NOVEL SOLUTION FOR LEG MOTION WITH 5-LINK BELT MECHANISM

E.Ch. LOVASZ[*], C. POP, F. POP and V. DOLGA
Department of Mechatronics, Faculty of Mechanical Engineering
Politehnica University of Timisoara, ROMANIA
E-mails: erwin.lovasz@upt.ro, florina.pop@upt.ro,
cristian.pop@upt.ro, valer.dolga@upt.ro

From the analysis of Theo Jansen walking mechanism and of the path curve that it describes the reduced capability for crossing over obstacles of the Jansen leg (1 DOF) is pointed out. By using a 5 link belt mechanism with 2 DOF can be adapted for generating similar Jansen mechanism path curve, where the step height of this path can be increased. For this purpose a mathematical model is conceived in order to analyse and determine the parameters for driving and control of the operation of the novel walking leg solution.

Key words: walking leg, Jansen leg, 5-link belt mechanism, path generation, kinematic analysis.

1. Introduction

The variety of walking robots prototypes that are developed around the world for different activities in domains like civil, military research, agriculture and entertainment are based on different types of leg configurations. The leg mechanism used as support for obtaining certain mobility for the walking robots represents the main objective for optimal construction analysis offering a series of advantages regarding manoeuvrability, static and dynamic stability.

Almost all of the existing walking robots created till now include in their mechanical structure a type of a walking mechanism. There are a variety of mechanisms developed during time from simple to more complex configurations used as support for generating walking movement. In order to design new concepts that are able to reproduce the walking characteristics of their biological inspiration models the type of walking of animals, reptiles, insects and humans is studied with attention (Doroftei and Adascalitei, 2012). A new attempt of a bio-inspired leg mechanism was conceived by Jansen (2007). The scalable design, reduced degree of mobility and low energy consumption are the main advantages that Jansen type of walking mechanism possess in comparison with other similar configurations (Nansai *et al*., 2013).

Studies regarding kinematics and dynamics of this configuration were done by Nansai *et al*. (2013), Giesbrecht and Wu Qiong (2012) and Moldovan (2013). Various optimization methods including genetic algorithms and adaptive network based fuzzy inference system are used in order to improve the existent link lengths and trajectory shape in Nansai *et al*. (2013). Studies for adapting the mechanism for climbing possibilities are conducted by Komoda and Wagatsuma (2011). The Jansen leg mechanism was studied by authors in Moldovan (2013), were the planar linkage was submitted to kinematic analysis and synthesis, CAD design and simulation.

Silva and Machado deal with a general overview regarding legged locomotion and the types of walking mechanisms used in the construction of walking robots (Silva and Machado, 2007). In order to develop new improved artificial walking systems a variety of leg configurations were studied and developed.

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[*] To whom correspondence should be addressed

Several configurations are based on well-known mechanisms (pantograph, Cebisev and parallel linkages) or specially designed (Jansen and Klann linkage).

Studies regarding motion characteristics described by a leg mechanism composed of a pantograph linkage combined with a Cebisev mechanism used for the design of biped robot with 1-DOF are conducted in LARM by Liang *et al.* (2008). The possibility for the mechanism to describe a trajectory shape similar to an ovoid curve is analyzed.

A new possibility for integrating a parallel mechanism in the configuration of a quadruped robot for assisting the people with disabilities is presented in Wang *et al.* (2013). The parallel leg mechanism has a modular configuration with 3 DOF formed by 3 serial parallel units.

Another variant of a walking mechanism is presented in Bałchanowski *et al.* (2009) specially implemented on a wheelchair based upon a four bar mechanism and combined with wheels.

The Klann linkage was studied and implemented in the structure of a new eight legged robot prototype in order to test its capabilities and the type of walking algorithm (Lokhande and Emche, 2013).

From the experiments conducted for determining the characteristics of walking (path curve for the characteristic point, step length and height) in the case of Jansen leg configuration, it has been concluded that for crossing over larger obstacles it will be necessary to increase the step height that this mechanism describes. To this end other solutions are proposed in order to modify the link length(s) through a supplementary actuator or to increase the number of leg mechanism DOF's and controlling the movement of the leg path.

