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Novel Relay Selection Strategies for Cooperative NOMA

Zheng Yang, Zhiguo Ding, *Senior Member, IEEE*, Yi Wu, *Member, IEEE*, and Pingzhi Fan, *Fellow, IEEE*

Abstract—In this paper, we consider non-orthogonal multiple access (NOMA) relaying networks, where one base station communicates with two mobile users with the aid of multiple relays. We propose a two-stage relay selection strategy for NOMA networks with decode-and-forward (DF) and amplify-and-forward (AF) relaying protocols with different quality of service requirements at the users, respectively. Then, the outage probabilities of the NOMA two-stage DF and AF schemes are obtained in closed-form expressions, and the diversity order is determined based on their asymptotic expressions at high signal-to-noise ratio. Both of the developed analytical results and carried out computer simulations show that NOMA two-stage DF (AF) relaying is superior to existing relay selection strategies developed for cooperative NOMA and orthogonal multiple access networks.

Index Terms—Cooperative non-orthogonal multiple access, amplify-and-forward, decode-and-forward, outage probability.

I. INTRODUCTION

Because of its superior spectral efficiency, non-orthogonal multiple access (NOMA) has lately attracted much attention, and NOMA has been recognized as a promising multiple access candidate scheme for the next generation mobile communication networks [1] and [2]. Furthermore, downlink NOMA has recently been adopted by 3GPP long term evolution (LTE) [3], namely, multiple user superposition transmission (MUST).

The basic concept of NOMA is to exploit the power domain and channel quality difference among users to achieve multiple access, where users can be served at the same time/frequency/code [2]. Meanwhile, successive interference cancellation (SIC) is implemented at the users to decode the signals for those users whose channel conditions are poor, then remove these decoded messages, and lastly detect their own information [4].

The outage probability and average sum rate for downlink NOMA with partial channel information have been studied in [5] and [6], where users are randomly placed in a cell.

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These analytical results draw a conclusion that NOMA outperforms traditional orthogonal multiple access (OMA). Another conclusion is that in contrast to OMA, the strong user in NOMA can significantly strengthen its performance, but the performance gain for the weak user is relatively small. A natural idea is that combining relay transmission with NOMA can enhance the weak user's performance. Cooperative NOMA was firstly proposed in [7], where the user having strong channel conditions can act as a relay to help the user suffering from weak channel conditions because SIC is implemented at the strong user.

Recall that there are two well known relay protocols, e.g., decode-and-forward (DF) and amplify-and-forward (AF), which have been commonly used in cooperative communication networks [8] and [9]. In [10], the authors have studied the performance of the scenario in which one source communicates with multiple receivers via an AF relay by using NOMA under Nakagami-m fading, while one source communicates with two users through multiple NOMA AF relaying under Rayleigh fading is considered in [11]. It can be concluded that NOMA relaying networks can yield better performance than OMA relaying systems when users' power allocation coefficients and target data rates are well chosen [10] [11]. Furthermore, the NOMA AF relaying networks in [10] have been extended to a case with one source and multiple receivers each equipped with multiple antennas [12]. A NOMA DF relaying model has been proposed in [13], in order to enhance cell edge user performance. Following the cooperative NOMA model in [7], the concept of simultaneous wireless information and power transfer has been applied at the strong user to aid the base station for serving the weak user [14]. Recently, a two-stage max-min relay selection scheme was proposed for cooperative NOMA in [15], i.e., the first stage is to find a subset of the relays which can successfully decode the message intended to the user with a low rate requirement, then the max-min scheme is applied to select a relay from the subset in order to serve the user which requires a high rate. The outage performance for the two-stage max-min relay selection approach is superior to that of the conventional max-min strategy. However, the qualified selected relays in the first stage cannot always correctly decode the message intended to the user with a high data rate requirement, which will lead to a loss performance gain. Furthermore, fixed power allocation was considered in [15], which leads to a situation that the outage probability always occurs if one chooses a wrong choice for the power allocation coefficients.

In this paper, we consider a cooperative NOMA system with one transmitter communicates with two receivers via multiple relays. The major contributions of this paper are organized as follows.

- A new two-stage relay selection scheme is proposed in

cooperative NOMA networks, where one user's quality of service (QoS) can be strictly satisfied in the first stage, while maximizing its partner's rate in the second stage. We then combine DF and AF relaying with this two-stage approach, respectively. Unlike the two-stage max-min scheme in [15], the users' power allocation coefficients are related to the relay-destination channels in two-stage DF relaying, and they are functions of the source-relay and relay-destination channels in two-stage AF relaying.

- According to the proposed power allocation factors, we obtain the exact expressions for the outage probabilities for the users in two-stage DF and AF relaying NOMA, respectively, as well as their asymptotic expressions at high signal-to-noise ratio (SNR) in order to analyze the diversity gain. The analytical expressions demonstrate that the outage performance for NOMA two-stage DF (AF) relaying is better than those in conventional OMA DF (AF) relaying, although they realize the same diversity order.
- Recall that the coverage probability in the NOMA two-stage max-min strategy will be always one without choosing the users' target rates and power allocation factors in a correct way [15]. This new NOMA two-stage AF relaying can avoid such a disadvantage and is applicable with more flexible parameter selection. Furthermore, this new two-stage AF relaying can achieve the best outage probability among those existing relay selection schemes for cooperative NOMA.

This paper is organized as follows. In Section II, the system model and two relay selection schemes are introduced. Section III and Section IV propose the two-stage DF and AF relay selection scheme for cooperative NOMA, respectively, and investigate the outage probability is achieved by the proposed schemes. Numerical results are presented in Section V to demonstrate the outage performance of the proposed schemes. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

Consider a cooperative NOMA system which contains one source, M relays and two users. Assume that all nodes are equipped with a single antenna, and there is no direct link between the source and the users. The source communicates with the users through a chosen DF or AF relay from the M available relays. Furthermore, assume that all links undergo quasi-static Rayleigh fading. Recall that the users are ordered by the quality of their channels from the relays to users in cooperative NOMA [10] and [12]. Unlike these existing works, we assume that channel conditions between the relays and users are statistically the same, i.e., the relay-user channel fading gains are assumed to be independent and identically distributed, and the users are not ordered based on their channel conditions. Instead, the users are ordered by their different QoS requirements [15] and [16], e.g., the users are distinguished not by their channel conditions, but their QoS requirements. Without loss of generality, assume that the user U_1 has a low target rate but requires to be served quickly, such as an Internet of Things sensor or a healthcare device. On the

other hand, the user U_2 would like to download a movie with high resolution. The two-phase signal transmission schemes for DF and AF relaying are presented in the following two subsections.

A. DF Relaying

In the first time slot, the source transmits a superposition signal, $x_s = \sum_{i=1}^2 \sqrt{\gamma_i} x_i$, to M relays, where x_i is the signal for U_i , γ_i denotes the power allocation factor for message x_i with $\sum_{i=1}^2 \gamma_i = 1$ and $\gamma_i > 0$. Therefore, the received signal at relay n , $1 \leq n \leq M$, is given by

$$y_n^r = \sqrt{P} \sum_{i=1}^2 \sqrt{\gamma_i} x_i g_n + \omega_n^r, \quad (1)$$

where P and ω_n^r denote the transmission power at the source and the additive white Gaussian noise (AWGN) with zero mean and variance σ^2 , and g_n is the channel coefficient from the source to relay n .

