

NOZZLE FLOWS AT LOW AND MODERATE REYNOLDS NUMBERS

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ABSTRACT

Flow past long radius nozzles is studied in the Reynolds number range of 1 to 10,000 based on experiments in an oil recirculation system. The ratio of nozzle outlet diameter to pipe diameter is varied from 0.2 to 0.8. Aspects studied include the coefficient of discharge, sensitivity of the discharge coefficient to marginal shifts in pressure tap location, loss coefficient, loss as function of the meter pressure differential, critical Reynolds number corresponding to the origin of turbulence downstream of the nozzle, settling length, and nature of development of pressure and velocity fields. The contribution contained in the paper fills the gap that exists in literature on the performance of long radius nozzles at low and moderate Reynolds numbers.

Keywords: Nozzles; Flow meters; Losses; Critical Reynolds number; Flow development.

1. INTRODUCTION

A flow nozzle has a curved entrance leading smoothly to a cylindrical throat. Various standard designs are available and two of the most common forms are the ASME long radius nozzle and the circular arc nozzle. In the circular arc nozzle the inlet consists of two circular arcs of different radii. In the long radius flow nozzle [1] with which the present studies are concerned, the approach to the throat is a quadrant of an ellipse.

While studies [2, 3, 4] are available on discharge characteristics of flow nozzles, there appears to be hardly any investigations on their other characteristics. Even these studies are confined to higher Reynolds numbers only. The objective of the present study is to fill the gap that exists in literature

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regarding the long radius nozzle performance at low and moderate Reynolds numbers and provide information on aspects such as coefficient of discharge, loss coefficient and settling length. In the present investigations, integrated studies are made on flow past long radius nozzles covering metering aspects, loss characteristics and development of pressure and velocity fields downstream of the nozzle. The studies cover the range of pipe Reynolds number $R (= \bar{u}D/\nu$ where \bar{u} is the average pipe velocity, D is pipe diameter and ν is kinematic velocity) of 1 to 10,000. Four nozzles are studied with nominal β ratios (ratio of nozzle outlet diameter to pipe diameter) of 0.2, 0.4, 0.6 and 0.8. The studies form part of an extensive experimental project on sharp and quadrant-edged orifices and nozzles [5].

2. ASPECTS STUDIED

Aspects considered in the study include the coefficient of discharge, sensitivity of discharge coefficient to marginal shifts in pressure tap location, the loss coefficient, the loss as a fraction of the meter pressure differential, the critical Reynolds number at which turbulence originates downstream of the nozzle, the pressure and velocity field settling length and the nature of the development of pressure and velocity fields. The determination of the various parameters of interest are based on experimental investigations conducted in an oil recirculation system.

Referring to Fig. 1, the coefficient of discharge C is given by

$$Q = \frac{CA_d \sqrt{2gh_m}}{\sqrt{1-\beta^4}} \quad (1)$$

where Q is the discharge rate, A_d is the nozzle outlet area, h_m is the pressure head drop across the nozzle corresponding to the $D - D/2$ taps (Fig. 1) and g is gravitational acceleration.

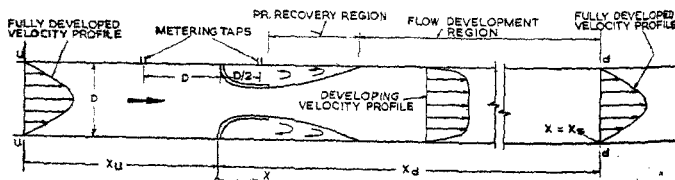


Fig. 1. Schematic diagram for nozzle flow.

In a constriction meter, a strict tolerance is generally called for in the location of the downstream pressure tap. A slight shift in this location

could significantly affect the C value. The experimental results were used to determine the sensitivity factor S defined by

$$S = \frac{\Delta C/C}{\Delta x/D} = \frac{\sqrt{h_m/h'_m} - 1}{\Delta X} \quad (2)$$

where ΔC is the variation in C value corresponding to a small shift Δx in the downstream metering tap location, $\Delta x/D$ is the nondimensional shift denoted by ΔX , h_m and h'_m are the pressure head drops across the nozzle for standard tap location and the slightly shifted tap location respectively.