This paper proposes a novel solution for improving the capability of walking mechanisms to cross over larger obstacles. From studies made for the trajectory described by the Jansen mechanism it appeared that the step height is not enough for crossing obstacles (Giesbretch and Wu Qiong, 2012). The novel solution consists in using of a 2 DOF 5-link belt mechanism that has the ability to reproduce multiple variants of path curves. By taking the Jansen mechanism path curve as the target trajectory and adapting the 5 link belt mechanism in order to describe similar trajectory with variable step height, the capabilities of it can be improved.

2. Kinematic analysis of Jansen walking mechanism

In order to compute the Jansen leg path it is necessary to perform the geometrical analysis. This study onvolves computing of the angular positions of each mechanism's elements and the coordinates of the coupler point M with its trajectory.

From the structural point of view the investigated mechanism is composed of eight bars connected through revolute joints and is assigned a single degree of freedom. It can be divided in three main closed loops of linkages: A_0AB_0B (first four bar linkage), A_0AEB_0 (second four bar linkage) and final B_0CDE (five bar linkage). The crank (2) accomplishes a complete rotation ($\varphi \in [0, 2\pi]$) while the end point M describes the trajectory presented in Fig.1. The origin of the coordinate system is set in A_0 fixed joint.

For computing the parametric coordinates of point M through the whole walking cycle a mathematical model for geometrical analysis was developed. The equations were developed using the complex number method.

The equations of the closed loops are

$$l_2 \cdot e^{i \cdot \varphi} + l_3 \cdot e^{i \cdot \vartheta_1} = \left(x_{B_0} + i \cdot y_{B_0} \right) + l_4 \cdot e^{i \cdot \psi_1},$$

$$l_2 \cdot e^{i \cdot \varphi} + l_5 \cdot e^{i \cdot \vartheta_2} = \left(x_{B_0} + i \cdot y_{B_0} \right) + l_6 \cdot e^{i \cdot \psi_2}, \tag{2.1}$$

$$l_{41} \cdot e^{i \cdot (\alpha_0 + \psi_1)} + l_7 \cdot e^{i \cdot \gamma} = l_6 \cdot e^{i \cdot \psi_2} + l_8 \cdot e^{i \cdot \delta}.$$
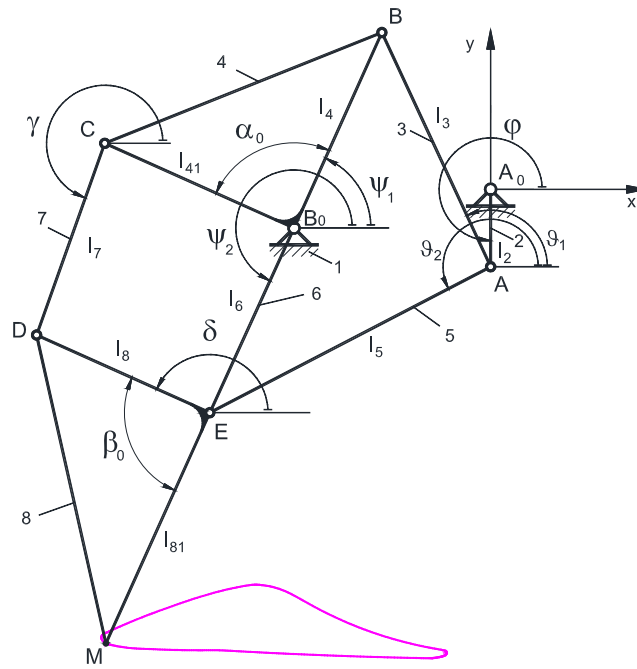
where x_{B_0}, y_{B_0} are known coordinates.

Fig.1. Jansen mechanism.

