Nuclear Forces and Bosons*)<br>_The Sakata Model and One-Boson-Exchange-Potentials-_

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#### Abstract

Existence of bosons with strangeness zero besides pion is expected in the full symmetry theory of the Sakata model and $\rho-, \omega-, \cdots$ mesons have been recently found. It is investigated whether these bosons could explain the nuclear forces, especially, the spin-orbit coupling interaction. Two cases are discussed: One is to identify the phenomenological potential with a sum of one-boson-exchange-potentials, and the other is to explain the phenomenological potential by the one-pion-exchange-potential, two-pion-exchange-potential, together with a sum of one-boson-exchange-potentials. The analyses show that the presence of an $I=1$ vector boson (mainly interacting through tensor coupling) and an $I=0$ vector boson (mainly interacting through vector coupling) for both cases. They may be identified as the $\rho$-and $\omega$-meson respectively. In addition, an. $I=0$ scalar boson is needed for the first case, and an $I=0$, scalar or pseudoscalar boson is required for the second case. (Existence of other bosons is not necessarily excluded.) Consistency of the obtained results with other experimental evidence and with the mass level scheme of the Sakata model is discussed.


## § 1. Introduction and summary

The essential aim of research on nuclear forces, espeically that of the Japanese group, has been to attack elementary particles from their outside according to the Taketani theory, ${ }^{2)}$ and much success has actually been obtained. ${ }^{3)}$ On the other hand, properties of nuclear forces for the states with isospin $T=1$ have become fairly clear, ${ }^{4,5) 6(* *)}$ as a result of accumulation of experimental data on proton-proton scattering. Among these the following three properties of nuclear forces may be worth noticing, since they might be unable to be simply explained by the pion field theory:
i) Presence of a hard-core-like repulsive force of the radius of about $2 / M$ in the ${ }^{1} S_{0}$-state, where $M$ is the nucleon mass, is known from the fact that the phase shift ${ }^{1} \partial_{0}$. changes its sign at about $250 \mathrm{Mev}^{7}{ }^{7}$ and also from an analysis of the low energy scattering length and effective range by assuming the tail of the

[^0]one-pion-exchange-potential (hereafter abbreviated as OPEP)..$^{87}$
ii) Spin-orbit coupling potential was first introduced phenomenologically by Gammel and Thaler ${ }^{9}$ to explain splitting of triplet $P$-phase shifts at 310 Mev , and after that several versions of the potential shape were proposed. ${ }^{485}$ Such phenomenological spin-orbit coupling potentials are very strong in the inner region: Theytare all about -50 Mev at $x=0.7$.*) On the other hand, the two-pion-exchange-potential (hereafter abbreviated as TPEP) gives a weak tail of the spin-orbit coupling, even if the recoil effect is fully taken into account. ${ }^{6110)}$ Namely, the tail with the $\exp (-2 x)$-dependence is -6.5 Mev at $x=0.7$ for the $p v$-coupling case and quite negligible in the $p s$-coupling case.
iii) Relations among singlet even phase shifts with different angular momenta, ${ }^{1} \partial_{0},{ }^{1} \dot{\partial}_{2},{ }^{1} \delta_{4}$ at $310($ and 210$) M e v^{7}$ can not be interpreted in terms of a simple central potential. This fact was first clearly pointed out by Hamada ${ }^{4)}$ and later discussed in detail by Yoder and Signell. ${ }^{11)}$

We think that the above three properties of nuclear forces may serve as clues to investigate the dynamics of the Sakata model. ${ }^{12)}$ In the present paper we will discuss mainly the spin-orbit coupling interaction and some related problems.

