Novel Statistical Approach to Blind Recovery of Earth Signal and Source Wavelet using Independent Component Analysis

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Abstract: - This paper provides a new statistical approach to blind recovery of both earth signal and source wavelet given only the seismic traces using independent component analysis (ICA) by explicitly exploiting the sparsity of both the reflectivity sequence and the mixing matrix. Our proposed blind seismic deconvolution algorithm consists of three steps. Firstly, a transformation method that maps the seismic trace convolution model into multiple inputs multiple output (MIMO) instantaneous ICA model using zero padding matrices has been proposed. As a result the nonzero elements of the sparse mixing matrix contain the source wavelet. Secondly, whitening the observed seismic trace by incorporating the zero padding matrixes is conducted as a pre-processing step to exploit the sparsity of the mixing matrix. Finally, a novel logistic function that matches the sparsity of reflectivity sequence distribution has been proposed and fitted into the information maximization algorithm to obtain the demixing matrix. Experimental simulations have been accomplished to verify the proposed algorithm performance over conventional ICA algorithms such as Fast ICA and JADE algorithm. The mean square error (MSE) of estimated wavelet and estimated reflectivity sequence shows the improvement of proposed algorithm

Key-Words: - blind deconvolution, seismic signal processing, sparse ICA, information maximization algorithm fast ICA algorithm, JADE algorithm, zero padding matrixes

1 Introduction

In seismic exploration, a seismic wavelet is sent to the earth layers and seismic trace is recorded by a geophone or hydrophone at the surface due to the impedance mismatches between different geological layers which are a great concern to the geophysicist. The geophysical structure of the earth can be explored through an analysis of the reflectivity from deep layers of the earth .The true reflectivity signal , however, is not easily reached; as an alternative, the recorded seismic trace is a smeared version of the reflectivity sequence, caused by the reverberations due to the surface layers [18]. One of the essential goals is to undo the effects of the degradation in order to recover the true earth signal [17]. This usually necessitates a certain deconvolution technique. The main aim of seismic deconvolution is to remove the characteristics of the source wavelet from the recorded seismic trace, so that one is perfectly left with only the reflectivity sequence (earth signal). The blind approaches of seismic deconvolution can be considered in situations where the reflectivity sequence and the source wavelet, are unknown from given seismic traces. In seismology, the recorded seismic trace $x(t_i)$ is defined to be the linear convolution of the source wavelet $h(t)$ with the earth's reflection coefficients $r(t)$. Assuming no noise, the mathematically representation of this relationship is given in (1).

$$x(t_i) = \sum_j h(t_{i-j}) r(t_j) \tag{1}$$

To precisely estimate both the earth signal and wavelet source, it is critical for the deconvolution algorithm to incorporate as much prior knowledge about the reflectivity sequence and the wavelet as possible. From geophysics point of view the earth layers are more or less homogenous and separated by interfaces. This prior knowledge allows us to statistically model the reflectivity sequence as a Bernoulli Gaussian process [15, 16]. Furthermore, the convolution process gives rise to a sparse mixing matrix which will be exploited to obtain an efficient ICA parameter estimation [4]. There are many methods of seismic deconvolution that can be accomplished so that an optimal estimate can be made of the earth model. A common of the seismic deconvolution methods utilize the steady state Wiener digital filter that assumes a minimum phase wavelet [3, 9]. In 1990 similar methods were developed by Weinstein and Shalvi [10]. Recently Bayesian statistic framework approaches [2, 8, 11, 12] have been applied for blind seismic deconvolution. They explicitly modelled the sparseness of reflectivity sequence as Bernoulli Gaussian process where the location, amplitude and number of spikes are considered. Also Kaplan and Ulrych [4] introduced banded ICA algorithm to solve blind seismic deconvolution by incorporating the banded property of mixing matrix into an ICA algorithm as prior information. Our novel technique which can be summarised in figure (1) presents a novel method to solve blind seismic deconvolution problem using independent component analysis by exploiting the sparsity of both the reflectivity sequence and the mixing matrix.

This paper is systematized as follows. In section 2, the transformation method that maps the seismic trace convolution model into multiple inputs multiple output (MIMO) instantaneous ICA model will be explained in detail. Section (3) presents the mathematical analysis of blind seismic deconvolution algorithm. The experimental simulations that illustrate the improvement of estimated wavelet and earth signal using proposed techniques over Fast ICA algorithm [7] and JADE algorithm [14] will be presented in Section 4.

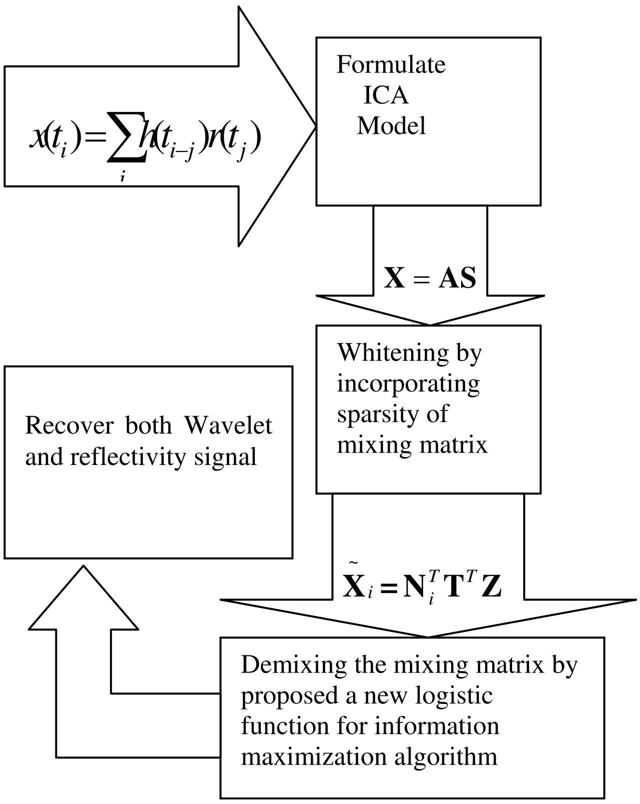

Fig. 1. Block diagram of the proposed algorithm

2 Proposed MIMO-ICA Model

Convolution model of discretely sampled seismic trace in equation (1) can be represented as ICA model using the zero padding matrices $\mathbf{N}_i$.

$$\mathbf{x} = \mathbf{A}\mathbf{s} \quad (2)$$

The $\mathbf{N}_i$ matrices plays an important role in constructing the $2l$ x $(3l-1)$ sparse mixing matrix $\mathbf{A}$ by mapping the wavelet vector $\mathbf{h}$ into the i^{th} row of mixing matrix as shown in equation (3) ,where l is the number of wavelet points.

