NOVEL *ZEROTH-ORDER RESONANCE* IN COMPOSITE RIGHT/LEFT-HANDED TRANSMISSION LINE RESONATORS

ATSUSHI SANADA, CHRISTOPHE CALOZ, and TATSUO ITOH
Electrical Engineering Department, University of California, Los Angeles
405 Hilgard Ave., Los Angeles, CA90095-1594, USA
E-mail: sanada@ieee.org, caloz@ee.ucla.edu, itoh@ee.ucla.edu

A novel resonator using a composite right/left-handed (CRLH) transmission line (TL) is presented and its extraordinary resonant characteristics with an infinite-wavelength wave are discussed. The resonance corresponds to *the zeroth-order* on the analogy of the conventional mode numbering. The zeroth-order resonant characteristics and loss mechanism are fully described by the circuit theory and it is shown that the resonant frequency of the zeroth-order resonator (ZOR) is independent of *the physical length* so that the resonator can be arbitrarily small under the condition that sufficient reactance is squeezed in a small length. Full-wave simulations and experiments for meta-structured ZORs implemented with the microstrip line technology agree well with the theoretical results and hence the validity of the theory is confirmed. A 61% size reduction from the conventional resonator with a good unloaded *Q* of 250 is obtained for a prototype ZOR made of 1.5-cell CRLH TL at 1.9GHz in the experiment.

1. Introduction

The composite right/left-handed (CRLH) transmission line (TL) [1] is a meta-structured TL [1-3] composed of unit cells with a series capacitance and a shunt inductance as well as a series inductance and a shunt capacitance. The series capacitance and the shunt inductance provide the left-handed (LH) nature[a] as the dominant mode, whereas the series inductance and the shunt capacitance provide the right-handed (RH) nature at higher frequencies. The CRLH TL can well describe the nature of actually implemented structures which have inevitable parasitic series inductance and shunt capacitance, in contrast to the idealized LH TLs with only a series capacitance and a shunt inductance in the unit cell. More importantly, the CRLH TL has a unique feature of supporting *an infinite-wavelength wave* at a finite and non-zero frequency.

In this paper, a novel type of resonator using *the infinite-wavelength wave* in a CRLH TL is demonstrated. The resonator is referred to as *a zeroth-order resonator* (ZOR) based on the analogy of the conventional TL resonant mode numbering. The zeroth-order resonant characteristics are fully described based on the dispersion relation of the CRLH TL obtained by the Bloch & Floquet theory, and independence of the resonant frequency with the physical length is discussed. In addition, the loss mechanism in the zeroth-order resonant state is discussed while the unloaded *Q* of the ZOR is theoretically obtained. Full-wave simulations and experiments of ZORs implemented with the microstrip line technology are carried out and their resonant characteristics are compared with the theory.

[a] The backward (LH) wave is a wave that has antiparallel phase and group velocities. In contrast, an ordinary wave with parallel phase and group velocities is referred to as the right-handed (RH) wave.

2. Theory

Let us consider an open- or short-ended resonator with an ideal CRLH TL [1] with a physical length of *l*. The resonance occurs when the condition

$$\beta_n = \frac{n\pi}{l} \quad (n = 0, \pm 1, \pm 2, \ldots) \qquad (1)$$

is satisfied, where β_n is the phase constant of the CLRH TL for the resonant mode *n* as shown in Fig. 1. *n* can take negative values (in this case,

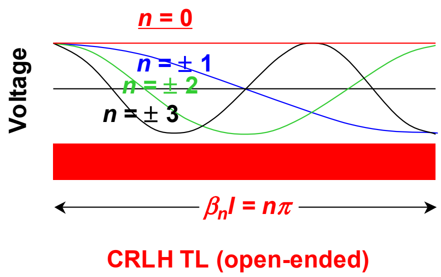
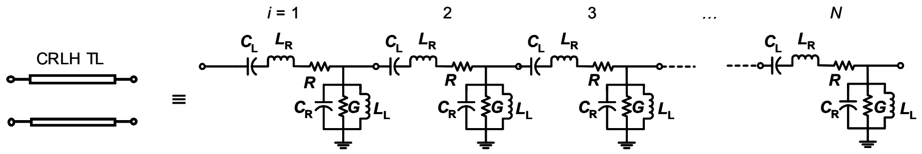
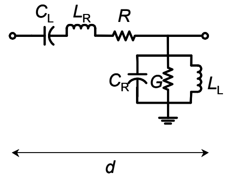


Fig. 1 Resonant modes in the CRLH TL resonator.



