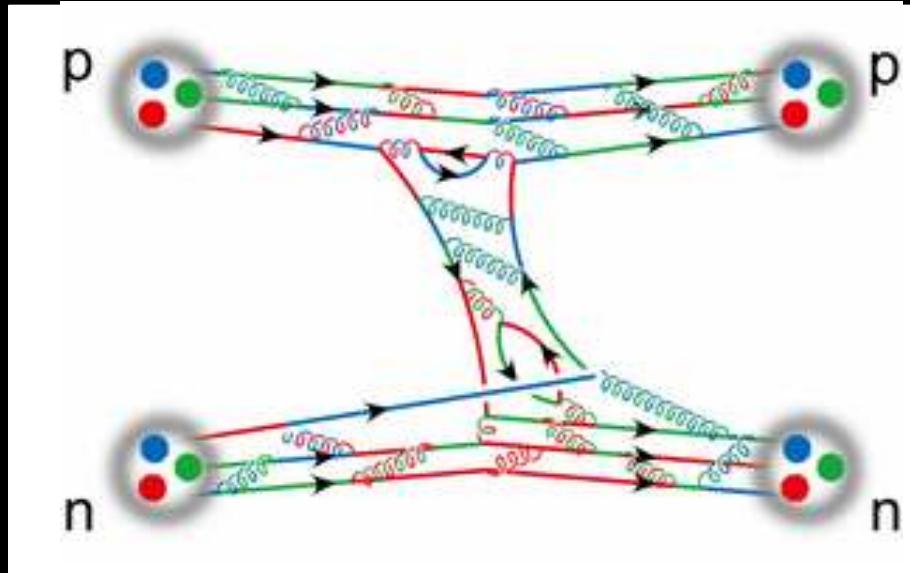


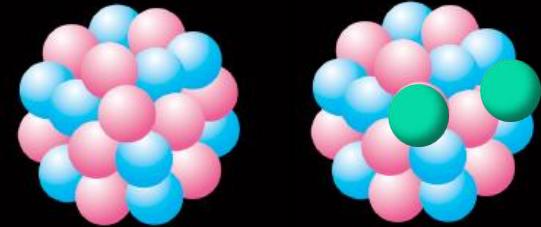
Nuclear Force from Lattice QCD



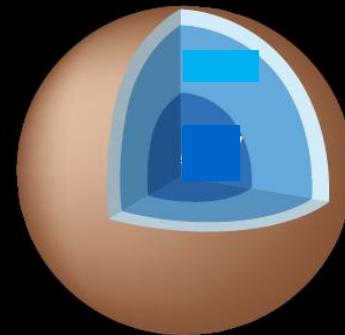
- [1] Why nuclear force now ?
- [2] NN force from lattice QCD
- [3] BB and BM forces from lattice QCD
- [4] Summary and Future

The nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei



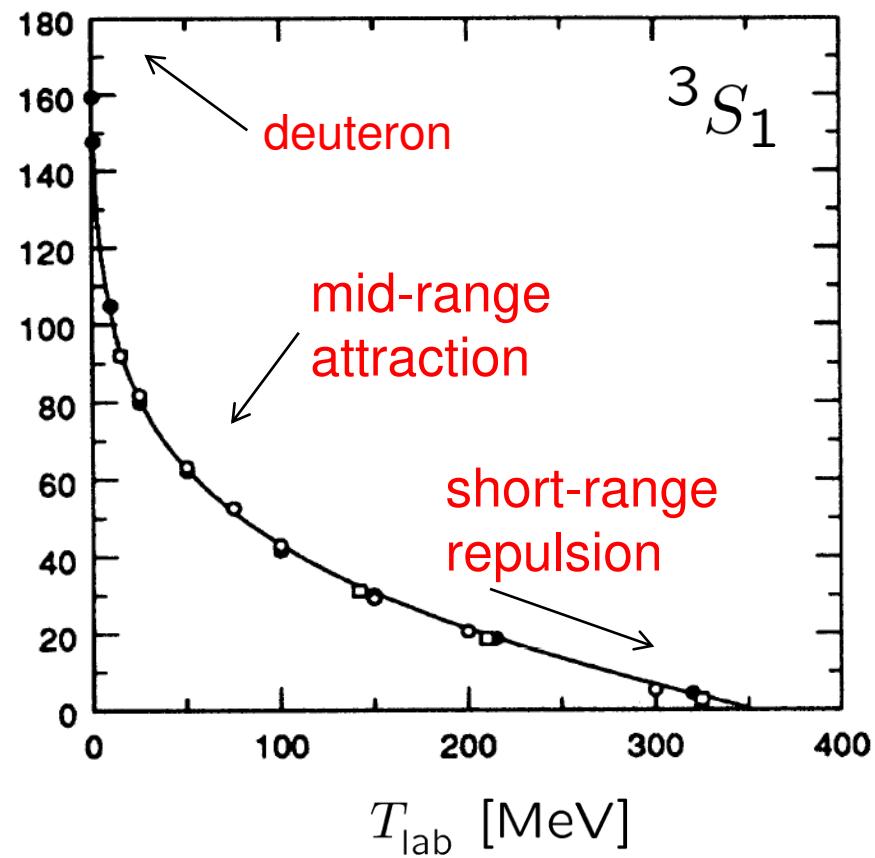
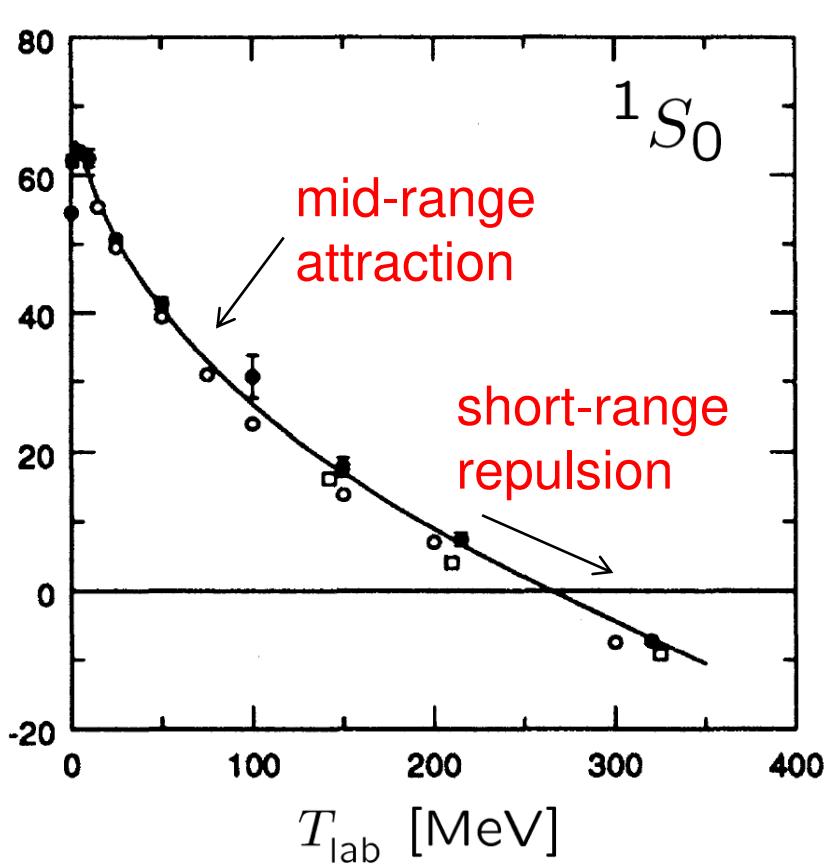
- Structure of neutron stars



- Ignition of Type II supernovae



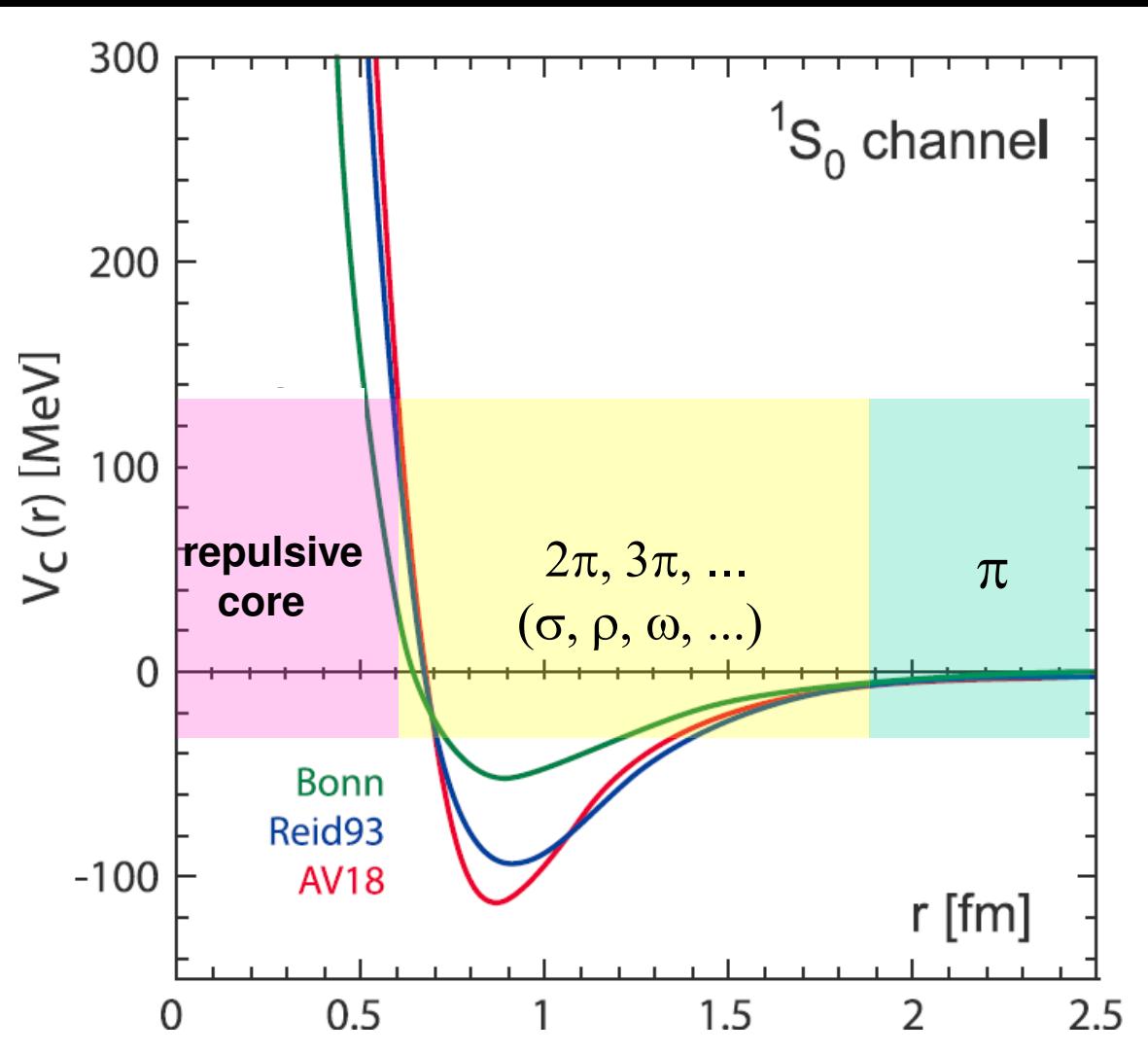
NN phase shifts



Nijmegen partial-wave analysis,
Stoks et al., Phys.Rev. C48 (1993) 792

Phenomenological NN potentials

(~40 parameters to fit 5000 phase shift data)



One-pion exchange
by Yukawa (1935)

Multi-pions
by Taketani et al. (1951)

Repulsive core
by Jastrow (1951)

Repulsive core : history

Phys. Rev. 81 (1951) 165

On the Nucleon-Nucleon Interaction*

ROBERT JASTROW**

Institute for Advanced Study, Princeton, New Jersey

(Received August 18, 1950)

A charge-independent interaction between nucleons is assumed, which is characterized by a short range repulsion interior to an attractive well. It is shown that it is then possible to account for the qualitative features of currently known $n-p$ and $p-p$ scattering data. Some of the implications for saturation are discussed.



Phys. Rev. 106 (1957) 1366

Possible Existence of a Heavy Neutral Meson*

YOICHIRO NAMBU

*The Enrico Fermi Institute for Nuclear Studies,
The University of Chicago, Chicago, Illinois*

(Received April 25, 1957)



ρ^0 would contribute a repulsive nuclear force of Wigner type and short range ($\lesssim 0.7 \times 10^{-13}$ cm), more or less similar to the phenomenological hard core.

ω-meson

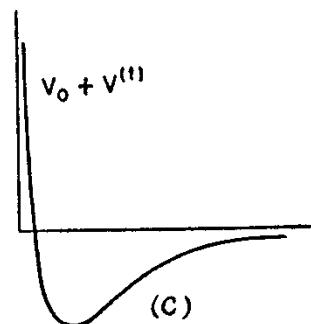
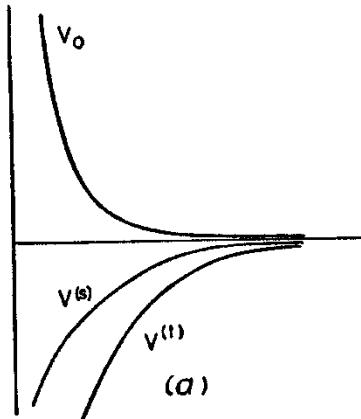
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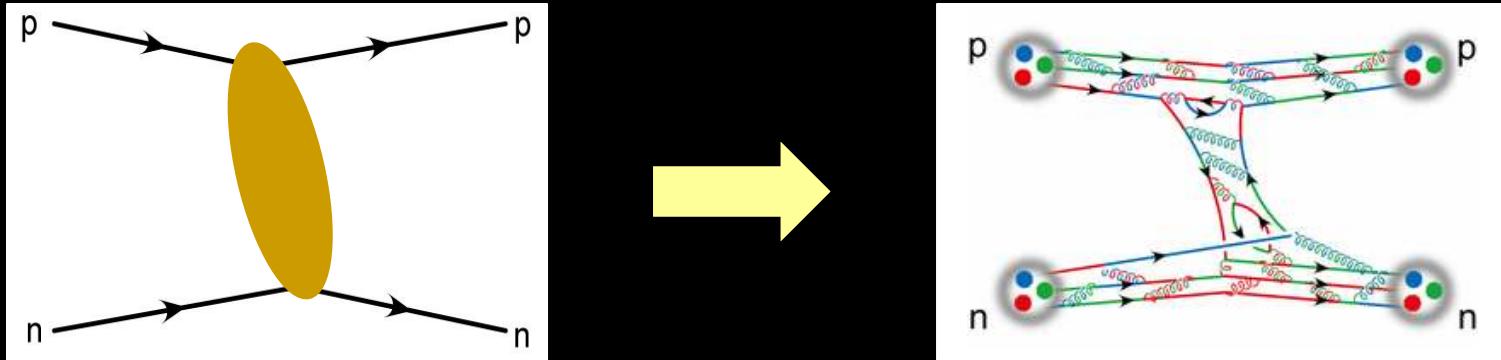


So I got up in the question period and I said, "Maybe the reason is that inside the nuclear force of attraction, which holds nuclei together, there's a very strong short-range force of repulsion, like a little hard sphere inside this attractive Jell-O."

I'll never forget, Oppenheimer got up, he liked to needle the young fellows and he said, very dryly, "Thank you so much for, we are grateful for every tiny scrap of help we can get." But

I ignored his needle and pursued my idea, and actually calculated the scattering of neutrons by protons. I showed that it fit the data very well. Oppenheimer read my paper for the Physical Review and took back his criticisms. This work became a permanent element of the literature of physics.

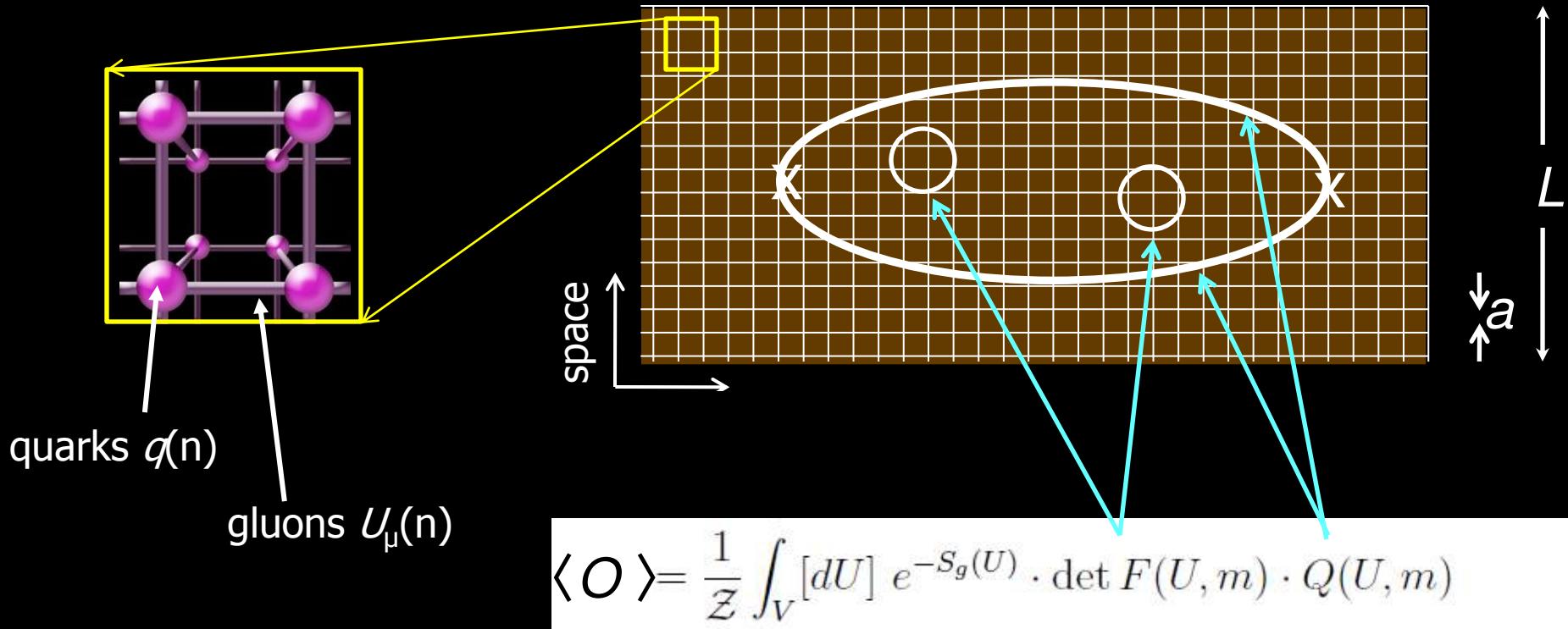
From Phenomenology to First Principle



Y. Nambu, "Quarks : Frontiers in Elementary Particle Physics", World Scientific (1985)

"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."