$$\mathbf{A}=\begin{bmatrix}\mathbf{a}_0\\ \mathbf{a}_1\\ \vdots\\ \mathbf{a}_{2l-1}\end{bmatrix}=\begin{bmatrix}[\mathbf{hN}_0]\\ [\mathbf{hN}_1]\\ \vdots\\ [\mathbf{hN}_{2l-1}]\end{bmatrix}=\begin{bmatrix}h(t_1) & \cdots & h(t_l) & 0 & 0 & 0\\ 0 & h(t_1) & \cdots & h(t_l) & 0 & 0\\ 0 & 0 & \ddots & & \ddots & 0\\ 0 & 0 & 0 & h(t_1) & \cdots & h(t_l)\end{bmatrix} \quad (3)$$

The zero padding matrices with dimension l x $(3l-1)$ can be represented as

$$\mathbf{N}_0=\begin{bmatrix}1&0&0&0&\cdots&0\\0&1&0&0&\cdots&0\\0&0&1&0&\cdots&0\end{bmatrix}\cdots\mathbf{N}_{2l-1}=\begin{bmatrix}0&\cdots&0&1&0&0\\0&\cdots&0&0&1&0\\0&\cdots&0&0&0&1\end{bmatrix} \quad (4)$$

As a result, the rows of matrix $\mathbf{A}$ contain the delayed versions of the same wavelet vector as shown in fig (2).

Where $\mathbf{s}$ and $\mathbf{x}$ are the reflectivity sequence vector and the seismic trace vector respectively. n is the number of reflectivity sequence samples

$$\mathbf{s} = \begin{bmatrix} r(t_1) & r(t_2) & r(t_3)\cdots\cdots\cdots & r(t_n)\end{bmatrix} \quad (5)$$

$$\mathbf{x} = \begin{bmatrix} x(t_1) & x(t_2) & x(t_3)\cdots\cdots\cdots & x(t_n)\end{bmatrix} \quad (6)$$

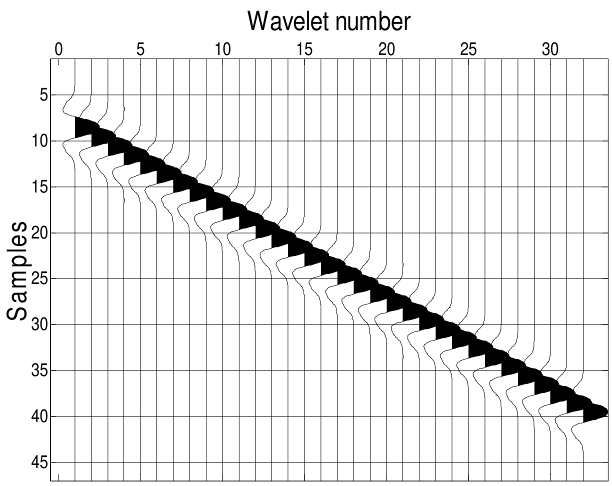

Figure 2. Delayed versions of the seismic wavelet that the mixing matrix contained

In other words, the single input single output (SISO) convolution model in equation (1) is transformed to (SISO) instantaneous ICA model (2).

The ICA model in equation (2) provides only one realization of each of the reflectivity sequences and seismic wavelet. This is insufficient to characterize the corresponding statistics and hence it is inadequate for ICA. However, the available information can be rearranged such that organizing the reflectivity sequence vector of $\mathbf{s}$ as a matrix $\mathbf{S}$ with dimension $(3l-1)\,\mathrm{x}\,n$, as shown in figure (3) ,where the first row contains all the values of the reflectivity vector and the second row contains the delayed version of the same reflectivity vector by the delay operator (z) and so on until we reach to the delayed version by $(3l-1)$ [4]

$$\mathbf{S}=\begin{bmatrix}\mathbf{s}\\ z\mathbf{s}\\ \vdots\\ z^{n-3l}\mathbf{s}\\ z^{n-3l-1}\mathbf{s}\end{bmatrix}=\begin{bmatrix} r(t_1) & r(t_2) & \cdots & \cdots & \cdots & \cdots & \cdots & r(t_n)\\ 0 & r(t_1) & r(t_2) & \cdots & \cdots & \cdots & \cdots & r(t_{n-1})\\ 0 & \ddots & \ddots & \ddots & \ddots & \cdots & \cdots & \vdots\\ \vdots & \ddots & 0 & r(t_1) & r(t_2) & \cdots & \cdots & r(t_{n-3l})\\ 0 & 0 & 0 & 0 & r(t_1) & r(t_2) & \cdots & r(t_{n-3l-1})\end{bmatrix} \tag{7}$$

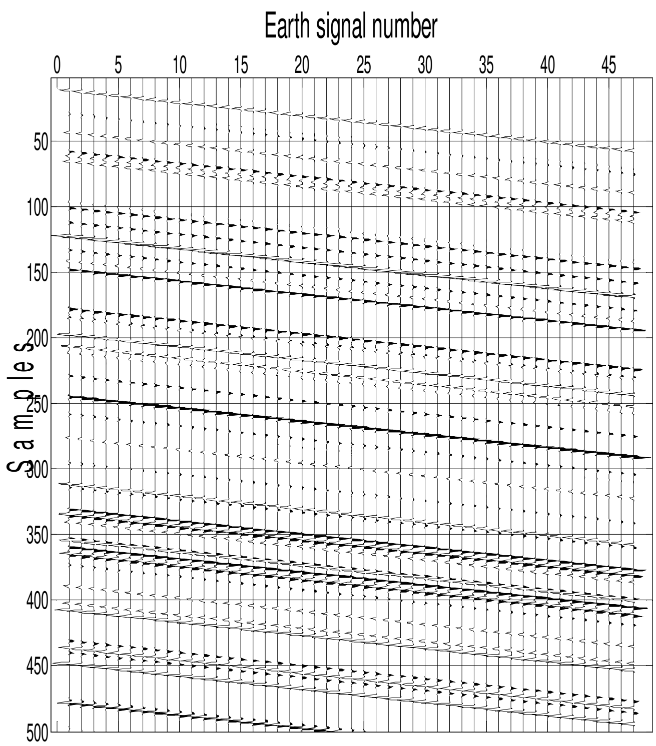

Figure 3. Shifted versions of the earth signal that the source matrix contained

From (3) and (7) the seismic trace vector can be rewritten as a matrix $\mathbf{X}$ with dimension $2l\,\mathrm{x}\,n$ so it can be shown from the figure (4) that the observed seismic trace matrix contains shifted versions of the same seismic trace vector.

$$\mathbf{X}=\begin{bmatrix}\mathbf{x}\\ z\mathbf{x}\\ \vdots\\ z^{n-2l+1}\mathbf{x}\\ z^{n-2l}\mathbf{x}\end{bmatrix}=\begin{bmatrix} x(t_1) & x(t_2) & \cdots & \cdots & \cdots & \cdots & \cdots & x(t_n)\\ 0 & x(t_1) & x(t_2) & \cdots & \cdots & \cdots & \cdots & x(t_{n-1})\\ 0 & \ddots & \ddots & \ddots & \ddots & \cdots & \cdots & \vdots\\ \vdots & \ddots & 0 & x(t_1) & x(t_2) & \cdots & \cdots & x(t_{n-2l+1})\\ 0 & 0 & 0 & 0 & x(t_1) & x(t_2) & \cdots & x(t_{n-2l})\end{bmatrix} \tag{8}$$

In other words the single input single output convolution model (SISO) in equation (1) can be represented as multiple input multiple output (MIMO) instantaneous mixing model

$$\mathbf{X}=\mathbf{AS} \tag{9}$$

Consequently the blind seismic deconvolution problem of single channel is presented in a way that it can be solved using instantaneous ICA; so that the time delayed arrival of the captured signal at the geophones can be handled more efficiently.

