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## Radiation



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## Nuclear Cross Sections for 95 Mev Neutrons <br> James DeJuren and Norman Knable June 10, 1949

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Nuclear Cross Sections for 95 Mev Neutrons<br>James DeJuren and Norman Knable<br>Radiation Laboratory, University of California, Berkeley, California June 10, 1949


#### Abstract

The total cross sections of twelve different elements were measured using the neutron beam from the 184 -inch cyclotron operating with deuterons. Bismuth fission ionization chambers were employed as both monitor and detector in conventional "good geometry" attenation measurements in the neutron flux emerging from the three inch diameter collimating port in the 10 foot thick concrete shielding. The mean energy of detection of the neutrons in this experiment is estimated to be 95 Mev 。

Measurements were also made with a monitor and detector placed inside the concrete shielding where we could obtain an intense neutron flux over a large area. Attenuators of four different elements were placed in front of the detector in a "poor geometry" arrangement so that attenuation was due essentially to inelastic collisions which degrade the neutron energy below the fission threshold. A second detector was placed outside the concrete shielding in the collimated neutron bean in line with the neutron source, absorber, and first detector. Attenuation in it is caused by both inelastic and elastic scattering. By this arrangement the ratio of inelastic to total cross section can be determined directly in one experiment.


The nuclear radii as calculated from the observed cross section using the theory of the transparent nucleus vary as $1.38 \times 10^{\infty 13} \mathrm{~A}^{1 / 3} \mathrm{~cm}$ 。 In this energy range the ratios of the inelastic to total cross sections are all less than one malf.

Nuclear Cross Sections for 95 Mev Neutrons<br>James DeJuren and Norman Knable<br>Radiation Laboratory，University of California，Berkeley，Califormia June 10， 1949

## INTRODUCTION

Several experiments have been conducted to measure the cross sections of nuclei with neutrons．They have been spaced at intervals corresponding to the periods when an increase in the energy of the bombarding neutrons was made aqailable．It was thought that as the available energies increased the measured cross sections would tend to become independent of the energy of the bombarding particle。．The results of these previous measurements are sumarized in a recent paper of Cooks MoMillan，Peterson and Sewell（1）which describes a measurement of
（1）L。Cook， $\mathrm{E}_{\mathrm{o}}$ McMillan，Jo Peterson，and Do Sewell，Physical Review 75， 7 （1949）
cross sections using the neutrons produced by 190 Mev deuterons of the 184 －inch cyclotron．

The threshold energy of the reaction used for detecting the neutrons $\mathrm{C}^{12}(\mathrm{n} ; 2 \mathrm{n}) \mathrm{Cll}^{11}$ is 20 Mev and an average cross section is obtained for neutrons having the average energy of the distribution produced in the eyclotron，modi－ fied by the（ $n, 2 n$ ）cross section energy dependence．This work showed the ex－ pected $A^{1 / 3}$ variation of the nuclear radius，calculated from an opaque nuclear model；for heavy nuclei but clearly indicated that the light nuclei（carbon and lighter）had smaller apparent radii than would be expected from an $A / 3$ law． This effect was attributed to the transparency of the iight nuclei for neutrons of short wave lengths。

These results indicated the need for a similar experiment to be done at higher energies in order to investigate the energy dependence of the effect． The estimated average energy of detection in the early experiment was 83 Mevo In the experiment to be described below，the same source of neutrons was em－
ployed but a bismuth fission ionization chamber (2) with a threshold of about 50
(2) C. Wiegand, Review of Scientific Instruments 19s 790 (1948)

Mev and a mean detection energy of 95 Mev served as the detector. In addition to raising the average detection energy this high threshold served a second purposes it allowed us to measure the cross sections for inelastic processes.

Such a measurement was necessary for one could no longer assume that the cross sections for inelastic scattering and for elastic scattering were equal to each other and to onewhalf the total cross sections as would be expected from an opaque model of the nucleus. The nuclear transparency would cause the ratio of inelastic and elastic scattering cross sections to be energy dependent.

Fernbach, Serber, and Taylor ${ }^{(3)}$ have, by assuming a model of the nucleus
S. Fernbach, R。Serber, and ToBo Taylor, Physical Review 75, 1352 (1949) which has an absorption coefficient and an index of refraction arrived at a formula for the radius of the nucleus in terms of the total cross section which for the experimental cross sections given below shows the radii of the nuclei to be directly proportional to $\mathrm{A}^{1 / 3}$ within the limits of experimental error. The same good fit has been made with the data of the previous experiment using the index of refraction and absorption coefficient corresponding to the neutron momentum of the average energy of detection in that experiment.