Lattice QCD

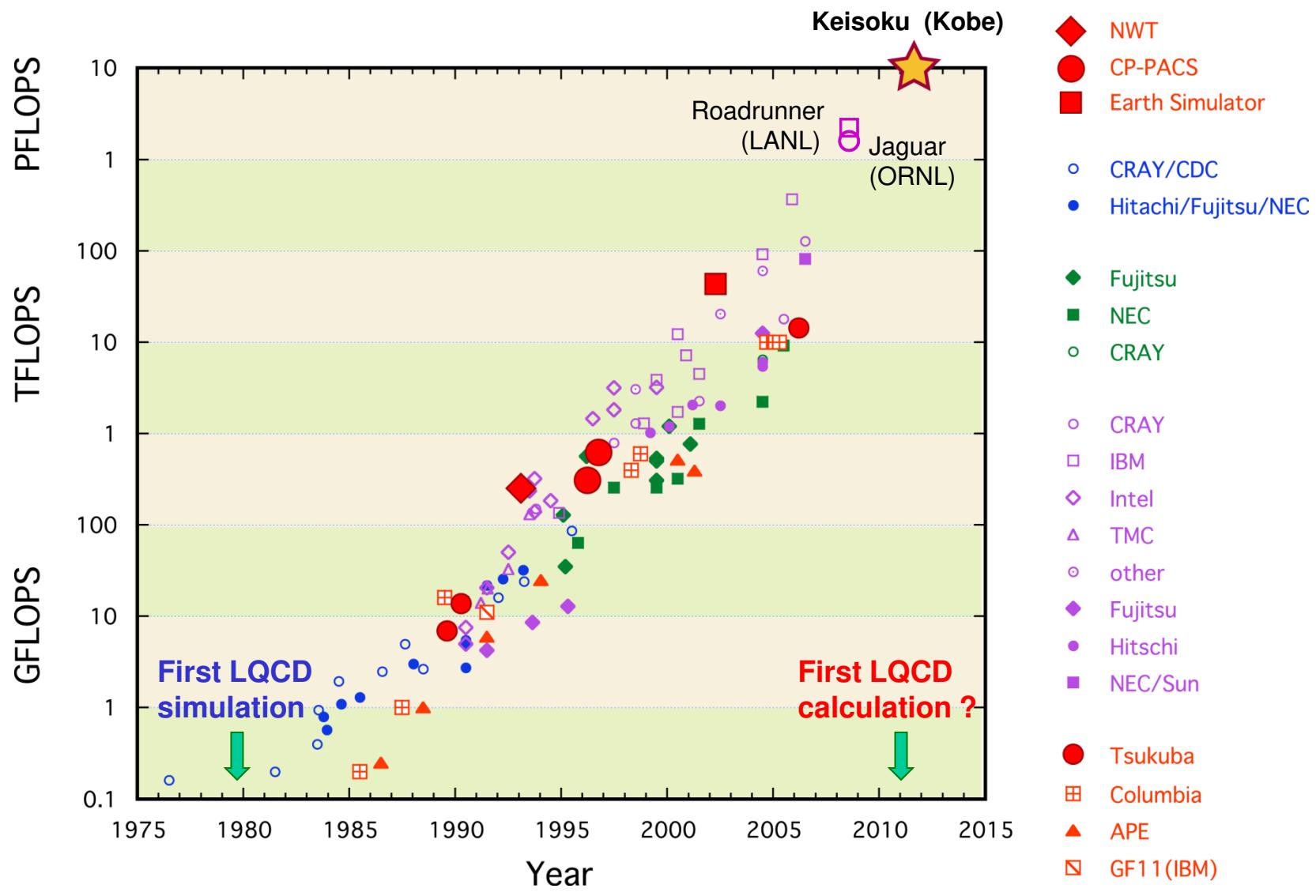


- well defined statistical system (finite a and L)
- gauge invariant
- fully non-perturbative

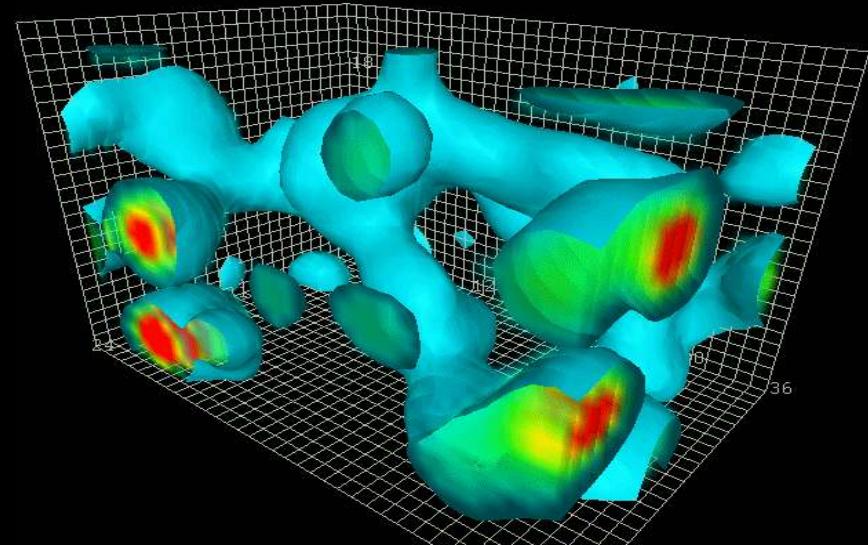
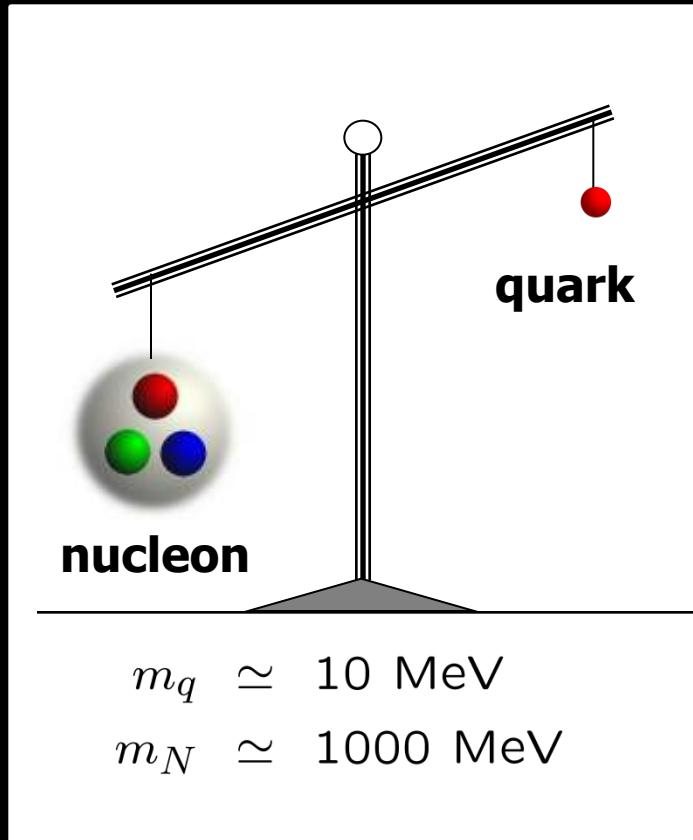


Monte Carlo simulations

Supercomputer peak performance

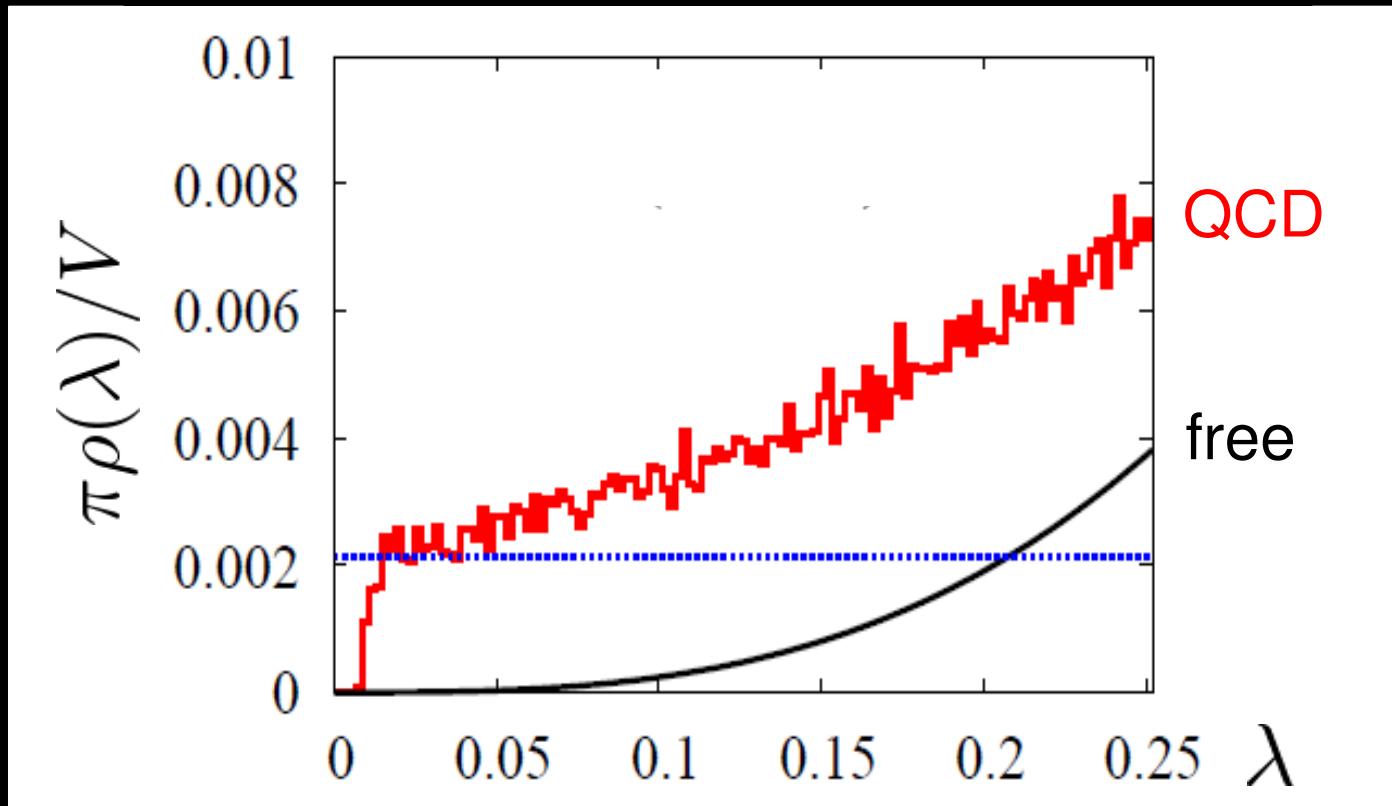


QCD Vacuum and Hadron Masses from Lattice QCD



- Chiral condensate
- Dynamical quark mass
- Hadron masses

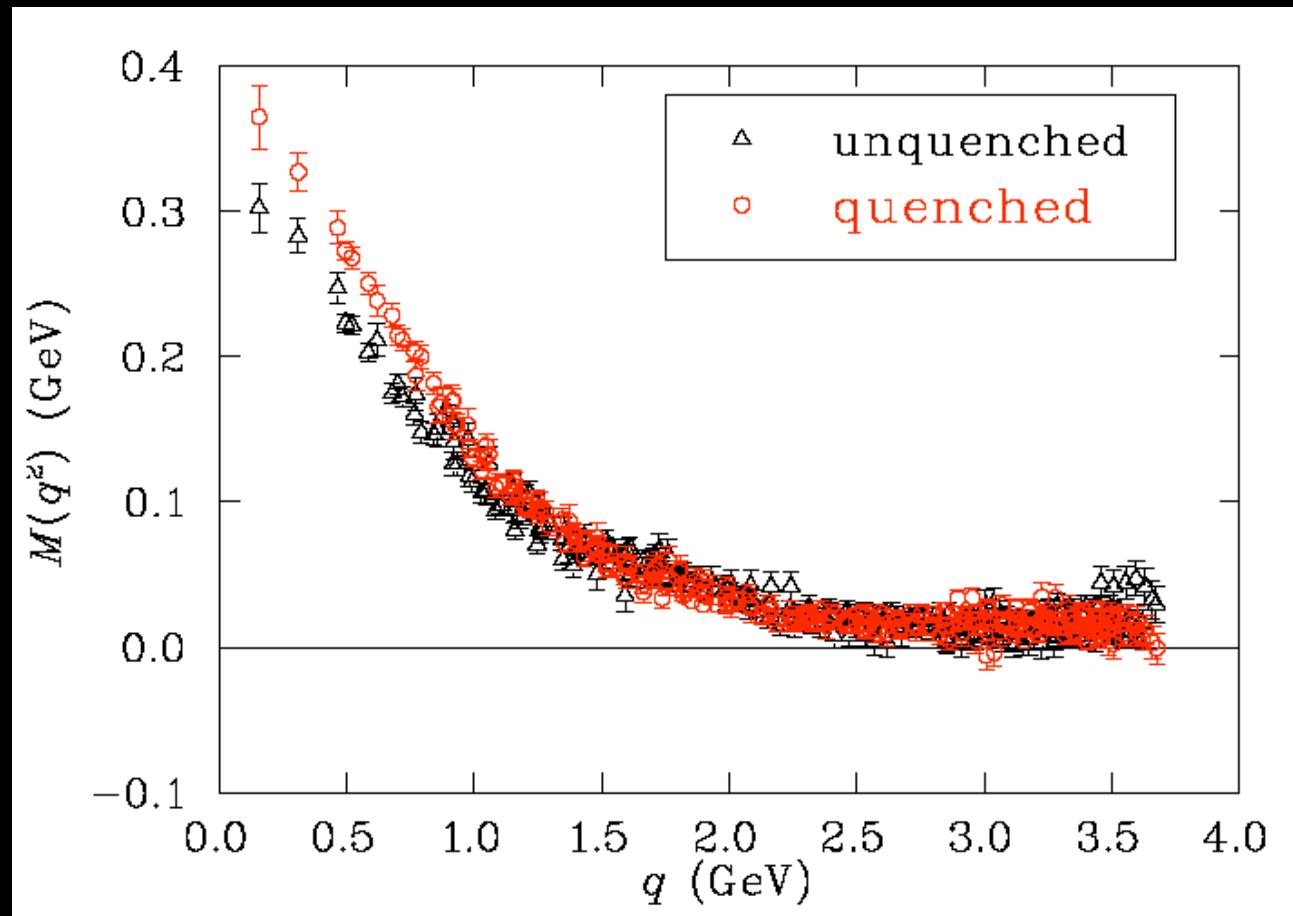
Chiral condensate in lattice QCD



$$\langle\bar{q}q\rangle = -(251 \pm 7 \pm 11 \text{ MeV})^3 \quad \text{at 2 GeV}$$

Dynamical quark-mass in lattice QCD

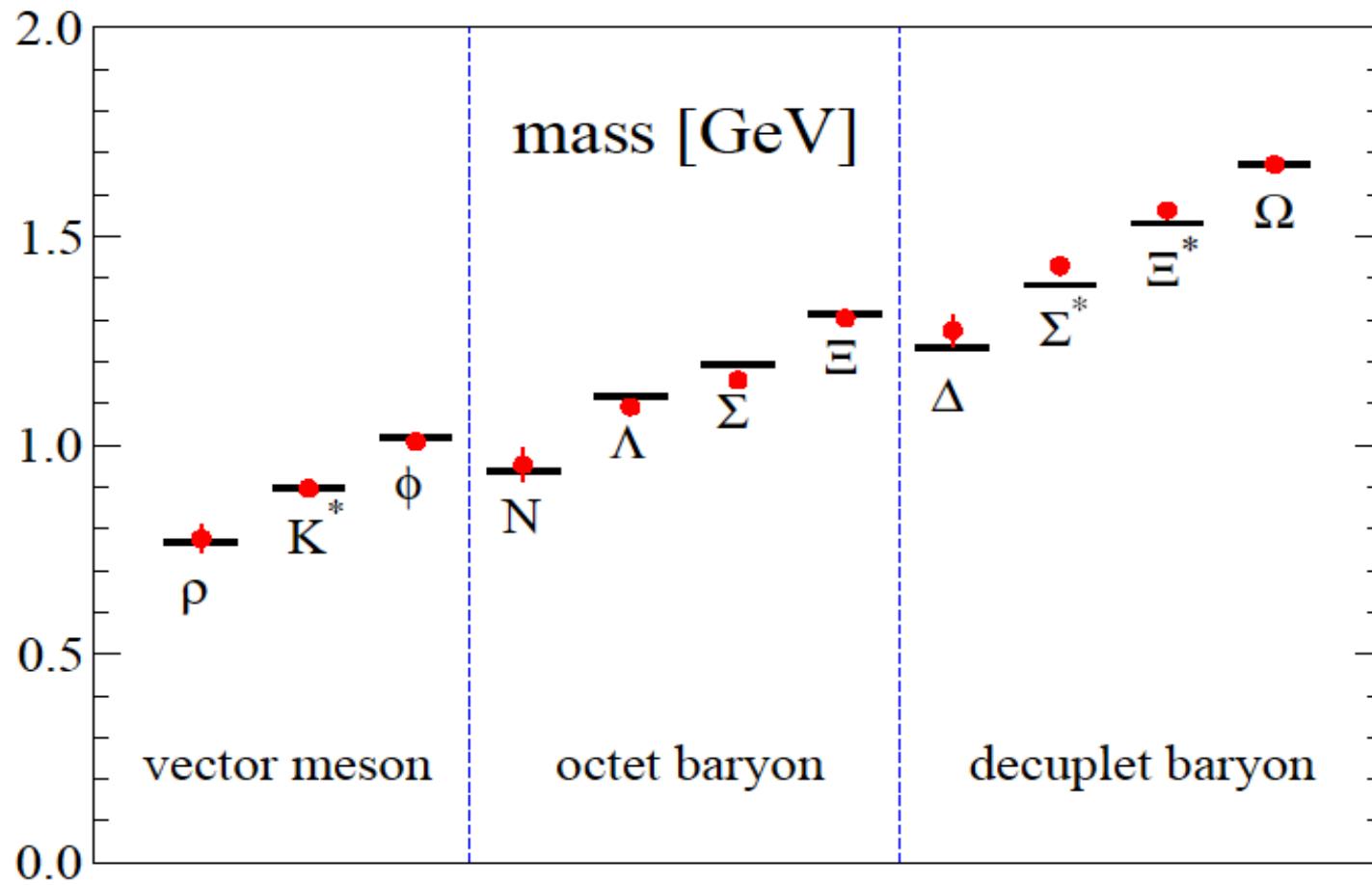
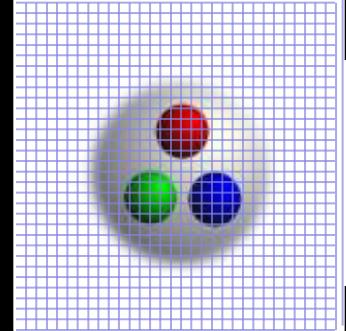
$$S^{-1}(q) = Z^{-1}(q) (i\gamma \cdot q + M(q))$$



$M(0) \simeq 300$ MeV

Landau gauge, Improved KS fermion, (2+1)-flavor
P. O. Bowman et al., Phys. Rev. D71 (2005) 054507

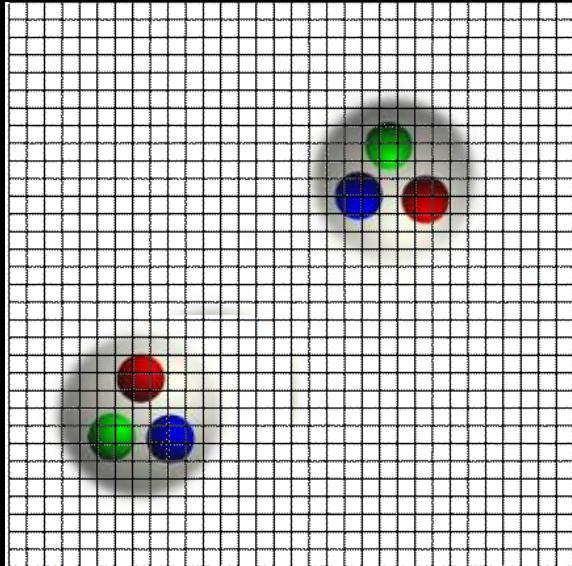
Hadron masses in lattice QCD



3% accuracy at present

Improved-Wilson, (2+1)-flavor,
 $L=2.9$ fm, $a=0.09$ fm, $32^3 \times 64$, (π, K, Ω) inputs,
 $m_{ud}(\text{min.}) = 3.5$ MeV $\Leftrightarrow m_\pi(\text{min.}) = 156$ MeV
PACS-CS Coll., Phys. Rev. D79 (2009) 034503

Nambu-Bethe-Salpeter Amplitude and Nuclear Forces in Lattice QCD



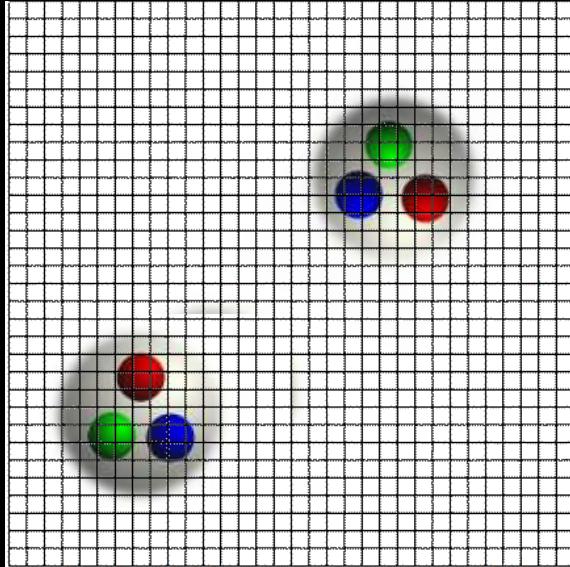
How to extract
the NN force at low energies
in lattice QCD ?