Due to the fact that the QoS requirements of U_1 are much less demanding than those in U_2 , we assume that the decoding order is always from U_1 to U_2 . Therefore, the achievable rates for relay n to decode the signals x_1 and x_2 are given by [15]

$$R_1^n = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_1 \rho |g_n|^2}{1 + \gamma_2 \rho |g_n|^2} \right), \quad (2)$$

and

$$R_2^n = \frac{1}{2} \log_2 (1 + \gamma_2 \rho |g_n|^2), \quad (3)$$

respectively, where $\rho = \frac{P}{\sigma^2}$ denotes the transmission SNR. Note that the R_2^n in (3) is achievable under the condition that $R_1^n \geq R_1$, i.e., the SIC is successfully carried out at relay n to remove the signal x_1 , where R_1 denotes the target data rate of U_1 .

If relay n can successfully decode both the signals x_1 and x_2 in the first phase, the relay n can use NOMA to broadcast a superimposed mixture to the users in the second phase. Assume the superimposed mixture is $x_n^r = \sum_{i=1}^2 \sqrt{\alpha_i} x_i$, where α_i is the power allocation coefficient for U_i with $\alpha_1 + \alpha_2 = 1$ and $\alpha_i > 0$. Therefore, the observation signals at U_k , $k = 1, 2$ can be written as follows:

$$y_k^d = \sqrt{P} h_{nk} \sum_{i=1}^2 \sqrt{\alpha_i} x_i + \omega_k^d, \quad (4)$$

where h_{nk} is the channel coefficient from relay n to U_k , P denotes the transmission power at relay n , and ω_k^d denotes the AWGN at U_k , i.e., $\omega_k^d \sim \mathcal{CN}(0, \sigma^2)$.

According to (4), the instantaneous rate for U_1 to detect its signal x_1 is given by

$$R_1^d = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_1 \rho |h_{n1}|^2}{1 + \alpha_2 \rho |h_{n1}|^2} \right). \quad (5)$$

The SIC is carried out at U_2 to remove the signal for U_1 , therefore the instantaneous rate for U_2 to detect the signal x_1 is given by

$$R_{1 \rightarrow 2}^d = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_1 \rho |h_{n2}|^2}{1 + \alpha_2 \rho |h_{n2}|^2} \right). \quad (6)$$

Under the condition that $R_{1 \rightarrow 2}^d \geq R_1$, the observation at U_2 in (4) can remove the signal x_1 before detecting its own message. Therefore, the achievable rate for U_2 to decode the message x_2 is given by

$$R_2^d = \frac{1}{2} \log_2(1 + \alpha_2 \rho |h_{n2}|^2). \quad (7)$$

Note that the decoding order of most existing works about NOMA, such as [5] and [10], relies on using the users' channel conditions. The shortcoming is that all the relays need to know the relay-user channel information, which will significantly increase the system overhead, especially in cooperative NOMA networks with a large number of users or relays. However, in this paper, the user ordering is based on the users' QoS requirements, which means NOMA can be implemented even if users have statistically the same channel conditions and the relays do not need to know the CSI for the channels between the relays and the users.

B. AF Relaying

Recall that the signal received by relay n in the first time slot is y_n^r in (1). In the second time slot, relay n amplifies the received signal y_n^r by a coefficient β , and then broadcasts it to the users in AF relaying. Therefore, the received signal at U_k can be expressed as follows:

$$y_k^d = \beta \sqrt{P} g_n (\sqrt{\gamma_1 P} x_1 + \sqrt{\gamma_2 P} x_2) h_{nk} + \beta \sqrt{P} h_{nk} \omega_n^r + \omega_k^d, \quad (8)$$

where the amplify factor is defined as follows:

$$\beta = \frac{1}{\sqrt{P|g_n|^2 + \sigma^2}}. \quad (9)$$

Similar to cooperative NOMA with DF relaying, U_1 in NOMA AF relaying will firstly decode its signal x_1 . Therefore, the received signal-to-interference-plus-noise ratio (SINR) for U_1 to decode message x_1 based on (8) and (9), is given by

$$\text{SINR}_1 = \frac{\gamma_1 \rho^2 |g_n|^2 |h_{n1}|^2}{\rho^2 |g_n|^2 |h_{n1}|^2 \gamma_2 + \rho |g_n|^2 + \rho |h_{n1}|^2 + 1}. \quad (10)$$

Furthermore, the received SINR for U_2 to decode message x_1 is given as follows:

$$\text{SINR}_{1 \rightarrow 2} = \frac{\gamma_1 \rho^2 |g_n|^2 |h_{n2}|^2}{\rho^2 |g_n|^2 |h_{n2}|^2 \gamma_2 + \rho |g_n|^2 + \rho |h_{n2}|^2 + 1}. \quad (11)$$

Note that signal x_1 can be correctly decoded by U_2 , if $\frac{1}{2} \log_2(1 + \text{SINR}_{1 \rightarrow 2}) \geq R_1$, i.e., x_1 can be removed from y_2^d . Therefore, U_2 detects its own message with the following SINR:

$$\text{SINR}_2 = \frac{\gamma_2 \rho^2 |g_n|^2 |h_{n2}|^2}{\rho |g_n|^2 + \rho |h_{n2}|^2 + 1}. \quad (12)$$

By using the proposed two-stage DF and AF relay selection schemes in Section II A and B, the outage performance of cooperative NOMA networks with the proposed schemes will be studied in the following two sections. The outage probability is considered as the performance evaluation criterion due to the following reason.

The outage probability of cooperative DF relaying networks is defined as the probability that neither the instantaneous rates at the relays nor the instantaneous rates at the destinations are larger than or equal to the users' data target rates. For cooperative AF relaying networks, the outage probability is directly determined by the event that the instantaneous rates at the destinations are less than the users' targeted data rates. Note that in delay-sensitive networks, the throughput mainly depends on the outage probability, since a fixed rate is transmitted by the transmitter. Therefore, the outage probability is a fundamental performance metric of delay-sensitive cooperative NOMA networks.

On the other hand, given the use of optimal channel coding with an infinite coding length, the outage probability provides a tight bound on the error probability of decoding [17]. While the bit error rate (BER) is another commonly used criterion, it is worth pointing out that evaluating the BER requires to specify the used channel coding and modulation.

III. COOPERATIVE NOMA WITH TWO-STAGE DF RELAY SELECTION

In this section, we firstly propose a two-stage DF relay selection scheme, then investigate its outage performance.

For a given relay n , only if the instantaneous rates for U_1 and U_2 are larger or equal to their target data rates R_1 and R_2 , respectively, the relay can successfully decode the messages x_1 and x_2 , i.e.,

$$\begin{cases} \frac{1}{2} \log_2 \left(1 + \frac{\gamma_1 |g_n|^2}{\rho + \gamma_2 |g_n|^2} \right) \geq R_1, \\ \frac{1}{2} \log_2(1 + \gamma_2 \rho |g_n|^2) \geq R_2, \end{cases} \quad (13)$$

Based on the rate requirements of the users, we can choose different values for R_1 and R_2 , and we will demonstrate how the R_1 and R_2 affect the outage performance in the numerical result section.