The loss coefficient K is defined as

$$K = \frac{h_L}{\bar{u}^2/2g} \quad (3)$$

where h_L is the excess head loss due to the nozzle. Referring to Fig. 1 the loss coefficient is obtained from energy considerations as

$$K = \frac{h_u - h_d}{\bar{u}^2/2g} - f \frac{(x_u + x_d)}{D} \quad (4)$$

where h_u and h_d refer to the pressure heads at sections sufficiently upstream and downstream of the nozzle at which the flow is fully established, and f is the friction factor. The second term eliminates the normal friction losses from the total losses to determine the excess loss due to the nozzle. The excess loss may also be given as a fraction G of the pressure drop across the meter. Thus from eqs. (1) and (3), G is given by

$$G = \frac{h_L}{h_m} = \frac{\beta^4}{1 - \beta^4} KC^2 \quad (5)$$

Beyond a particular Reynolds number the flow just downstream of the nozzle becomes turbulent even though the approach flow is laminar. This critical Reynolds number R_c , is estimated from indirect evidences. For Reynolds numbers less than the critical value, the flow is in the purely laminar regime with laminar flow upstream and downstream of the nozzle. For Reynolds numbers between R_c and 2000, the flow downstream of the nozzle is of relaminarising type with the turbulence that originates immediately downstream of the nozzle decaying due to viscous forces. For $R > 2000$, the flows upstream and downstream of the nozzle are of a turbulent nature in a normal engineering pipe system.

For purely laminar flow, the rate of jet expansion downstream of the nozzle becomes more gradual with increase in Reynolds number and conse-

quently the length of the pressure recovery region also increases with Reynolds number. This experimental observation is confirmed by numerical solution of Navier-Stokes equations [6]. However, with the origin of turbulence downstream of the nozzle, the rate of jet expansion increases suddenly with a consequent upstream shift in the reattachment point and a reduction in the pressure recovery length. This upstream shift of the jet reattachment point is confirmed by Johansen's flow visualisation studies for an orifice [7]. Thus the critical Reynolds number for the nozzles is estimated as the Reynolds number corresponding to maximum length of the pressure recovery region. It is also estimated from the variation of G with R as at the critical Reynolds number G has a minimum value increasing clearly with the origin of turbulence. These indirect estimates of critical Reynolds number are also confirmed by observations of centreline velocity at some distance downstream of the jet reattachment point.

The settling length from the pressure field observations x_s is obtained by a direct comparison of the pressure drop between successive pressure taps with the corresponding pressure drop for fully developed flow. The flow is taken to be settled at a pressure tap s , beyond which the loss between successive taps agrees with the fully developed flow loss within experimental accuracies. Care was taken to ensure that downstream of the tap s , any slight deviation within experimental accuracy between the experimental profiles and the corresponding fully developed profiles was not of a cumulative nature. Several pressure profile plots showed that the settling length determined by this approach corresponds to a flow settlement to within 1 per cent of the fully developed flow.

3. EXPERIMENTAL ARRANGEMENT

The experiments were conducted in an oil recirculation system with a steel pipe of 54.4 mm inner diameter [5]. A straight disturbance free approach length of $176 D$ was provided before the test reach which is more than sufficient to ensure fully established approach flow. The test reach consisted of a length of $30 D$ upstream of the nozzle and $240 D$ downstream. Thirty-two pressure taps were located in the test reach with spacing varying from $0.5 D$ near the nozzle to $20 D$ well downstream of it. Twelve pitot type velocity probes were also provided with spacing varying from $7 D$ near the nozzle to $40 D$ well downstream. The pressure differences were measured by a system of oil-air and oil-mercury manometers while the discharge was measured by a calibrated collecting tank. Because of viscous effects on pitot tubes, velocity observations were restricted to

pipe Reynolds numbers of 600 and above and even for these observations corrections for viscous effects were made based on Macmillan's experimental results [8]. Four oils were used as fluid medium with kinematic viscosities of 20 cs (oil A), 350 cs (oil B), 100 cs (oil C) and 10.5 cs (oil D) at 30°C. The flowing oil temperature was maintained steady by means of a water cooling arrangement. Verification studies without any nozzle were conducted and good agreement was found with well-known solutions of fully developed laminar and turbulent flows [5].