Y. Nambu,
“Force Potentials in Quantum Field Theory”,
Prog. Theor. Phys. 5 (1950) 614.

K. Nishijima,
“Formulation of Field Theories for Composite
Particles”, Phys. Rev. 111 (1958) 995.

Nambu-Bethe-Salpeter Amplitude and Nuclear Forces in Lattice QCD



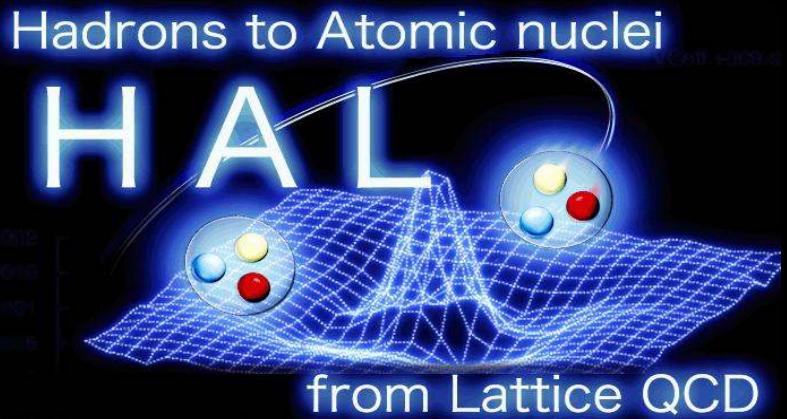
How to extract
the NN force at low energies
in lattice QCD ?



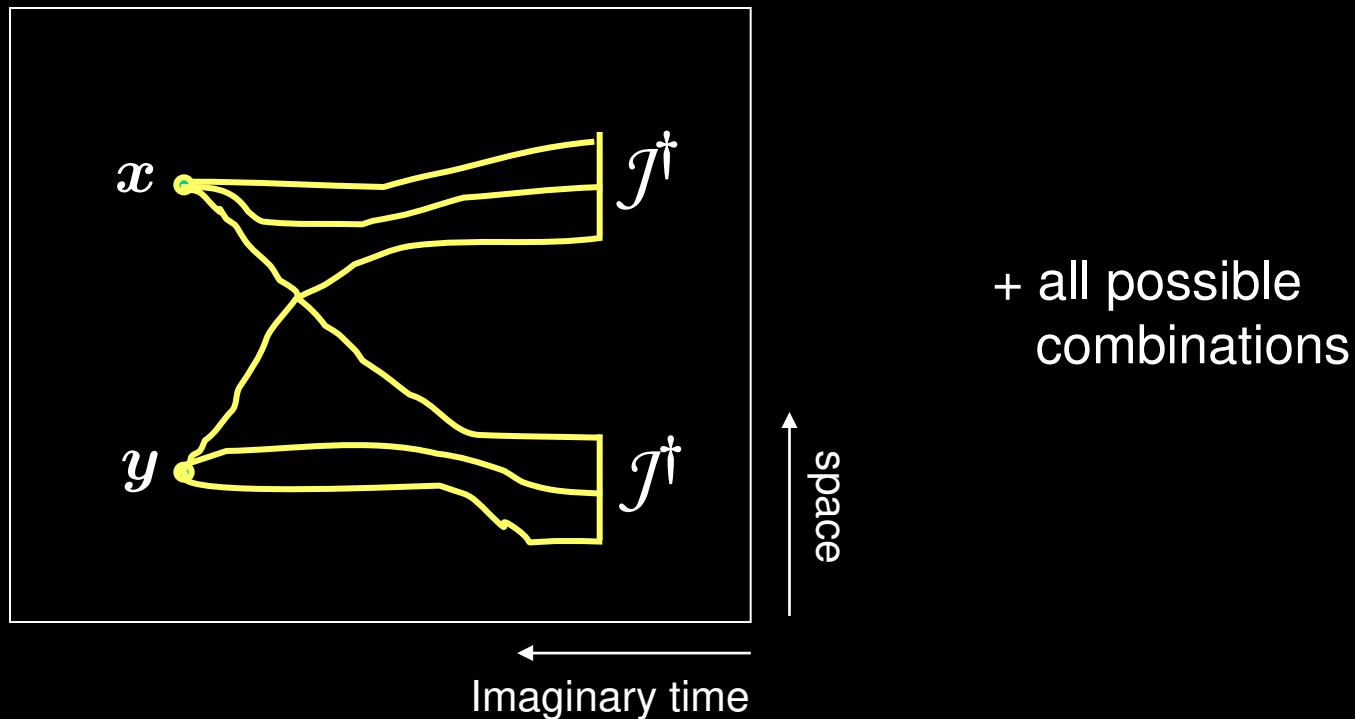
T. Hatsuda, Y. Ikeda, N. Ishii (Tokyo)

S. Aoki, T. Doi, T. Inoue,
K. Murano, K. Sasaki (Tsukuba)

H. Nemura (Tohoku)



Equal-time NBS amplitude $\phi(\mathbf{r})$ in lattice QCD



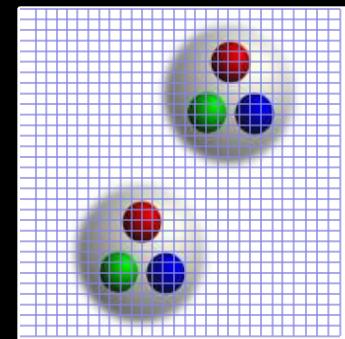
$$\begin{aligned} C_4(\mathbf{r}; t) &= \langle N_1(\mathbf{x}, t) N_2(\mathbf{y}, t) \mathcal{J}_1^\dagger(0) \mathcal{J}_2^\dagger(0) \rangle \\ &= \sum_n \langle 0 | N_1(\mathbf{x}) N_2(\mathbf{y}) | n \rangle A_n e^{-E_n t} \longrightarrow \phi(\mathbf{r}) A_0 e^{-E_0 t} \end{aligned}$$

$\phi(\mathbf{r} > R) \rightarrow$ phase shift : Luscher, Nucl. Phys. B354 (1991) 531

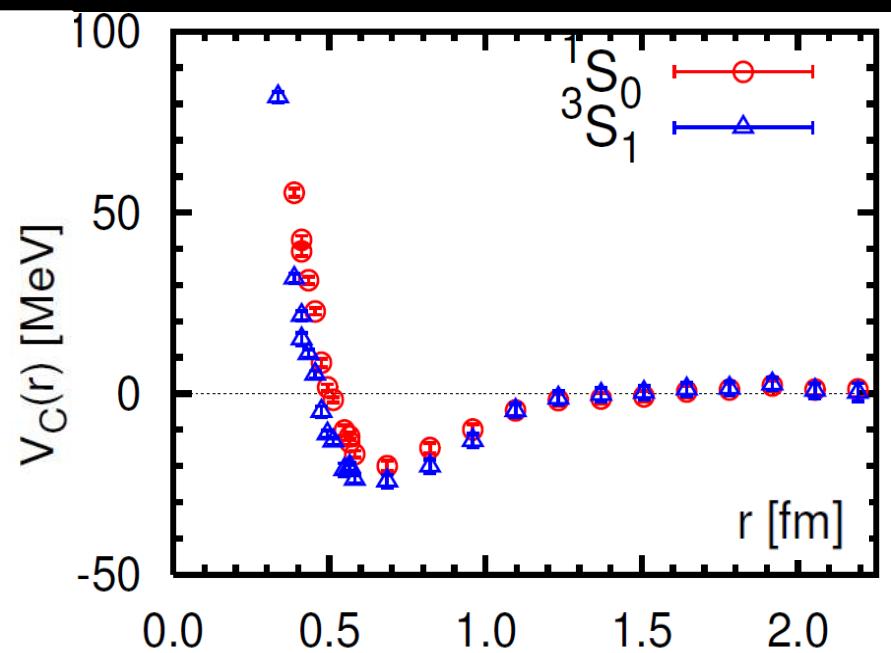
$\phi(\mathbf{r} < R) \rightarrow$ potential : Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

Central NN potentials from lattice QCD

wave function
↓
NN potential

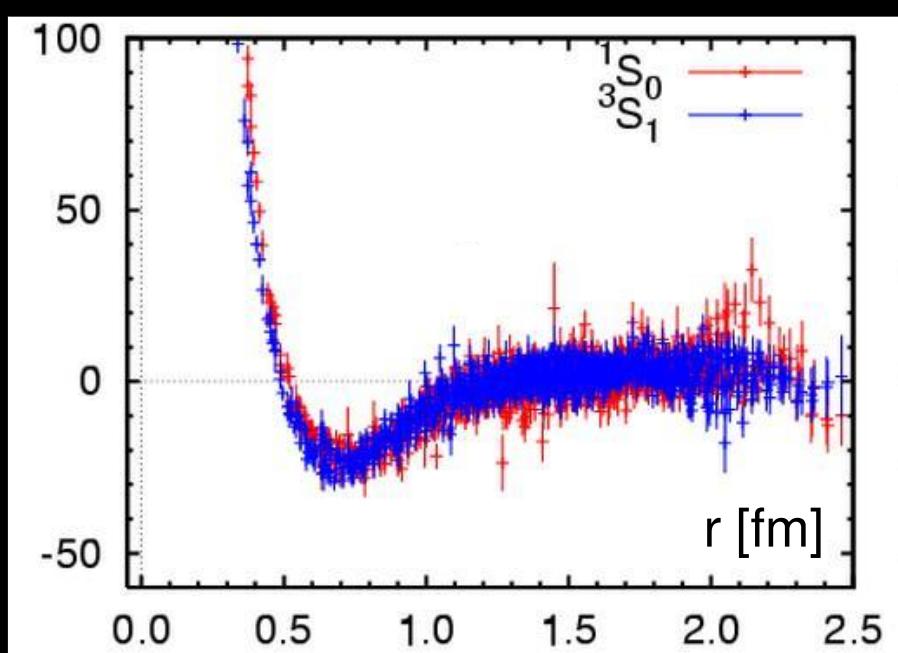


Quenched QCD
($m_\pi=530\text{MeV}$, $L=4.4 \text{ fm}$)



Ishii, Aoki & Hatsuda,
PRL 99 (2007) 022001

Full QCD
($m_\pi=570\text{MeV}$, $L=2.9 \text{ fm}$)



Ishii, Aoki & Hatsuda,
arXive 0903.5497 [hep-lat]

Systematic procedure to define the NN potential in lattice QCD

Full details: Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat], PTP 123 (2010) 89-128

(i) Choose your favorite operator: e.g. $N(x) = \epsilon_{abc} q^a(x) q^b(x) q^c(x)$

- observables do not depend on the choice
- yet the local operator is useful

Nishijima, Haag, Zimmermann (1958)

(ii) Measure the NBS amplitude: $\phi(\vec{r}) = \langle 0 | N(\vec{x} + \vec{r}) N(\vec{x}) | 6q \rangle$

(iii) Define the non-local potential: $(E - H_0)\phi(\vec{r}) = \int U(r, \vec{r}')\phi(\vec{r}')d^3r'$

(iv) Velocity expansion : $U(\vec{r}, \vec{r}') = V(\vec{r}, \nabla)\delta^3(\vec{r} - \vec{r}')$

$$V(\vec{r}, \nabla) = V_C(r) + S_{12}V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{V_D(r), \nabla^2\} + \dots$$

LO

LO

NLO

NNLO

Okubo-Marshak (1958), Tamagaki-Watari (1967)

(v) Calculate observables : phase shifts, binding energies etc

Key channels in NN scattering ($^{2s+1}L_J$)

$$V(\vec{r}, \nabla) = V_C(r) + S_{12}V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{V_D(r), \nabla^2\} + \dots$$

LO

LO

NLO

NNLO

1S_0

Central force \longleftrightarrow nuclear BCS pairing

Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

3S_1 - 3D_1

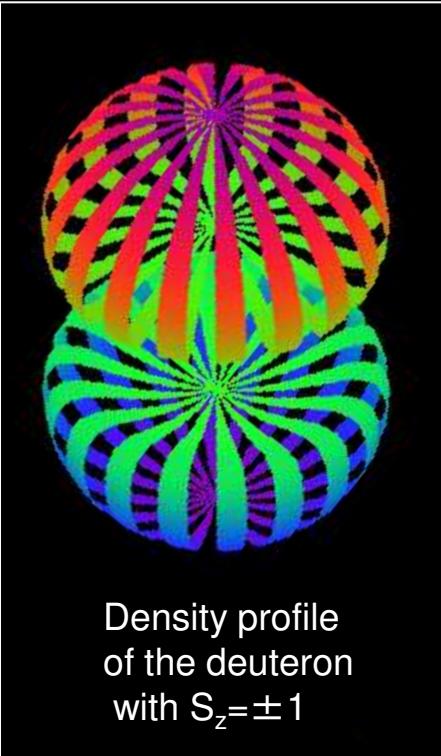
Tensor force \longleftrightarrow deuteron binding

Pandharipande et al., Phys. Rev. C54 (1996)

3P_2 - 3F_2

LS force \longleftrightarrow neutron superfluidity
in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)



[Q1] Operator dependence of the lattice potential

[Q2] Energy dependence of the lattice potential

[A1] $(N(x), U(x,x'))$ is a combination to define observables

- remember,

QM : $(\Phi, U) \sim (\Phi', U) \rightarrow$ observables

QFT : (asymptotic field, vertices) \rightarrow observables

EFT : (choice of field, vertices) \rightarrow observables

- local operator = a convenient choice for reduction formula

[A2] $U(x,x')$ is E-independent by construction

- non-locality can be determined order by order in velocity expansion (c.f. ChPT)

LO potential (central & tensor) in lattice QCD

- For $J^P = 1^+$, $|\phi\rangle$ comprises **S-wave** and **D-wave**,

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

where,

$$|\phi_S\rangle = \mathcal{P} |\phi\rangle = (1/24) \sum_{\mathcal{R} \in O} \mathcal{R} |\phi\rangle$$

$$|\phi_D\rangle = \mathcal{Q} |\phi\rangle = (1 - \mathcal{P}) |\phi\rangle$$

- Therefore, we have 2-component Schrödinger eq.

S-wave:

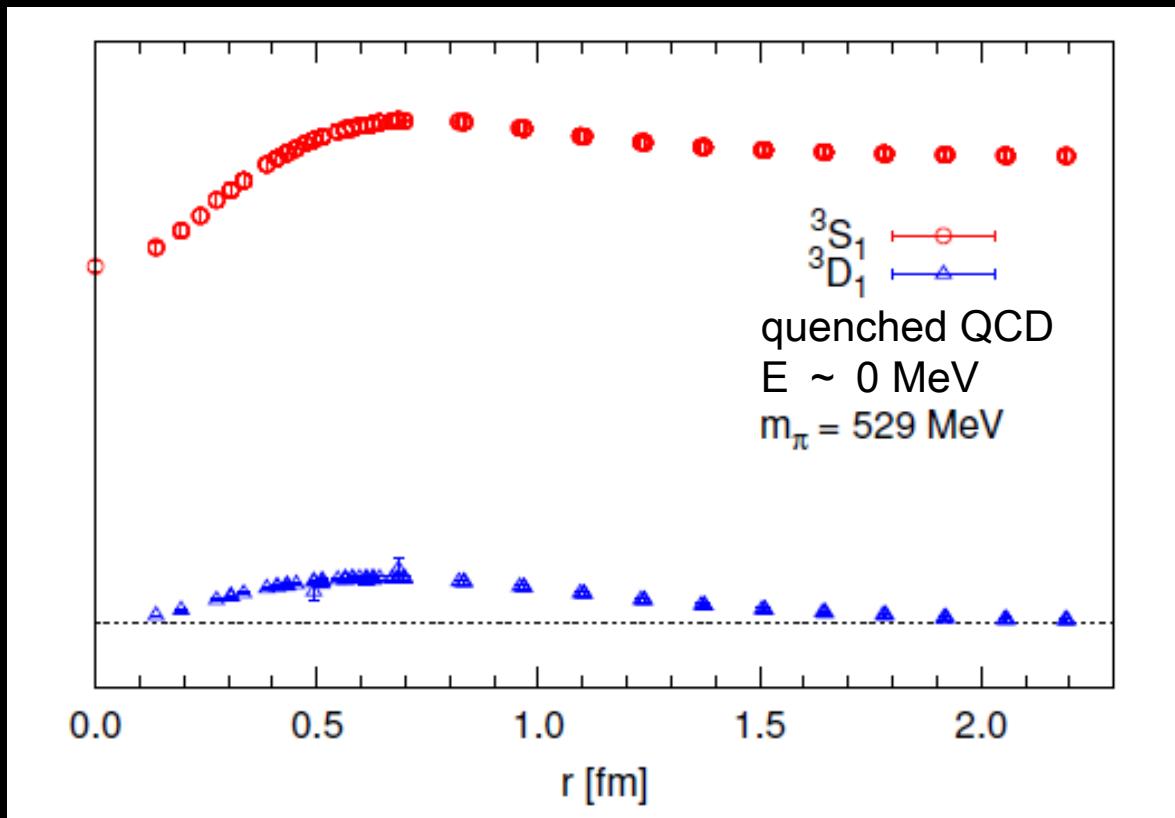
$$\mathcal{P} (T + V_C + V_T S_{12}) |\phi\rangle = E \mathcal{P} |\phi\rangle$$