Based on (13), we can find that ¹

$$|g_n|^2 > \frac{\eta}{\rho}, \quad (14)$$

where $\eta = \max \left\{ \frac{\varepsilon_1}{\gamma_1 - \gamma_2 \varepsilon_1}, \frac{\varepsilon_2}{\gamma_2} \right\}$, $\varepsilon_1 = 2^{2R_1} - 1$, $\varepsilon_2 = 2^{2R_2} - 1$. Note that if we use (14) as the constraint, which can ensure that the relay n can correctly decode both x_1 and x_2 .

Recall that U_1 has the priority to be served, i.e., the target rate requirements of U_1 need to be met as follows:

$$\frac{1}{2} \log_2 \left(1 + \frac{\alpha_1 \rho |h_{n1}|^2}{1 + \alpha_2 \rho |h_{n1}|^2} \right) \geq R_1. \quad (15)$$

Based on (15), we can find that the maximal power allocation factor α_2 can be expressed as follows:

$$\alpha_2 = \max \left\{ 0, \frac{\rho |h_{n1}|^2 - \varepsilon_1}{\rho |h_{n1}|^2 (1 + \varepsilon_1)} \right\}. \quad (16)$$

Note that if $|h_{n1}|^2 > \frac{\varepsilon_1}{\rho}$, the condition in (15) is always possible. That is to say, U_1 can always successfully detect its own message under the condition $|h_{n1}|^2 > \frac{\varepsilon_1}{\rho}$.

¹Please note that the probability for the event that this constraint cannot be satisfied will be taken into consideration for the outage probability calculation.

According to (14)–(16), a set of active relays which can satisfy the QoS requirements of U_1 can be defined as follows:

$$S_n = \left\{ n : |g_n|^2 > \frac{\eta}{\rho}, |h_{n1}|^2 \geq \frac{\varepsilon_1}{\rho} \right\}. \quad (17)$$

In S_n , only the best relay is selected to serve U_2 , which can be defined as follows:

$$n^* = \arg \max_{n \in S_n} \{|h_{n2}|^2\}. \quad (18)$$

Furthermore, in order to ensure U_2 can achieve a high data rate, we impose a constraint that the SIC can be successfully carried out at U_2 , i.e., $\frac{1}{2} \log_2 \left(1 + \frac{\alpha_1 \rho |h_{n^*2}|^2}{1 + \alpha_2 \rho |h_{n^*2}|^2} \right) \geq R_1$. Again, please note that the impact of this constraint will be taken into consideration for the outage probability. Then the maximal power allocation factor α_2 can be written as follows:

$$\alpha_2 = \max \left\{ 0, \frac{\rho |h_{n^*2}|^2 - \varepsilon_1}{\rho |h_{n^*2}|^2 (1 + \varepsilon_1)} \right\}. \quad (19)$$

Based on (16)–(19), the power allocation factor α_2 for U_2 can be rewritten as follows:

$$\alpha_2 = \min \left\{ \frac{\rho |h_{n^*1}|^2 - \varepsilon_1}{\rho |h_{n^*1}|^2 (1 + \varepsilon_1)}, \max \left\{ 0, \frac{\rho |h_{n^*2}|^2 - \varepsilon_1}{\rho |h_{n^*2}|^2 (1 + \varepsilon_1)} \right\} \right\}. \quad (20)$$

It can be observed from (20) that the power allocation coefficient α_2 depends on ε_1 and the channel fading gains from the best relay to the two users. Furthermore, note that if $|h_{n^*2}|^2 < \frac{\varepsilon_1}{\rho}$, the power allocation factor becomes zero, $\alpha_2 = 0$, since the relay allocates all the power to serve U_1 , especially when the target rate of the U_1 is high or the channel condition between the best relay and U_2 is poor.

According to the above discussion of the two-stage DF relay selection scheme, we provide an exact expression for the outage probability and its diversity gain achieved by this scheme as follows.

Theorem 1: The outage probability for U_2 in cooperative NOMA with the proposed two-stage DF relaying scheme is given by

$$P_2 = e^{-\frac{a_1}{\rho}} \sum_{k=0}^M \binom{M}{k} e^{-\frac{ak}{\rho}} (1 - e^{-\frac{a}{\rho}})^{M-k} (1 - e^{-\frac{a_2}{\rho}})^k + \frac{a_1}{\rho} \sum_{k=0}^M \binom{M}{k} e^{-\frac{ak}{\rho}} (1 - e^{-\frac{a}{\rho}})^{M-k} q(k), \quad (21)$$

where $a_1 = \varepsilon_2(\varepsilon_1 + 1)$, $a_2 = a_1 + \varepsilon_1$, $a = \eta + \varepsilon_1$, $q(k) = \int_0^1 e^{-\frac{a_1 t}{\rho}} \left(1 - e^{-\frac{t}{\rho}(\frac{\varepsilon_1}{t} + a_1)} \right)^k dt$. The diversity gain of the U_2 is given by

$$d = - \lim_{\rho \rightarrow \infty} \frac{P_2}{\rho} = M. \quad (22)$$

Proof: See Appendix A. ■

One can observe from Theorem 1 that the diversity gain of cooperative NOMA with the two-stage DF relaying scheme is proportional to the number of relays, which is the same as traditional OMA with max-min and DF relaying selection. Note that the numerical results in section V show that compared to OMA with DF relaying, the NOMA two-stage DF scheme

can realize better outage behaviour. The main reason is that the users can simultaneously share the same time/frequency/code orthogonal resource, which leads to the improvement of the spectral efficiency.

IV. COOPERATIVE NOMA WITH TWO-STAGE AF RELAY SELECTION

In this section, we focus on how to combine AF protocol with the two-stage strategy, and apply two-stage AF relaying into cooperative NOMA. We then study the outage probability for such NOMA with two-stage AF relaying.

Since U_1 needs to be served quickly with low QoS requirements, which means $\text{SINR}_1 \geq \varepsilon_1$ should be hold, if possible. On the other hand, recall that SIC is carried out at U_2 to remove the signal from U_1 , but the channel fading gains between the relay and the users, i.e., h_{n1} and h_{n2} , are not ordered. In order to ensure U_2 can achieve a high data rate, we impose the following constraint $\text{SINR}_{1 \rightarrow 2} \geq \varepsilon_1$, which means SIC can be successfully implemented by U_2 to remove the signal intended to U_1 . Therefore, based on (10) and (11), the maximal transmission power factor of γ_2 is given by

$$\gamma_2 = \min \left\{ \max\{0, \gamma_{21}\}, \max\{0, \gamma_{22}\} \right\}, \quad (23)$$

where

$$\gamma_{21} = \frac{\rho^2 |g_n|^2 |h_{n1}|^2 - \varepsilon_1 \rho |g_n|^2 - \varepsilon_1 \rho |h_{n1}|^2 - \varepsilon_1}{\rho^2 |g_n|^2 |h_{n1}|^2 (1 + \varepsilon_1)}, \quad (24)$$

and

$$\gamma_{22} = \frac{\rho^2 |g_n|^2 |h_{n2}|^2 - \varepsilon_1 \rho |g_n|^2 - \varepsilon_1 \rho |h_{n2}|^2 - \varepsilon_1}{\rho^2 |g_n|^2 |h_{n2}|^2 (1 + \varepsilon_1)}. \quad (25)$$

In order to ensure that both U_1 and U_2 can always correctly decode signal x_1 , the conditions $\rho^2 |g_n|^2 |h_{n1}|^2 - \varepsilon_1 \rho |g_n|^2 - \varepsilon_1 \rho |h_{n1}|^2 - \varepsilon_1 > 0$ and $\rho^2 |g_n|^2 |h_{n2}|^2 - \varepsilon_1 \rho |g_n|^2 - \varepsilon_1 \rho |h_{n2}|^2 - \varepsilon_1 > 0$ should be satisfied at the same time. Therefore, we can get the following constraint conditions

$$|g_n|^2 > \frac{\varepsilon_1}{\rho}, |h_{n1}|^2 > G_1(|g_n|^2), |h_{n2}|^2 > G_1(|g_n|^2), \quad (26)$$

where $G_1(|g_n|^2) = \frac{\varepsilon_1(\rho |g_n|^2 + 1)}{\rho(\rho |g_n|^2 - \varepsilon_1)}$.