The total cross sections of 12 different nuclei were measured employing bismuth fission ionization chambers as both monitor and detector in conventional "good geometry" attenuation measurements in the neutron flux energing from a three inch diameter collimating port in the ten feet thick concrete shielding.

Measurements were also made with a monitor and detector placed inside the concrete shielding where we could obtain an intense neutron flux over a large area. Attenuators of four different elements were placed in front of the detector so that the angle subtended by the attenuator was greater than the angle at which the differential scattering cross section of the attenuator nuclei fell
to one percent of its value in the forward direction. This arrangement simulated the situation in which a plane wave of neutrons strikes a slab of material infinite in area so that attenuation is due solely to inelastic collisions. Measurements with higher threshold detectors gave results which agreed within the probable errors, thus confirming our assumption that an inelastic collision reduces the neutron energy below the threshold for bismuth fission.

A second detector was placed outside the concrete shielding in the collimated beam in line with the neutron source, attenuator, and first detector. Attenuation of the beam for the last detector is caused by both elastic and inelastic scattering。 By this arrangement the ratio of inelastic to total cross sections can be measured directly in one experiment.

COUNTERS AND NEUTRON DETECTION

## Counters

The ionization chambers (2) employed outside the concrete shielding contained 31 aluminum discs $1 / 32$ inch thick and two inches in diameter (Figure 1 ). Alternate discs were coated with thin layers of bismuth and the separation between adjacent discs was $3 / 8$ inch. The discs of the chambers used as monitors were plated with bismuth 1.0 to $1.5 \mathrm{mg} / \mathrm{cm}^{2}$ thick; the detector disos with bismuth $10 \mathrm{mg} / \mathrm{cm}^{2}$ thick. For exposure to equal neutron flux the detector counting rate is three to four times that of the monitors. The chambers employed in the "poor geometry" measurements inside the shielding were of a shallow design, containing three plates 4.5 inches in diameter, $3 / 4$ inch apart. Thin bismuth layers were used (as in the monitors described above). The fragments produced in fission of bismuth by the high energy neutrons experience their greatest rate of energy loss at the beginning of their ranges since they are initially multiply ionized and recombine with electrons as their velocities decrease. Hence most of the ionization in the argon gas occurs near the negative bismuth coated plates. The electrons formed are collected on the aluminum plates (field strength $500 \mathrm{v} / \mathrm{cm}$ ) and the resulting pulses are amplified by means of linear
amplifiers.

## Pulse Characteristics

The number of ions produced by a bismuth fission fragment is much greater than that by an alpha particle or proton given of by a spallation reaction in the chamber. Consequently it will take coincidences of several spallation reactions to give a pulse as large as a fission pulse. The gas used in the chambers was argon with 3 percent carbon dioxide, and the pressure was adjusted so that the renge of the fission fragments was approximately equal to the separation of the plates, since at a higher pressure the total ionization in the gas from a fission fragment would be unchanged but the ionization from an alpha particle would increase proportionally with pressure. If at a given neutron intensity the logarithm of the counting rate is plotted against the discriminator voltage (Figure 2) a curve with a broad plateau is obtained. The variation of the counting rate on the plateau is usually one to two percent, per volt change of dism criminator voltage. If the logarithm of the ratio of the number of counts of the chamber to a fixed monitor is plotted against discriminator voltage a similar curve results. If the neutron intensity is increased, the rapidly varying "pileup" region of the curve, due to pulses from coincident spallation reactions is shifted to the right, but the remainder of the curve is unchanged. This agrees with what we would expect from theoretical considerations, which show the number of "pileups" to be a much more rapidly increasing function of the neutron intensity than is the number of fissions. Variation of neutron intensity thus affords a criterion for distinguishing fission pulses from "pileups" of spallation products.