D-wave:

$$\mathcal{Q} (T + V_C + V_T S_{12}) |\phi\rangle = E \mathcal{Q} |\phi\rangle$$

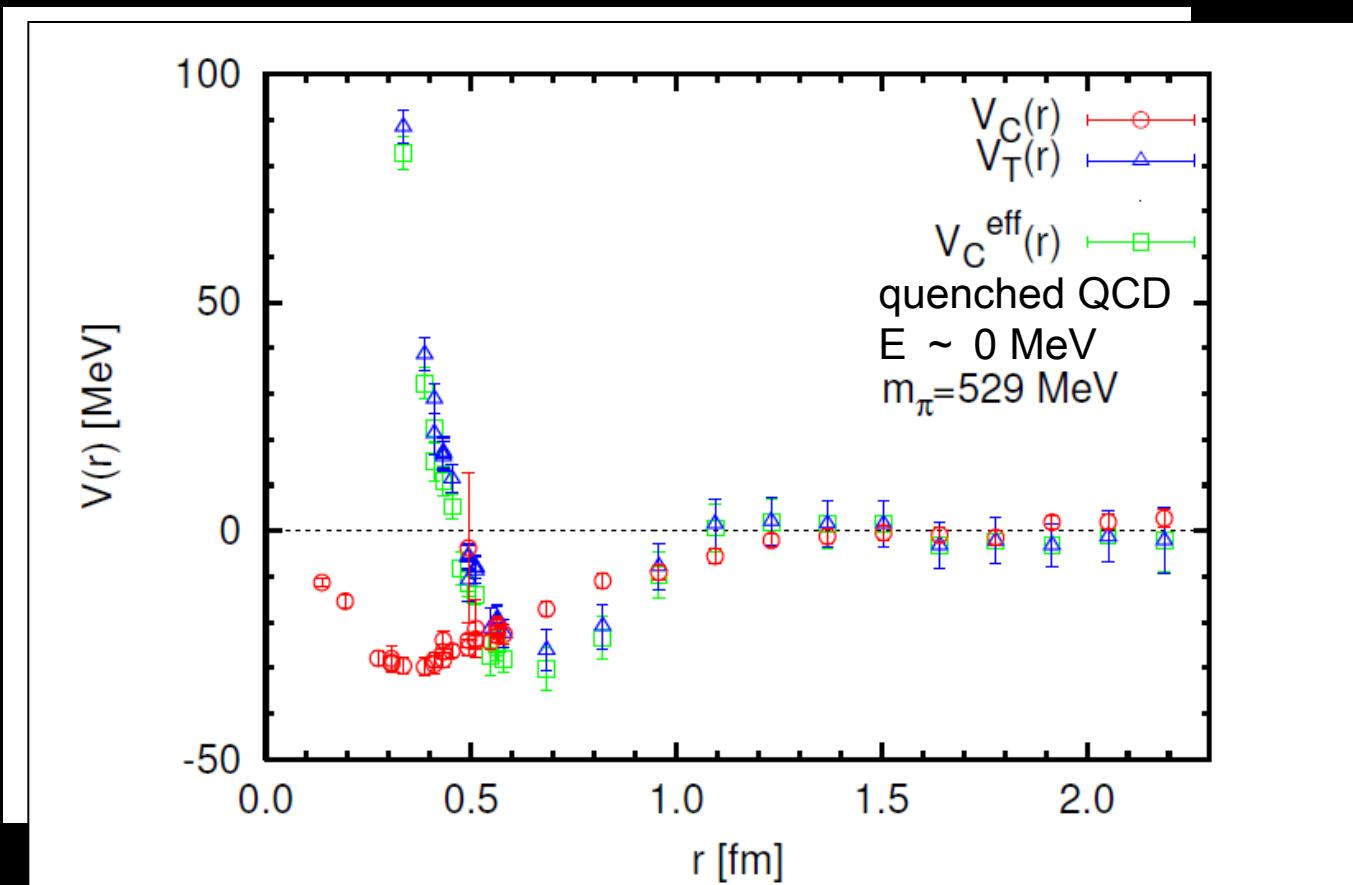
Central & tensor potentials : $V_C(r)$ & $V_T(r)$

Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
PTP 123 (2010) 89-128



Central & tensor potentials : $V_C(r)$ & $V_T(r)$

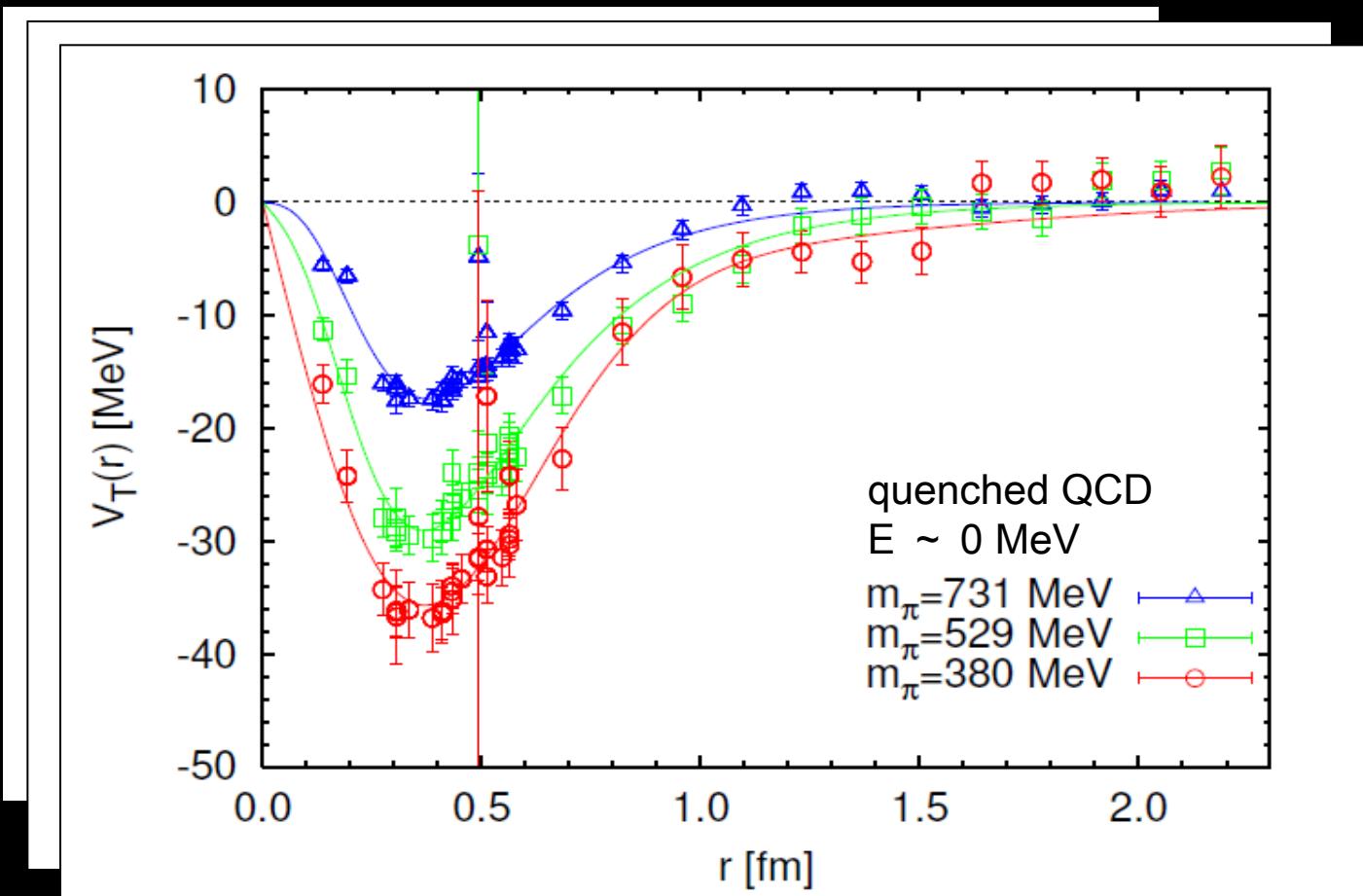
Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
PTP 123 (2010) 89-128



$V_c(r \rightarrow 0) \sim (\log r)^\beta / r^2$, $V_T(r \rightarrow 0) \rightarrow 0$
from operator product expansion
(Aoki, Balog & Weisz, arXiv:1002.0977)

Central & tensor potentials : $V_C(r)$ & $V_T(r)$

Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
PTP 123 (2010) 89-128



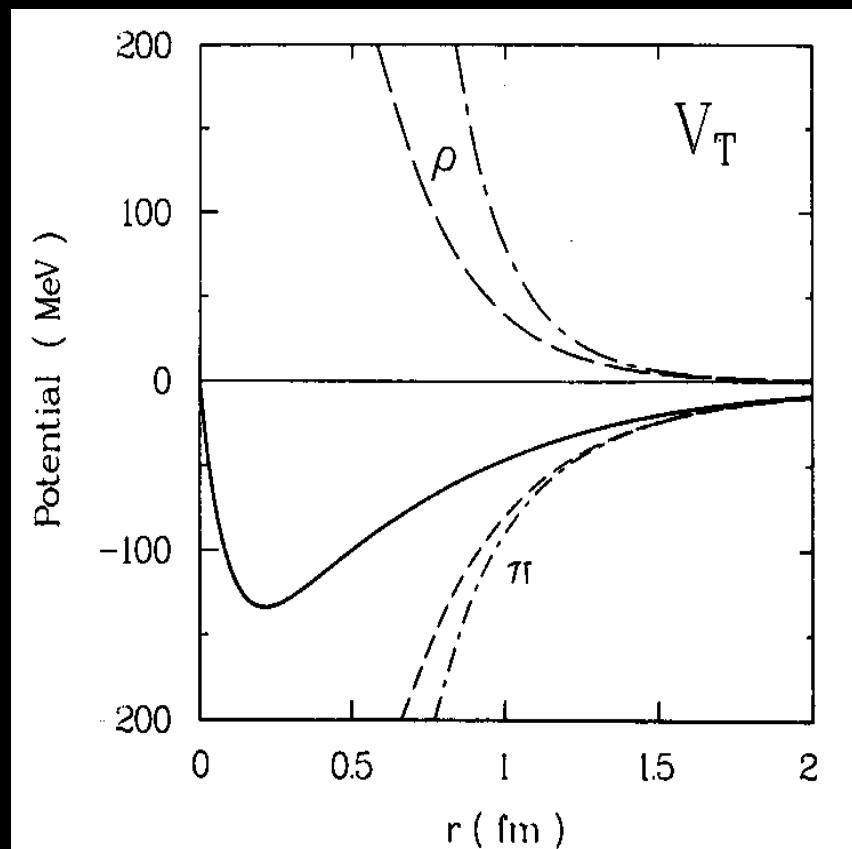
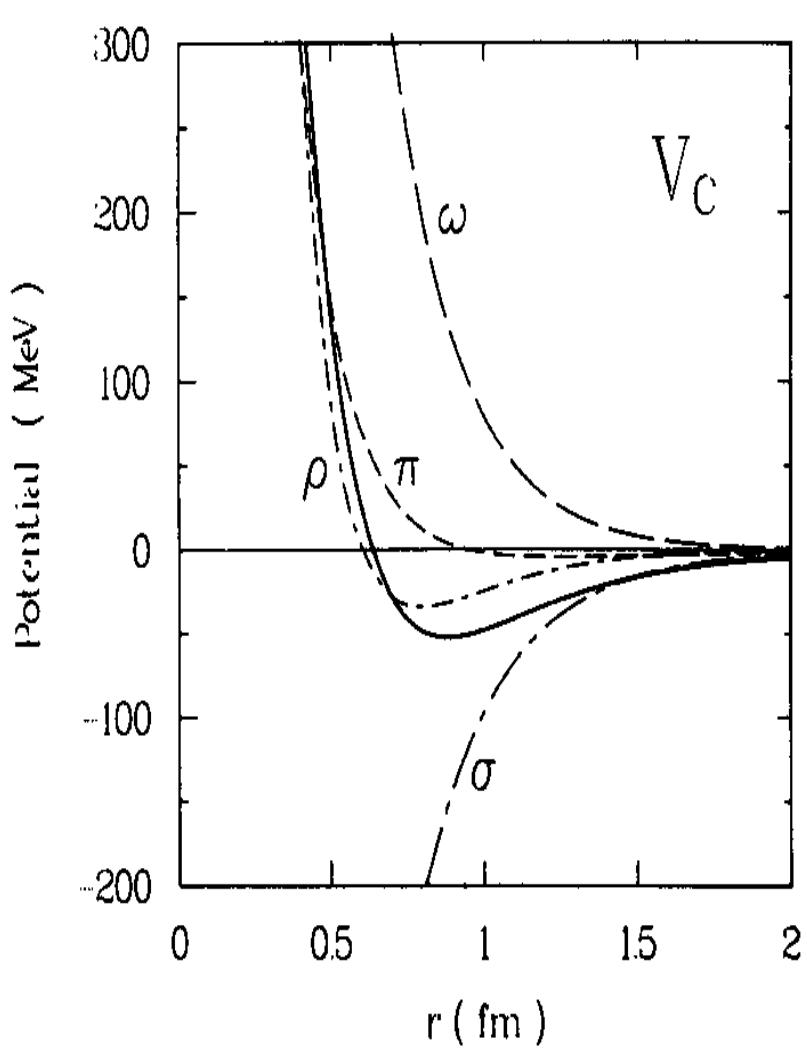
- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

fit function

$$V_T(r) = b_1(1 - e^{-b_2 r^2})^2 \left(1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right) \frac{e^{-m_\rho r}}{r}$$

$$+ b_3(1 - e^{-b_4 r^2})^2 \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{e^{-m_\pi r}}{r},$$

One boson exchange model : $V_C(r)$ & $V_T(r)$

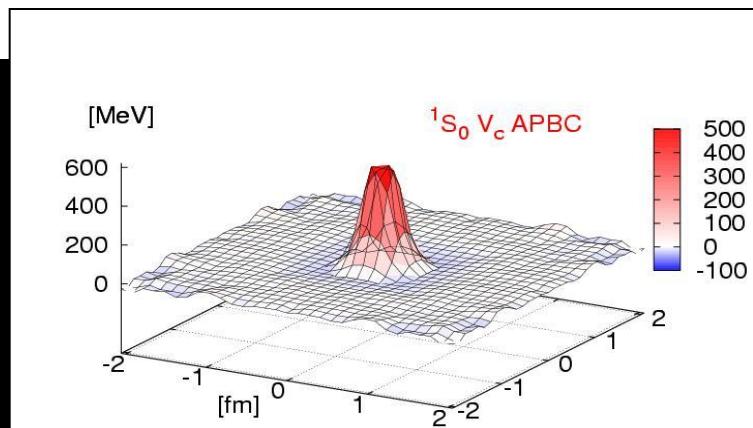
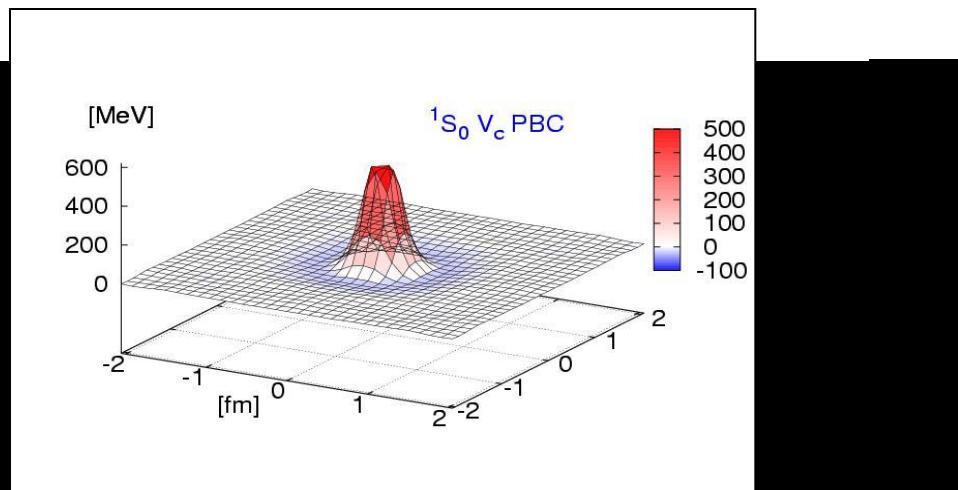
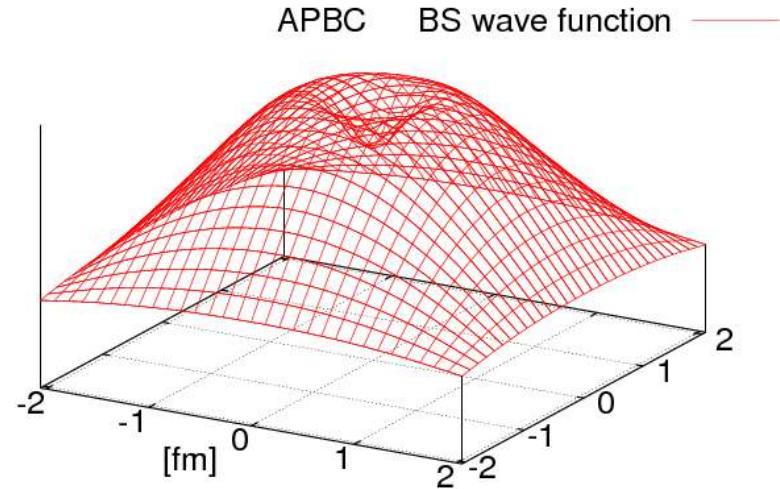
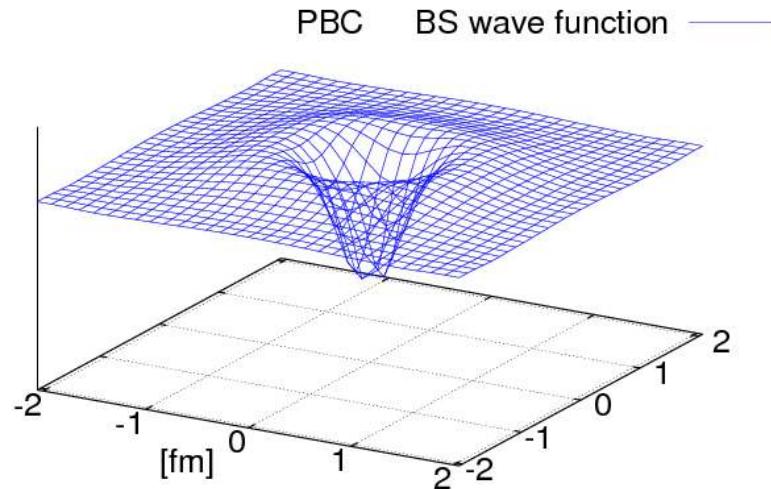


velocity dependence of the potential (NNLO)

$$V(\vec{r}, \nabla) = V_C(r) + S_{12}V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{V_D(r), \nabla^2\} + \dots$$

● PBC (E~0 MeV)

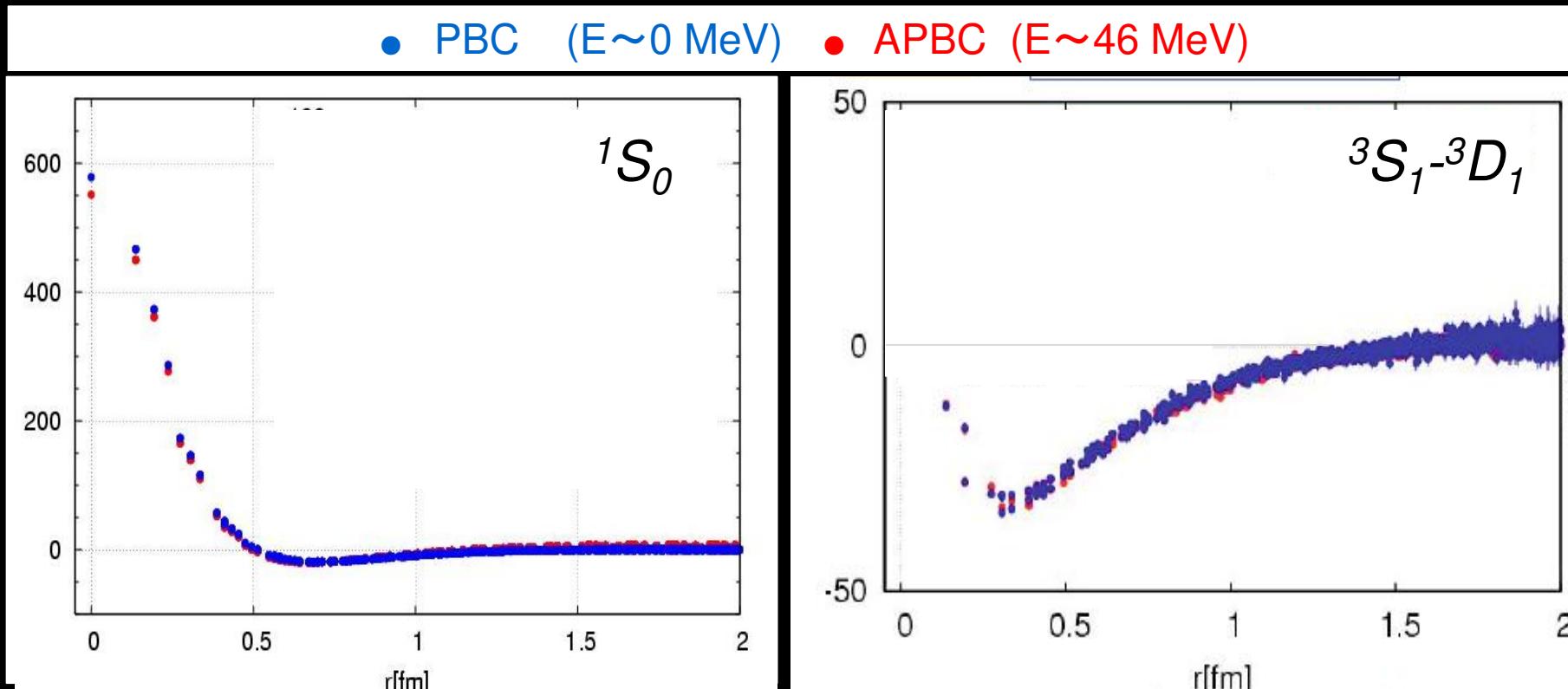
● APBC (E~46 MeV)



velocity dependence of the potential (NNLO)

$$V(\vec{r}, \nabla) = V_C(r) + S_{12} V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{V_D(r), \nabla^2\} + \dots$$