Based on (26), the target rate requirement can be realized of U_1 , and the SIC can be correctly carried out at U_2 to remove the message for U_1 . Therefore, the two-stage AF relaying selection scheme is defined as follows:

$$S_n = \left\{ n : |g_n|^2 > \frac{\varepsilon_1}{\rho}, \min\{|h_{n1}|^2, |h_{n2}|^2\} > G_1(|g_n|^2) \right\}. \quad (27)$$

Note that γ_2 in (23) can be rewritten by applying (27) as follows:

$$\gamma_2 = \min\{\gamma_{21}, \gamma_{22}\}, \quad (28)$$

where γ_{21} and γ_{22} are defined in (24) and (25), respectively. Compared to the two-stage DF relaying scheme, it is interesting to find that the power allocate factors in the two-stage AF relaying scheme are always positive. In Section V, both the analytical results and computer simulations show that the outage probability for users in cooperative NOMA networks

with two-stage AF relaying can achieve better performance than those with two-stage DF relaying.

According to (12) and (28), the instantaneous rate at U_2 can be expressed as follows:

$$R_2^n = \min \left\{ \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{21} \rho^2 |g_n|^2 |h_{n2}|^2}{\rho |g_n|^2 + \rho |h_{n2}|^2 + 1} \right), \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{22} \rho^2 |g_n|^2 |h_{n2}|^2}{\rho |g_n|^2 + \rho |h_{n2}|^2 + 1} \right) \right\}. \quad (29)$$

One best relay is selected from (27) to achieve the maximal rate for U_2 , i.e.,

$$n^* = \arg \max_n \{R_2^n, n \in S_n\}. \quad (30)$$

The following theorem provides an exact expression for the outage probability achieved by the two-stage AF relay selection in cooperative NOMA, as well as its diversity gain.

Theorem 2: The outage probability for U_2 in cooperative NOMA with the proposed two-stage AF relaying scheme is given by

$$P_2 = \sum_{k=0}^M \binom{M}{k} (1 - P_s)^{M-k} (P_s - q_1 - q_2)^k, \quad (31)$$

where $q_1 = \int_0^1 \frac{\varepsilon_1 e^{-\frac{g_1(t)}{\rho}}}{\rho t^2} \sqrt{\frac{g_2(t)}{\rho^2}} \mathbf{K}_1 \left(\sqrt{\frac{g_2(t)}{\rho^2}} \right) dt$, $q_2 = 2 \int_0^1 \frac{\varepsilon_1 (a_2 + 1) e^{-\frac{g_1(t)}{\rho}}}{\rho^2 t^2} \mathbf{K}_0 \left(\sqrt{\frac{g_2(t)}{\rho^2}} \right) dt$, $g_1(t) = a_2 + \varepsilon_1 \left(\frac{1}{t} + \frac{a_1}{\varepsilon_1} \right) (t + 1)$, $g_2(t) = 4\varepsilon_1 (a_2 + 1) \left(\frac{1}{t} + \frac{a_1}{\varepsilon_1} \right) (t + 1)$, $P_s = e^{-\frac{3\varepsilon_1}{\rho} \sqrt{\frac{8\varepsilon_1(1+\varepsilon_1)}{\rho^2}} \mathbf{K}_1 \left(\sqrt{\frac{8\varepsilon_1(1+\varepsilon_1)}{\rho^2}} \right)}$. The diversity order of U_2 is M .

Proof: See Appendix B. ■

It can be seen from Theorem 2 that a diversity order of M can be achieved by cooperative NOMA with the two-stage AF relaying scheme, which is the same as the traditional OMA AF relaying strategy. However, the advantage of this new scheme for cooperative NOMA is that it allows the two users to share the same spectrum resource, and user U_1 with low target data rate requirements can be always served.

V. NUMERICAL RESULTS

This section provides computer simulations to evaluate the correctness of the outage performance of cooperative NOMA with the two-stage DF/AF relaying scheme. Suppose that the channels between any two nodes are independent and identically distributed complex Gaussian distributed, i.e., $\mathcal{CN}(0, 1)$.

Fig. 1 and Fig. 2 plot the outage probability for cooperative NOMA with two-stage DF relaying and AF relaying, respectively, where cooperative OMA is also shown as a cooperative scheme. Observing the Fig. 1, one can conclude that compared to cooperative NOMA with two-stage max-min relaying, the proposed two-stage DF scheme can realize better outage performance. Furthermore, both Fig. 1 and Fig. 2 manifest that cooperative NOMA can remarkably enhance the outage performance compared to cooperative OMA although all of them achieve the same diversity gain. In addition, when

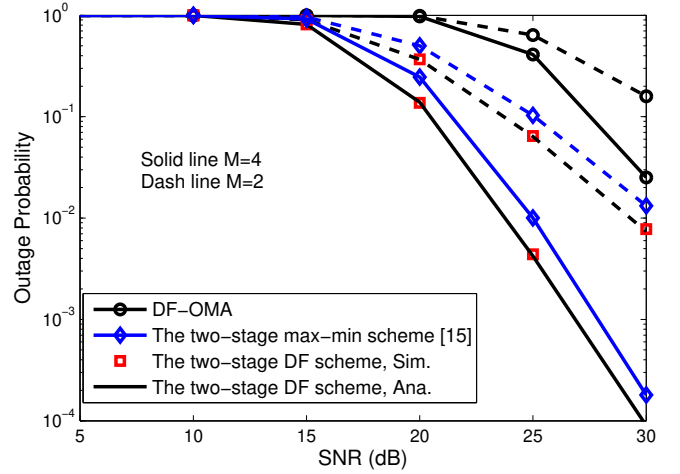


Fig. 1. Impact of M on the outage performance for cooperative NOMA and OMA with different DF relaying schemes, where $R_1 = 0.5 \text{ bits/s/Hz}$, $R_2 = 2 \text{ bits/s/Hz}$, the power allocation factors of two-stage max-min relay selection for cooperative NOMA are $\gamma_1 = \alpha_1 = \frac{3}{4}$ [15], while the power allocation coefficient of two-stage DF relaying networks is $\gamma_1 = \frac{3}{4}$ in Theorem 1.