Four nozzles with nominal β ratios of 0.2, 0.4, 0.6 and 0.8 were studied. The exact β ratios were 0.206, 0.404, 0.609 and 0.805. The nozzles were made as per the ASME specifications [1]. For $\beta = 0.2$, specifications of the low β series were adopted and for the other three β ratios, specifications of the high β series were adopted (Fig. 2).

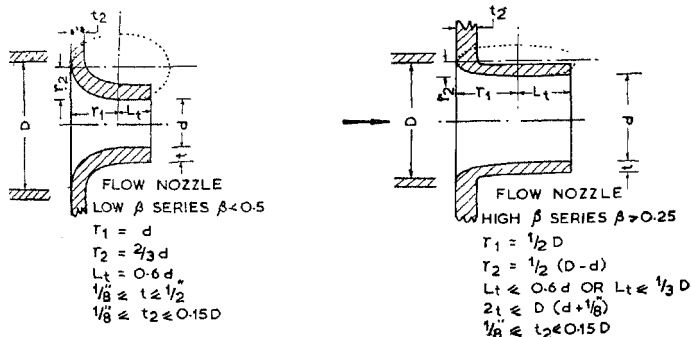


FIG. 2. Schematic diagram of nozzles.

4. COEFFICIENT OF DISCHARGE

The results for the coefficient of discharge based on eq. (1) are presented in Fig. 3. The curves upto $R = 10,000$ are based on the present experimental results and the curves are extended for $R > 10,000$ from ASME results [1]. For higher Reynolds numbers in the present experimental range, where the ASME data is available, there was good comparison between the present results and the ASME values [5]. Unlike for sharp-edged orifices, the coefficient of discharge for a nozzle increases monotonically with Reynolds number in the whole range from 1 to 10^5 .

Using continuity and energy equations, the coefficient of discharge for a nozzle may be shown to be

$$C = \frac{\sqrt{1 - \beta^4}}{\sqrt{\alpha_1 + K_f - \alpha_0 \beta^4}} \quad (6)$$

where α_0 and α_1 are the kinetic energy coefficients for the approach flow and for the flow just downstream of the nozzle outlet and K_f is the loss coefficient corresponding to the energy loss between the metering taps in terms of the nozzle velocity head. As K_f decreases with R , C increases with R . The increase is rapid at lower Reynolds number corresponding to the rapid change in K_f .

The data points for $\beta = 0.6$ and 0.8 indicated a small local drop in the C value at $R = 2000$ corresponding to the approach flow becoming turbulent. Considering the change in the nature of the approach flow which might change the friction losses in the nozzle, a local drop in C at $R = 2000$ is not unexpected. As this influence is more predominant for high β ratios, this local drop in C is noted for $\beta \geq 0.6$. However, smooth curves have been drawn through the data points (Fig. 3) in view of the relatively small magnitude of this local change and also in view of lack of any confirmation from previous investigations. In general studies on nozzles at low Reynolds numbers are scanty.

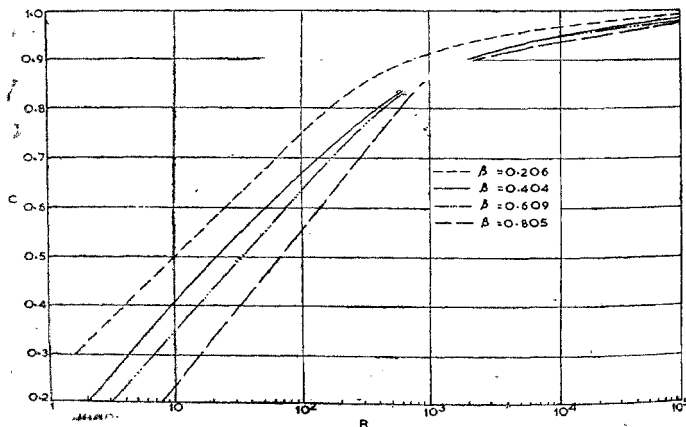


FIG. 3. Variation of coefficient of discharge.