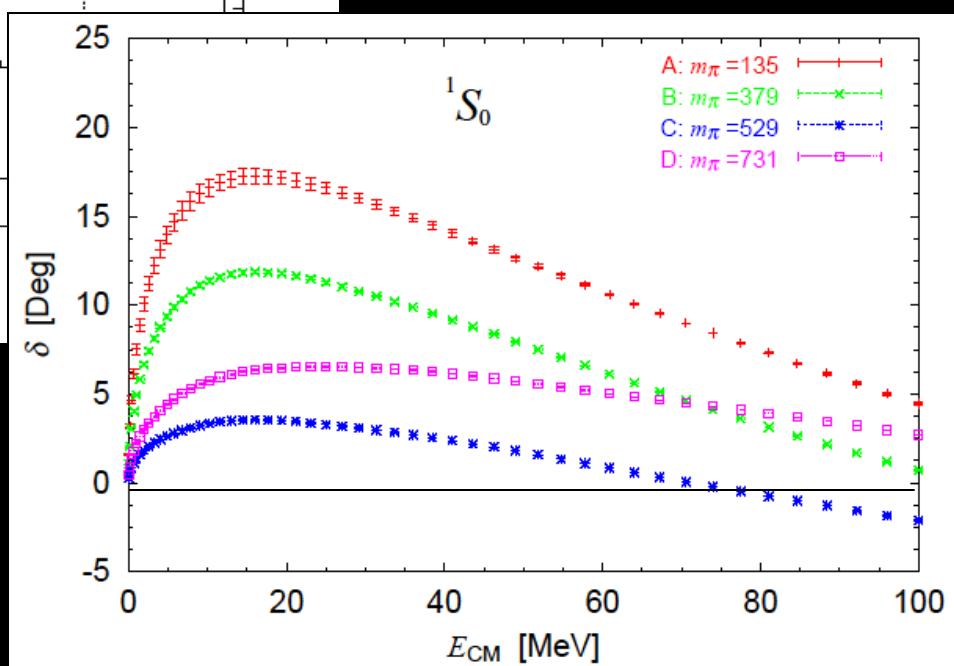
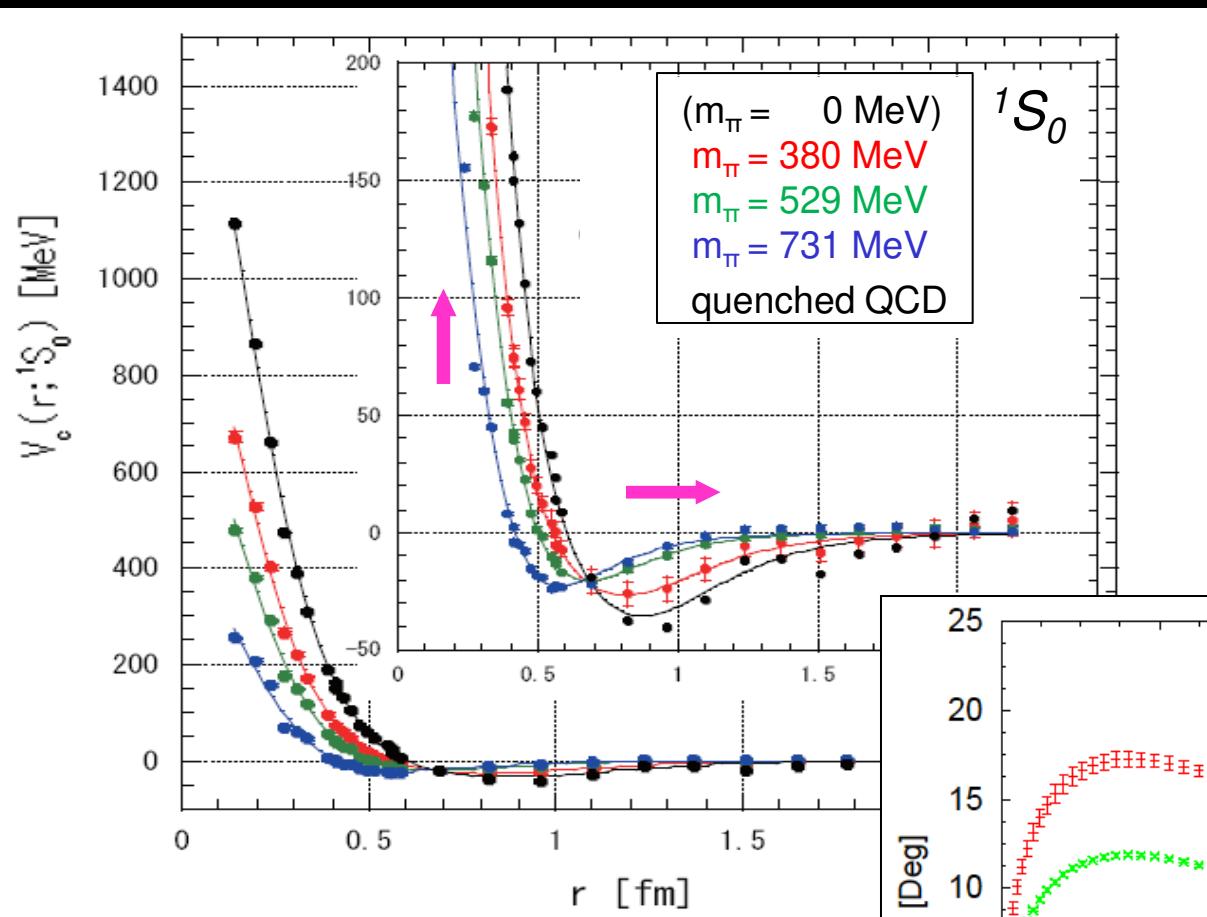
NNLO



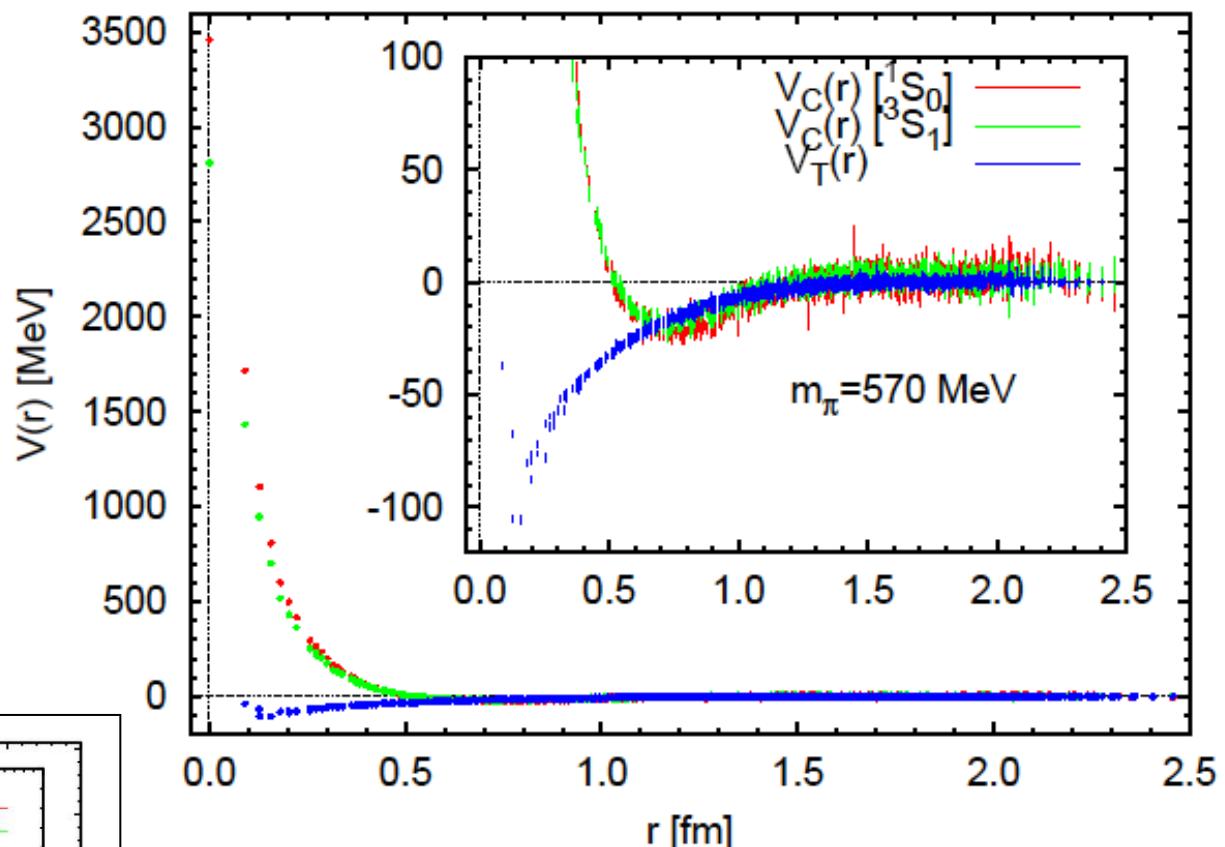
- NNLO can be determined from $\varphi(r)$ for different E
- NNLO is small at least up to $E_{cm} \sim 46$ MeV ($T_{lab} \sim 100$ MeV)

quenched QCD
 $m_\pi = 529$ MeV

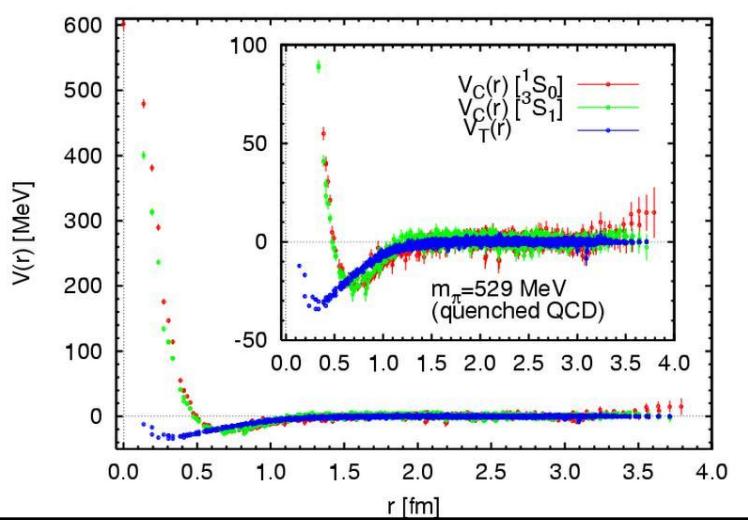
Phase shift $\delta_0(k)$ from $V_c(r)$ in 1S_0



$V_C(r)$ and $V_T(r)$ in full QCD ($m_\pi=570$ MeV, $L=2.9$ fm)

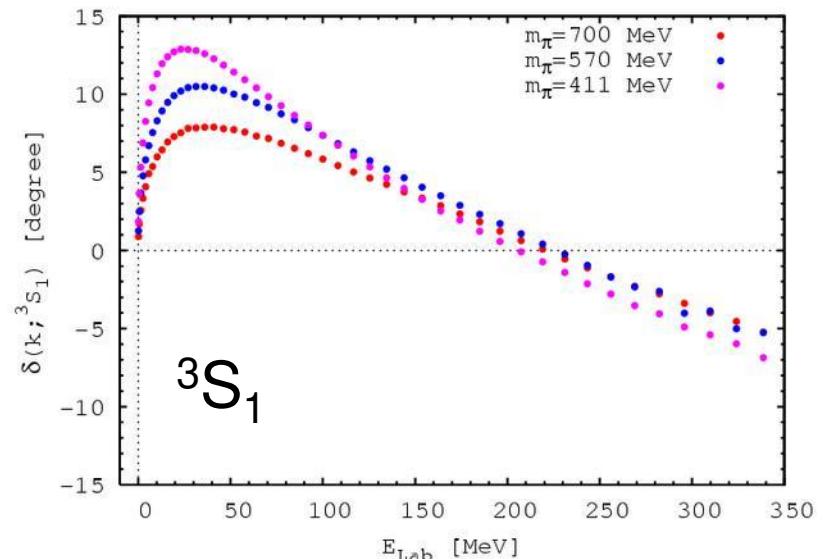
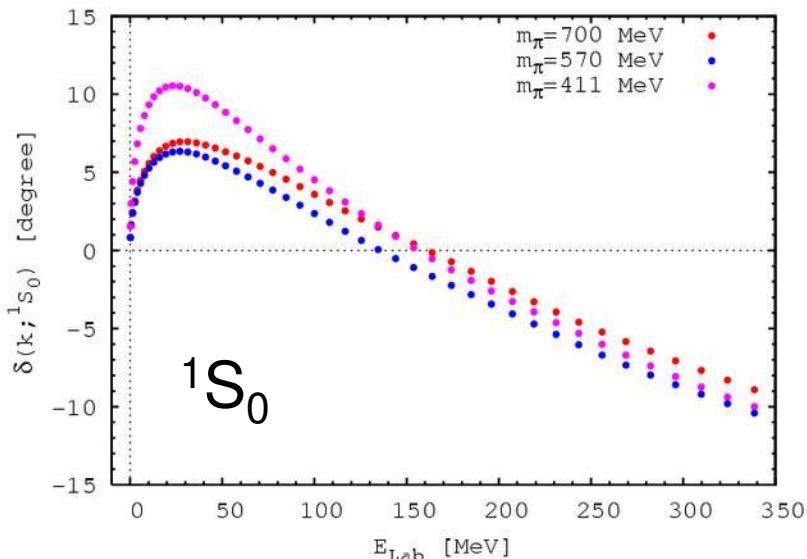
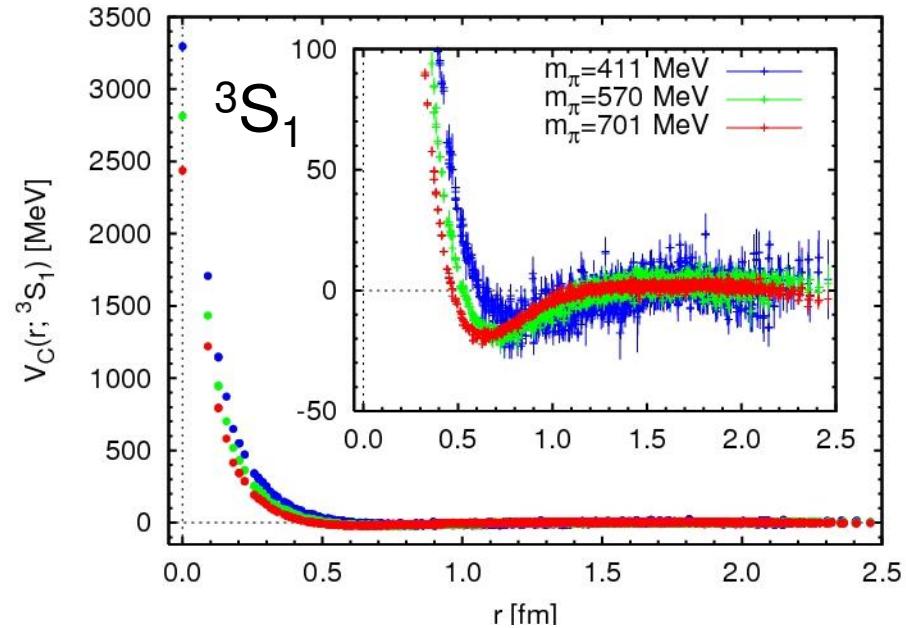
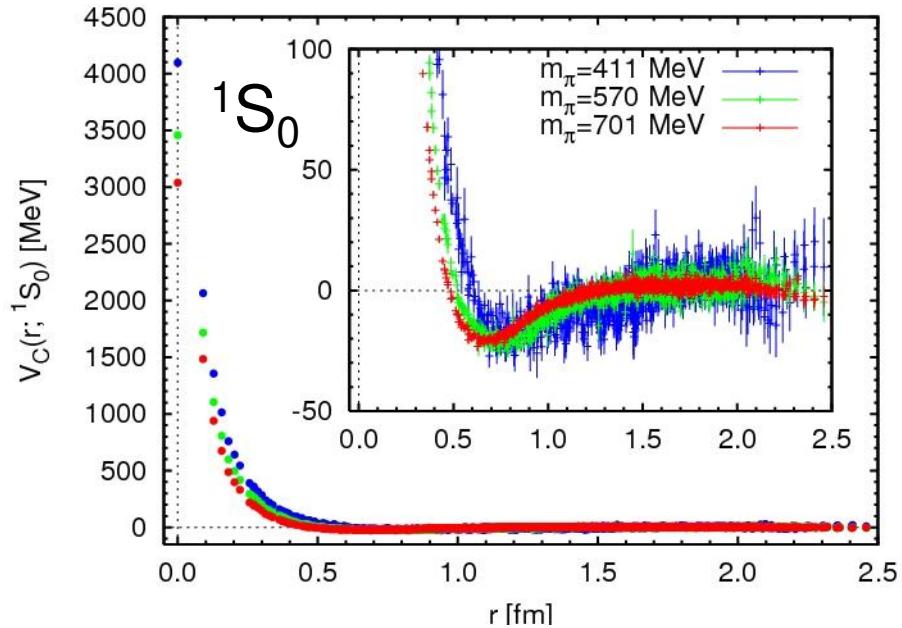