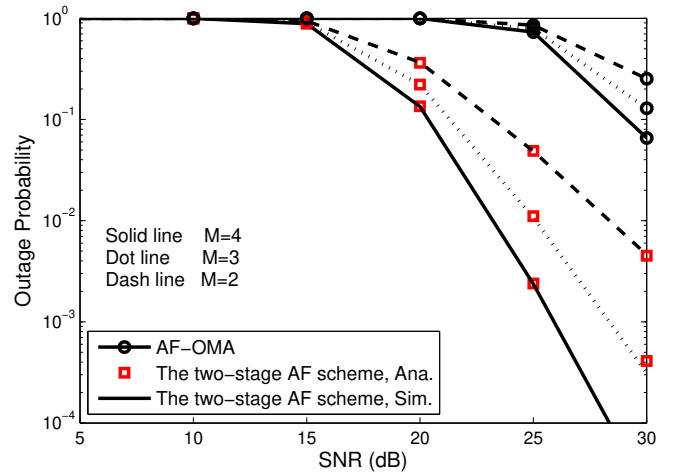
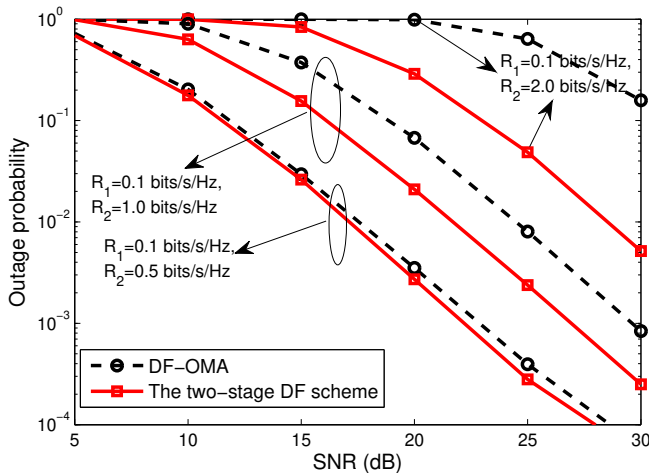


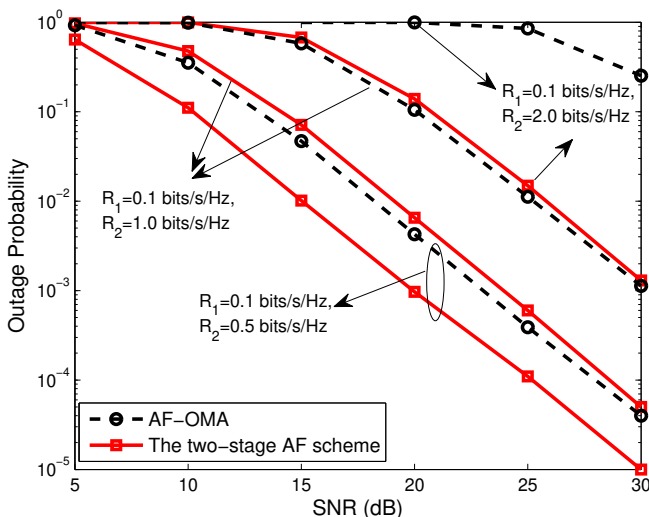
Fig. 2. Effect of M on the outage performance for cooperative OMA and NOMA with AF relaying strategy, where $R_1 = 0.5 \text{ bits/s/Hz}$, $R_2 = 2 \text{ bits/s/Hz}$. The analytical results are based upon Theorem 2.

increasing the number of the relays M , the gap between the outage probabilities achieved by cooperative NOMA and OMA becomes large for both DF and AF relaying. In addition, it should be noticed that the analytical results in (21) and (31) are aligned with the Monte Carlo simulations in Fig. 1 and Fig. 2, respectively.

In Fig. 3, we investigate how the users' target data rates affect the outage probabilities of cooperative OMA with DF (AF) relaying and cooperative NOMA when two-stage DF (AF) relaying is used, respectively. It can be seen from Fig. 3 that in contrast to OMA with DF (AF) relaying, the outage probability for NOMA with two-stage DF (AF) relaying is significantly smaller, as increasing the gap between R_1 and R_2 . The main reason for this phenomenon is that compared to OMA, the two users can be served at the same time in NOMA systems, which leads to an improvement of the



(a) two-stage DF relaying NOMA.



(b) two-stage AF relaying NOMA.

Fig. 3. Effect of users' target rates on cooperative two-stage AF/DF relaying NOMA with $M = 2$.

spectral efficiency. Furthermore, the user U_2 in cooperative NOMA with two-stage relaying selection can achieve a higher data rate, since the SIC is likely to be successfully carried out at U_2 to remove the signal to U_1 . Therefore, the two-stage relaying schemes for cooperative NOMA can be adapted to meet different QoS requirements.

In Fig. 4, we compare the outage performance for cooperative NOMA with two-stage AF/DF relaying to the two-stage max-min scheme by using different users' target rates and different numbers of relays. It can be observed from Fig. 4 that both two-stage AF and DF relaying schemes can achieve better outage performance than the two-stage max-min scheme [15] for different choices of M and R_2 . Furthermore, we can also see that the outage performance is enhanced significantly with low target data rates as M increases. In addition, Fig. 4 also demonstrates that the two-stage AF relaying scheme can achieve the best outage performance among the three different relaying schemes.

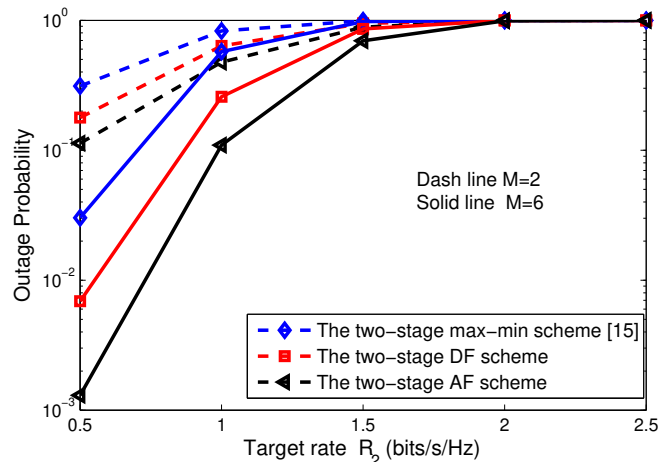


Fig. 4. Outage probability comparison between cooperative NOMA with DF and AF relaying, where $R_1 = 0.1$ bits/s/Hz, $SNR = 10$ dB, the power allocation coefficients of two-stage DF relaying is $\gamma_1 = \frac{3}{4}$. The power allocation coefficients of two-stage max-min are $\gamma_1 = \alpha_1 = \frac{3}{4}$ [15].

VI. CONCLUSIONS

In this paper, we have proposed two kinds of two-stage DF and AF relay selection schemes for cooperative NOMA. For both two-stage DF and AF relaying schemes, we have derived exact and asymptotic analytical expressions for outage probability. The proposed two-stage cooperative NOMA schemes not only achieve full diversity order for both DF and AF relaying, but also yield better outage performance than cooperative OMA. In addition, compared to two-stage max-min relay selection [15], the proposed two-stage DF or AF relaying schemes can further improve the outage probability for cooperative NOMA.

In this paper, the unicast traffic and two users are considered for cooperative NOMA networks, one can extend the cooperative NOMA networks to the case with multiple users and multicast-unicast transmission [18]. In this case, for both the two-stage DF and AF relaying schemes, the rates of the multicast users are restricted to be higher than the target rate of the multicast message, then the best relay is selected to serve the unicast user. On the other hand, the proposed two-stage AF/DF relaying scheme is superior to the two-stage max-min scheme, but it cannot achieve the optimal outage performance in cooperative NOMA networks. For the two-stage DF relaying scheme, if full CSI is available at the source and the relays, the maximal ratio combining (MRC) strategy can be applied at the selected relays to achieve the optimal performance [19]. However, the disadvantage of the MRC strategy is that the global CSI at the relays and destinations should be fed to the source and the relays, respectively, which will significantly increase the system overhead. We will consider this interesting relaying scheme in the future, but it is out the scope of this paper.