By separating of term which contains the angle (ϑ_1 or ϑ_2) in the first, or, second Eqs (2.1), respectively, and through multiplication with its complex conjugate relationship an equation in the form below results

$$A_i(\varphi) \cdot \cos\psi_i + B_i(\varphi) \cdot \sin\psi_i + C_i(\varphi) = 0, \qquad i = 1, 2, \tag{2.2}$$

where

$$A_1(\varphi) = 2l_4 x_{B_0} - 2l_2 l_4 \cdot \cos\varphi, \qquad B_1(\varphi) = 2l_4 y_{B_0} + 2l_2 l_4 \cdot \sin\varphi,$$

$$C_1(\varphi) = x_{B_0}^2 + y_{B_0}^2 + l_2^2 - l_3^2 + l_4^2 - 2l_2 x_{B_0} \cdot \cos\varphi - 2l_2 y_{B_0} \cdot \sin\varphi,$$

$$\tag{2.3}$$

$$A_2(\varphi) = 2l_6 x_{B_0} - 2l_2 l_6 \cdot \cos\varphi, \qquad B_2(\varphi) = 2l_6 y_{B_0} + 2l_2 l_6 \cdot \sin\varphi,$$

$$C_2(\varphi) = x_{B_0}^2 + y_{B_0}^2 + l_2^2 - l_5^2 + l_6^2 - 2l_2 x_{B_0} \cdot \cos\varphi - 2l_2 y_{B_0} \cdot \sin\varphi,$$

and next from Eq.(2.2) we get the expression for position angle ψ_i ($i=1, 2$)

$$\psi_i(\varphi) = 2 \cdot a\tan \frac{B_i(\varphi) - \sqrt{A_i(\varphi)^2 + B_i(\varphi)^2 - C_i(\varphi)^2}}{A_i(\varphi) - C_i(\varphi)}. \tag{2.4}$$

A similar procedure is applied for the third closed loop for computing the position angles (δ)

$$A_3(\varphi) \cdot \cos\delta + B_3(\varphi) \cdot \cos\delta + C_3(\varphi) = 0,$$

(2.5)

$$\delta(\varphi) = 2 \cdot a\tan \frac{B_3(\varphi) - \sqrt{A_3(\varphi)^2 + B_3(\varphi)^2 - C_3(\varphi)^2}}{A_3(\varphi) - C_3(\varphi)}.$$

where

$$A_3(\varphi) = 2l_6 l_8 \cdot \cos\psi_2 - 2l_{41} l_8 \cdot \cos(\psi_1 + \alpha_0),$$

$$B_3(\varphi) = -2l_6 l_8 \cdot \sin\psi_2 + 2l_{41} l_8 \cdot \sin(\psi_1 + \alpha_0),$$

(2.6)

$$C_3(\varphi) = l_{41}^2 + l_6^2 - l_7^2 + l_8^2 + 2l_6 l_{41} \cdot \cos(\psi_1 + \alpha_0).$$

The parametric coordinates of the point M are described through the following relations

$$x_M + i \cdot y_M = x_{B_0} + i \cdot y_{B_0} + l_6 \cdot e^{i \cdot \psi_2(\varphi)} + l_{81} \cdot e^{i \cdot (\delta(\varphi) + \beta_0)},$$

(2.7)

$$\begin{cases} x_M(\varphi) = x_{B_0} + l_6 \cdot \cos\psi_2(\varphi) + l_{81} \cdot \cos(\delta(\varphi) + \beta_0), \\ \\ y_M(\varphi) = y_{B_0} + l_6 \cdot \sin\psi_2(\varphi) + l_{81} \cdot \sin(\delta(\varphi) + \beta_0). \end{cases}$$

(2.8)