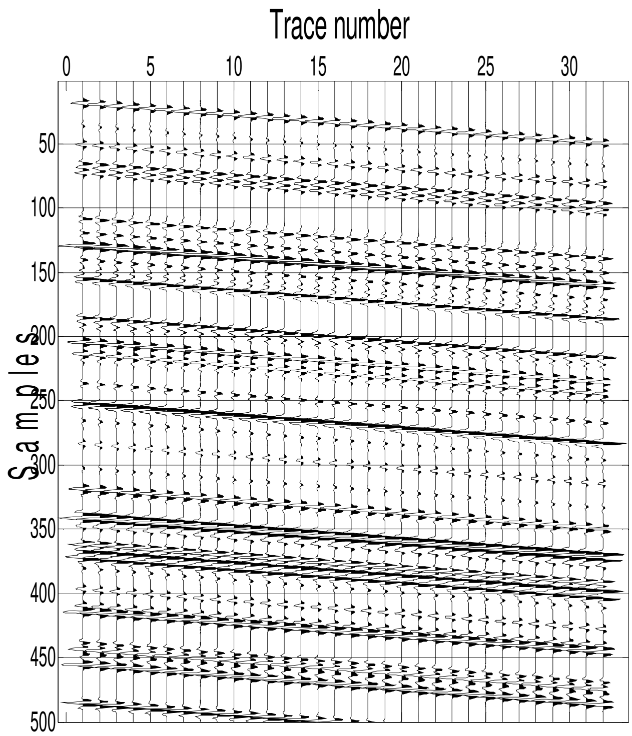

Figure 4. Shifted versions of the seismic trace that the observed seismic trace matrix contained

3 Proposed Parameter Estimation Algorithm

The proposed algorithm consists of two main steps. The first step is the preprocessing step. The Second step is the application of the information maximization algorithm to whitened seismic trace.

3.1 Preprocessing step

Whitening the observed seismic trace $\mathbf{X}$ using Eigen-value decomposition (EVD) of the covariance matrix $\mathbf{R}_{xx}$ by incorporating the zero padding matrixes $\mathbf{N}_i$ is proposed as a pre-processing strategy to exploit the sparsity of banded matrix $\mathbf{A}$ before applying the information maximization algorithm. a new observed seismic trace $\tilde{\mathbf{X}}$ can be obtained which is white, this means that its components are uncorrelated and their variances equal unity. In other words, the covariance matrix $\mathbf{R}_{xx}$ equals the identity matrix $\mathbf{I}$ [13].

$$\mathbf{R}_{xx} = \mathbf{X}\mathbf{X}^T / n = \mathbf{I} \tag{10}$$

From [4] the whitened seismic mixture matrix can be written as

$$\tilde{\mathbf{X}}_i = \mathbf{N}_i^T \mathbf{T}^T \mathbf{Z} \tag{11}$$

Where $\mathbf{T} = \mathbf{D}^{-1/2}\mathbf{E}^T$ and $\mathbf{Z} = \mathbf{TX}$

$$\text{Hence, } \tilde{\mathbf{X}}_i = \mathbf{N}_i^T [\mathbf{D}^{-1/2}\mathbf{E}^T]^T \mathbf{D}^{-1/2}\mathbf{E}^T \mathbf{X} \tag{12}$$

Where $\mathbf{E}$ and $\mathbf{D} = diag[d_1,..........,d_n]$ are the eigenvectors and eigenvalues respectively of covariance matrix $\mathbf{R}_{xx}$,so the whitened mixture can be considered as a new set of seismic mixtures with l x n dimension matrix also it can be seen from (12) that the zero padding matrix $\mathbf{N}_i$ enforces the sparse property of the mixing matrix during the whiting pre-process step. In other words $\mathbf{N}_i$ is prior information. As shown in figure (5) incorporating the zero padding matrices in the preprocessing step will result in reducing the dimension of the whitening seismic trace to l x n .

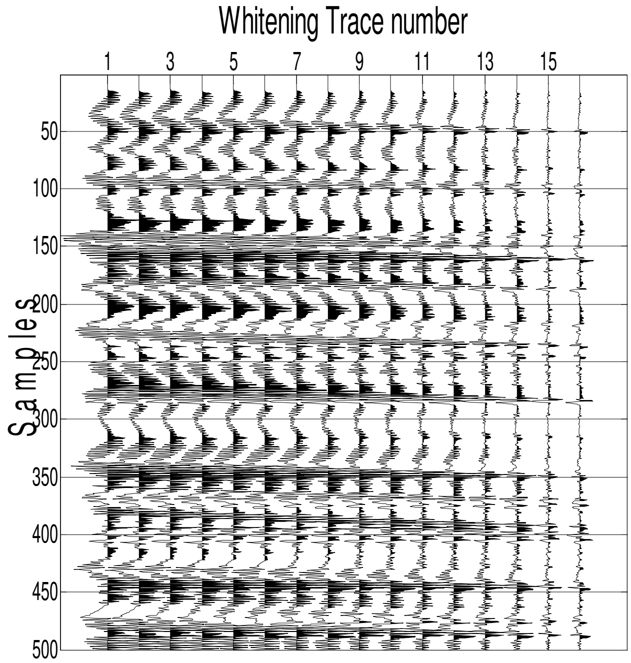

Figure 5. Whitened seismic trace

3.2 Information maximization algorithm

Applying the information maximization algorithm [1] to the whitened mixture $\tilde{\mathbf{X}}$ will result in demixing matrix $\mathbf{W}$. This algorithm which is modified by Amari [5] using a natural gradient method to avoid matrix inversions during ICA training, does not assume any knowledge of input distribution. However, in our case it is well known that the distribution of input reflectivity sequence can be modelled as Bernoulli Gaussian distribution [2] as follows

$$p(u_i) = p_i\delta(u_i) + \frac{1-p_i}{\sqrt{2\pi\sigma_2^2}} e^{-\left(\frac{u_i^2}{2\sigma_2^2}\right)} \tag{13}$$

In which p_i is the probability that reflection occurs, if p_i=0 it indicates the position of high reflector and small reflector position is given by p_i=1. Where both p_i and the variance σ^2 can be estimated using maximum likelihood approach.

$$\hat{\theta} = \arg\max_{\theta} \ln p(\mathbf{X};\theta) \text{ ,where } \theta = (p_i, \sigma^2) \tag{14}$$

Incorporating the prior information in (13) into information maximization algorithm [1] will result in a new blind seismic deconvolution algorithm.