APPENDIX A

PROOF OF THEOREM 1

Based on (7), (17), and (18), the coverage probability of U_2 can be obtained as follows:

$$\begin{aligned} \bar{P}_2 &= \sum_{k=1}^M \Pr \left\{ \frac{1}{2} \log_2 (1 + \alpha_2 \rho |h_{n^*2}|^2) \geq R_2, |S_n| = k \right\} \\ &= \sum_{k=1}^M \underbrace{\Pr \left\{ \alpha_2 \geq \frac{\varepsilon_2}{\rho |h_{n^*2}|^2} \middle| |S_n| = k \right\}}_{Q_1} \Pr \{ |S_n| = k \}. \end{aligned} \quad (32)$$

For a given k , the above probability Q_1 can be evaluated by applying (20) as follows:

$$\begin{aligned} Q_1 &= \Pr \left\{ |h_{n^*2}|^2 > \frac{\varepsilon_1}{\rho}, \frac{|h_{n^*1}|^2 - \frac{\varepsilon_1}{\rho}}{|h_{n^*1}|^2(\varepsilon_1 + 1)} \geq \frac{\varepsilon_2}{\rho |h_{n^*2}|^2}, \right. \\ &\quad \left. \frac{|h_{n^*2}|^2 - \frac{\varepsilon_1}{\rho}}{|h_{n^*2}|^2(\varepsilon_1 + 1)} \geq \frac{\varepsilon_2}{\rho |h_{n^*2}|^2} \middle| n^* \in S_n \right\} \\ &= \Pr \left\{ |h_{n^*1}|^2 \geq \frac{\varepsilon_1 |h_{n^*2}|^2}{\rho |h_{n^*2}|^2 - a_1}, \right. \\ &\quad \left. |h_{n^*2}|^2 > \frac{a_2}{\rho} \middle| |h_{n^*1}|^2 > \frac{\varepsilon_1}{\rho} \right\} \\ &= \frac{\Pr \left\{ |h_{n^*1}|^2 \geq \frac{\varepsilon_1 |h_{n^*2}|^2}{\rho |h_{n^*2}|^2 - a_1}, |h_{n^*2}|^2 > \frac{a_2}{\rho} \right\}}{\Pr \{ |h_{n^*1}|^2 > \frac{\varepsilon_1}{\rho} \}} \\ &= e^{\frac{\varepsilon_1}{\rho}} \int_{\frac{a_2}{\rho}}^{\infty} e^{-\frac{\varepsilon_1 y}{\rho y - a_1}} f_{|h_{n^*2}|^2}(y) dy. \end{aligned} \quad (33)$$

By using partial integration and the CDF of $|h_{n^*2}|^2$, e.g., $F_{|h_{n^*2}|^2}(y) = (1 - e^{-y})^k$, Q_1 in (33) can be further evaluated as follows:

$$\begin{aligned} Q_1 &= e^{\frac{\varepsilon_1}{\rho} - \frac{\varepsilon_1 y}{\rho y - a_1}} F_{|h_{n^*2}|^2}(y) \Big|_{\frac{a_2}{\rho}}^{\infty} \\ &\quad - \varepsilon_1 a_1 e^{\frac{\varepsilon_1}{\rho}} \int_{\frac{a_2}{\rho}}^{\infty} \frac{e^{-\frac{\varepsilon_1 y}{\rho y - a_1}}}{(\rho y - a_1)^2} F_{|h_{n^*2}|^2}(y) dy. \end{aligned} \quad (34)$$

Let $t = \frac{\varepsilon_1}{\rho y - a_1}$, and the above Q_1 can be expressed as follows:

$$\begin{aligned} Q_1 &= 1 - e^{-\frac{a_1}{\rho}} (1 - e^{-\frac{a_2}{\rho}})^k \\ &\quad - \frac{a_1}{\rho} \int_0^1 e^{-\frac{a_1 t}{\rho}} \left(1 - e^{-\frac{1}{\rho}(\frac{\varepsilon_1}{t} + a_1)} \right)^k dt. \end{aligned} \quad (35)$$

The probability of the event that one random relay is selected as an active relays in S_n is given by

$$\Pr \left\{ |g_n|^2 \geq \frac{\eta}{\rho}, |h_{n1}|^2 \geq \frac{\varepsilon_1}{\rho} \right\} = e^{-\frac{\alpha}{\rho}}. \quad (36)$$

Then, the probability for the event that there are k active relays in S_n can be evaluated as follows:

$$\Pr \{ |S_n| = k \} = \binom{M}{k} e^{-\frac{\alpha}{\rho} k} (1 - e^{-\frac{\alpha}{\rho}})^{M-k}. \quad (37)$$

Substituting (35) and (37) into (32), and using the fact that the outage probability $P_2 = 1 - \bar{P}_2$, the first part of the theorem is proved.

In order to study the diversity order of the outage probability for U_2 , assume $\rho \rightarrow \infty$ in the following part.

When $x \rightarrow 0$, the exponential function can be approximated by applying Taylor expanded as follows:

$$e^{-x} \approx 1 - x. \quad (38)$$

Based on (38), the first term in (21) can be approximated as follows:

$$\begin{aligned} &\sum_{k=1}^M \binom{M}{k} e^{-\frac{\alpha}{\rho} k} (1 - e^{-\frac{\alpha}{\rho}})^{M-k} e^{-\frac{a_1}{\rho}} (1 - e^{-\frac{a_2}{\rho}})^k \\ &\approx \sum_{k=1}^M \binom{M}{k} \frac{a^{M-k}}{\rho^{M-k}} \frac{a_2^k}{\rho^k} = \frac{1}{\rho^M} \sum_{k=1}^M \binom{M}{k} a^{M-k} a_2^k. \end{aligned} \quad (39)$$

By using Gauss-Chebyshev integral [20], we can obtain the following integral approximation

$$\begin{aligned} &\int_0^1 e^{-\frac{a_1 t}{\rho}} \left(1 - e^{-\frac{1}{\rho}(\frac{\varepsilon_1}{t} + a_1)} \right)^k dt \\ &\approx \frac{\pi}{2N} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| e^{-\frac{a_1 t_i}{\rho}} \left(1 - e^{-\frac{1}{\rho}(\frac{\varepsilon_1}{t_i} + a_1)} \right)^k, \end{aligned} \quad (40)$$

where $t_i = \frac{1}{2} \left(1 + \cos \frac{2i-1}{N} \right)$, and N is the Gauss-Chebyshev integral approximated sum term.

Based on (38) and (40), the last term in (21) can be approximated as follows:

$$\begin{aligned} &\frac{a_1}{\rho} \sum_{k=1}^M \binom{M}{k} e^{-\frac{\alpha k}{\rho}} (1 - e^{-\frac{\alpha}{\rho}})^{M-k} \\ &\quad \times \int_0^1 e^{-\frac{a_1 t}{\rho}} \left(1 - e^{-\frac{1}{\rho}(\frac{\varepsilon_1}{t} + a_1)} \right)^k dt \\ &\approx a_1 \sum_{k=1}^M \binom{M}{k} \frac{a^{M-k}}{\rho^{M-k+1}} \frac{\pi}{2N} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| \left(\frac{\varepsilon_1}{t_i} + a_1 \right)^k \\ &= \frac{\pi a^{M-k}}{2N} \frac{a_1}{\rho^{M+1}} \sum_{k=1}^M \binom{M}{k} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| \left(\frac{\varepsilon_1}{t_i} + a_1 \right)^k. \end{aligned} \quad (41)$$

Substituting (39), and (41) into (21), the second part of the theorem is proved.