The stability of the amplification and discrimination volt ages was periodically checked by feeding in fixed voltage pulses and it was determined that the experimental errors so incurred were less than the statistical errors of counting。

## EFFECTIVE NEUTRON DETECTION ENERGY

The neutron beam was produced by bombarding a 0.5 inch Be target with 190 Mev deuterons in the 284－inch cyclotron。 The neutrons produced by＂stripping＂ have an energy distribution theoretically predicted by Serber ${ }^{(4)(5)}$ and checked
（4）Ro Serber，Physical Review 72， 1008 （1947）
（5）A。CoHelmholz，E。MoMillan，and Do Sewell，Physical Review 72， 1003 （1947） Angular distribution．
experimentally with proton recoil counters $(6)$ and by proton recoils in a cloud
（6）JoHadley，E。Kelly，CoLeith，E。Segrè，Co Wiegand，and Ho Yorkg Fhysical Review 75， 357 （1949）
chamber。（7）The variation of the fission cross section with neutron energy was
（7）K。Bruecker，WoHartsough，E。Hayward，and W。Powell，Physical Review 75， 555 （1949）
investigated by Wiegand and Kelly（8）by varying the radius of the target probe
（8）E．L：Kelly and Co Wiegand，Physical Review 73，1135（1948）
in the 184 minch cyclotron．As there is a spread in neutron energies for each radius and the more energetic neutrons in each energy spectrum have the highest fission efficiency，the curve of bismuth fission cross section as a function of neutron energy may be even steeper than indicated．If the fission efficiency and neutron distribution for corresponding energy intervals are multiplied a curve results proportional to the detection efficiency as a function of neutron energy（Figure 3）．The curve is similar to the original neutron distribution； both curves have an energy spread of about 26 Mev at half maximum，but the averm age energy of the neutrons detected by fission is about 95 Mev．

EXPERIMENTAL ARRANGEUENTS

## Arrangement for Total Cross Sections

The monitors and detectors employed in measuring total cross sections were aligned outside the concrete shielding by means of $x$－ray film（which was exposed
to $\gamma$-rays formed when deuterons struck the terget). The attenuators were placed behind the monitor at a distance of 100 inches from the detector (Figure 4). so that the detector subtended a solid angle of 00003 steradian at the attenuator. A steel collimator 18 inches long with a 1 inch x I. 5 inch cross sectional aperture was employed in all but the first few experiments. The correction for small angle scattering was negligible when the collimator was used and the maximum correction without the collimator was only 2.2 percent for the Pb cross section.

## Arrangement for Inelastic Cross Sections

The two shallow type chambers mentioned earlier were equipped with cathode followers that would function in the stray magnetic field inside the concrete shielding. These chambers could be placed in the broad neutron beam (5) outside the cyclotron tank. One chamber was placed in the central portion of the beam in line with the probe and collimator (Figure 4). Absorber slabs could be placed in front of this chamber in "poor geometry" fashion. The second chamber was placed to one side of the absorbers to serve as a monitor. The absorbers were stacked in a conical array as shown in Figure 4. The angular distribution of elastically scattered neutrons from nuclei is peaked in the forward direction and even for carbon the fraction singlyascattered more than 450 is negligible; so the conical half angle of 450 employed permitted essentially all the elasti= cally scattered neutrons to enter the poor geometry detector that would have entered even if the slabs were infinite in area. The attenuation therefore, is a measure of the cross section for inelastic collision. A detector was also placed outside the shielding, as previously mentioned, in line with the collimated beam, so measurements of total and inelastic cross sections could be measured simultaneously.

The total thickness of attenuators used was restricted by the condition that the flux should be uniform over the first disc (largest in area). A survey of the uniformity was made by moving a shallow fission chamber with 2 inch
diameter bismuth-coated plates over the region where the poor geometry attenuam tors were placed. The neutron flux was uniform (within 95 percent of the peak value) over an area 7 inches in radius. Discs greater than 8 inches in radius could not be used because the cyclotron deflector housing reduced the beam intensity at that radius. The attenuation was exponential over more than two inelastic mean free paths of copper, with no transition effects. Magnitude of Attenuation Employed

As the attenuation was exponential, within the probable errors, for both the "good" and "bad" geometry experiments, it was not necessary to measure the slope of a semi $\log$ plot of the intensity versus thickness. Measurements were made with between one and two mean free paths of material.

For the experiments performed outside the shielding the monitor and detector counting rates were nomally about equal with no attenuator present. The statistical error in the value of the cross section measured is equal to the statistical error of counting divided by the number of mean free paths of absorber. The principal counting error results from the detector reading with attenuator present. Slightly over two mean free paths of material was the most efficient length for a cross section determination, but to minimize diffraction scattering into the detector usually less than two mean free paths were employed.