Quenched QCD



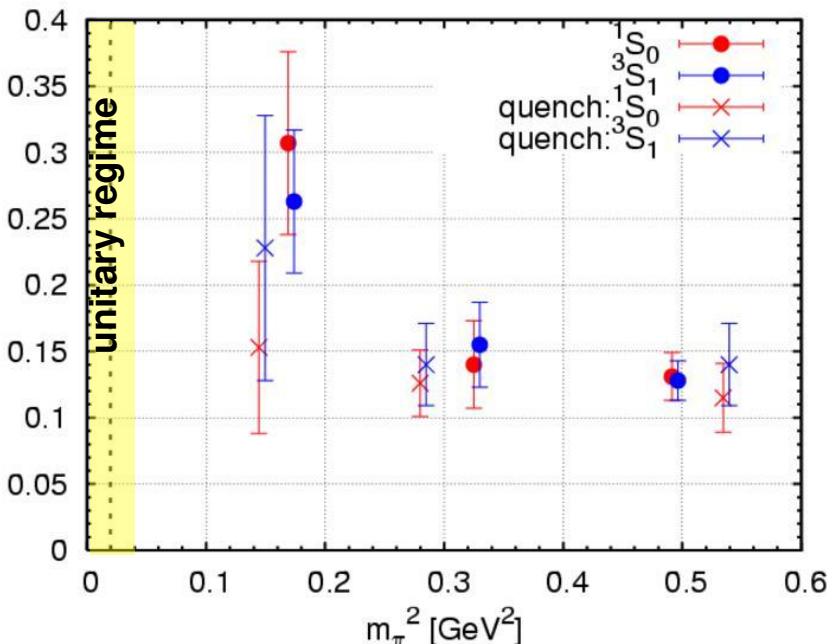
Full QCD

Phase shift from $V(r)$ in full QCD

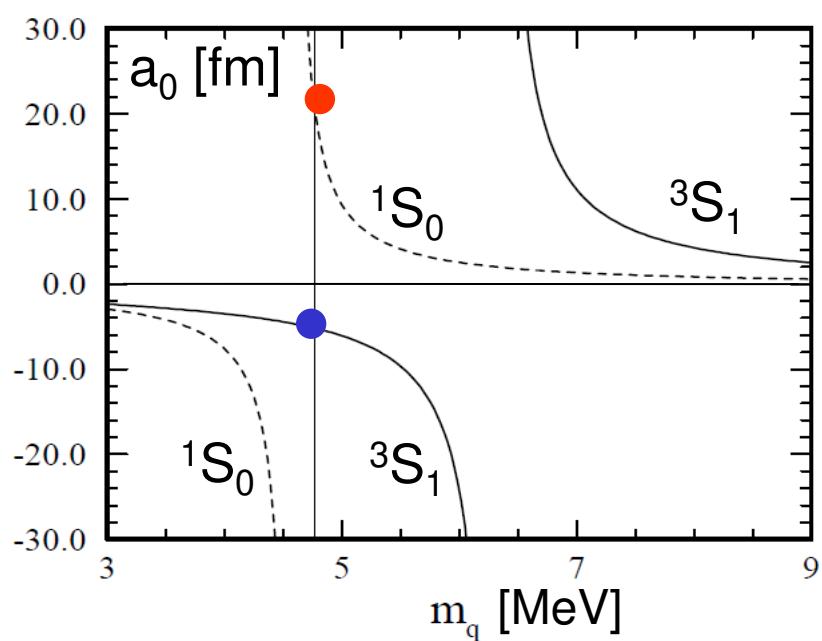


NN scattering lengths in full QCD

BS wave func. $\rightarrow E \rightarrow$ Luscher's formula
(CP-PACS method, 2005)



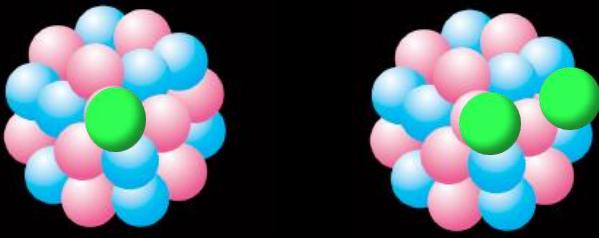
- Overall attraction
- Still far from “unitary regime”



YN and YY interactions in lattice QCD

$$\begin{array}{c} 8 \\ \square \end{array} \otimes \begin{array}{c} 8 \\ \square \end{array} = \begin{array}{c} 27 \\ \square \end{array} \oplus \begin{array}{c} 10^* \\ \square \end{array} \oplus \begin{array}{c} 1 \\ \square \end{array} \oplus \begin{array}{c} 8 \\ \square \end{array} \oplus \begin{array}{c} 10 \\ \square \end{array} \oplus \begin{array}{c} 8 \\ \square \end{array}$$

The diagram shows the tensor product of two 8-dimensional representations (8x8 grid) resulting in a direct sum of several representations. The 27 and 10* representations have red boxes at their bottom-left corners.



- no phase shifts available for YN and YY scatterings
- plenty of hyper-nucleus data will soon be available at J-PARC



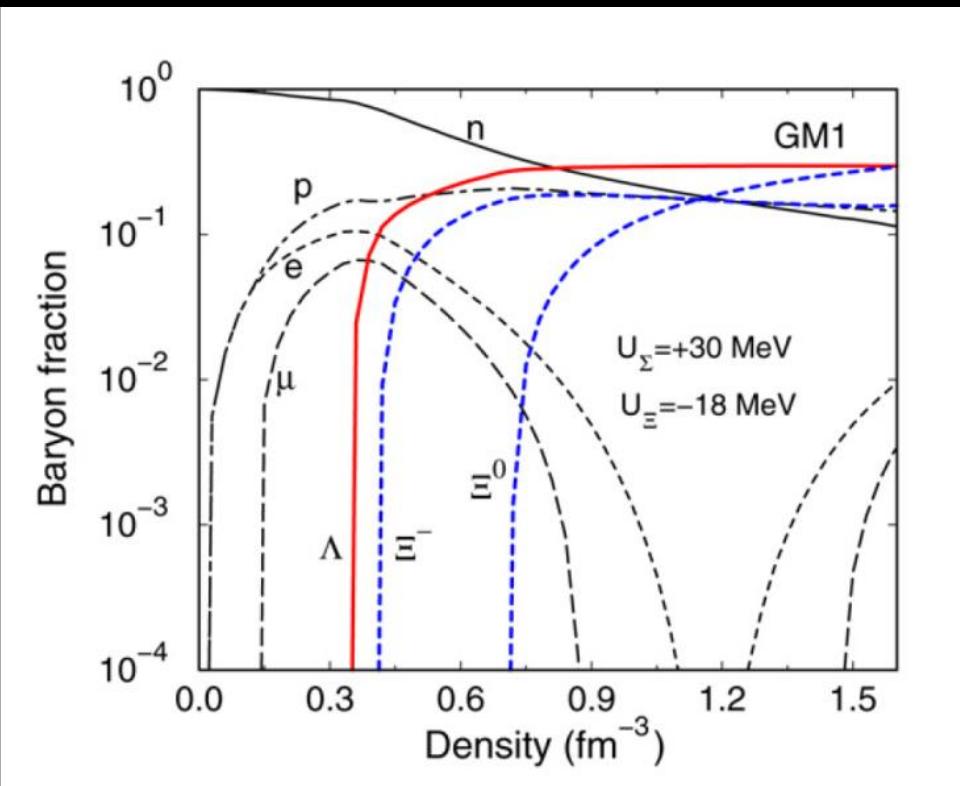
J-PARC (Tokai, Japan)



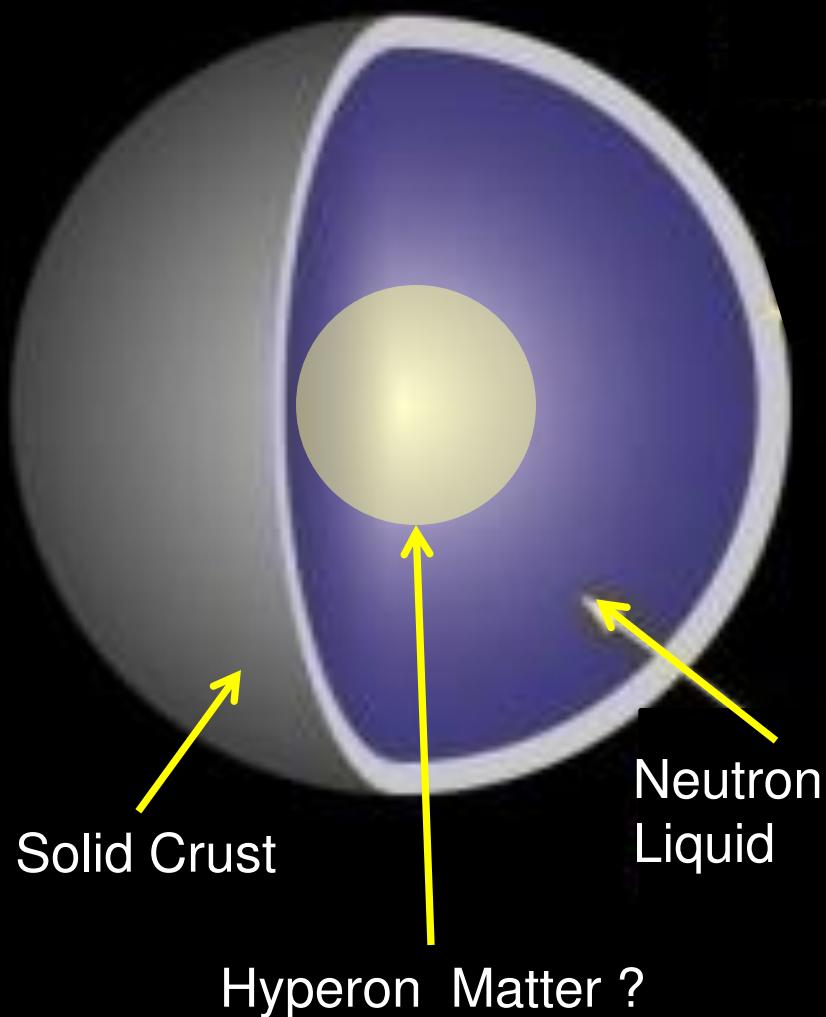
- predictions from lattice QCD
- difference between NN and YN ?

Hyperon Core of Neutron Stars

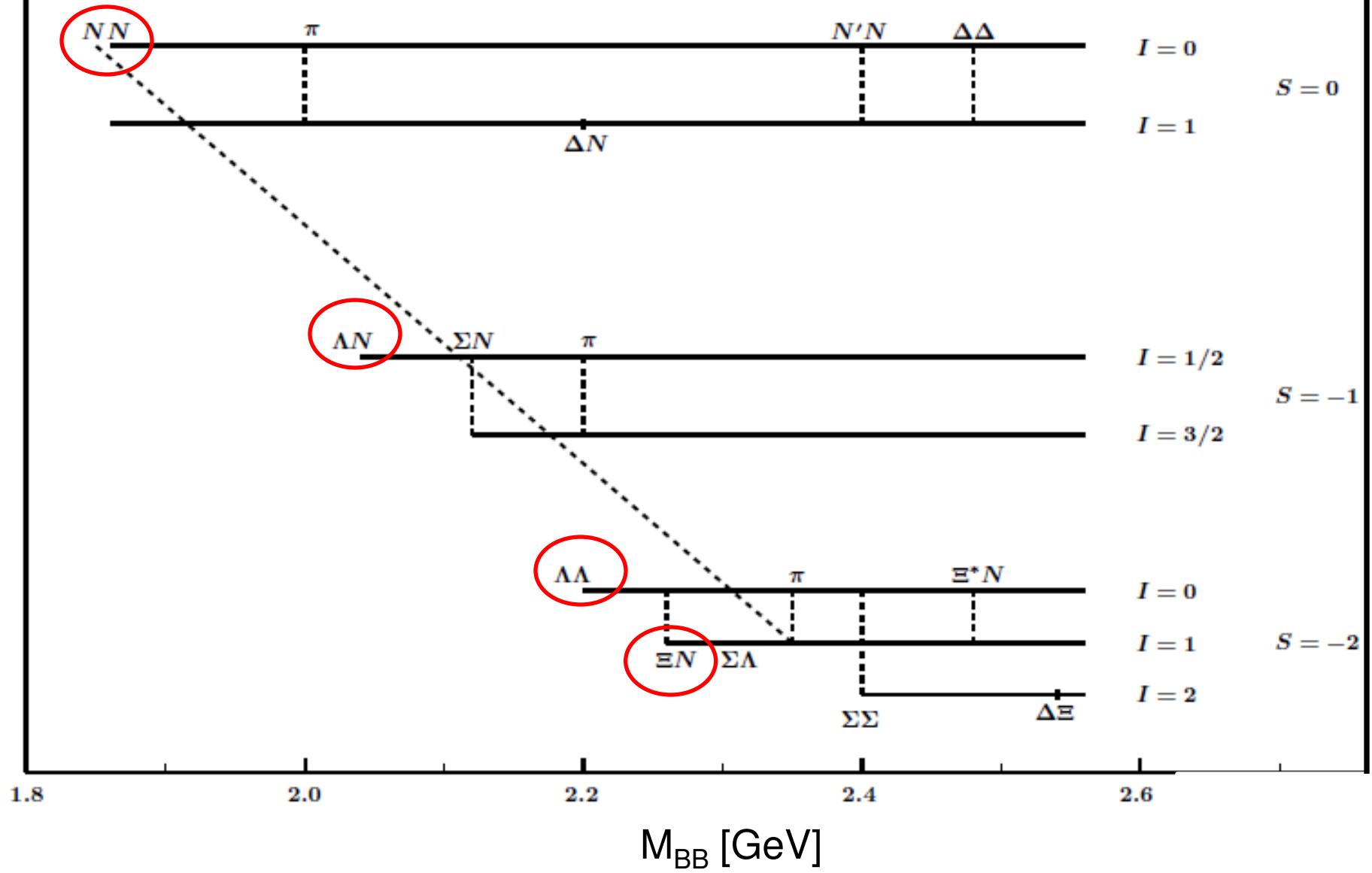
Radius ~ 10 km
Mass \sim solar mass
Central density $\sim 10^{12}$ kg/cm³



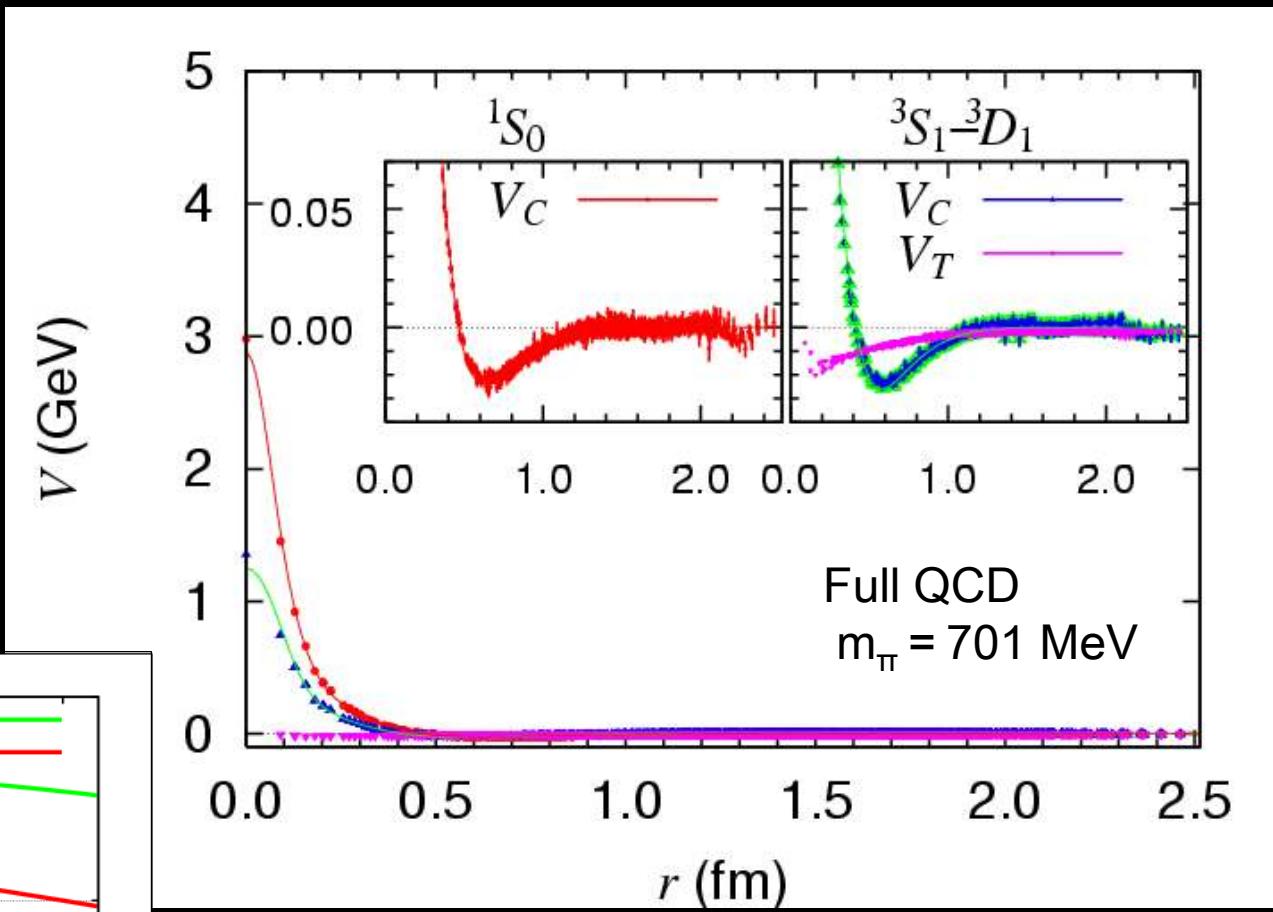
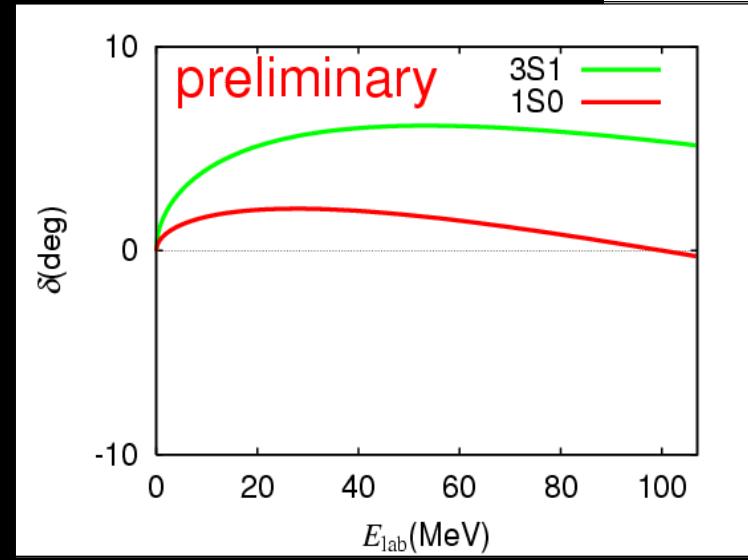
Schaffner-Bielich, NP A804 (2008).