In deriving mass formula of the Sakata model, it is usually assumed that the force between two fundamental baryons is opposite in sign to that between a fundamental baryon and its antiparticle. ${ }^{1314) 158)}$ The hard-core like repulsion of the radius of about $2 / M$ in the two-nucleon system may be understood in terms of the repulsion between two fundamental baryons. Machida first proposed such an idea ${ }^{16)}$ and argued that the assumption of the presence of the repulsion in all nucleon states leads to results which are in disagreement with nucleon-antinucleon annihilation data. Otsuki argued, in connection with Ohnuki's assumption**) of classifying mesons into two classes, a possibility of relating the ${ }^{1} S_{0}$-state repulsion to pseudoscalar mesons and the (less repulsive) ${ }^{3} S_{1}$-state one to vector mesons which are the bound states composed of a fundamental particle and a fundamental antiparticle, and, at the same time, of explaining the nuclear saturation and $A$-particle-nucleon forces. ${ }^{17}$ )

The relations among ${ }^{1} \delta_{0},{ }^{1} \delta_{2}$ and ${ }^{1} \delta_{4}$ can be explained either by taking into account nonstatic effects due to the $p v$-coupling interaction ${ }^{6) 18)}$ or by introducing a drastic energy and/or angular momentum variations in nuclear force at very small distances. In the former case, the renormalizability cannot be regarded as one of the guiding principles of the quantum field theory, and this fact may support the point of view of the Sakata model that an elementary particle cannot be a point particle but has its own structure. ${ }^{19)}$ In the latter case the variation in the hard core region might be attributed to phenomenological properties of the forces between fundamental baryons.

We think that the origin of the spin-orbit coupling interaction may be as follows: In the usual pion field theory, pions are virtually exchanged between nucleons. In the Sakata model these pions are to be dissociated into fundamental baryon and its antibaryon. On the other hand, many pairs of fundamental

[^1]baryon anti-baryon are to be virtually created around nucleons. When two nucleons approach each other, a number of fundamental particles generated through such dissociation and pair creation processes are to be rearranged into bosons (stable or quasi-stable bound states of fundamental particles) with various transformation properties and exchanged as such entities between two nucleons. With the assumption of the full symmetry among the fundamental baryons in the Sakata model, ${ }^{20}$ each boson corresponds to a basis vector of an irreducible representation of the $U(3)$-group (the Ikeda-Ogawa-Ohnuki theory). ${ }^{\text {a1) }}$ It may be probable that among such bosons there are scalar and vector bosons which give a spin-orbit coupling interaction. ${ }^{22)}$

It utterly depends on the dynamical laws governing the fundamental particles of the Sakata model, what type of boson is easiest to be born as a result of the dissociation, pair creation and rearrangement, and at what distances the exchange of boson begins between two nucleons. Therefore we may make two contrasting approaches :
I. One is to assume the dissociation and rearrangement "from the first". Namely, two or more pions should not be exchanged freely as in the case of the usual perturbation theory, but be exchanged as correlating to each other in a form of one boson of some type or other and the nuclear potentials are represented as a sum of thus induced one-boson-exchange-potentials. Such a treatment implies an extreme approximation for the dynamical effects of the pion field theory and may be called a " particle theory ". In $\S 2$, the properties of bosons that are required are discussed from such a point of view.
II. The other approach is to assume that the dissociation, pair creation and rearrangement may happen only when two nucleons are very close together. In this case the outer part of nuclear forces is explained by the pion field theory, namely, by the nonstatic OPEP and TPEP including the ( $3 / 2,3 / 2$ )-resonance effect, ${ }^{6,}$ and the spin-orbit coupling interaction is mainly attributed to the bosons. In $\S 3$, properties of such bosons as are allowed to be present are discussed on the assumption that the TMO- or KMO-potential is the "true" pion theoretical one.