## ATTENUATOR MATERIALS

## Physical Properties

The metals used in the experiments were usually machined cylinders and were measured by two different observers with micrometer or vermier calipers and carefully weighed. The liquids employed were weighed in a 500 ml volumetric flask, which was first calibrated by weighing distilled water at 19.10. The measured value of the density was $.9980 \mathrm{~g} / \mathrm{ml}$ while the accepted value is . 9984 $\mathrm{g} / \mathrm{ml}$.

The liquids were held in a 3 inch inner diameter cylindrical brass holder of variable length with $1 / 16$ inch thick brass end caps. During an experiment
several counting intervals were employed with the holder alternately filled and empty.

The nitrogen cross section was derived from melamine, $\mathrm{C}_{3} \mathrm{H}_{6} \mathrm{~N}_{6}$. The melamine was carefully tamped in the brass holder in an effort to make the density as uniform as possible. An alternate empty holder with similar caps was employed during the "blank" counting intervals.

When the holder was used the beam from the 1 inch $\times 1.5$ inch collimator was always carefully centered by means of x-ray film to minimize scattering from the brass walls.

## Purity of Attenuators

Chemically pure materials were used, and the effect of minute impurities on the values obtained is negligible.

Because of its theoretical importance the total cross section of hydrogen was measured carefully on three separate occasions using pentane-carbon differences. Six graphite cylinders were machined to fit in the brass liquid holder. The length of the holder was adjusted ( 43 inches) so that the mass per unit area of carbon in the graphite was equal to the carbon in the pentane employed. The graphite cylinders were distributed along the length of the holder to simulate the distribution of carbon in the pentane. A counting interval was first taken with graphite in the holder. When the pentane was substituted the attenuation in the detector over the previous case was due solely to the hydrogen in the pentane. Hence the counting errors are due to these cycles only and the hydrogen cross section is directly obtained without the need of blank cycles, which would introduce added statistical errors. As only 0.5 mean free path of hydrogen was present and the statistical error of the cross section is the counting error divided by the mean free path, it was important to use the counting time efficiently.

The graphite and pentane were analyzed by the Pacific Chemical Laboratories for impurities which were as follows:


The error introduced by these impurities is not significant.
The difference between the deuterium and hydrogen cross sections was determ mined directly in a similar experiment by filling the holder altermately with equal numbers of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ molecules. The heavy water was 99.87 percent $\mathrm{D}_{2} \mathrm{O}$ by weight.

## RESULTS

## Total Cross Sections

The data for the total cross sections measured outside the shielding are given in Table lo Corrections for diffraction scattering are included in a few cases where this is significant. Statistical errors given are in terms of standard deviations. The effect of background was investigated by placing fourteen mean free paths of various absorbers between detector and monitor. Without absorbers 950 detector counts were obtained for 1000 monitor counts. With absorbers no detector counts were obtained for 3600 monitor counts. Even a few detector counts would have meant negligible error from backgroundo. As the chambers have excellent high-voltage and discriminator charactaristics, the error introo duced by the equipment is probably less than one percent. Repeated measurements have given differences that appear to be purely statistical.

## Inelastic Cross Sections

The inelastic cross section measurements are subject to greater sources of error. The "poor geometry" detectors count inelastically scattered neutrons or neutrons ejected from nuclei if their energies are above the threshold for bis: muth fission. The inelastic cross-sections obtained, therefore, represent a lower limit to the true values. However, it is not expected that the contribution of neutrons over threshold energy from inelastic processes is large enough