5. SENSITIVITY FACTOR

The variation of sensitivity factor S defined in eq. (2) was evaluated for determining the effect on C of a small downstream shift in the downstream metering tap location. The results showed that the nozzles are much less sensitive to slight changes in the metering tap location than the sharp-edged orifices. While the effect of a slight shift in the tap location does not have any noticeable influence on C for small β ratios, for high β ratios such as 0.8, the sharp-edged orifice is very severely affected by tap location. For instance, at $R = 10,000$, while a downstream shift of $0.05 D$ in the downstream tap location changes C by 2.5 per cent for a sharp-edged orifice with $\beta = 0.8$, the corresponding change for a nozzle is 0.1 per cent. In the case of a nozzle, one may expect a fairly constant pressure in the whole annular space between the nozzle and the pipe wall which is essentially determined by the frictional losses in the nozzle. This explains the low values of S for the nozzles.

6. LOSS COEFFICIENT

The loss coefficient values given by eq. (4) and determined from pressure measurements are given in Fig. 4. It is seen that at very low Reynolds numbers, K is inversely proportional to R . The limiting value of R upto which the linear relationship holds good is below 2 for $\beta = 0.2$ and 9, 17 and 90 for $\beta = 0.4, 0.6$ and 0.8 respectively. For $\beta = 0.2$ and 0.4 , the $K-R$ curves become gradually flatter from the initial 45° line till K become constant at sufficiently high Reynolds numbers in turbulent flow. For $\beta = 0.6$ and 0.8 , the K curves display a minimum value at $R = 300$ and 1100 respectively. These correspond to the critical Reynolds number at which turbulence originates downstream, as confirmed by studies on velocity observations and pressure recovery characteristics. There is also a local drop in K at $R = 2000$. This reduction in K value is associated with the reduction in excess loss in the flow developing region downstream of the pressure recovery region, corresponding to the reduction in settling length for turbulent approach flow. These effects are not significantly felt for $\beta = 0.2$ and 0.4 in view of the small relative magnitude of losses in the developing region for low β ratios. The results in Fig. 4 show that the K value has reached near constancy at $R = 10,000$.

7. VARIATION OF G

The variation of G obtained from eq. (5) giving the excess loss as a percentage of the pressure drop across the nozzle, is given in Fig. 5. The

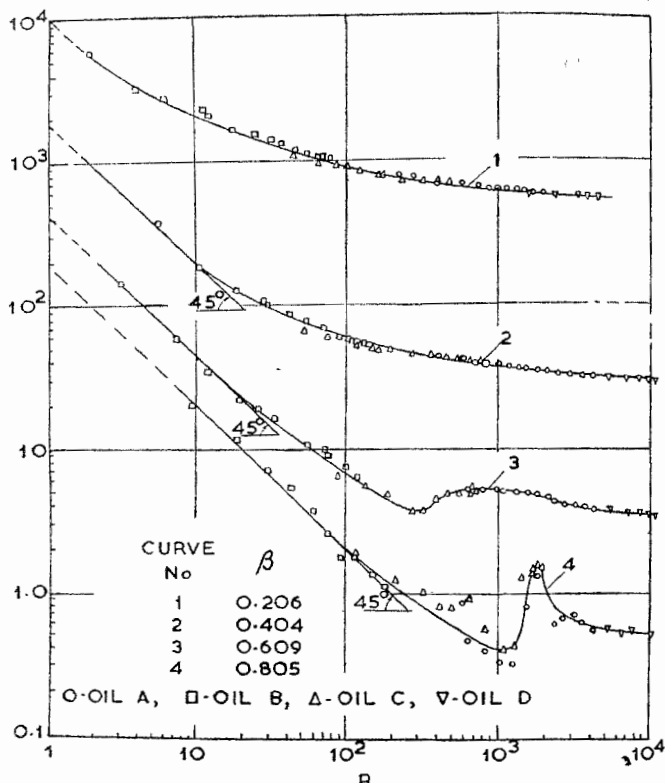


FIG. 4. Variation of loss coefficient.