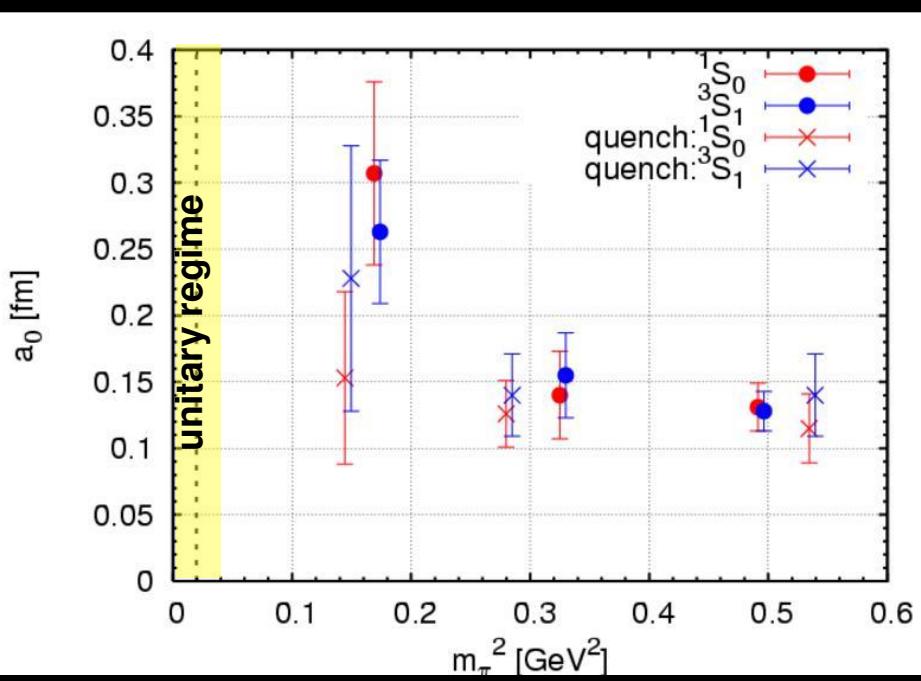
Baryon-Baryon Thresholds $S = 0, -1, -2$



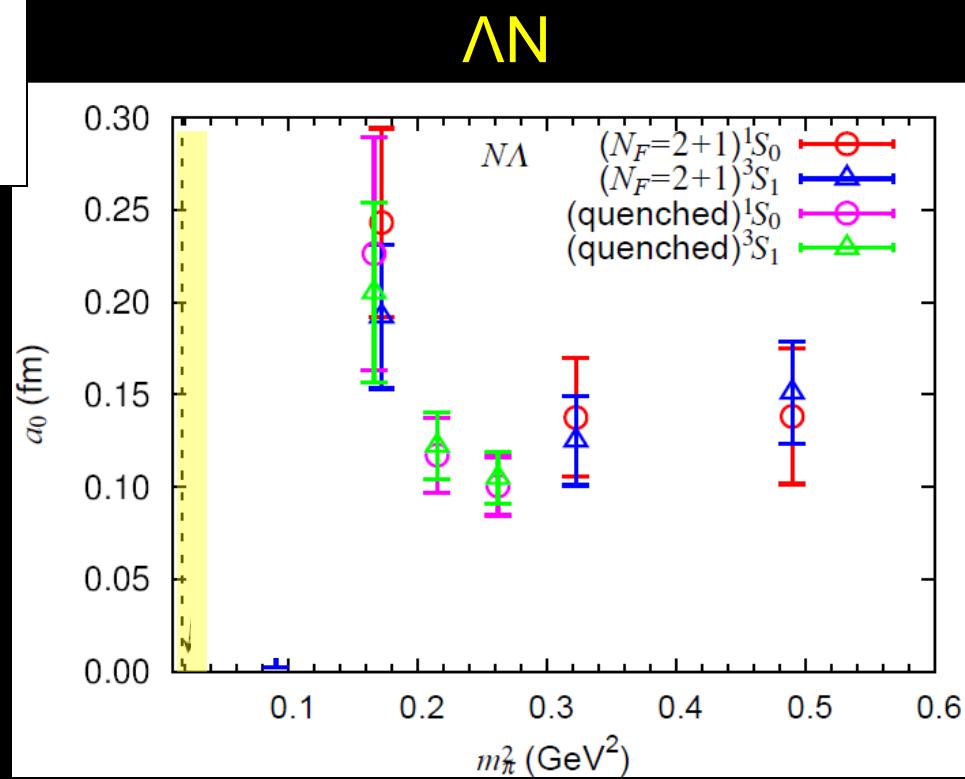
S=-1 system: ΛN interaction ($I=1/2$) in full QCD



NN and ΛN Scattering lengths in full QCD



NN

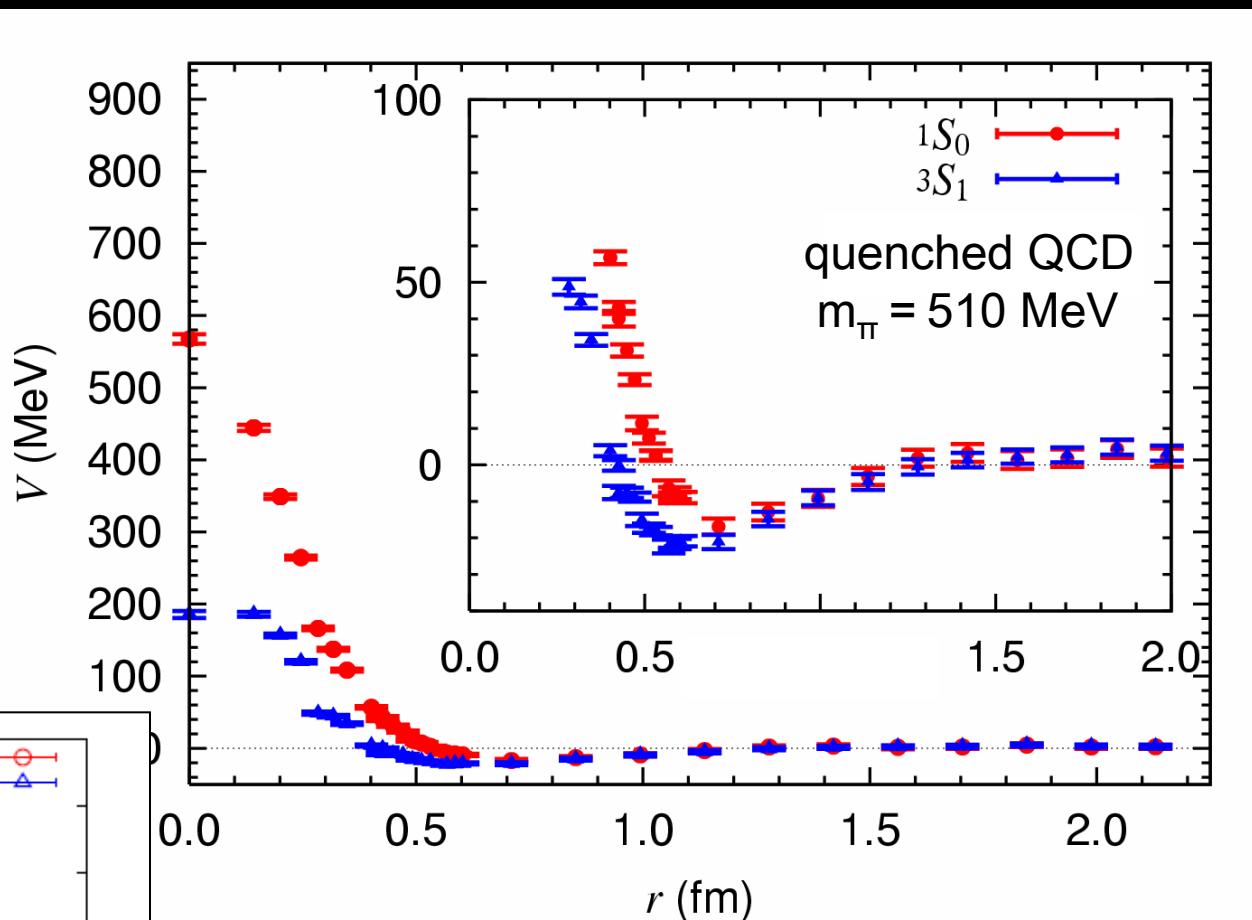
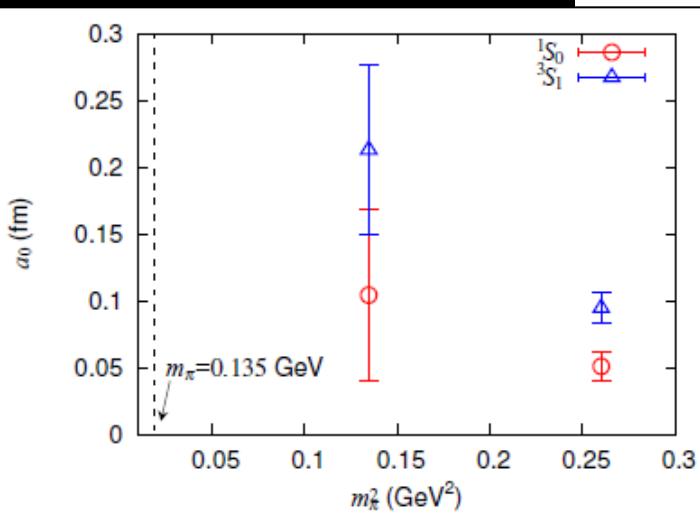


ΛN

S=-2 system: ΞN interaction ($I=1$)

J-PARC DAY-1 exp. :
 $^{12}\text{C}(\text{K}^-, \text{K}^+)^{12}\text{Be}_\Xi$

Nemura, Ishii, Aoki, T.H.,
 Phys.Lett. B673, 136 (2009)



1. Repulsive core + attractive well
2. Large spin dependence
3. Overall attraction

BB interaction

$$\begin{array}{c} \text{8} \\ \square \end{array} \otimes \begin{array}{c} \text{8} \\ \square \end{array} = \begin{array}{c} \text{27} \\ \square \end{array} \oplus \begin{array}{c} \text{10*} \\ \square \end{array} \oplus \begin{array}{c} \text{1} \\ \square \end{array} \oplus \begin{array}{c} \text{8} \\ \square \end{array} \oplus \begin{array}{c} \text{10} \\ \square \end{array} \oplus \begin{array}{c} \text{8} \\ \square \end{array}$$

The diagram shows the tensor product of two 8-electron configurations (8s) resulting in six possible configurations. The first configuration (27) has all electrons in the same orbital. The second (10*) has one electron in each of five orbitals. The third (1) has all electrons in different orbitals. The fourth (8) has two electrons in each of four orbitals. The fifth (10) has three electrons in each of three orbitals. The sixth (8) has four electrons in each of two orbitals. Red boxes highlight the last two configurations (10 and 8) in the final sum.

- We have **six** independent potentials for a given L.

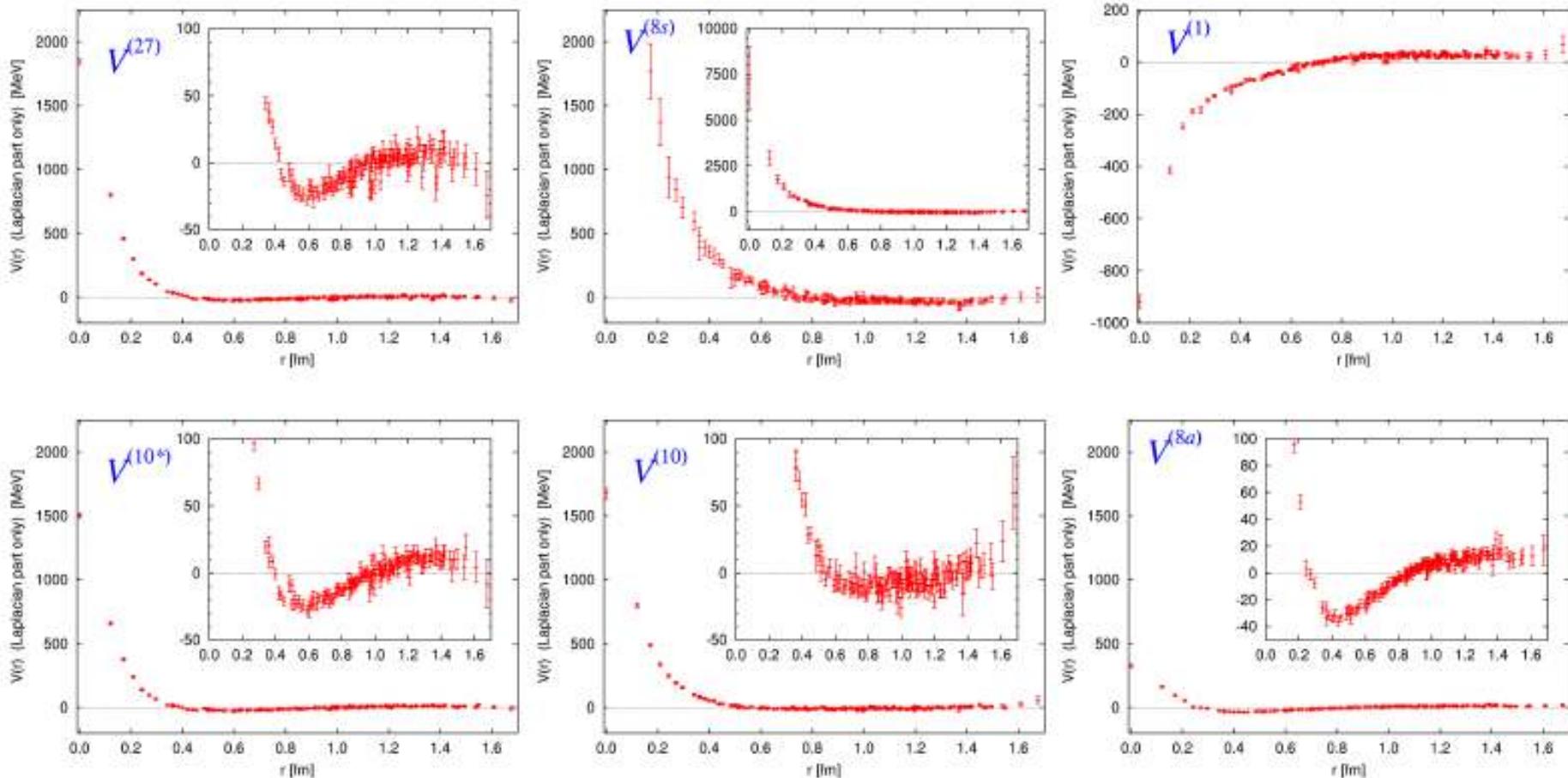
$$^1S_0 : V^{(27)}(r), V^{(8s)}(r), V^{(1)}(r)$$

$$^3S_1 : V^{(10*)}(r), V^{(10)}(r), V^{(8a)}(r)$$

What makes the difference among these six ?

Pauli principle at work

1S_0

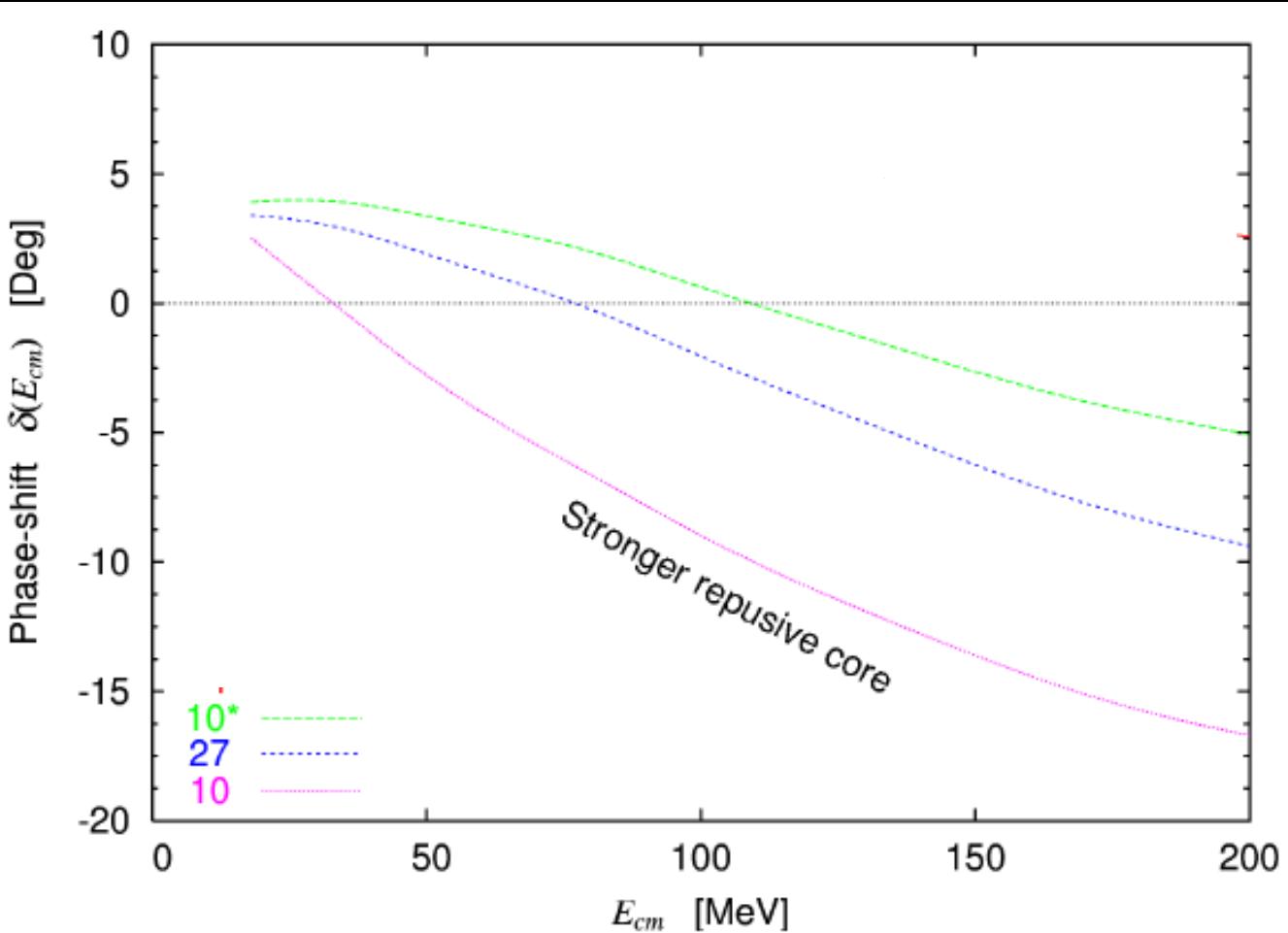


3S_1

16³ × 32, full (CP-PACS/JLQCD config.) $a=0.12$ fm, $L=2$ fm
SU(3) limit: $m_\pi=m_K=835$ MeV, $m_B=1745$ MeV

Inoue et al. (HAL QCD Coll.)

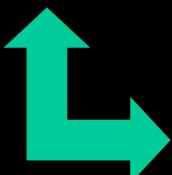
BB phase shift from BB potential in full QCD



3-flavor full QCD
CP-PACS/JLQCD
configurations
 $a=0.12\text{fm}$, $L=2.0\text{fm}$
 $m_{\pi,K}=835\text{ MeV}$

Inoue (HAL QCD)

Oka-Yazaki



Pauli principle

- 27-plet and 10^* -plet get **mild** repulsion.
- 10-plet get **strong** repulsion.
- 1-let get **no** repulsion.

Quark model prediction

Summary of the eigenvalues of the normalization kernel, the adiabatic potential V at $R = 0$ due to the color magnetic interaction and the effective hard core radius r_c .