TOTAL CROSS SECTIONS FOR 95 MEV NEUTRONS MEASURED WITH BISHUTH FISSION CHAMBERS

| Element | Density $\mathrm{g} / \mathrm{cm}^{3}$ | Atomic Number 2 | Mass Number A | $A^{2 / 3}$ | Cross Saction $\sigma_{t} \times 10^{24} \mathrm{~cm}^{2}$ | Radius. $\mathrm{R} \times 10^{13} \mathrm{~cm}$ from model |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fiydrogen |  | 1 | 1 | 1.00 | $.0745 \neq .002$ $.073 \mp .003$ $.071 \mp .002$ $.073 \mp .0015$ | $1.69 \pm .02$ |
| Deuterium |  | 1 | 2 | 1.26 | $.104 \pm .004$ | $1.89 \pm .03$ |
| BeryIIium | 1.847 | 4 | 9 | 2.08 | $.396 \pm .004$ | $2.94 \pm .015$ |
| Carbon | 1.580 | 6 | 12 | 2.29 |  | $3.20 \pm .01$ |
| Nitrogen |  | 7 | 14 | 2.41 | . $570 \pm .007$ | $3.35 \pm .02$ |
| Oxygen |  | 8 | 16 | 2.52 | $.663 \pm .007$ | $3.52 \pm .02$ |
| Aluminum | 2.714 | 13 | 27 | 3.00 | $.993 \pm .011$ | $4.11 \pm .02$ |
| Chlorine |  | 17 | 35.46 | 3.29 | $1.28 \pm .02$ | $4.55 \pm .03$ |
| Copper | 8.90 | 29 | 63.57 | 3.99 | $\begin{aligned} & 2.00 \\ & 2.00 \end{aligned} \pm .02$ | $5.53 \pm .03$ |
| Tin | 7.28 | 50 | 118.7 | 4.92 | $\begin{gathered} 3.18 \\ (3.13) \end{gathered} \pm .03$ | $6.81 \pm .03$ |
| Lead | 11.33 | 82 | 207.2 | 5.92 | $\begin{gathered} 4.48 \\ (4.38) \end{gathered} \pm .03$ | $8.14 \pm .03$ |
| Uranium | 18.89 | 92 | 238.1 | 6.20 | $\begin{gathered} 4.92 \\ (4.89) \end{gathered} \pm .06$ | $8.58 \pm .0 .5$ |
| DeuteriumHydrogen |  | 0 | 1 | 1.00 | $0.031 \pm .002$ |  |

conpounds

| $\mathrm{H}_{2} \mathrm{O}$ | .998 |
| :--- | :--- |
| $\mathrm{D}_{2} \mathrm{O}$ | 1.104 |

Pentane
$\mathrm{C}_{5} \mathrm{H}_{12}$
$\mathrm{CCl}_{4}$
1.592

Melamine $\mathrm{C}_{3} \mathrm{H}_{6} \mathrm{~N}_{6}$ Values in parentheses are not corrected for diffraction scattering
to modify the present interpretations．
Elastically scattered neutrons will have a greater path length than undevi－ ated neutrons in both the absorber and the bismuth layer of the detector．The relative probability of detection for different neutron paths is discussed in the appendix．Table II contains the experimental results．The ratio of $\frac{\sigma_{i}}{\sigma_{t}}$ is given，since it is independent of density determinations of the material used and is measured directly．The total cross sections listed in Table II were measured with＂better＂geometry than those in Table $I$ ，and therefore require no corrections for diffraction．

Table II

| Element | $\frac{\sigma_{i}}{\sigma_{t}}$ | $\sigma_{t} \times 10^{24} \mathrm{~cm}^{2}$ | Total MoF。P． Used |
| :---: | :---: | :---: | :---: |
| Carbon | $.460 \pm .016$ | $.496 \pm .012$ | 0.60 |
|  | $.457 \pm .026$ | ． 489 士．018 |  |
|  | ． 434 士．023 | ． 497 士。017 |  |
|  | ．431 士．024 | ． 498 士．017 |  |
|  | $.454 \pm .022$ | $.502 \pm .016$ |  |
| Aluminum | $.42 \pm .015$ | $0.995 \pm .02$ | 0.94 （dural） |
| Copper | $.39 \pm .005$ | $2.005 \pm .02$ | 2.5 |
| Lead | $.40 \pm .01$ | $4.46 \pm .06$ | 1.9 |
|  | ． 38 壬：01 | 4.46 玉．06 |  |
|  | ． 40 玉．01 |  |  |
| Carbon（ave．） | $.45 \pm .015$ | $.497 \pm .008$ |  |

Actually dural was used，instead of aluninum，but the correction for the impur－ ities（mainly copper）is negligible for the ratio of $\frac{\sigma_{i}}{\sigma_{t}}$ ．

An attempt was made to measure the ratio of $\frac{\sigma_{i}}{\sigma_{t}}$ for carbon and copper using gold fission for neutron detection since gold presumably has a higher fission threshold than bismuth．The values obtained were：

$$
\frac{\sigma_{i}}{\sigma_{t}}
$$

$\sigma_{t}$

$$
.42 \pm .04
$$

$$
0.48 \pm 0.35
$$

Copper

$$
.38 \pm .015
$$

$$
1.92 \pm .06
$$

The counting rate was about a factor of ten less for gold fission than for bismuth. From the total cross sections obtained, the average energy of neutrons detected is estimated to be between 100 and 105 Mev . Evidently the use of a higher threshold did not observably increase the ratio of $\frac{\sigma_{i}}{\sigma_{t}}$ above the bismuth fission values. Both methods of detection agree, within the statistical errors.