Some simplifications are made for both cases of our approach to avoid tiresome numerical discussions. For example, the nuclear forces are discussed by making use of the potential concept, whose validity may not always be guaranteed at small distances. The masses of bosons other than the pion are assumed as $m=4^{* * * *)}$ Furthermore, parameters are so adjusted that the resultant nuclear potential is similar to Hamada's potential. ${ }^{4}$ However, such simplifications would

[^2]not alter our conclusions.
It is interesting to notice that although the above two approaches are of very different character from each other, the obtained results for the types of bosons which are needed are very similar. Namely an $I=1$ vector boson interacting with a nucleon mainly through the tensor coupling and an $I=0$ vector boson interacting mainly through the vetcor coupling, are present in both approaches. They may be identified as the $\rho$ - and $\omega$-meson respectively. Furthermore, an $I=0$ scalar boson is present except for the case where the KMO potential ${ }^{133}$ is assumed for TPEP in the approach II. In the latter case, an $I=0$ pseudoscalar boson should be present instead. (Existence of other bosons is not necessarily excluded.)

Examples of the coupling constants of these bosons with the nucleon are: In approach I,

| $I=1$ vector : | $g_{v}^{\prime}{ }^{2} / 4 \pi=0.31$ | (vector coupling), |
| :--- | :--- | :--- |
|  | $f_{v}^{\prime}{ }^{2} / 4 \pi=1.1$ | (tensor coupling), |
| $I=0$ vector : | $g_{v}{ }^{2} / 4 \pi=2.06$ | (vector coupling), |
|  | $f_{v}{ }^{2} / 4 \pi=1.1$ | (tensor coupling), |
| $I=0$ scalar : | $g_{s}{ }^{2} / 4 \pi=5.0$. |  |

In approach II, to explain the spin-orbit coupling interaction in the $T=1$ states,

$$
\begin{aligned}
& I=0 \text { and/or } I=1 \text { vector: } g_{v}{ }^{2} / 4 \pi+g_{v}^{\prime}{ }_{v}^{2} / 4 \pi=12 \text { (vector coupling), } \\
& I=0 \text { and/or } I=1 \text { scalar : } g_{s}{ }^{2} / 4 \pi+g_{s}^{\prime 2} / 4 \pi=12 .
\end{aligned}
$$

In approach II, when the TMO potential is assumed,

$$
\begin{array}{lll}
I=1 \text { vector : } & g_{v}{ }^{\prime 2} / 4 \pi=2.3 & \text { (vector coupling), } \\
& f_{v}{ }^{\prime 2} / 4 \pi=0.5 & \text { (tensor coupling), } \\
I=0 \text { vector : } & g_{v}{ }^{2} / 4 \pi=6.0 & \text { (vector coupling), } \\
& f_{v}{ }^{2} / 4 \pi=0 & \text { (tensor coupling), } \\
I=0 \text { scalar : } & g_{s}{ }^{2} / 4 \pi=6, & \\
I=1 \text { pseudoscalar : } & f_{p s}^{\prime \prime} / 4 \pi=0.25 . &
\end{array}
$$

In approach II, when the KMO potential is assumed,

$$
\begin{array}{lll}
I=1 \text { vector: } & g_{v}{ }^{\prime 2} / 4 \pi=0.4 & \text { (vector coupling), } \\
& f_{v}{ }^{\prime 2} / 4 \pi=2.4 & \text { (tensor coupling), } \\
I=0 \text { vector : } & g_{v}{ }^{2} / 4 \pi=7.6 & \text { (vector coupling), }
\end{array}
$$

$$
\begin{array}{ll}
f_{v}^{2} / 4 \pi=0.1 & \text { (tensor coupling), } \\
I=0 \text { and/or } I=1 \text { pseudoscalar : } & \left(f_{p s}^{2} / 4 \pi+f_{p s}^{\prime 2} / 4 \pi\right)=12 .
\end{array}
$$

In $\S 4$, other experimental evidences on the bosons than the nuclear forces are discussed. They are consistent with our results obtained in $\S 2$ and $\S 3$. It is also shown that the existence of the bosons is consistently explained in the frame of the Sakata model if we take the mass level scheme of bosons of the full symmetry theory.