I	J	BB	Eigenvalue	$V(R = 0)$ [MeV]	r_c [fm]
$\frac{1}{2}$	0	NA	1	381	0.44
		$N\Sigma$	$\frac{1}{3}$	303	0.72
$\frac{1}{2}$	1	NA	1	264	0.37
		$N\Sigma$	1	215	0.30
$\frac{3}{2}$	0	$N\Sigma$	$\frac{10}{9}$	391	0.40
$\frac{3}{2}$	1	$N\Sigma$	$\frac{2}{9}$	346	0.77
0	1	$N\Xi$	$\frac{8}{9}$	93	0.29
1	0	$N\Xi$	$\frac{4}{9}$	342	0.68
		$\Lambda\Sigma$	$\frac{6}{9}$	298	0.56

Oka, Shimizu, Yazaki
Nucl. Phys. A464 (1987)

the eigenvalue for $8s = 0$
pure-forbidden!!

semi-forbidden and
hence strong repulsive
10-plet

8a-plet both quark-antisym
and OGE are weak

irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}} \overline{\Lambda}\overline{\Lambda} - \sqrt{\frac{1}{40}} \overline{\Sigma}\overline{\Sigma} + \sqrt{\frac{12}{40}} \overline{N}\overline{\Xi} \quad \text{or} \quad +\sqrt{\frac{1}{2}} \overline{p}\overline{n} + \sqrt{\frac{1}{2}} \overline{n}\overline{p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}} \overline{\Lambda}\overline{\Lambda} - \sqrt{\frac{3}{5}} \overline{\Sigma}\overline{\Sigma} + \sqrt{\frac{1}{5}} \overline{N}\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}} \overline{\Lambda}\overline{\Lambda} + \sqrt{\frac{3}{8}} \overline{\Sigma}\overline{\Sigma} + \sqrt{\frac{4}{8}} \overline{N}\overline{\Xi} \quad \text{with}$$

$$\overline{\Sigma}\overline{\Sigma} = +\sqrt{\frac{1}{3}} \overline{\Sigma^+}\overline{\Sigma^-} - \sqrt{\frac{1}{3}} \overline{\Sigma^0}\overline{\Sigma^0} + \sqrt{\frac{1}{3}} \overline{\Sigma^-}\overline{\Sigma^+}$$

$$\overline{N}\overline{\Xi} = +\sqrt{\frac{1}{4}} \overline{p}\overline{\Xi^-} + \sqrt{\frac{1}{4}} \overline{\Xi^-}\overline{p} - \sqrt{\frac{1}{4}} \overline{n}\overline{\Xi^0} - \sqrt{\frac{1}{4}} \overline{\Xi^0}\overline{n}$$

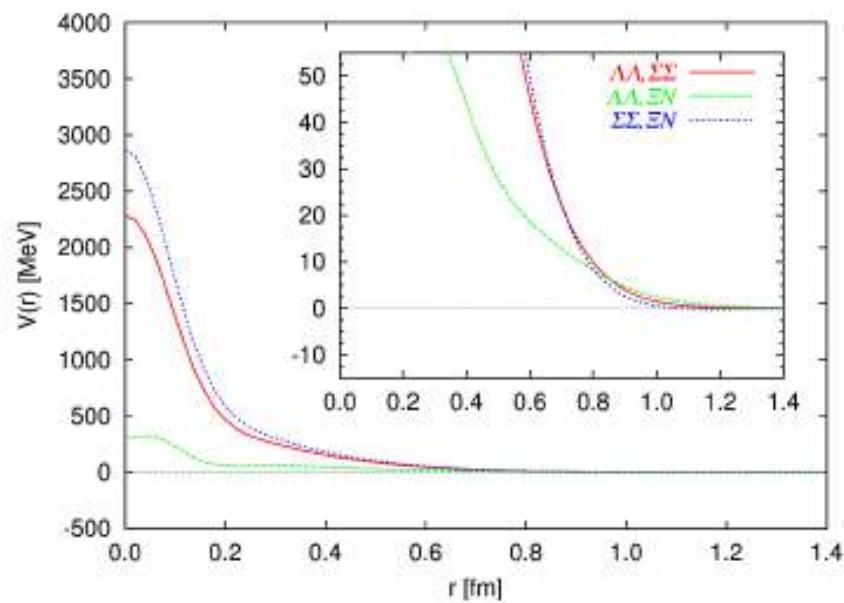
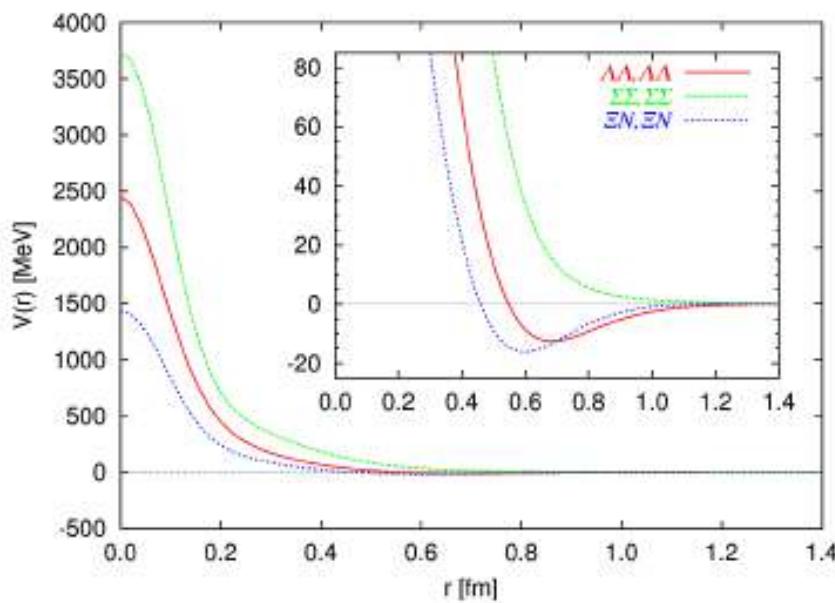
$$\overline{BB^{(10*)}} = +\sqrt{\frac{1}{2}} \overline{p}\overline{n} - \sqrt{\frac{1}{2}} \overline{n}\overline{p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}} \overline{p}\overline{\Sigma^+} - \sqrt{\frac{1}{2}} \overline{\Sigma^+}\overline{p}$$

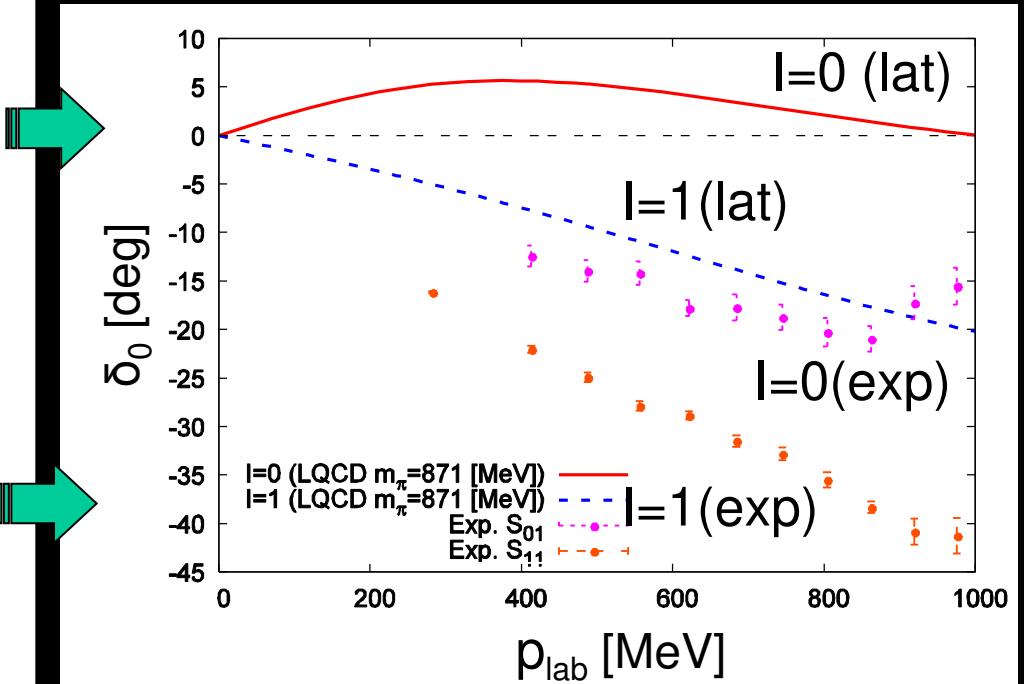
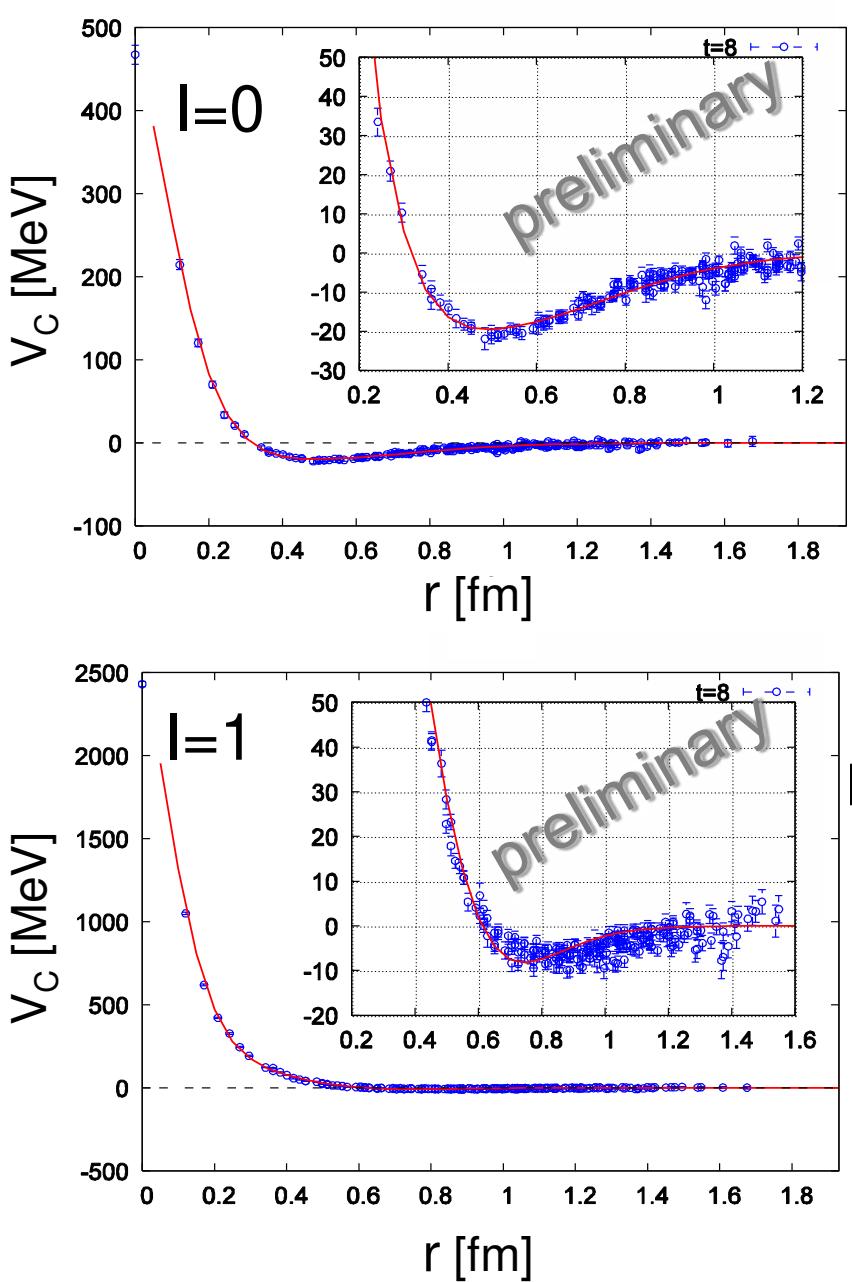
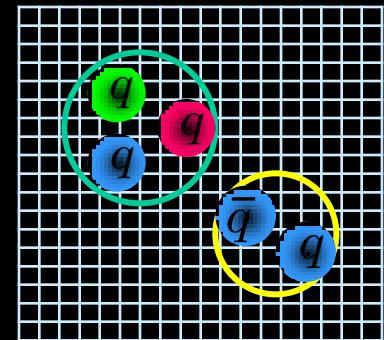
$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}} \overline{p}\overline{\Xi^-} - \sqrt{\frac{1}{4}} \overline{\Xi^-}\overline{p} - \sqrt{\frac{1}{4}} \overline{n}\overline{\Xi^0} + \sqrt{\frac{1}{4}} \overline{\Xi^0}\overline{n}$$

$S=-2, I=0$ BB 1S_0 potential

$$\begin{pmatrix} \Lambda\Lambda \\ \Sigma\Sigma \\ \Xi N \end{pmatrix} = U \begin{pmatrix} |27\rangle \\ |8\rangle \\ |1\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V_{\Sigma\Sigma}^{\Lambda\Lambda} & V_{\Xi N}^{\Lambda\Lambda} \\ V_{\Sigma\Sigma}^{\Lambda\Lambda} & V_{\Sigma\Sigma}^{\Sigma\Sigma} & V_{\Xi N}^{\Sigma\Sigma} \\ V_{\Xi N}^{\Lambda\Lambda} & V_{\Xi N}^{\Sigma\Sigma} & V^{\Xi N} \end{pmatrix}$$

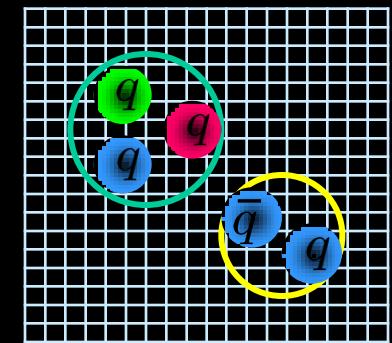
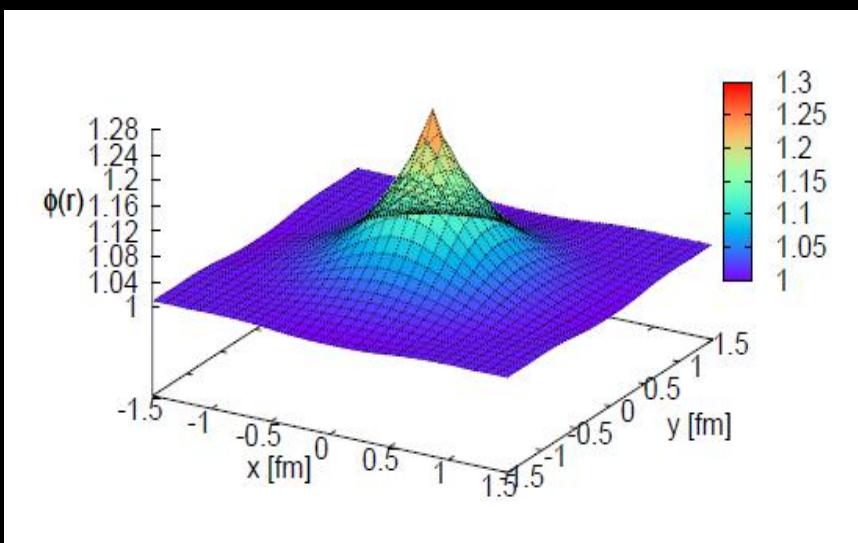


S-wave $N-K^+$ (us^{bar}) interaction



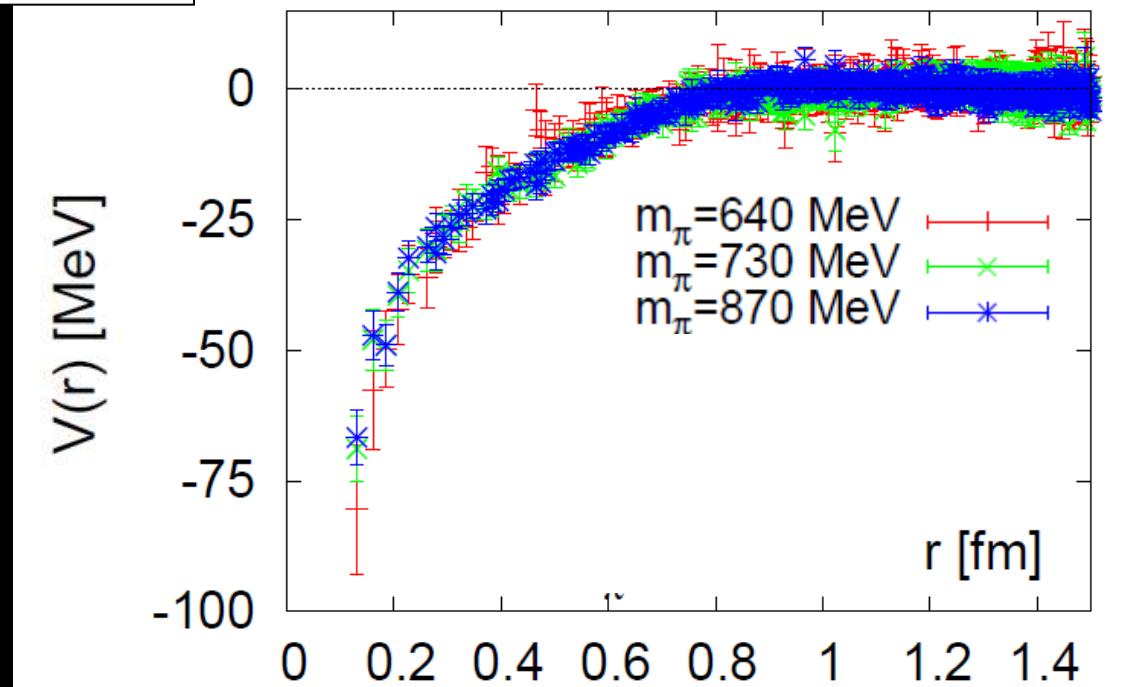
(2+1)-flavor full QCD
 CP-PACS/JLQCD configurations
 $a=0.12$ fm, $L=1.93$ fm
 $m_\pi=871$ MeV
 Ikeda et al. (HAL QCD Coll.)

S-wave $N\text{-}\eta_c$ ($cc^{\bar{b}a}$) interaction



Quenched QCD
 $32^3 \times 48$, $a = 0.093$ fm

Kawanai & Sasaki (2010)

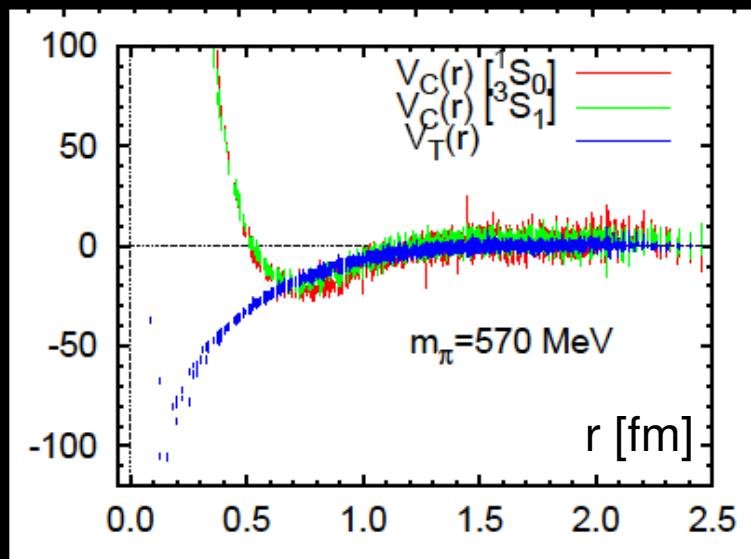


Summary and Future

- Nuclear forces from LQCD (HAL QCD strategy)
 - BS amplitude → YN, YY potentials → observables
→ exact few body calculations
- Full QCD with $m_\pi=140$ MeV is our ultimate goal
 - current : PACS-CS config. ($N_f=2+1$) with $L=2.9\text{fm}$ & $m_\pi = 156\text{-}701$ MeV
 - in 1-2 years: PACS-CS config. ($N_f=2+1$) with $L=5.8\text{fm}$ & $m_\pi = 140$ MeV
 - in 5 years: new full QCD config. on 10 Pflops machine at Kobe (2012-)

○ Current and Future targets of HAL QCD

- tensor force and π -N coupling
- LS force
- 3B forces
- light nuclei from LQCD potentials
- B-M interactions



Some Recent References

- **NN force in quenched QCD:**
Ishii, Aoki & T.H., Phys. Rev. Lett. 99 (2007) 022001 [nucl-th/0611.096].
- **Theoretical foundation of the HAL formalism:**
Aoki, T.H. & Ishii, Prog. Theo. Phys 123 (2010) 89–128 [arXive:0909.5585 [hep-lat]].
- **YN force in quenched QCD:**
Nemura, Ishii, Aoki & T.H., Phys. Lett. B673 (2009) 136 [arXiv:0806.1094 [nucl-th]].
- **NN force in full QCD:**
Ishii, Aoki & T.H. (for PACS-CS Coll.), LATTICE08 Proc. arXiv: 0903.5497 [hep-lat]
- **YN force in full QCD:**
Nemura, Ishii, Aoki & T.H. (for PACS-CS Coll.), LATTICE08 Proc. arXiv: 0902.12251 [hep-lat]