constant G values recommended by ASME [1] and valid for higher Reynolds numbers are also shown in the figure and a good comparison is seen with the present values at higher Reynolds numbers.

For purely laminar flow, G decreases with increase in Reynolds number corresponding to increased pressure recovery with more gradual expansion of the jet downstream of the nozzle. However, with the origin of turbulence

downstream of the nozzle, the eddy losses in the pressure recovery region increase with a consequent increase in G value (Fig. 5). Thus the Reynolds

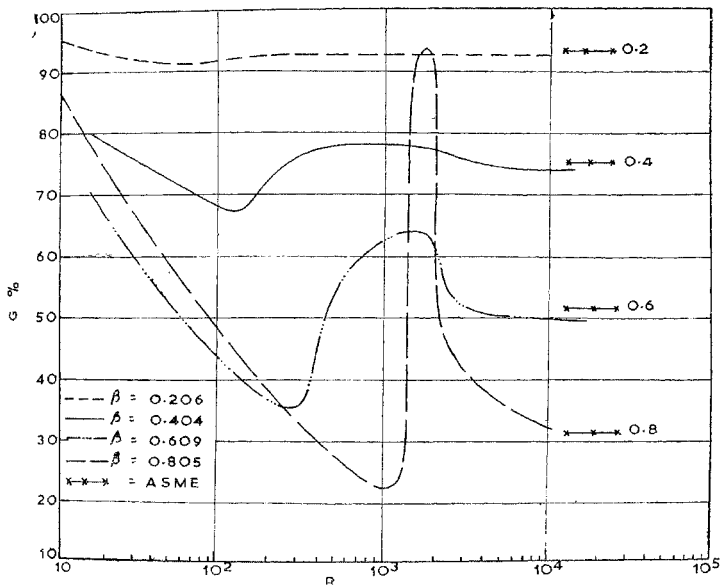


FIG. 5. Variation of G .

number value corresponding to the minimum value of G may be taken as the critical Reynolds number. The rise in G value after the critical Reynolds number is very pronounced for high β ratios. The G curves also show a local drop at $R = 2000$ and this drop is extremely pronounced for $\beta = 0.8$. This is associated with the drop in K values (Fig. 4) and the drop in C values at $R = 2000$ as discussed earlier.

8. CRITICAL REYNOLDS NUMBER

The critical Reynolds numbers corresponding to the origin of turbulence downstream of the nozzle were obtained from the G curves (Fig. 5)

and from a study of the pressure recovery length. The R values corresponding to the minimum values of G are 70, 140, 300 and 1080 for $\beta = 0.2, 0.4, 0.6$ and 0.8 respectively. The R values of 300 and 1100 for $\beta = 0.6$ and 0.8 corresponding to minimum values of K (Fig. 4) agree with the R_c values from the G curves.

The variation of the nondimensional pressure recovery length X_r is given in Fig. 6 for the nozzle with $\beta = 0.2$. X_r increases gradually with

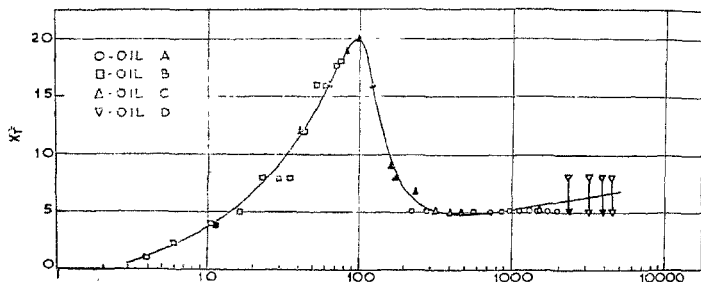


FIG. 6. Variation of pressure recovery length.

