatible (*i.e.* commuting) sets of projection operators,

$$P_\pm = \frac{1}{2}\gamma^\mp\gamma^\pm = \frac{1}{2}(1 \pm \alpha_3) \tag{1}$$

$$\Lambda_\pm = \frac{1}{2}(1 \pm \sigma_3) \tag{2}$$

$$\chi_\pm = \frac{1}{2}(1 \pm \gamma_5) \tag{3}$$

$P_\pm$ projects on the "good" and "bad" light–cone components of the quark field, respectively. $\Lambda_\pm$ and $\chi_\pm$ project on positive and negative helicity and chirality states respectively. It is easy to show, then, that the good light–cone component of the quark field with positive (negative) helicity, $\psi_{\uparrow+} \equiv \Lambda_+ P_+ \psi$ ($\psi_{\downarrow+} \equiv \Lambda_- P_+ \psi$), has positive (negative) chirality. In contrast, the bad light–cone component with positive (negative) helicity, $\psi_{\uparrow-} \equiv \Lambda_+ P_- \psi$ ($\psi_{\downarrow-} \equiv \Lambda_- P_- \psi$), has *negative* (*positive*) chirality.

Our studies have shown[5, 9] that

(1) Quark fragmentation functions of the form shown in Fig. [1] and the equivalent gluon fragmentation functions (without further active parton lines) are sufficient to characterize hadron production in hard processes, provided: (i) one studies leading twist (twist-two) $(\mathcal{O}(1/Q^0))$ in any hard process, or (ii) one studies an effect in deep inelastic lepton scattering at the lowest twist at which it arises, and one ignores QCD radiative corrections.

(2) Each appearance of a bad component of the quark field costs one power of $\sqrt{Q^2}$ in the deep inelastic limit (*i.e.* it increases the twist by unity);

(3) For produced hadrons of spin-1/2, helicity differences are observed in longitudinal spin asymmetries; helicity flip is observed in transverse spin asymmetries;

(4) Perturbative QCD cannot flip quark chirality (except through quark mass insertions which we assume to be negligible for light quarks) so chirally–odd quark distribution and fragmentation functions must occur in pairs.

The first two rules emerge from a detailed study of the operator product expansion [10] or equivalently the collinear expansion of Feynman diagrams. [11] Rule (1.i) is well-known and corresponds to the usual probabilistic formulation of the parton model at twist-two. Rule (1.ii) is a new result presented in detail in Ref. [9]. As examples consider two *distribution* functions to which the rule (1.ii) applies: transverse polarization (g_2) or longitudinal (F_L) effects in deep inelastic lepton scattering. In the absence of QCD radiative corrections, these effects first appear at twist-three $\mathcal{O}(1/\sqrt{Q^2})$ and twist-four $\mathcal{O}(1/Q^2)$ respectively. There are several multiquark/gluon distribution functions which cannot be reduced to Fig. [1] which might be expected enter g_2 or F_L. In the case of g_2 it is well known since the work of Shuryak and Vainshteyn [12] that all contributing operators at twist-three can be arranged by careful use of the QCD equations of motion in the form of a quark-quark correlation function evaluated in the target state. The same result applies to F_L, in this case at twist-four. This result allows us to use the properties of two-particle forward amplitudes to catalogue the quark distribution and fragmentation functions which control hadron production at the leading non-trivial twist in

deep inelastic scattering. Rule (3) is a simple consequence of quantum mechanics: transversely polarized states are linear combinations of helicity eigenstates. The final rule is obvious since QCD and the electroweak interactions are all chirally invariant in perturbation theory neglecting mass insertions.

We now combine the above classification of quark fields with these rules to enumerate and characterize quark fragmentation. Fragmentation functions can be labelled uniquely by specifying the helicity of quarks and hadrons and the light cone projection of the quarks in Fig. [1]: $\hat{A}^{ab}_{hH;h'H'}$, where a and b are the quark light–cone projections, either $+$ or $-$. Parity invariance of QCD requires: $\hat{A}^{ab}_{hH;h'H'} = \hat{A}^{ab}_{-h-H;-h'-H'}$. Time reversal invariance, which further reduces the number of independent quark *distribution functions* does not generate relationships among the $\{\hat{A}\}$ because it changes the *out*–state $(PX)_{\text{out}}$ in Fig. [1] to an *in*–state.