When the whitened seismic mixture is to be passed through a logistic function, maximum information transmission can be achieved when the sloping part of the logistic is optimally lined up with the high density part of the input distribution.

A new logistic function is proposed to match the sparsity of the input signal. This function can be modelled as the integral of the input distribution.

$$\mathbf{Y} = g(\mathbf{U}) = \int p(\mathbf{U})du \quad (15)$$

$$\mathbf{U}_i = \mathbf{W}\tilde{\mathbf{X}}_i + \mathbf{W}_0 \quad (16)$$

Where the whitened seismic trace $\tilde{\mathbf{X}}_i$ is multiplied by a weight matrix $\mathbf{W}$ and added to a bias weight $\mathbf{W}_0$, the above evaluate as

$$\mathbf{Y}_i = p_i \int \delta(\mathbf{U}_i)du + \frac{1-p_i}{\sqrt{2\pi\sigma_2^2}} \int e^{-\left(\frac{\mathbf{U}_i^2}{2\sigma_2^2}\right)} du \quad (17)$$

From integration table [6],It can be shown that

$$\int e^{ax^2} dx = \frac{-j\sqrt{\pi}}{2\sqrt{a}} erf(jx\sqrt{a})$$

$$\int \delta(x)dx = H(x) \approx 0.5 + 0.5\tanh(x)$$

$$\mathbf{Y}_i = p_i H(\mathbf{U}_i) + \frac{1-p_i}{\sqrt{2\pi\sigma_2^2}} \times \left[\frac{-j\sqrt{\pi}}{2\sqrt{\frac{-1}{2\sigma_2^2}}} erf(j\mathbf{U}_i\sqrt{\frac{-1}{2\sigma_2^2}}) \right] \quad (18)$$

Where $j = \sqrt{-1}$ and $erf(-\mathbf{X}) = -erf(\mathbf{X})$

$$\mathbf{Y}_i = p_i H(\mathbf{U}_i) + \frac{1-p_i}{2\sqrt{\pi}} erf(\frac{\mathbf{U}_i}{\sqrt{2\sigma_2^2}}) \quad (19)$$

$$\mathbf{Y}_i = p_i(0.5 + 0.5\tanh(\mathbf{U}_i)) + \frac{1-p_i}{2\sqrt{\pi}} erf(\frac{\mathbf{U}_i}{\sqrt{2\sigma_2^2}}) \quad (20)$$

Fitting in the proposed logistic function $\mathbf{Y}$ into Information maximization algorithm [5] will result in a new blind seismic deconvolution algorithm. So a weight matrix can be proposed as

$$\mathbf{W}_i = \mathbf{W}_i + (\mathbf{I} - \mathbf{Y}_i\mathbf{U}_i - \mathbf{U}_i\mathbf{U}_i^T)\mathbf{W}_i \quad (21)$$

a bias vector can be written as $\Delta\mathbf{w}_0 = \mathbf{1} - 2\mathbf{y}$ .Given only the recorded seismic trace, the proposed blind deconvolution algorithm produces the demixing matrix $\mathbf{W}$ which contains l rows, each of them represent an estimated wavelet as shown in figure (6), in other words l wavelets are recovered , $\tilde{\mathbf{h}}_1$ .... $\tilde{\mathbf{h}}_l$.

The best estimated wavelet is extracted from the pool of l candidate solution according to the following criteria.

$$\psi(c_i) = \left\| \mathbf{x}_n - c_i \mathbf{h}_{\tilde{i}} * \mathbf{d}_{\tilde{i}} \right\|_2^2 \quad (22)$$

Where $\mathbf{d}_i = \mathbf{W}\tilde{\mathbf{x}}_i$ is the realization of i^{th} independent component, $i = 1,\dots\dots,l$. It is easily verified that equation (22) has it is extreme points when

$$c_i = c_i^{(*)} = \frac{\mathbf{x}_n^T (\mathbf{h}_{\tilde{i}} * \mathbf{d}_{\tilde{i}})}{(\mathbf{h}_{\tilde{i}} * \mathbf{d}_{\tilde{i}})^T (\mathbf{h}_{\tilde{i}} * \mathbf{d}_{\tilde{i}})} \quad (23)$$