After implementing these equations in a MathCad program based on input data characterized by the link lengths in *mm* (l_2 =15, l_3 = 50, l_4=41.5, l_{41}=40.1, l_5=61.9, l_6=39.3, l_7=39.4, l_8=36.7, l_{81}=49, α_0=90^0, β_0=90^0), the coordinates for A_0 (x_{A0}=0, y_{A0}=0), B_0 (x_{B0}=-38, y_{B0}=-7.5), input values of the angle φ from the crankshaft (2) and position of B_0 fixed couple, the coordinates of the end point M were calculated. The trajectory shape described by the leg mechanism based on the coordinates achieved is presented in Fig.2.
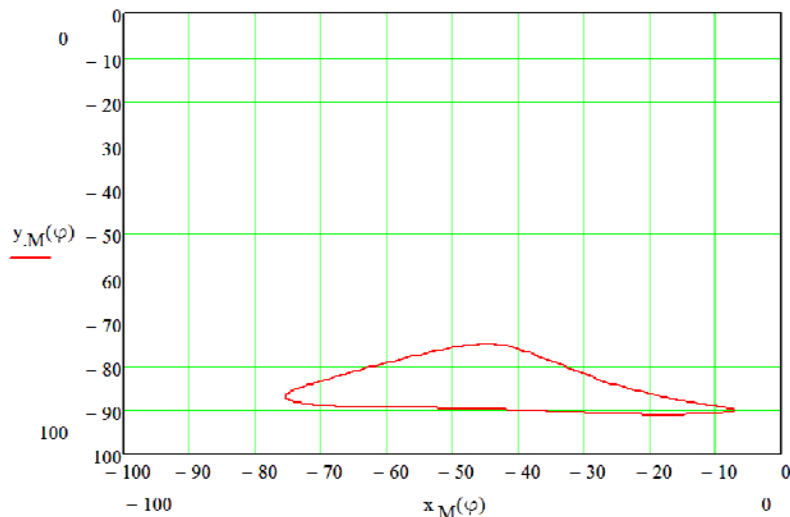

Fig.2. View of Jansen leg mechanism's trajectory.

3. Kinematic analysis of the 5-link belt mechanism

A special form of a planar mechanism with 2 DOF called 5-link belt mechanism is proposed to be used as a leg mechanism configuration in order to increase the step height.

This type of mechanism can be used for generating special trajectories. It has a variable link length, based on a belt as a flexible and inextensible element that is enveloped or developed on a circular wheel (see Fig.3).
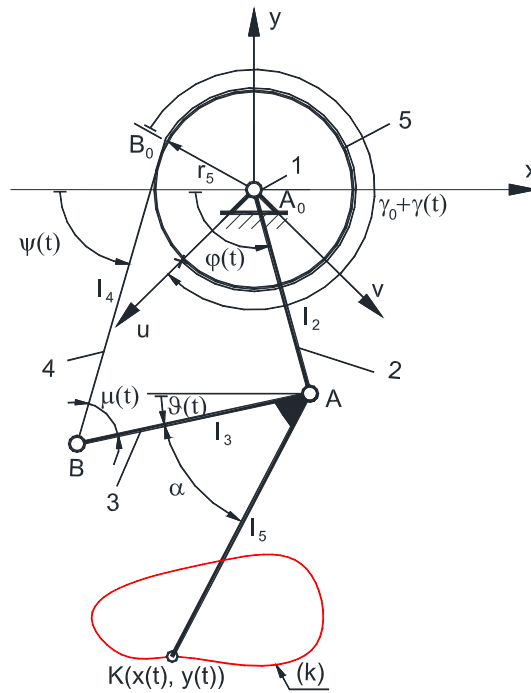

Fig.3. Kinematic schema of 5-link belt mechanism.

The link (2) and the circular wheel (2.5) are driving elements. The trajectory for the characteristic point K is given in a parameterized form, with $t \in [0,1]$

$$K = x(t) + i \cdot y(t),$$ (3.1)

and the same trajectory (2.1) can be described with the coupler point K of the 5-link belt mechanism by the relationship, written in complex numbers

$$K = l_2 \cdot e^{i \cdot \varphi(t)} + l_5 \cdot e^{i \cdot (\alpha + \vartheta(t))}.$$ (3.2)

From Eqs (3.1) and (3.2) equalized and by eliminating the exponent $\alpha + \vartheta(t)$ we get

$$\varphi(t) = 2 \cdot \arctan \frac{B(t) \pm \sqrt{A^2(t) + B^2(t) - C^2(t)}}{A(t) - C(t)}$$ (3.3)

where

$$A(t) = 2l_2 \cdot x(t) \qquad B(t) = 2l_2 \cdot y(t) \qquad C(t) = l_5^2 - x^2(t) - y^2(t) - l_2^2 \,. \tag{3.4}$$