R at low values of R corresponding to the more gradual expansion of the jet. But with the origin of turbulence, the jet reattachment point shifts upstream suddenly with a reduction in X_r . Once the jet becomes turbulent, there is only a gradual shift in the jet reattachment point and X_r . Thus the critical Reynolds number corresponds to the Reynolds number at which X_r is maximum. The R_c values corresponding to maximum values of X_r are 98, 180, 400 and 1760 for $\beta = 0.2, 0.4, 0.6$ and 0.8 respectively. Thus R_c values based on X_r are higher than those based on G . The range given by these two critical Reynolds numbers may be treated as the range of transition. This was confirmed for $\beta = 0.8$ by centreline velocity observations at a distance of $12 D$ downstream of the nozzle. Upto $R = 1100$, the centreline velocity was nearly $2 \bar{u}$ confirming the existence of a practically fully developed profile at this location. The nondimensional centreline velocity dropped steeply to a value of 1.4 in the Reynolds number range of 1100 to 1750 and thereafter changed only gradually, suggesting the onset of turbulence in this range of Reynolds number.

9. SETTLING LENGTH

The settling length variation for $\beta = 0.2$ is given in Fig. 7. All the points except one are based on pressure field observations. There is a

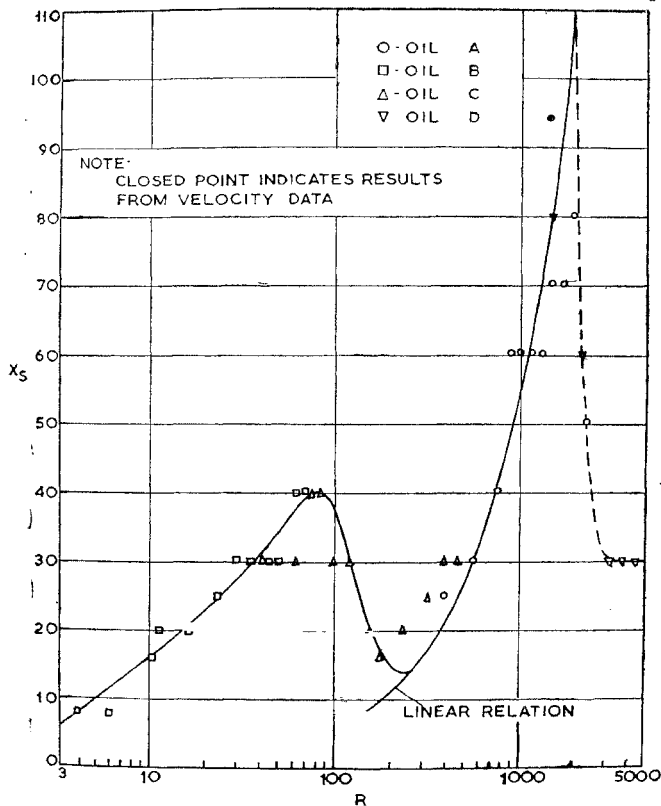


FIG. 7. Settling length for nozzle, $\beta = 0.206$.

local peak in the settling length at about the critical Reynolds number. This peak is associated with the extended length of the pressure recovery region

(Fig. 6) at the critical Reynolds number. For $\beta = 0.2$, critical Reynolds number is very low and at these low Reynolds numbers, the pressure recovery length forms a considerable part of the total settling length. With the origin of turbulence, the pressure recovery length reduces and the settling length also correspondingly reduces. After this local drop, the settling length increases linearly with Reynolds number in the relaminarisation regime and it drastically drops to $30D$ for transitional and turbulent approach flows.