APPENDIX B

PROOF OF THEOREM 2

Based on (12), (27), and (30), the outage probability for U_2 can be expressed as follows:

$$\begin{aligned} P_2 &= \sum_{k=1}^M \Pr \{ R_2^{n^*} < R_2, |S_n| = k \} + \Pr \{ |S_n| = 0 \} \\ &= \sum_{k=1}^M \Pr \{ R_2^{n^*} < R_2 \middle| |S_n| = k \} \Pr \{ |S_n| = k \} \\ &\quad + \Pr \{ |S_n| = 0 \}. \end{aligned} \quad (42)$$

The term $\Pr \{ |S_n| = 0 \}$ in (42) can be expressed as follows:

$$\Pr \{ |S_n| = 0 \} = (1 - P_s)^M, \quad (43)$$

where P_s can be calculated as follows:

$$\begin{aligned} P_s &= \Pr\left\{|g_n|^2 > \frac{\varepsilon_1}{\rho}, |h_{n1}|^2 > G_1(|g_n|^2), |h_{n2}|^2 > G_1(|g_n|^2)\right\} \\ &= \int_{\frac{\varepsilon_1}{\rho}}^{\infty} e^{-\frac{2\varepsilon_1(\rho z+1)}{\rho(\rho z-\varepsilon_1)}-z} dz \\ &= e^{-\frac{3\varepsilon_1}{\rho}} \sqrt{\frac{8\varepsilon_1(1+\varepsilon_1)}{\rho^2}} \mathbf{K}_1\left(\sqrt{\frac{8\varepsilon_1(1+\varepsilon_1)}{\rho^2}}\right), \end{aligned} \quad (44)$$

where the last equation follows from the fact that $\int_0^{\infty} \exp(-\frac{\beta}{4x} - \gamma x) dx = \sqrt{\frac{\beta}{\gamma}} \mathbf{K}_1(\sqrt{\beta\gamma})$ in [21, eq. (3.324)].

For a relay n which is selected randomly from S_n in (27), the coverage probability of R_2^n in (29) can be evaluated by applying (28) as follows:

$$\begin{aligned} &\Pr\{R_2^n \geq R_2 | n \in S_n\} \\ &= \Pr\left\{\frac{\varepsilon_2(\rho|g_n|^2 + \rho|h_{n2}|^2 + 1)}{\rho^2|g_n|^2|h_{n2}|^2} \leq \min\{\gamma_{21}, \gamma_{22}\} \mid n \in S_n\right\} \\ &= \Pr\left\{|h_{n2}|^2 > G_2(|g_n|^2), |h_{n1}|^2 > G_3(|g_n|^2, |h_{n2}|^2), \right. \\ &\quad \left. |g_n|^2 > \frac{a_2}{\rho} \mid |g_n|^2 > \frac{\varepsilon_1}{\rho}, |h_{n1}|^2 > G_1(|g_n|^2), \right. \\ &\quad \left. |h_{n2}|^2 > G_1(|g_n|^2)\right\} \\ &= \frac{1}{P_s} \Pr\left\{|h_{n2}|^2 > G_2(|g_n|^2), |h_{n1}|^2 > G_3(|g_n|^2, |h_{n2}|^2), \right. \\ &\quad \left. |g_n|^2 > \frac{a_2}{\rho}\right\} \\ &= \frac{1}{P_s} \int_{\frac{a_2}{\rho}}^{\infty} \int_{\frac{a_2(\rho z+1)}{\rho(\rho z-a_2)}}^{\infty} e^{-\frac{\varepsilon_1 y(\rho z+1)}{\rho^2 z y - a_1 \rho z - a_2 \rho y - a_1} - y} dy e^{-z} dz, \end{aligned} \quad (45)$$

where P_s is defined in (44), $G_2(|g_n|^2) = \frac{a_2(\rho|g_n|^2+1)}{\rho(\rho|g_n|^2-a_2)}$, $G_3(|g_n|^2, |h_{n2}|^2) = \frac{\varepsilon_1|h_{n2}|^2(\rho|g_n|^2+1)}{\rho|h_{n2}|^2(\rho|g_n|^2-a_2)-a_1(\rho|g_n|^2+1)}$.

Let $s = \rho z - a_2$ and $t = \frac{\varepsilon_1(\rho z+1)}{\rho y(\rho z-a_2)-a_1(\rho z+1)}$, the double integral in (45) can be further evaluated as follows:

$$\begin{aligned} &\int_{\frac{a_2}{\rho}}^{\infty} \int_{\frac{a_2(\rho z+1)}{\rho(\rho z-a_2)}}^{\infty} e^{-\frac{\varepsilon_1 y(\rho z+1)}{\rho^2 z y - a_1 \rho z - a_2 \rho y - a_1} - y} dy e^{-z} dz \\ &= \int_0^{\infty} \int_0^1 \frac{\varepsilon_1(s+a_2+1)}{\rho^2 s t^2} e^{-(1+t)(\frac{1}{t} + \frac{a_1}{\varepsilon_1}) \frac{\varepsilon_1(s+a_2+1)}{\rho s}} dt \\ &\quad \times e^{-\frac{s+a_2}{\rho}} ds \\ &= \underbrace{\int_0^1 \frac{\varepsilon_1 e^{-\frac{g_1(t)}{\rho}}}{\rho^2 t^2} \int_0^{\infty} e^{-\frac{s}{\rho} - \frac{g_2(t)}{4\rho s}} ds dt}_{q_1} \\ &\quad + \underbrace{\int_0^1 \frac{\varepsilon_1(a_2+1) e^{-\frac{g_1(t)}{\rho}}}{\rho^2 t^2} \int_0^{\infty} \frac{e^{-\frac{s}{\rho} - \frac{g_2(t)}{4\rho s}}}{s} ds dt}_{q_2}, \end{aligned} \quad (46)$$

where $g_1(t) = a_2 + \varepsilon_1(\frac{1}{t} + \frac{a_1}{\varepsilon_1})(t+1)$, $g_2(t) = 4\varepsilon_1(a_2+1)(\frac{1}{t} + \frac{a_1}{\varepsilon_1})(t+1)$.

q_1 in (46) can be calculated by applying $\int_0^{\infty} \exp(-\frac{\beta}{4x} -$

$\gamma x) dx = \sqrt{\frac{\beta}{\gamma}} \mathbf{K}_1(\sqrt{\beta\gamma})$ in [21, eq. (3.324)] as follows:

$$q_1 = \int_0^1 \frac{\varepsilon_1 e^{-\frac{g_1(t)}{\rho}}}{\rho t^2} \sqrt{\frac{g_2(t)}{\rho^2}} \mathbf{K}_1\left(\sqrt{\frac{g_2(t)}{\rho^2}}\right) dt. \quad (47)$$

q_2 in (46) can be calculated by applying $\mathbf{K}_\nu(z) = \frac{1}{2}(\frac{z}{2})^\nu \int_0^{\infty} \frac{\exp(-\frac{z^2}{4t}-t)}{t^{\nu+1}} dt$ in [21, eq. (8.432)] as follows:

$$q_2 = 2 \int_0^1 \frac{\varepsilon_1(a_2+1) e^{-\frac{g_1(t)}{\rho}}}{\rho^2 t^2} \mathbf{K}_0\left(\sqrt{\frac{g_2(t)}{\rho^2}}\right) dt. \quad (48)$$