An experiment was made to determine the angular distribution of elastically scattered neutrons from a carbon sphere. A fission chamber mounted on a radial arm pivoted below the sphere was used as the detector. The distribution is plotted in Figure 7.

DISCUSSION OF RESULTS

## The Transparent Nucleus Model

Fembach, Serber, and Taylor ${ }^{(3)}$ in a recent paper, have assumed a model for nuclei bombarded by high energy neutrons, in which the nucleus is regarded as a sphere of material characterized by an absorption coefficient and index of rem fraction, both of which are functions of the neutron momentum. Using their notation, $k_{1}$ is the increase of the propagation vector of the neutron wave upon entering the nucleus and $\mathbb{K}$ is the absorption coefficient. A wave going a distance $X$ in nuclear matter will have its amplitude and relative phase changed by a factor $e^{\left(-\frac{1}{2} K+i k_{l}\right) X}$. The solutions for the inelastic and elastic cross sections are respectively:

$$
\begin{aligned}
& \sigma_{i}=\pi R^{2}\left\{1-\frac{1-(1+2 K R) e^{-2 K R}}{2 K^{2} R^{2}}\right\} \\
& \sigma_{e}=2 \pi \int_{0}^{R} \mid 1-e^{\left.\left(-K+2 i k_{j}\right) s\right|^{2} s d s}
\end{aligned}
$$

The form for $\sigma_{e}$ is merely indicated as the integrated expression is quite com plex. $R$ is the radius of the nucleus and $\sigma_{t}=\sigma_{i}+\sigma_{e}$

Taylor has determined $\mathrm{k}_{1}=2.85 \times 1012 \mathrm{~cm}^{-1}$ and $\mathrm{K}=3.0 \times 10^{12} \mathrm{~cm}^{-1}$ to give the best straight line fit for the radii calculated from the total cross sections when $R$ is plotted as a function of $A^{1 / 3}$. The equation for the line of
best fit is $R=1.38 \mathrm{~A}^{1 / 3} \times 10^{-13} \mathrm{~cm}$ 。
The ratio of $\frac{\sigma_{i}}{\sigma_{t}}$ for the afore-mentioned values of $K$ and $k_{1}$ is given as a function of $K R$ in Figure 9 and the experimental values are shown.

## Comparison with Data at 83 Mev

Cook, McMillan, Peterson, and Sewell (1) have measured total cross sections of the same elements in this laboratory using carbon discs, which have a 20 MeV threshold for the $c^{12}(n, 2 n) c^{l l}$ reaction, for detectors. The average energy of the neutrons detected in their experiments is estimated to be 83 Mev as compared to 95 Mev for bismuth fission. The 95 Mev cross sections are roughly 10 percent smaller than those derived using carbon discs, indicating greater transparency at the higher energy.

## ACKNOWLEDGMENTS

The authors wish to express their gratitude to Dr. Burton Jo Moyer for his help and suggestions in carrying out the program, to the cyclotron crew for their cooperation in performing the experiments, and to Dr。E.Segre, E. Kelly, and C. Wiegand for use of their vacuum evaporator for plating the chamber discs with bismuth and gold. The work was performed under the auspices of the Atomic Energy Commission.

## APPENDIX

## Effect of Path Length on Inelastic Cross Section Measurements

If $I_{0}$ be the number of neutrons $/ \mathrm{cm}^{2}$ incident on a plane absorber of thickness $L$, the number of neutrons $/ \mathrm{cm}^{2}$ suffering $p$ elastic scattering events in traversing the material is given by:
A)

$$
I_{p L}=I_{0}\left(\frac{L}{\lambda_{e}}\right)^{p} \frac{1}{p!} e^{\frac{-L}{\lambda_{t}}}
$$

where $\lambda_{\theta}$ and $\lambda_{t}$ are the mean free paths for elastic scattering, and for any type of collision, respectively.