The settling length variations for $\beta = 0.4, 0.6$ and 0.8 are shown in Fig. 8. While the trend of X_s variation for $\beta = 0.4$ is similar to that for $\beta = 0.2$, for $\beta = 0.6$ the critical Reynolds number influence is not apparent while for $\beta = 0.8$, X_s increases sharply at about the critical Reynolds number. These apparent discrepancies are explained when the relative magnitude of critical Reynolds numbers are considered. For low values of β such as 0.2 and 0.4 , R_c is low and at these low values of Reynolds numbers any turbulence that originates decays quickly and hence flow settles in a short distance downstream of the pressure recovery region. The local peak in X_s corresponds to the local peak in X_r . For high β ratios such as 0.8 , R_c is in the range of 1500 and once turbulence originates a considerable distance is required for its decay in view of smaller viscous forces at these moderate Reynolds numbers. The discontinuity in X_s curve at about R_c for $\beta = 0.8$ (Fig. 8) indicates that the process of relaminarisation is much slower than pure laminar flow development. Velocity profile observations for Reynolds numbers below and above the critical value substantiate this conclusion. For a moderate value of β such as 0.6 , there is no apparent effect of critical Reynolds number on X_s variation.

The experimental results show that in the range of relaminarisation, the settling length increases linearly with Reynolds number and the corresponding curves in Figs. 7 and 8 are least square fits. An equation

$$X_s = 0.049 R \quad (7)$$

is found to give a fairly good fit to the data for all the nozzles. Studies [5] showed that eq. (7) can be used for determining the settling length downstream of concentric, eccentric and quadrant-edged orifices also.

10. CHARACTERISTICS OF FLOW DEVELOPMENT

Figure 9 gives the pressure profiles in the region of pressure recovery and just downstream for $\beta = 0.2$ for a number of Reynolds numbers;

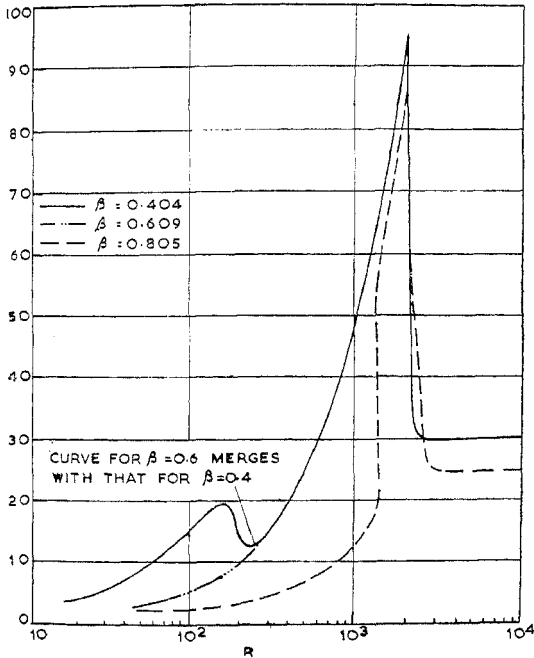


FIG. 8. Variation of settling length.

A refers to pressure tap location at $0.5 D$ downstream of the nozzle. The downstream shift of the end of the pressure recovery region with Reynolds number for $R < R_c$ is clearly seen with recovery extending upto $20 D$ at $R = 75$. The pressure peak shifts back to $14 D$ at $R = 120$ and it is located at about $5 D$ for $R = 1670$.

Based on pressure and velocity profile observations, the energy variations along the length were studied. The excess energy loss in the flow developing region downstream of the pressure recovery region was obtained for different Reynolds numbers. The results show that the excess loss downstream of the pressure recovery region is negligibly small for $R < R_c$. In