As a first example, consider production of a scalar meson like the pion. Through order $1/\sqrt{Q^2}$ there are three independent fragmentation functions: $\hat{A}^{++}_{\frac{1}{2}0;\frac{1}{2}0}$, $\hat{A}^{+-}_{\frac{1}{2}0;\frac{1}{2}0}$, and $\hat{A}^{-+}_{\frac{1}{2}0;\frac{1}{2}0}$. The first is twist-two and scales in the $Q^2 \to \infty$ limit, the latter two are twist-three and are suppressed by $1/\sqrt{Q^2}$ in the $Q^2 \to \infty$ limit. The first function, $\hat{A}^{++}_{\frac{1}{2}0;\frac{1}{2}0}$, is proportional to the traditional fragmentation function $D(z, Q^2)$. It has the same twist, light-cone, helicity and chirality structure as the familiar, spin-average quark distribution function, $f_1(x, Q^2)$, so to avoid an explosion of notation we denote it by $\hat{f}_1(z, Q^2)$ [We will follow the same convention for other fragmentation functions.]:

$$\hat{f}_1(z, Q^2) \propto \hat{A}^{++}_{\frac{1}{2}0;\frac{1}{2}0} \qquad (4)$$

If we were studying quark *distribution* functions, the latter two would be equal by time-reversal invariance. Here, there are two independent fragmentation functions.

$$\hat{e}_1(z, Q^2) \propto \hat{A}^{+-}_{\frac{1}{2}0;\frac{1}{2}0} + \hat{A}^{-+}_{\frac{1}{2}0;\frac{1}{2}0}$$
$$\hat{e}_{\bar{1}}(z, Q^2) \propto \hat{A}^{+-}_{\frac{1}{2}0;\frac{1}{2}0} - \hat{A}^{-+}_{\frac{1}{2}0;\frac{1}{2}0} \qquad (5)$$

The application to spin–1/2 is summarized in Table 1. The fragmentation functions described in Eqs. (4) – (5) (for spin-zero) and in Table 1 (for spin 1/2) are sufficient to describe quark fragmentation in processes to which Rule (1.i) or (1.ii) applies.

In order to relate particular deep inelastic processes to quark distribution and fragmentation functions and to study them in models of non-perturbative QCD, it is necessary to have operator representations for them. We presented this formalism for distribution functions in Ref. [5]. Since we are interested in pion production here, we study fragmentation functions which are independent of the final hadron's spin. The generalizions to spin–1/2 and spin–1 are presented in Ref. [9]. Generalizing the procedure in Refs. [5] and [13], we can define four fragmentation functions with quark fields alone,

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0| \gamma^\mu \psi(0) |PX_{\text{out}}\rangle\langle PX_{\text{out}}| \bar{\psi}(\lambda n) |0\rangle = 4[\hat{f}_1(z)p^\mu + \hat{f}_4(z)M^2 n^\mu] \tag{6}$$

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0| \psi(0) |PX_{\text{out}}\rangle\langle PX_{\text{out}}| \bar{\psi}(\lambda n) |0\rangle = 4M\hat{e}_1(z), \tag{7}$$

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0| \sigma^{\mu\nu} i\gamma_5 \psi(0) |PX_{\text{out}}\rangle\langle PX_{\text{out}}| \bar{\psi}(\lambda n) |0\rangle = 4M\epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta \hat{e}_{\bar{1}}(z), \tag{8}$$

where P is the four-momentum of the pion and p and n are two light-like vectors such that $p^2 = n^2 = 0$, $p^- = n^+ = 0$, $p \cdot n = 1$, and $P^\mu = p^\mu + n^\mu m_\pi^2/2$. All Dirac indices on quark fields are implicitly contracted. The mass M appearing in Eqs. (6) – (8) is a generic QCD mass scale, which we sometimes choose for convenience to be the nucleon mass. We avoid use of the produced hadron mass because of the singular behavior introduced in the chiral limit (the left hand side of Eq. (7) or (8) does not vanish as $m_\pi \to 0$). The summation over X is implicit and covers all possible states which can be populated by the quark fragmentation. The state $|PX_{\text{out}}\rangle$ is an out state between the pion and X. The renormalization scale dependence is suppressed in Eqs. (6) – (8). Here we work in $n \cdot A = 0$ gauge, otherwise gauge links have to be added to ensure the color gauge invariance. The gauge invariance and other issues of interpretation for equations like Eqs. (6)– (8) are discussed in detail in Ref. [13]. From a simple dimensional analysis, we see that

$\hat{f}_1(z)$, $\hat{e}_1(z)$ and $\hat{e}_{\bar{1}}(z)$, and $\hat{f}_4(z)$ are twist-two, -three, and -four, respectively (they contribute in order $1/Q^0$, $1/Q^1$, and $1/Q^2$ respectively); and from their γ-matrix structure, $\hat{f}_1(z)$ and $\hat{f}_4(z)$ are chirally even and $\hat{e}_1(z)$ and $\hat{e}_{\bar{1}}(z)$ are chirally odd. This assignment agrees with the results quoted above. Hermiticity guarantees that these fragmentation functions are real.