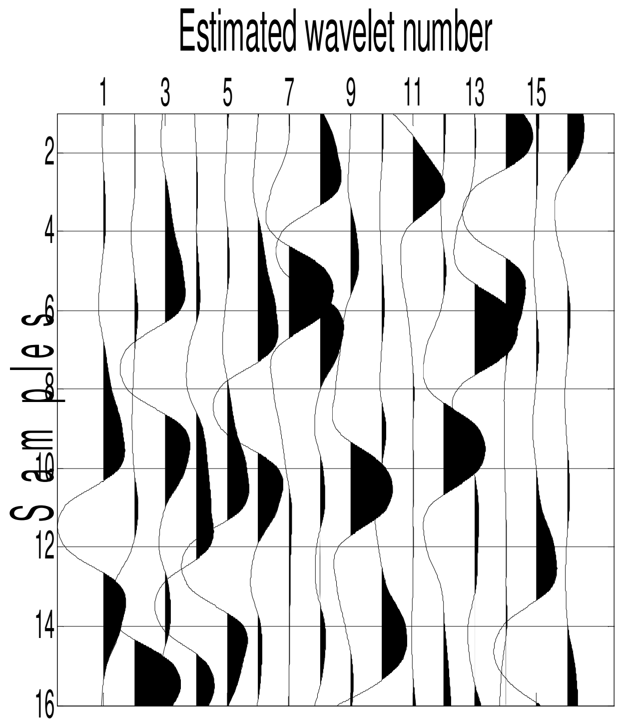

Figure 6. The estimated wavelets that the demixing matrix contained

4 Simulation and results analysis

Consider the seismic trace signal in figure (15), which can be generated by convolving 16 points of seismic wavelet with the 500 points of reflectivity sequence, as input to our proposed algorithm without any knowledge of both seismic wavelet and reflectivity sequence except that the earth signal distribution can be modelled as Bernoulli Gaussian process. The proposed algorithm is successfully able to recover both the earth signal (reflectivity sequence) and the seismic wavelet. This ability of the recovering seismic wavelet can be confirmed by comparing the estimated seismic wavelet with original source wavelet in figure (7) where it can be noticed that the 16 samples of the wavelet have been recovered accurately. Comparing this with the Fast ICA algorithm result in figure (8) and with JADE algorithm result in figure (9), the proposed algorithm yields an improvement by 88.5% over Fast ICA algorithm and 93% over JADE algorithm in terms of accuracy, shaping and scaling of estimated seismic wavelet. This means that our novel methodology solves scaling problems can be found in most ICA algorithms. From figure (11) it can be seen that four tests have been conducted and the results prove that the minimum square error of the recovered wavelet using the proposed algorithm has been minimised compared with Fast ICA and JADE algorithm, by exploiting the sparsity of both mixing matrix and reflectivity sequence. It is worthy to know that the threshold of MSE for wavelet estimation is 0.01 and any values above this threshold are considered poor. The enhanced resolution of recovered earth signal can be clearly seen in figure (12), which presents the comparison between the estimated and original reflectivity sequence (earth signal) using our proposed technique where it can been seen that the earth signal has been accurately recovered and matches the scaling of the original earth signal. By comparing the results in figure (12), (13) and (14),Our proposed algorithm shows an increased performance of estimating earth signal by 73% over the Fast ICA and up to 86.5% over JADE algorithm in terms of mean square error ,scaling and shaping. Results from figure (10) prove that the minimum square error of estimated reflectivity sequence by proposed technique has been statistically minimised compared to the FastICA algorithm and JADE algorithm.