Substituting (47) and (48) into (45), we have

$$\Pr\{R_2^n \geq R_2 | n \in S_n\} = \frac{q_1 + q_2}{P_s}. \quad (49)$$

By relying on (49), the first term in (42) can be evaluated as follows:

$$\begin{aligned} &\sum_{k=1}^M \Pr\{R_2^{*n} < R_2 | |S_n| = k\} \Pr\{|S_n| = k\} \\ &= \sum_{k=1}^M \left(1 - \Pr\{R_2^n \geq R_2 | n \in S_n\}\right)^k \Pr\{|S_n| = k\} \\ &= \sum_{k=1}^M \binom{M}{k} P_s^k (1 - P_s)^{M-k} \left(1 - \frac{q_1 + q_2}{P_s}\right)^k, \end{aligned} \quad (50)$$

where $\Pr\{|S_n| = k\} = \binom{M}{k} P_s^k (1 - P_s)^{M-k}$.

Substituting (43) and (50) into (42), and after some algebraic manipulations, we can obtain (31). The proof of the first part of the theorem is completed.

When $z \rightarrow 0$, the Bessel function $z\mathbf{K}_1(z)$ can be approximated by using series representation as follows [21, eq. (8.446)]:

$$\begin{aligned} z\mathbf{K}_1(z) &= 1 + \sum_{k=0}^{\infty} \frac{(\frac{z}{2})^{2k+1} z}{k! \Gamma(k+2)} (\ln \frac{z}{2} + C) \\ &\quad - \frac{1}{2} \sum_{l=0}^{\infty} \frac{(\frac{z}{2})^{2l+1} z}{l!(n+l)!} \left(\sum_{k=1}^l \frac{1}{k} + \sum_{k=1}^{l+1} \frac{1}{k}\right) \\ &\approx 1 + \frac{z^2}{2} \ln \frac{z}{2}. \end{aligned} \quad (51)$$

Based on (51) and $\rho \rightarrow \infty$, P_s in (44) can be approximated as follows:

$$\begin{aligned} P_s &= e^{-\frac{3\varepsilon_1}{\rho}} \sqrt{\frac{8\varepsilon_1(1+\varepsilon_1)}{\rho^2}} \mathbf{K}_1\left(\sqrt{\frac{8\varepsilon_1(1+\varepsilon_1)}{\rho^2}}\right) \\ &\approx \left(1 - \frac{3\varepsilon_1}{\rho}\right) \left(1 + \frac{2\varepsilon_1(1+\varepsilon_1)}{\rho^2} \ln \frac{2\varepsilon_1(1+\varepsilon_1)}{\rho^2}\right) \\ &\approx 1 - \frac{3\varepsilon_1}{\rho}. \end{aligned} \quad (52)$$

Based on (51), q_1 can be approximated as follows:

$$\begin{aligned} q_1 &= \int_0^1 \frac{\varepsilon_1 e^{-\frac{g_1(t)}{\rho}}}{\rho t^2} \sqrt{\frac{g_2(t)}{\rho^2}} \mathbf{K}_1\left(\sqrt{\frac{g_2(t)}{\rho^2}}\right) dt \\ &\approx \frac{\varepsilon_1 e^{-\frac{2a_2}{\rho}}}{\rho} \int_0^1 \frac{e^{-\frac{\varepsilon_1}{\rho t} - \frac{a_1 t}{\rho}}}{t^2} dt. \end{aligned} \quad (53)$$

Let $v = \frac{1}{t}$, the above integral can be further evaluated as follows:

$$\begin{aligned} & \int_0^1 \frac{e^{-\frac{\varepsilon_1}{\rho t} - \frac{a_1}{\rho}}}{t^2} dt \\ &= \int_0^\infty e^{-\frac{\varepsilon_1 v}{\rho} - \frac{a_1}{\rho v}} dv - \int_0^1 e^{-\frac{\varepsilon_1 v}{\rho} - \frac{a_1}{\rho v}} dv \\ &= \frac{\rho}{\varepsilon_1} \sqrt{\frac{4a_1 \varepsilon_1}{\rho^2}} \mathbf{K}_1 \left(\sqrt{\frac{4a_1 \varepsilon_1}{\rho^2}} \right) - \int_0^1 e^{-\frac{\varepsilon_1 v}{\rho} - \frac{a_1}{\rho v}} dv. \quad (54) \end{aligned}$$

The Gauss Chebyshev technology [20] can be applied to approximate the above integral as follows:

$$\begin{aligned} \int_0^1 e^{-\frac{\varepsilon_1 v}{\rho} - \frac{a_1}{\rho v}} dv &\approx \frac{\pi}{2N} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| e^{-\frac{\varepsilon_1 v}{\rho} - \frac{a_1}{\rho v}} \quad (55) \\ &\approx 1 - \frac{\pi}{2N} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| \left(\varepsilon_1 v_i + \frac{a_1}{v_i} \right) \frac{1}{\rho}. \end{aligned}$$

Substituting (54) and (55) into (53), q_1 can be approximated as follows:

$$q_1 \approx 1 - \frac{2a_2 + \varepsilon_1}{\rho}. \quad (56)$$

The series representation of Bessel function $\mathbf{K}_0(z)$ can be approximated with $z \rightarrow 0$ as follows [21, eq. (8.447)]:

$$\begin{aligned} \mathbf{K}_0(z) &= -\ln\left(\frac{z}{2}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{(k!)^2} + \sum_{k=0}^{\infty} \frac{z^{2k} \Psi(k+1)}{2^{2k} (k!)^2} \\ &\approx -\ln \frac{z}{2} = \ln \frac{2}{z}. \quad (57) \end{aligned}$$

To approximate q_2 , based on (57), we can formulate

$$\begin{aligned} q_2 &= 2 \int_0^1 \frac{\varepsilon_1 (a_2 + 1) e^{-\frac{g_1(t)}{\rho}}}{\rho^2 t^2} \mathbf{K}_0 \left(\sqrt{\frac{g_2(t)}{\rho^2}} \right) dt \\ &\approx \frac{\pi}{N} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| \frac{\varepsilon_1 (a_2 + 1) e^{-\frac{g_1(t_i)}{\rho}}}{\rho^2 t_i^2} \mathbf{K}_0 \left(\sqrt{\frac{g_2(t_i)}{\rho^2}} \right) \\ &\approx \frac{\pi}{2N} \sum_{i=1}^N \left| \sin \frac{2i-1}{N} \right| \frac{\varepsilon_1 (a_2 + 1)}{\rho^2 t_i^2} \ln \frac{4\rho^2}{g_2(t_i)}. \quad (58) \end{aligned}$$

Based on (52), (56) and (58), we can find that

$$P_s - q_1 - q_2 \approx \frac{2a_1}{\rho}. \quad (59)$$

Substituting (52) and (59) into (31), the second part of theorem is proved.

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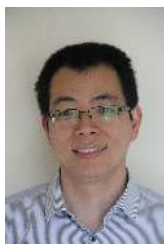
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