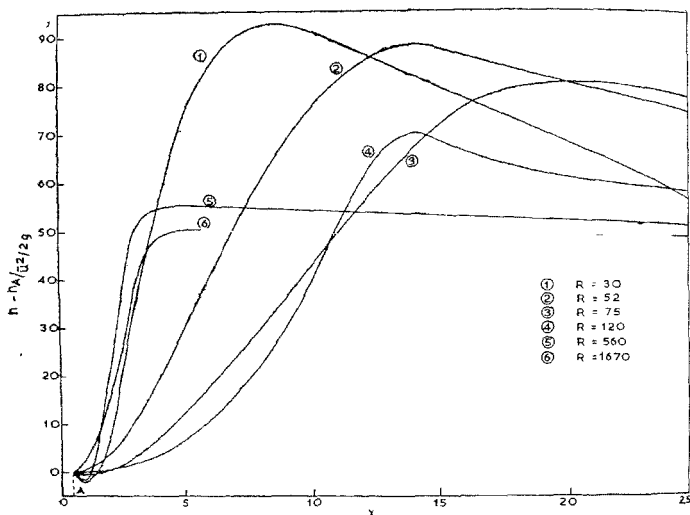


FIG. 9. Pressure recovery downstream of the nozzle $\beta = 0.206$.

the relaminarisation regime between R_c and 2000, the excess energy loss coefficient in the developing region was found to vary between 0.2 and 0.3 being higher for higher Reynolds numbers. While this loss is insignificant in relation to the total loss for $\beta = 0.2$, for a high β ratio such as 0.8, it forms about 25 per cent of the total excess loss. The excess loss coefficient in the developing region drops for turbulent flow to a value of about 0.05.

The development of velocity distribution at three Reynolds numbers are shown in Fig. 10 for the nozzle with $\beta = 0.4$; r is the radial coordinate. The data for $R = 620$ and 1520 are in the relaminarisation regime while the data for $R = 4150$ is in the turbulent flow regime. For $R = 620$, the velocity profile is practically identical to the parabolic profile at $40 D$ downstream. At $R = 1520$, the profile is nearly parabolic at $100 D$. This trend is in agreement with the now well established fact that relaminarisation is achieved over a longer distance at increasing Reynolds numbers. However, as turbulence observations were not made, detailed analysis of relami-

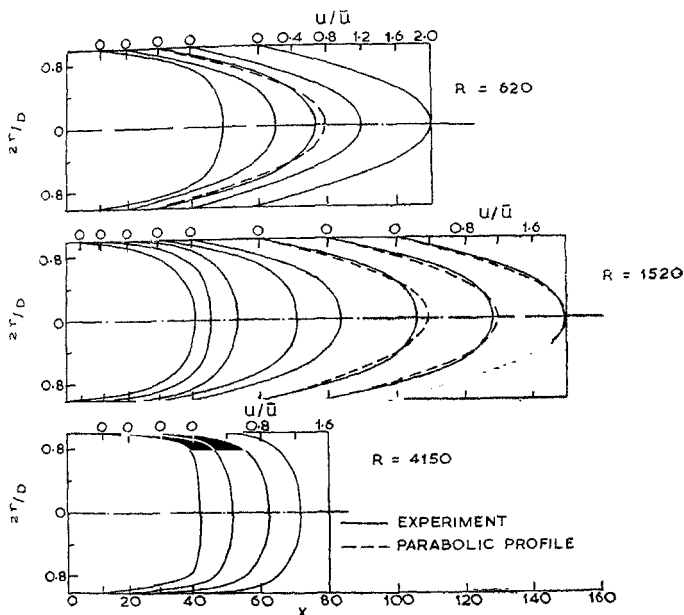


FIG. 10. Development of velocity profiles, $\beta = 0.404$.

narisation was not attempted. For turbulent approach flow ($R = 4150$) there is only a small change in the velocity profile, the profile in the early reaches of the development region being slightly more uniform than the fully developed profile. The nondimensional centreline velocity changes from 1.20 to 1.29 within $30 D$. Observations showed that for similar nature of flow, whether relaminarising or turbulent, the nature of flow development was similar for all β ratios.

11. CONCLUSION

The variations with Reynolds number of coefficient of discharge, loss coefficient, loss as a percentage of the pressure drop across the meter and settling length are studied for long radius flow nozzles with $\beta = 0.2, 0.4,$

0.6 and 0.8. The critical Reynolds number at which turbulence originates downstream of the nozzle is determined and the influence of the different flow regimes is discussed. The developments of pressure and velocity fields downstream of the nozzles are also studied.

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