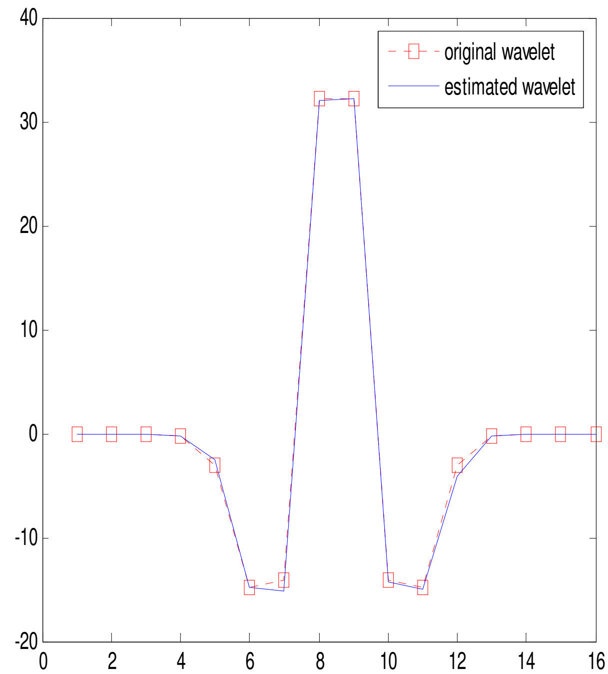

Figure.7 original and estimated wavelet using proposed

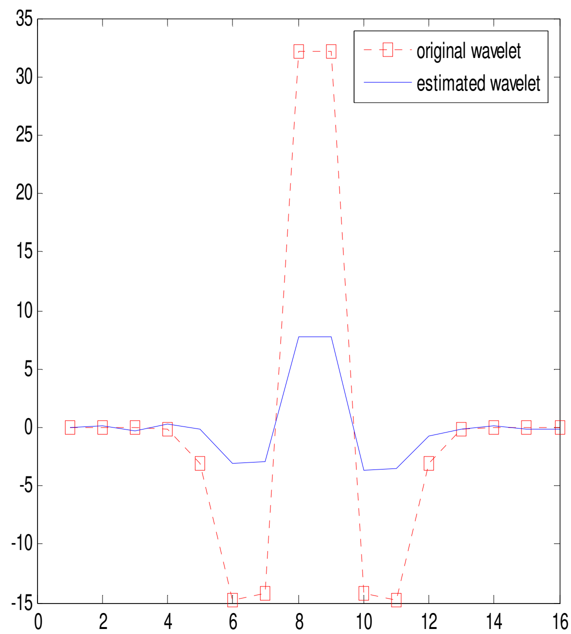

Figure.8 original and estimated wavelet using Fast ICA algorithm

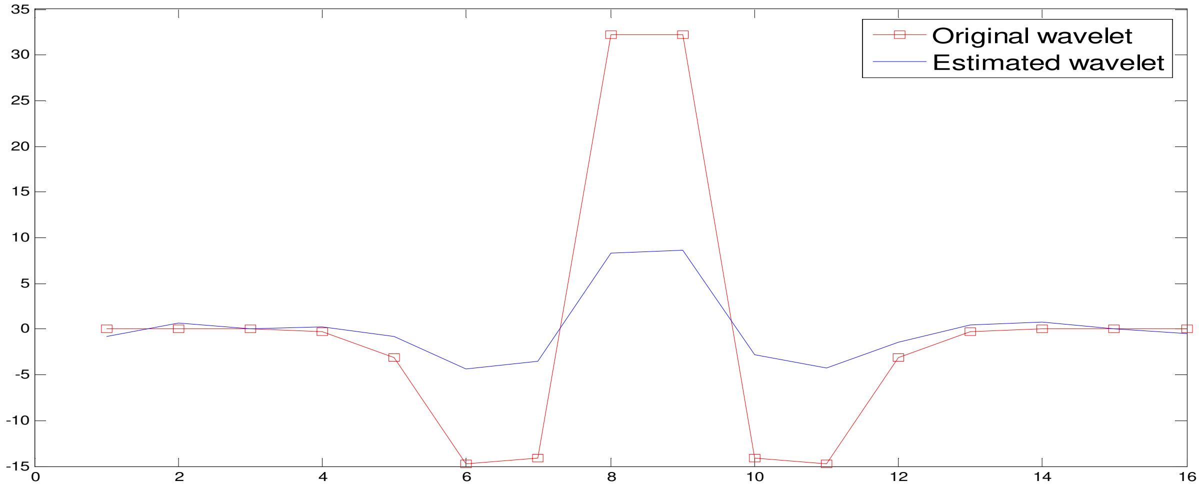

Figure. 9 Estimated wavelet using JADE algorithm

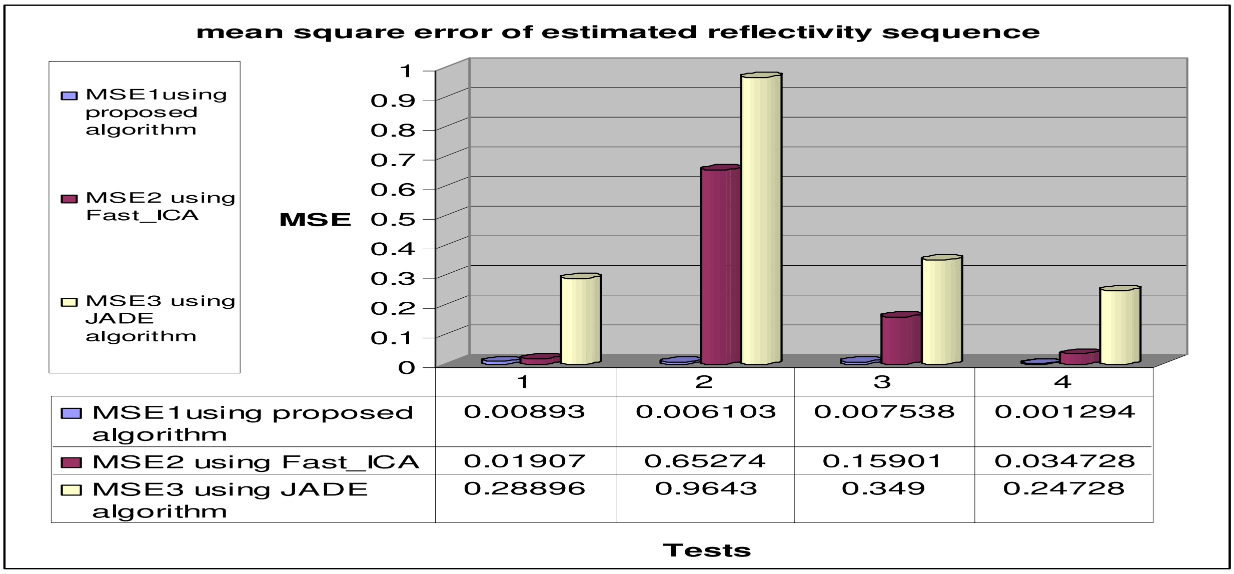

Figure. 10 MSE of estimated reflectivity sequence

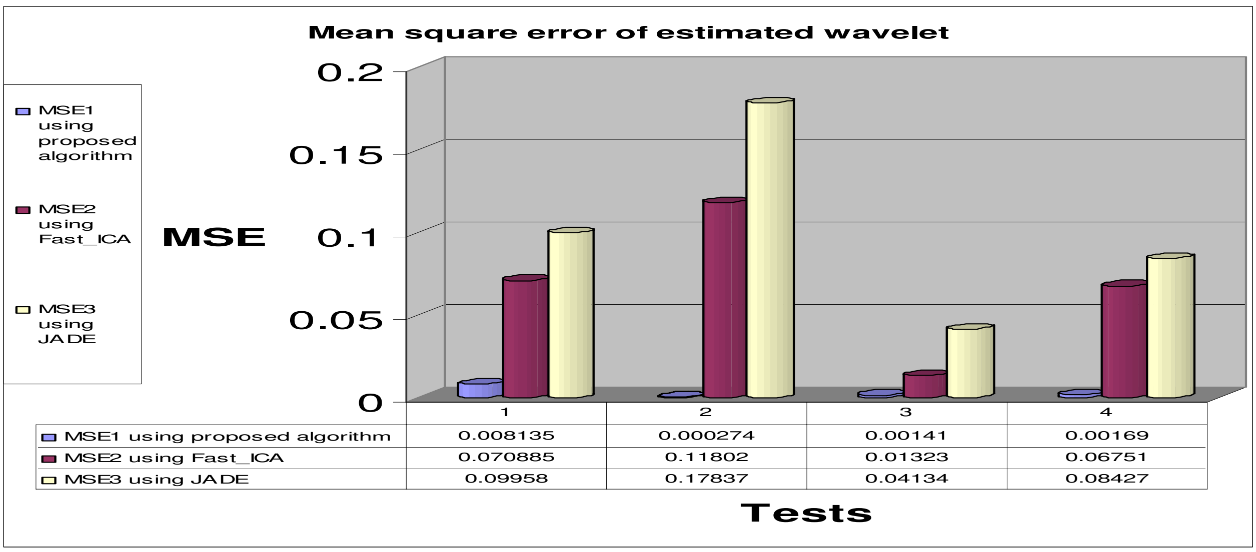

Figure. 11 MSE of estimated wavelet

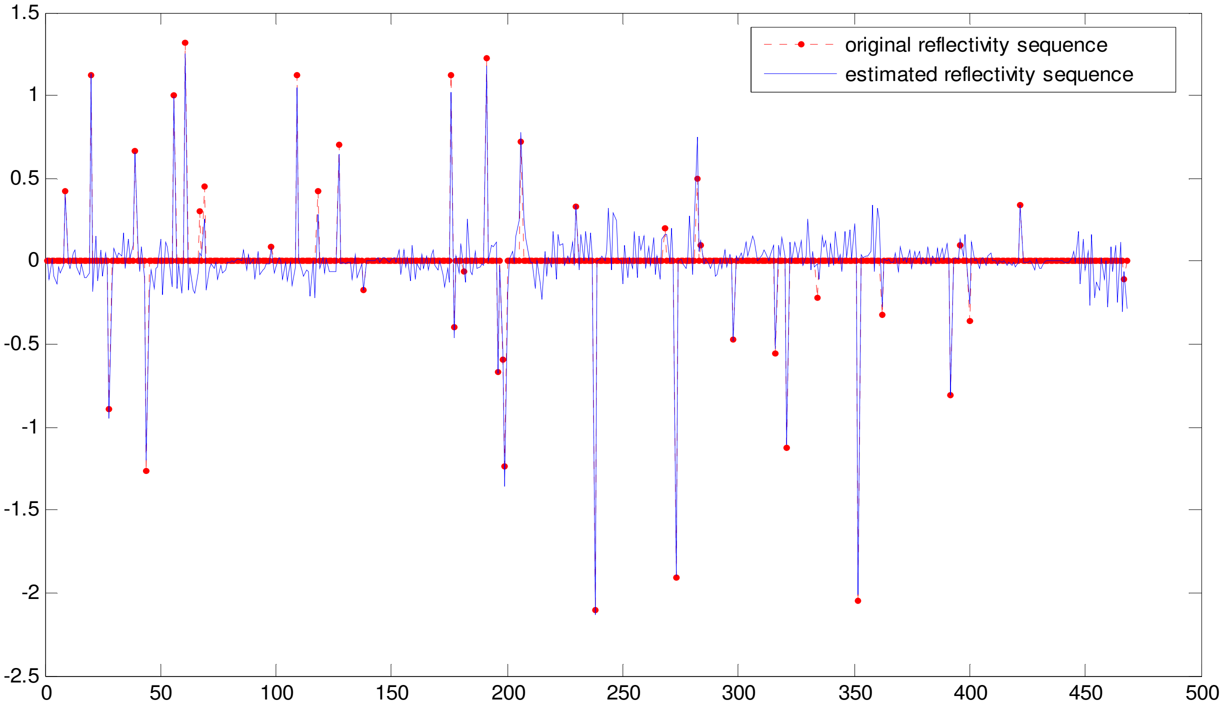

Figure. 12 Original and recovered earth signal using new algorithm

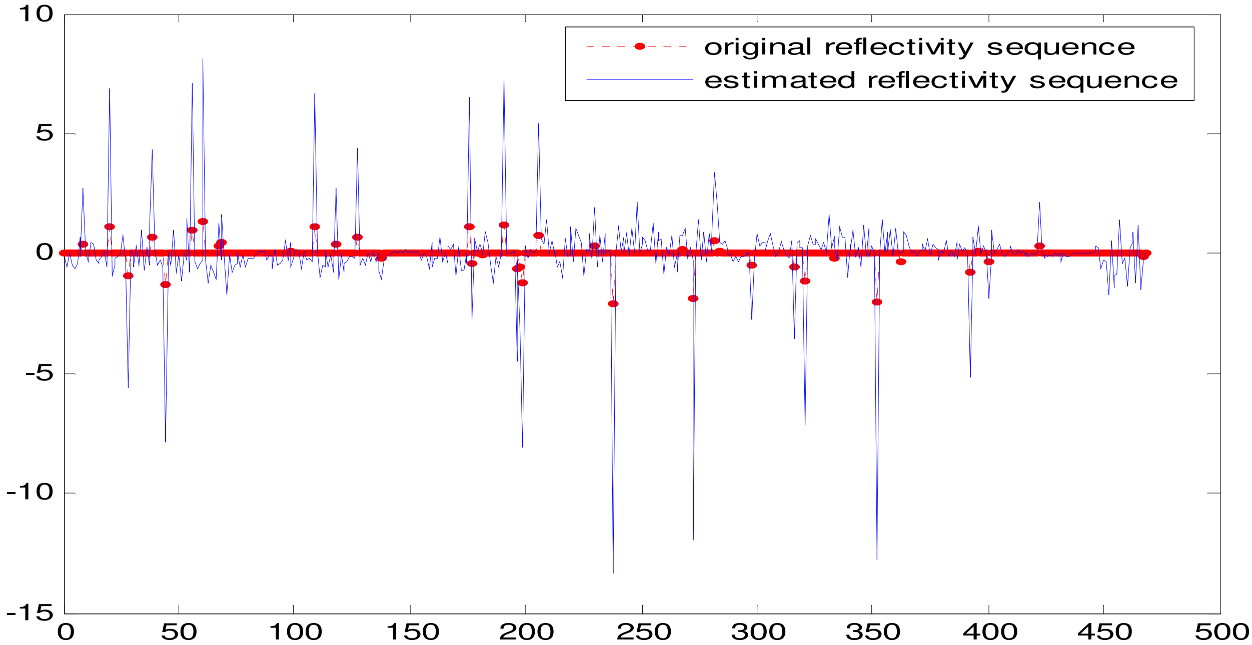

Figure. 13 original and recovered earth signal using FastICA

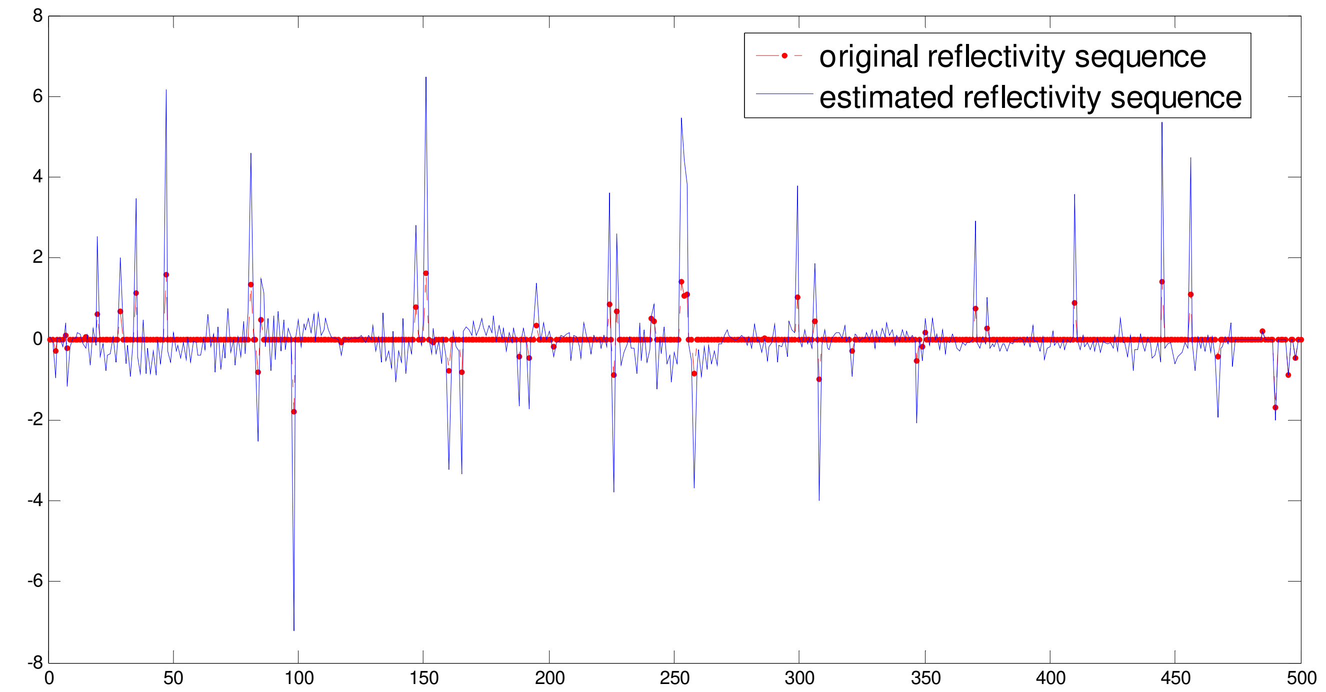

Figure.14 estimated earth signal using JADE

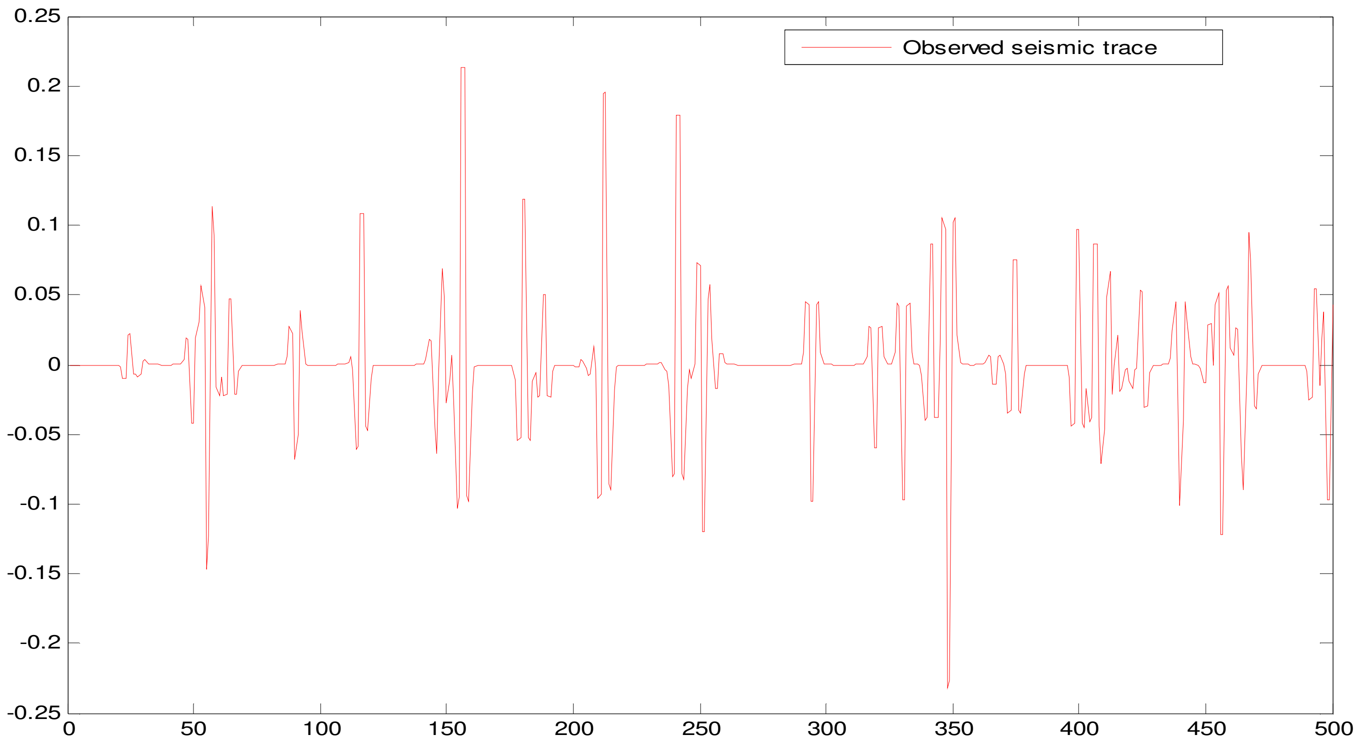

Figure. 15 synthesised seismic trace

5 Conclusion

A new technique for blind deconvolution of seismic signal has been proposed and developed. Simulations results of the blind estimation of the source wavelet and earth signal given only by the seismic trace signal as input has expressed the effectiveness of the new algorithm over the FastICA algorithm. The technique differs from many blind deconvolution algorithms as it uses independent component analysis to solve blind deconvolution problem by exploiting the sparsity of both the reflectivity sequence and the mixing matrix. Although it is computationally intensive our novel algorithm gives significant performance efficiency over FastICA in terms of shape and scaling. As a result the proposed technique has the ability to be used as a post stack improvement process to offer datasets for use in combination with the typically processed reflectivity data.