(a) CRLH TL.



(b) Unit cell.

Fig. 2 Equivalent circuit.

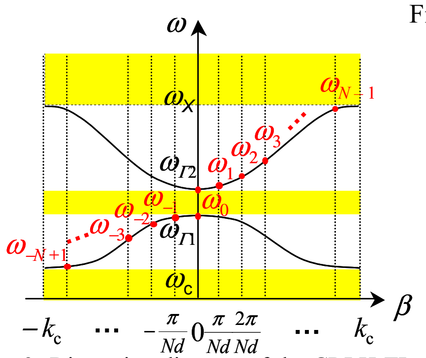
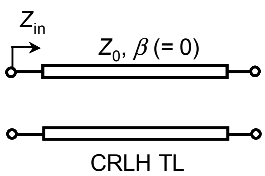
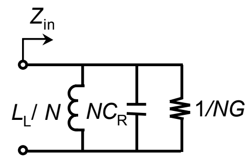


Fig. 3 Dispersion diagram of the CRLH TL and the resonant frequency of the ZOR.



(a) ZOR of CRLH TL.



(b) Equivalent circuit.

Fig.4 ZOR in the resonance state.

the wavelength $\lambda_g = 2\pi/|\beta_n|$) for the LH modes. The key part in our resonator is that n can even be *zero* with the infinite-wavelength wave $\lambda_g = \infty$ (i.e., $\beta_0 = 0$) with the CRLH TL.

Now, let us introduce a realizable CRLH TL model synthesized by a series of unit cells, the equivalent circuit of which is shown in Fig. 2. The resonant frequencies can be found as solutions of the dispersion relation of the CRLH TL obtained by applying the Bloch-Floquet theory to the unit cell in Fig. 2 (b) as

$$\beta_n d = \frac{n\pi d}{l} = \frac{n\pi}{N}$$
$$= \cos^{-1}\left\{1 - \frac{1}{2}\left[\frac{\omega_L^2}{\omega_n^2} + \frac{\omega_n^2}{\omega_R^2} - \left(\frac{\omega_L^2}{\omega_{se}^2} + \frac{\omega_L^2}{\omega_{sh}^2}\right)\right]\right\},$$
$$(n = 0, \pm 1, \pm 2, \ldots, \pm N-1), \quad (2)$$

where $\omega_L = \frac{1}{\sqrt{L_L C_L}}$, $\omega_R = \frac{1}{\sqrt{L_R C_R}}$, $\omega_{se} = \frac{1}{\sqrt{L_R C_L}}$, $\omega_{sh} = \frac{1}{\sqrt{L_L C_R}}$, and d is a length of the unit cell, N (= l/d) is the number of the unit cells in the resonator. The CRLH TL is assumed to be lossless (R = 0, G = 0). The solutions of (2) are depicted in Fig. 3. The resonant frequencies are distributed on the dispersion curves (the solid lines in Fig. 3) with a constant spacing of π/N along the β axis as shown with dots in Fig. 3.

In the zeroth-order state ($\beta \to 0$) of an open-ended ZOR, the input impedance Z_{in} from one of the open-ends toward the other end (see Fig. 4) is given as

$$Z_{in} = -jZ_0 \cot \beta l \overset{\beta \to 0}{=} -jZ_0 \frac{1}{\beta l}$$
$$= -j\sqrt{\frac{Z'}{Y'}}\left(\frac{1}{-j\sqrt{Z'Y'}}\right)\frac{1}{l} = \frac{1}{Y'}\frac{1}{Nd} = \frac{1}{NY}. \quad (3)$$