In an analog procedure the positional angle of the coupler (2.3) gives

$$\vartheta(t) = \varphi(t) - \alpha + \arccos\left(\frac{x^2(t) + y^2(t) - l_2^2 - l_5^2}{2l_2 l_5}\right). \tag{3.5}$$

Further we present the calculus of the drive angle $\theta(t)$ of the wheel and the calculus of the wheel angle $\gamma(t)$ in the mobile system of the wheel and belt angle $\psi(t)$ in the frame system

$$\theta(t) = \psi(t) - \gamma(t) \tag{3.6}$$

where

$$\gamma(t) = \frac{l - \sqrt{l_2^2 + l_3^2 - r_5^2 + 2l_2 l_3 \cdot \cos(\varphi(t) - \vartheta(t))}}{r_5} - \gamma_0 \,, \tag{3.7}$$

$$\psi(t) = 2\arctan\frac{r_5 \mp \sqrt{r_5^2 + l_4(t)^2 - \left(l_2 \cdot \cos\varphi(t) + l_3 \cdot \cos\vartheta(t)\right)^2}}{l_4(t) + l_2 \cdot \cos\varphi(t) + l_3 \cdot \cos\vartheta(t)}\,. \tag{3.8}$$

with: l - the total length of the belt, r_5 - the radius of the circular wheel and γ_0 the initial enveloped belt angle for $t = 0$.

In order to avoid the singularities, it is necessary to have the transmission angle always positive and $\mu(t) > \mu_{min}$, where

$$\mu(t) = \psi(t) - \vartheta(t)\,. \tag{3.9}$$

4. Numerical example for the novel of the 5-link belt mechanism

Starting with the parameterization for the Jansen mechanism path curve obtained through the parameterized coordinates (2.8) that lead to dependency of parameter $t \in [0,1]$ instead of φ, the trajectory is divided into seven sub-curves. For each sub-curve an interpolation function is applied based on a second order spline. This method is chosen in order to have a unitary form for each polynomial expression corresponding to the sub-curve and to avoid the global forms of interpolation that usually do not converge. The new parametric equations obtained will be further used for generating new trajectories which are different from the initial one.

The parametric equations for the curve are expressed depending of the parameter t as in

$$x_{M,i}(t) = x_i + m_{xi}(t - t_i) + a_{xi}(t - t_i)^2, \qquad y_{M,i}(t) = y_i + m_{yi}(t - t_i) + a_{yi}(t - t_i)^2, \tag{4.1}$$

$$m_{xi} + 2 \cdot a_{xi} \cdot (t_{i+1} - t_i), \qquad m_{yi} + 2 \cdot a_{yi} \cdot (t_{i+1} - t_i), \tag{4.2}$$

$$a_{xi} = \frac{x_{i+1} - x_i}{\left(t_{i+1} - t_i\right)^2} - \frac{m_{xi}}{t_{i+1} - t_i}, \qquad a_{yi} = \frac{y_{i+1} - y_i}{\left(t_{i+1} - t_i\right)^2} - \frac{m_{yi}}{t_{i+1} - t_i} \tag{4.3}$$

where i represents the interpolation range, m_{xi}, m_{yi} are the slope of the segment curve and the second order coefficients a_{xi}, a_{yi}.

The interpolation values and input geometrical data used in the following example for the 5-link belt mechanism are presented in Tab.1 and Tab.2.

Table 1. Interpolation values for the Jansen leg trajectory.

m_{x1}	103.487			m_{y1}	-7.874			
i	1	2	3	4	5	6	7	8
t_i	0	0.389	0.553	0.639	0.764	0.833	0.958	1
x_i	-74.363	-26.179	-7.636	-10.581	-39.419	-59.112	-75.319	-74.363
y_i	-87.965	-90.811	-90.395	-85.984	-46.329	-45.890	-86.849	-87.965

Table 2. Input geometric data of the 5-link belt mechanisms.