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References:

[1] A.J Bell, and T.J. Sejnowski, “An information maximization approach to blind separation and blind deconvolution,” *Neural Computation*, vol. 7, pp. 1129–1159, 1995

[2] O. Rosec, J.M.Boucher, B.Nsiri, Th.Chonavel, “Blind marine seismic deconvolution using statistical MCMC methods*, IEEE Oceanic Engineering*, vol. 28, pp. 502-512, 2003

[3] R. A. Wiggins, “Minimum entropy deconvolution,” *Geoexploration*, vol. 16, pp. 21-35, 1978.

[4] S.T. Kaplan, and T.J. Ulrych, “Blind deconvolution and ICA with a banded mixing matrix,”. *ICA2003, 4th Int. Symp. On ICA and Blind Signal Separation,* Nara, Japan, pp. 223-228.

[5] S. Amari, A. Cichocki, and H.H. Yang. “A new learning algorithm for blind signal separation.” In G. Tesauro, D.S. Touretzky, and T.K. Leen, editors, *Advances in Neural Information Processing Systems* 8, pp. 757-763, Cambridge, MA, 1996.

[6] E.Kreyszig, *Advanced engineering mathematics*. John Wiley & Sons, 1999.

[7] A. Hyvarinen, “Fast and robust fixed-point algorithms for independent component analysis,” *IEEE Transactions on Neural Networks*, vol. 10, no. 3, pp. 626–634, May 1999.

[8] Kjetil F. Kaaresen and Tofinn Taxt, “Multichannel blind deconvolution of seismic signals,” *Geophysics*, vol. 63, no. 6. pp. 2093–2107, 1998.