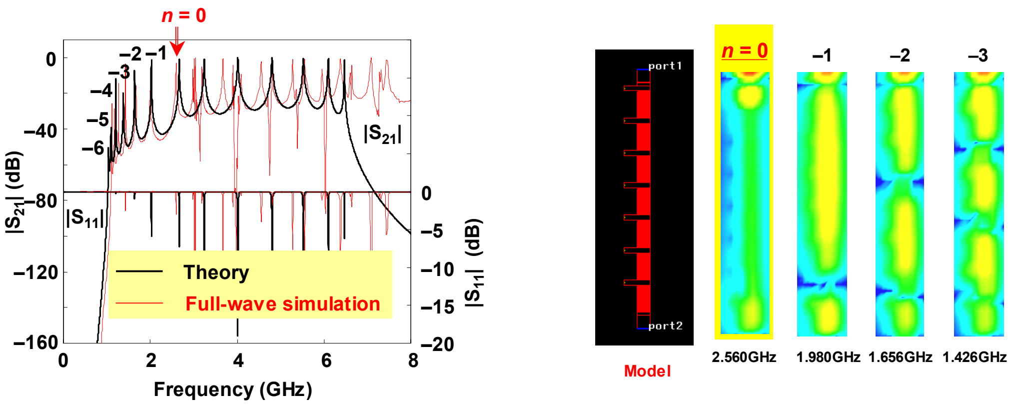
Here, $Z' = j(\omega L_L - 1/\omega C_R)/d$, $Y' = j(\omega L_R - 1/\omega C_L)/d$ and $Y = Y'd$. Z_{in} exhibits the impedance of the LC anti-resonant tank circuit with an inductance of L_L/N and a capacitance of NC_R as shown in Fig. 4(b). The resonant frequency of the ZOR, therefore, is given as

$$\omega = \frac{1}{\sqrt{(L_L/N)\cdot NC_R}} = \frac{1}{\sqrt{L_L C_R}} = \omega_{sh} \quad (4)$$

and the ZOR resonates at ω_{sh}. This suggests that the resonant angular frequency *depends only on the resonant frequency of the shunt anti-resonant tank circuit* with the inductance of L_L and the capacitance of C_R of the unit cell, *not the physical length l of the ZOR*.

The unloaded Q of the ZOR is obtained by considering the unloaded Q of the equivalent circuit shown in Fig. 4(b) as

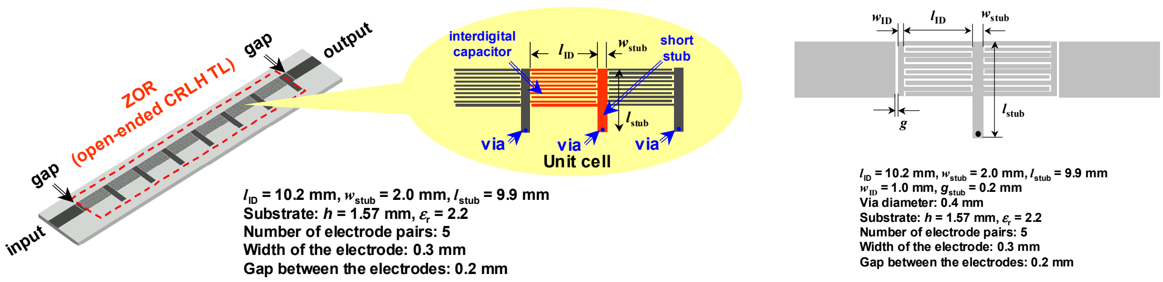
$$Q_0 = \frac{1/NG}{\omega_{sh} L/N} = \frac{1/G}{\omega_{sh} L}\left(= \omega_{sh}(1/G)C\right). \quad (5)$$



(a) Transmission and reflection characteristics of the ZOR. (L_L = 2.15nH, C_L = 1.41pF, L_R = 3.39nH, C_R = 2.21pF)

(b) Electric field distribution.

Fig.6 Resonant characteristics of the ZOR.



(a) 7-cell ZOR

(b) 1.5-cell ZOR

Fig.5 ZORs implemented on microstrip line.

It is noted that from the result of (5) that the unloaded Q is identical to that of the unit cell alone. This suggests that *the unloaded Q of the ZOR is independent of the number of the unit cells*. In addition, Eq. (5) suggests that, at the resonant frequency ω_{sh}, *no power is dissipated by the series resistance R*. This is because the voltages at every nodes of the ZOR are identical due to the infinite-wavelength wave and no current flows along the series resister R. Therefore, the ZOR might be able to exhibits higher unloaded Q with an optimum design.