Frame length	$l_1 = 0\ mm$	Coupler length	$l_3 = 10\ mm$	Belt length	$l = 650\ mm$	Coupler angle	$\alpha° = 135°$
Crank length	$l_2 = 55\ mm$	Coupler length	$l_5 = 50\ mm$	Wheel radius	$r_5 = 20\ mm$		

After introducing and computing the mathematical relations for the 5-link belt mechanism with the chosen parameters in different positions corresponding to the control points from the given curve we obtained the drive angles $\theta(t)$, $\varphi(t)$, wheel angle $\gamma(t)$, belt angles $\psi(t)$ and transmission angles $\mu(t)$ presented in Figs 4, 5 and 6. This operations were done for three different paths with various heights ($h = 30, 42, 55\ mm$).
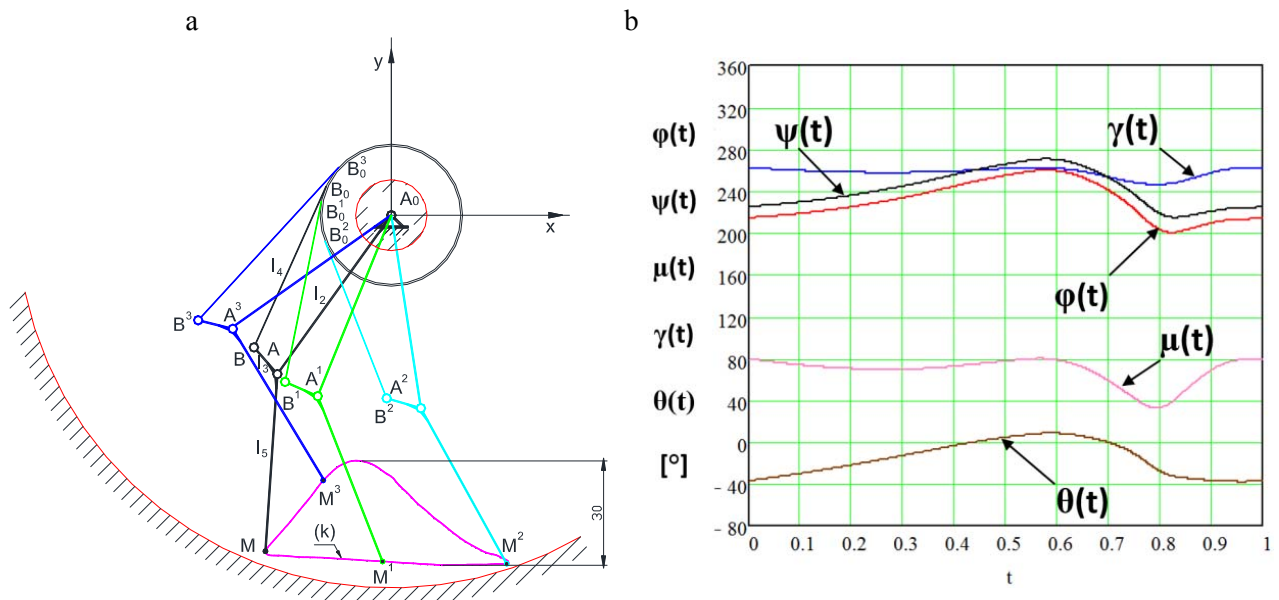