As an important application of the new fragmentation functions introduced above, we consider deep-inelastic scattering with longitudinally polarized leptons on polarized nucleon targets, focusing on pion production in the current fragmentation region. As we shall show below, this process allows us to gain access to the nucleon's transversity distribution.

The simplest cut diagram for the process is shown in Fig. [2], where a quark struck by the virtual photon fragments into an observed pion plus other unobserved hadrons. The cross section of the process is proportional to a trace and integral over the quark loop which contains the quark distribution function and fragmentation function. Due to chirality conservation at the hard (photon) vertex, the trace picks up only the products of the terms in which the distribution and fragmentation functions have the same chirality (Rule (4) above). When the nucleon is longitudinally polarized (with respect to the virtual-photon momentum), the twist-two, chirally even distribution $g_1(x)$ can couple with the twist-two chirally even fragmentation function $\hat{f}_1(z)$, producing a leading contribution $\mathcal{O}(1/Q^0)$ to the cross section. On the other hand, in the case of a transversely polarized nucleon, there is no leading-order contribution. At the next order, the nucleon's transversity distribution $h_1(x)$ can combine with the twist-three chirally odd fragmentation function $\hat{e}_1(z)$, and similarly $g_T(x)$ can combine with the chirally even transverse-spin distribution $\hat{f}_1(z)$. Both couplings produce $1/\sqrt{Q^2}$ contributions to the cross section.

It is simple to see, however, that Fig. [2] alone does not produce an electromagnetically-gauge-invariant result. This is a typical example of the need to consider multi-quark/gluon processes beyond twist-two.[10,11] In the present case (twist-three),

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however, the contributions from coherent scattering can be expressed, with novel use of QCD equations of motion, in terms of the distributions and fragmentation functions defined from quark bilinears. This is a specific example of Rule (1.ii). The combined result is gauge invariant, as can be seen from the resulting nucleon tensor,

$$\hat{W}^{\mu\nu} = -i\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha}{\nu}[(S \cdot n)p_\beta \hat{G}_1(x,z) + S_{\perp\beta}\hat{G}_T(x,z)] \tag{9}$$

where S is the polarization vector of the nucleon $(S^\mu = (S \cdot n)p^\mu + (S \cdot p)n^\mu + S_\perp^\mu)$, p and n are light-cone vectors defined with respect to the virtual-photon momentum q. The two structure functions in $\hat{W}^{\mu\nu}$ are related to parton distributions and fragmentation functions,

$$\hat{G}_1(x,z) = \frac{1}{2}\sum_a e_a^2 g_1^a(x)\hat{f}_1^a(z)$$

$$\hat{G}_T(x,z) = \frac{1}{2}\sum_a e_a^2 \Big[g_T^a(x)\hat{f}_1^a(z) + \frac{h_1^a(x)}{x}\frac{\hat{e}^a(z)}{z}\Big] \tag{10}$$

where the summation over a includes quarks and antiquarks of all flavors.

To isolate the spin-dependent part of the deep-inelastic cross section we take the difference of cross sections with left-handed and right-handed leptons, we use

$$\frac{d^2\Delta\sigma}{dE'd\Omega} = \frac{\alpha_{\rm em}^2}{Q^4}\frac{E'}{EM_N}\Delta\ell^{\mu\nu}\hat{W}_{\mu\nu} \tag{11}$$

where $Q^2 = -q^2$, $k = (E, \mathbf{k})$ and $k' = (E', \mathbf{k}')$ are the incident and outgoing momenta of the lepton, and $\Delta\ell^{\mu\nu}$ is the spin-dependent part of the lepton tensor, $\Delta\ell^{\mu\nu} = -{\rm Tr}[\gamma^\mu \not{k}' \gamma^\nu\gamma_5 \not{k}] = -4i\epsilon^{\mu\nu\alpha\beta}q_\alpha k_\beta$. It is convenient to express the cross section in terms of scaling variables in a frame where lepton beam defines the $\hat{e}_3$-axis and the $\hat{e}_1 - \hat{e}_3$ plane contains the nucleon polarization vector, which has a polar angle α. In this system, the scattered lepton has polar angles (θ, ϕ) and therefore the momentum transfer $\mathbf{q}$ has angles $(\theta, \pi - \phi)$. Then,

$$\frac{d^4\Delta\sigma}{dx\,dy\,dz\,d\phi} = \frac{8\alpha_{\rm em}^2}{Q^2}\Big[\cos\alpha(1 - \frac{y}{2})G_1(x,z)$$

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$$+ \cos\phi \sin\alpha \sqrt{(\kappa - 1)(1 - y)} \left(G_T(x, z) - G_1(x, z)(1 - \frac{y}{2}) \right) \Big] \quad (12)$$

where $y = 1 - E'/E$ and $\kappa = 1 + 4x^2 M^2/Q^2$ in the second term signals the suppression by a factor of $1/Q$ associated with the structure function G_T. The existence of G_1 in the same term is due to a small longitudinal polarization of the nucleon relative to $\mathbf{q}$ when its spin is perpendicular to the lepton beam.