3. Simulations and Experiments

In order to validate the theory shown here, full-wave method of moment (MoM) simulations for the ZORs implemented in microstrip line, as shown in Fig. 5, have been carried out. Figure 6(a) shows that the transmission and reflection characteristics of the 7-cell ZOR coupled to two ports with gaps at the ends. The thick lines show corresponding theoretical results given from the equivalent circuit shown in Fig. 2 (a). The circuit parameters were extracted for the unit cell shown in Fig. 5 by full-wave simulations in advance. The thin lines are MoM results applied to the entire structure of the ZOR. The theoretical zeroth-order resonant frequency appears exactly at the frequency of 2.5GHz given by (4), which agrees well with the computed results within the numerical error range. The major error is due to the ignorance of the higher order modes in the equivalent element value extractions. Fig. 6(b) shows the electric field distributions at 1.5mm (= $0.013\lambda_0$) above the ZOR surface in the zeroth-order resonant state as well as some non-zero resonant states of n = –1, –2 and –3 for comparison. The zeroth-order state, i.e., the infinite-wavelength wave resonance state is clearly observed at the theoretically predicted resonant frequency. Experiments have also been carried out for the 7-cell and 1.5-cell ZORs shown in Fig. 5. Measured transmission and reflection characteristics are shown in Figs. 7 and 8. The zeroth-order resonance is observed at the predicted frequencies and the validity of the theory is confirmed. The size reduction of 61% is achieved with the 1.5-cell ZOR compared with a conventional half-wavelength resonator. The measured unloaded Q's of the 7-cell and

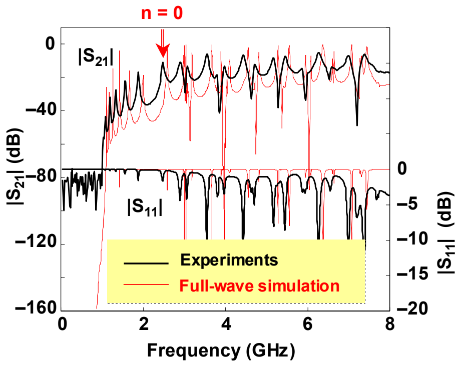


Fig.7 Resonant characteristics (7-cell ZOR).

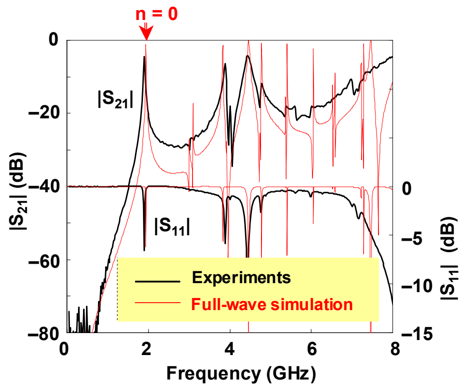


Fig.8 Resonant characteristics (1.5-cell ZOR).

1.5-cell ZORs calculated from the frequency characteristics are 280 and 250, respectively, which agree within the error range to the measured quality factor. In comparison, the unloaded Q of a typical conventional half-wavelength resonator with the same resonant frequency on the same substrate would be 200 ~ 300.

4. Conclusion

An unconventional resonator using CRLH TL is presented and its extraordinary zeroth-order resonance with an infinite-wavelength wave have been fully characterized and demonstrated. It has been shown that the resonant frequency of the ZOR is independent of *the physical length* so that the resonator can be *arbitrarily small* provided that sufficient reactance can be squeezed in a short length. Consequently, a drastic size reduction of resonators can be expected in microwave frequencies. The unusual loss mechanism of the zeroth-order resonance has also been explained. Numerical and experimental results agree well with the theoretical results and the validity of the theory has been confirmed. A 61% size reduction with a good unloaded Q of 250 is obtained for a prototype 1.5-cell ZOR at 1.9GHz in the experiment. Further size reduction and improvement of the unloaded Q can be expected with an optimized structure.

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