[9] David Donoh , “On minimum entropy deconvolution ,” in applied Time Series analysis II,D. Findley,Ed. Academic Press Inc., 1981

[10] Ofir Shalvi , Ehud Weinstein . “New criteria for blind deconvolution of nonminimum phase systems (channels),” IEEE Transactions on information theory ,vol.36, no. 2,March 1990

[11] Baziw, E. and Ulrych, T.J. Principle Phase Decomposition - A New Concept in Blind Seismic Deconvolution. IEEE Transactions on Geoscience and Remote Sensing (TGRS), vol. 44, No. 8, pp. 2271-2281, Aug. 2006

[12] Baziw, E. (2007). Implementation of the Principle Phase Decomposition Algorithm. IEEE Transactions on Geoscience and Remote Sensing (TGRS), vol. 45, No. 6, pp. 1775-1785, June. 2007

[13] Aapo Hyvärinen and Erkki Oja, “Independent Component Analysis: Algorithms and Applications”. *In Neural Networks* vol 13, no (4-5,) pp. 411-430, 2000

[14] J.F. Cardoso and A. Souloumiac. Blind beamforming for non-Gaussian signals. *IEE Proc. F.*, 140(6):771-774, December 1993

[15] T.4. Yu, P**.** C. Muller, *G.-Z.* Dai, and C.-F. Lin, “Minimum-variance deconvolution for noncausal wavelets,” *IEEE Trans. Geosci. Remote Sensing,* vol. 32, pp. 513-524, 1994.

[16] J. M. Mendel, *Optimal Seismic Deconvolution: An Estimation-Based Approach.* New York Academic, 1983.

[17] Cheng, Q., Chen, R., and Li, T.-H.. “Simultaneous wavelet estimation and deconvolution of reflection seismic signals*: IEEE Trans. Geosci. Remote Sensing,* ***34****, No. 2, 377–384. 1996*

[18] *E. A. Robinson and S. Treitel* “Geophysical Signal Analysis*”.Practice-Hall, 1980*