Fig.4. a) The 5-link belt mechanism generating a trajectory with $h = 30\ mm$; b) The drive angles $\varphi(t)$, $\theta(t)$, belt angles $\psi(t)$, wheel angle $\gamma(t)$ and transmission angles $\mu(t)$.
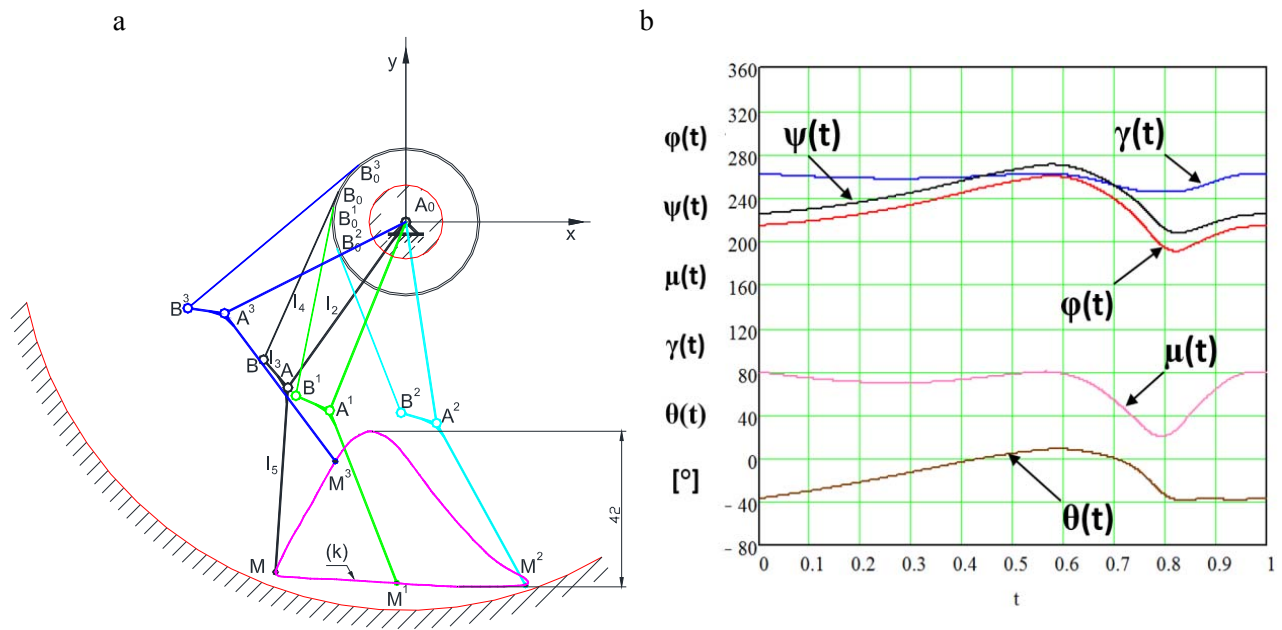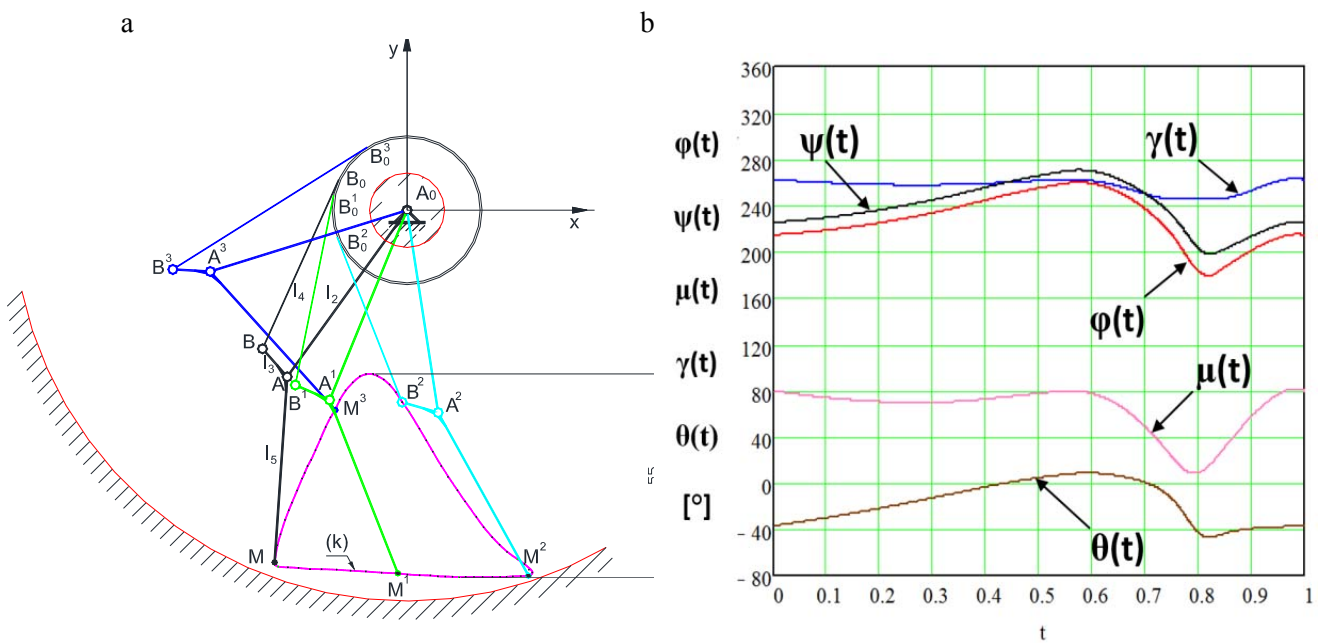
Fig.5. a) The 5 link belt mechanism generating a trajectory with *h = 42 mm*; b) The drive angles φ(*t*), θ(*t*), belt angles ψ(*t*), wheel angle γ(*t*) and transmission angles μ(*t*).