Eq. (12) is our main result. As a check, we multiply by z, integrate over it and sum over all hadron species. Using the well-known momentum sum rule, $\sum_{\text{hadrons}} \int dz\, z \hat{f}_1^a(z) = 1$, and the sum rule, $\sum_{\text{hadrons}} \int dz\, \hat{e}_1^a(z) = 0$, which is related to the fact that the chiral condensate vanishes in the perturbative QCD vacuum, we get the well known result for *total* inclusive scattering, given in Eq. (2.8) in Ref. [14] (if one neglects the terms of order $1/Q^2$ in the latter). [Eq (2.8) in Ref. [14] contains a sign error: the sign of the second term should be reversed corresponding to the replacement $\cos\phi \rightarrow \cos(\pi - \phi)$.] The similarity between the inclusive and semi-inclusive cross sections suggests that they can be extracted conveniently from the same experiment.

The aim of this example was to show that an unfamiliar fragmentation function ($\hat{e}_1$) could be employed to obtain a measurement of an interesting, if unfamiliar, distribution function (h_1). It is apparent from Eq. (12) that we have been only partially successful: although the h_1^a distribution for each quark flavour appears in Eq. (12), the sum over flavors couples it to the unknown flavor dependence of $\hat{e}_1^a$. Fortunately, flavor tagging can be used at large–z to identify the contributions of individual quark flavors. For x in the valence region (where one can ignore antiquarks in the nucleon), and $z \rightarrow 1$, the dominant fragmentation, $u \rightarrow \pi^+$, $d \rightarrow \pi^-$, $s \rightarrow K^-$, effectively allows one to trigger on the contributions of u, d and s quarks separately. One might be concerned that the unknown fragmentation function, $\hat{e}_1$, might not respect the dominant fragmentation selection rules, which have only been tested for the spin-averaged, twist-two fragmentation function, $\hat{f}_1$. However, the coherent gluon interactions which distnguish the twist-three $\hat{e}_1$ from $\hat{f}_1$ are flavor independent and should not alter the selection rules. More complicated

flavor structure does arise at higher twist where multiquark correlation functions appear. We have not attempted to make an estimate of the usefulness of this flavor tagging method in the manner of Ref. [15], which should precede the attempt to carry out this measurement.
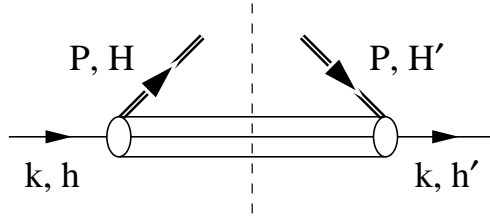
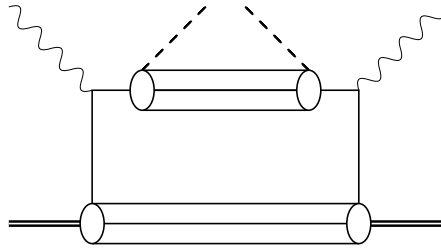Fig. 1.   Diagram for quark fragmentation functions.

Fig. 2.   Pion-production in deep-inelastic scattering.

Table I.   Quark fragmentation functions for spin-$\frac{1}{2}$ baryon.
Note: the functions with bar vanish if there are no final state interactions

	Twist-2 ++	Twist-3 +−(S)	Twist-3 +−(A)	Twist-4 −−
$\hat{A}_{\frac{1}{2}\frac{1}{2}\to\frac{1}{2}\frac{1}{2}} + \hat{A}_{\frac{1}{2}-\frac{1}{2}\to\frac{1}{2}-\frac{1}{2}}$	$\hat{f}_1$	$\hat{e}_1$	$\hat{e}_{\bar{1}}$	$\hat{f}_4$
$\hat{A}_{\frac{1}{2}\frac{1}{2}\to\frac{1}{2}\frac{1}{2}} - \hat{A}_{\frac{1}{2}-\frac{1}{2}\to\frac{1}{2}-\frac{1}{2}}$	$\hat{g}_1$	$\hat{h}_2$	$\hat{h}_{\bar{2}}$	$\hat{g}_3$
$\hat{A}_{\frac{1}{2}-\frac{1}{2}\to-\frac{1}{2}\frac{1}{2}}$	$\hat{h}_1$	$\hat{g}_2$	$\hat{g}_{\bar{2}}$	$\hat{h}_3$

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FIGURE CAPTIONS

1) Diagram for quark fragmentation functions.

2) Pion-production in deep-inelastic scattering.