If $\frac{d \sigma_{\theta}(\theta)}{d \omega}$ is the differential cross section per unit solid angle for elastic scattering at angle $\theta$, the number of neutrons/cm ${ }^{2}$ emerging with angles
between $\theta$ and $\theta+d \theta$ with respect to the incident neutrons is:
B) $d I \theta=I_{0} \sum_{p=1}^{\infty}\left(\frac{L}{\lambda_{e}}\right)^{p} \frac{1}{p!} e^{\frac{-L}{\lambda t}} \frac{1}{p} \frac{d \sigma_{e}\left(\frac{\theta}{\sqrt{p}}\right)}{d \omega} \frac{2 \pi \sin \theta d \theta}{\sigma_{e}}=I_{p L} \frac{1}{p} \frac{d \sigma_{e}\left(\frac{\theta}{\sqrt{p}}\right)}{d \omega} \frac{2 \pi \sin \theta d \theta}{\sigma_{e}}$;
where it has been assumed that the effect of $p$ elastic scatterings on the angular distribution is to broaden the distribution by a factor of $\sqrt{\mathrm{p}}$, and to decrease its height by $1 / \mathrm{p}$. Formulas $A$ and $B$ are only approximate since the path length is equal to $L$ for small angles only.

The probability of a neutron emerging at angle $\theta_{9}$ after sustaining an elastic collision into the $\theta$ direction within an element dx at depth $x$ is:

$$
e^{-\frac{x}{\lambda t}} \frac{d x}{\lambda_{e}} e^{-\left(\frac{L-x}{\lambda_{t} \cos \theta}\right)}
$$

The average attenuation experienced by neutrons following such a path is, from the above expression:

$$
\frac{1}{L} \int_{0}^{L} e^{-\frac{1}{\lambda t}}\left(x+\frac{L-x}{\cos \theta}\right)_{d x}
$$

The ratio of this attenuation at angle $\theta$ to the attenuation when $\theta=0$ is the relative probability of emergence for a neutron at angle $\theta$ compared with $\theta=0$ 。

After integratings the result is

$$
R_{A}=\frac{\lambda_{t}}{L} \frac{\cos \theta}{1-\cos \theta}\left[1-e^{\frac{-L}{\lambda_{t}}\left(\frac{1}{\cos \theta}-1\right)}\right]
$$

The chance of a neutron inducing fission in the bismuth layer is proportional to its path length in the layer or to $\frac{1}{\cos \theta}$ since the chamber plates are normal to the undeviated neutrons.

In an elastic scattering event in the attenuator the scattered neutron will lose an amount of energy given by

$$
\Delta E=\frac{2 m E(1-\cos \theta)}{M+m(1-\cos \theta)}
$$

where $m$ is the neutron mass, $M$ is the mass of the nucleus and $E$ is the energy of
the incident particle. Since the fission cross section is a function of energy the relative chance of producing a fission is

$$
\sigma_{f} \frac{(E-\Delta E)}{\sigma_{f}(E)}
$$

The probability of detecting a neutron scattered through angle $\theta$ relative to the probability of detecting an undeflected neutron is the product of the above factors
c)

$$
\frac{\pi_{\theta}}{\nabla_{0}}=\frac{1-e^{-n_{t}}\left(\frac{1-\cos \theta}{\cos \theta}\right)}{n_{t}(1-\cos \theta)} \quad \sigma_{f} \frac{(E-\Delta E)}{\sigma_{f}(E)}
$$

where $n_{t}=\frac{L}{\lambda_{t}}$, the number of total mean free paths of attenuator.
The change in the bismuth fission oross section is given approximately by

$$
\frac{d \sigma_{f}}{\sigma_{f}}=9 / 4 \frac{d E}{E}
$$

in the region of 90 Mev .
In our poor geometry experiments either $n_{t}$ was small or the average angle of deviation was small (due to forward peaking of elastically scattered particles from heavy nuclei); so the exponent in Equation $C$ may be expanded:

$$
\frac{\pi_{\theta}}{\pi_{0}}=\frac{1-\left[1-n_{t}\left(\frac{1-\cos \theta}{\cos \theta}\right)+\frac{n_{t}^{2}}{2}\left(\frac{1-\cos \theta}{\cos \theta}\right)^{2}+\cdots .0\right]}{n_{t}(1-\cos \theta} \frac{\sigma_{f}(E-\Delta E)}{\sigma_{f}(E)}
$$