Fig.6. The 5 link belt mechanism generating a trajectory with *h = 55 mm*; b) The drive angles φ(*t*), θ(*t*), belt angles ψ(*t*), wheel angle γ(*t*) and transmission angles μ(*t*).

5. Results and conclusions

The proposed novel leg mechanism solution using a 5-link belt mechanism is viable because it can properly reproduce the Jansen characteristic point trajectory and the control functions for the driving

elements can be easily implemented. As shown in Figs 4, 5 and 6 the controlled rotation angles do not have jumps in their variation towards the parameter *t*.

Also the transmission angle is in a positive range of values, which indicates that singularities are avoided throughout the entire control range.

It can be stated that, the novel mechanism structure through the control of the actuators (driving elements) can be adapted to follow both the Jansen trajectory and similar trajectories with different step heights. The step height can be improved in order to avoid different obstacles.

Nomenclature

$A_i(\varphi)$, $B_i(\varphi)$, $C_i(\varphi)$ – coefficients of the transmission equation with $i=1, 2$ (Eqs 2 and 3)
A_0, B_0 – fixed revolute joints (Fig.1)
A_0AB_0B – closed loop of the first four bar linkage (Fig.1)
A_0AEB_0 – closed loop of the second four bar linkage (Fig.1)
a_{xi}, a_{yi} – second order coefficients
B_0CDE – closed loop of the five bar linkage (Fig.1)
DOF – degree of freedom
h – height of the step [*mm*] (Figs 4a, 5a and 6a)
K – path generating point of the 5-link belt mechanism (Fig.3)
l – total length of the belt [*mm*] (Fig.3)
l_2 – crank length [*mm*] (Fig.3)
l_3 – coupler length [*mm*] (Fig.3)
l_4 – developed link length [*mm*] (Fig.3)
l_5 – coupler length [*mm*] (Fig.3)
$l_2,l_3,l_4,l_{41},l_5,$
l_6,l_7,l_8,l_{81} – link lengths [*mm*] (Fig.1)
M – path generating point of the Jensen leg mechanism
m_{xi}, m_{yi} – slopes of the segment curves
r_5 – wheel radius [*mm*] (Fig.3)
t – normalized parameter with $t=[0, 1]$
x_{B0}, y_{B0} – coordinates for B_0 fixed joint [*mm*]
x_M, y_M – coordinates for path generating point M [*mm*] (Fig.1)
$x(t)$, $y(t)$ – coordinates of the path generating point K [*mm*] (Fig.3)
α – fixed coupler angle [rad] (Fig.3)
α_0, β_0 – fixed angles of the tetrahedral links [rad]
γ_0 – initial enveloped belt angle [rad] (Fig.3)
$\gamma(t)$ – wheel angle in the mobile reference system [rad] (Fig.3)
$\theta(t)$ – drive angle of the 5-link belt mechanism wheel [rad] (Fig.3)
$\mu(t)$ – transmission angle [rad] (Fig.3)
ϑ_1, ϑ_2, Ψ_1,
Ψ_2, δ, γ – position angles of the links of Jansen leg mechanism [rad] (Fig.1)
φ – input crank angle of Jansen leg mechanism [rad] (Fig.1)
$\varphi(t)$ – drive angle the 5-link belt mechanism crank [rad] (Fig.3)
$\Psi(t)$ – belt angle in the frame reference system [rad] (Fig.3)

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