C ${ }^{8}$

$$
\cong\left[\frac{1}{\cos \theta}-\frac{n_{t}(1-\cos \theta)}{2 \cos ^{2} \theta}\right]^{\sigma_{f}(E-\triangle E)} \frac{\sigma_{f}(E)}{}
$$

If we know the angular distribution of elastically scattered neutrons we can correct the inelastic cross section data using $C^{\circ}$. For carbon the relative angular distribution of scattered neutrons in the annular region between $\theta$ and $\theta+d \theta$ is show in Figure 7. If the latter curve is multiplied by $C$ ' which has
been evalated for carbon, the area under the curve increases by about 1.2 percent. Hence the number of singly scattered neutrons detected in the poor geometry experiment was increased by 1.2 percent due to the increased probability of detection for $\theta>00$ 。

From A with $\left(\frac{L}{\lambda_{e}}\right)=.33$ the relative number of neutrons emerging that were elastically scattered $p$ times may be determined:

$$
\begin{aligned}
& p=0: 71 \text { percent } \\
& p=1: 25 \text { percent } \\
& p=2: 4 \text { percent }
\end{aligned}
$$

Those scattered once and twice will have a greater probability of detection and the increase in the total number of detector counts was roughly $.25 \times 1.2$ percent $+.04 \times 1.5$ percent (allowing a greater correction for double scattering) or 0.36 percent. This increase in the detector counts caused the inelastic cross section to be low by the increase divided by the number of inelastic mean free paths, $\frac{0.36 \%}{.27}$, or about 1.3 percent. Since the statistical error is 3.3
percent this correction is hardly significant.
The correction for the heavier nuclei is less, due to the greater forward peaking of the elastically scattered neutrons.

FIGURE CABTIONS
Figure I Internal Structure of Long Fission Ionization Chamber
2 Discriminator Curve: Number of Pulses Greater than Disc Voltage versus Disc Voltage
$3 \sigma_{F}-$ Fission Cross-Section of Bismuth
N(E) -- Energy Distribution of Neutrons
$N(E) \sigma_{F}(E) \propto$ Detection Efficiency
4 Arrangement of Apparatus
5 Fission Chambers and Absorbers Used in "Poor Geometry" Measurements
6 Absorption of Neutrons in Al
7 Angular Distribution of Elastically Scattered Neutrons from Carbon
8 Radii Calculated from Kodel Assuming
$K=3.00 \times 10^{12} \mathrm{~cm}^{-1}$ and $\mathrm{k}_{1}=2.85 \times 10^{12} \mathrm{~cm}^{-1}$ define straight line of $E q \cdot R=1.38 \times A^{1 / 3} \times 10^{-13} \mathrm{~cm}$
Dotted line is graph of radii calculated from $\sigma_{t}=2 \pi R^{2}$ (opaque nucleus)
9 Theoretical curve of $\frac{\sigma_{i}}{\sigma_{t}}$ for $K=3.00 \times 10^{12} \mathrm{~cm}^{-1}$ and $\mathrm{k}_{1}=2.85 \times 10^{-12} \mathrm{~cm}^{-1}$ with experimental points
Gen Par Re 317

FIG. 2

DOTTED CURVE IS AT NEUTRON INTENSITY HALF THAT OF SOLID CURVE

X OPERATING POINT

| 20 | 30 | 40 | 50 | 60 | 70 |
| :---: | :---: | :---: | :---: | :---: | :---: |



FIG. 4
ARRANGEMENT OF APPARATUS



FIG. 5
FISSION CHAMBERS AND ABSORBERS USED IN "POOR GEOMETRY"MEASUREMENTS.



FIG. 8
RADII CALCULATED FROM MODEL
ASSUMING $K=3.00 \times 10^{12} \mathrm{~cm}$ - $\quad$ AND $9 k_{1}=2.85 \times 10^{12} \mathrm{~cm}^{-1}$ DEFINE STRAIGHT

LINE OF EQ $R=138 \times A^{1 / 3} \times 10^{-13} \mathrm{~cm}$ DOTTED LINE IS GRAPH OF RADII CALCULATED $8 \quad \begin{aligned} & \text { DOTTED LINE IS GRAPH OF RAD } \\ & \text { FROM } \\ & =2 \pi R^{2} \text { (OPAQUE NUCLEUS) }\end{aligned}$