Our work may be regarded in a sense as a generalization of the works of Gupta or of Breit and Sakurai, who assumed scalar or vector boson respectively, or that of Fujii, who took into account the pion-pion interaction adjusting its parameter for the nucleon electromagnetic form factor. However, our motivation as discussed before is different from theirs. ${ }^{24) * \text { * }}$

Although the outer part of nuclear forces is one of the best established parts of the strong interaction, a small room for the effect of boson other than pion is not necessarily excluded. As it is very hard to experimentally detect a neutral boson, which may possibly be present in the framework of the Sakata model, the upper limit of the coupling constant of the boson is determined in §5 from the requirement that the established outer part of nuclear potential should not be masked by the one-neutral-boson-exchange-potential. Numerical results are summarized as follows:
For $m=2(3)$,

$$
\begin{array}{ll}
g_{s}{ }^{2} / g_{\pi}{ }^{2}<0.5(1.5) & : \text { scalar, } \\
f_{p s}^{2} / g_{\pi}{ }^{2}<2(10) & \text { : pseudoscalar, } \\
f_{v}{ }^{2} / g_{\pi}{ }^{2}<5(29) \text { and } g_{v}{ }^{2} \sim 0 & : \text { vector, } \\
g_{p v}^{2} \sim 0, f_{p v}^{2}=0 \text { or } g_{p v}^{2}=0, f_{p v}^{2} \sim 0 & : \text { pseudovector. }
\end{array}
$$

where $g_{\pi}{ }^{2}$ is the pion-nucleon coupling constants, $g_{\pi}^{2} / 4 \pi \approx 0.08$.

## § 2. Nuclear potential as a sum of one-boson-exchange-potentials

In this section we discuss approach I described in §1. The problem is to examine what types of bosons are required in order to explain the nuclear potential by a sum of one-boson-exchange-potentials, $\sum$ (OBEP). In the following discussions we are concerned with the minimum set of bosons to be needed. The case of coexistence of $p v(p t)$ boson and $p s$ boson, ${ }^{* *)}$ for example, is thus left out of consideration, because these bosons give just the same static potentials with opposite signs.

[^3]
## 2-1. Nuclear potential in the $T=1$ states

OPEP should be taken as the first term of $\sum$ (OBEP), because the outer part of the nuclear potential ( $x \geq 1.5$ ) due to one-pion exchange is now firmly established. ${ }^{3)}$ The remaining part of $\sum$ (OBEP) is identified with

$$
\begin{equation*}
V_{p h e n}-(\mathrm{OPEP}), \tag{1}
\end{equation*}
$$

where $V_{\text {phen }}$ means the phenomenological potential with the OPEP-tail deduced from available experimental data.

Features of the potential (1) in the $T=1$ states are summarized as follows:
i) In the singlet even state a very strong attractive force ( ${ }^{1} V_{C}^{+}$) must be present in order to fit the low-energy scattering parameters (scattering length and effective range) to the experimental values. ${ }^{88}$
ii) The depolarization experiments require a strong attractive*) spin-orbit coupling interaction ( $V_{\bar{L} s}$ ) of short range. The range of the interaction still remains uncertain, but it may not be as small as 0.4.
iii) A fairly strong attractive*) force is necessary in the tensor part ( $V_{\bar{r}}$ ) in the triplet odd state. Without this potential, the strong repulsive tensor force of OPEP alone gives too large a cross section for $p-p$ scattering above a few tens Mev. ${ }^{26)}$
iv) Any strong modification of OPEP is probably unnecessary in the central part of the triplet odd potential $\left({ }^{3} V_{c}^{-}\right)$. It might be favorable to add a weak attractive potential to OPEP.

We should like to note that the condition (i) is the most definite, (ii) the next, and so forth. Let us therefore introduce the boson fields which satisfy each condition in this order. A list of the nonstatic potentials necessary for the present discussion has been given by Lin, Machida and one of the authors (N.H.). ${ }^{22) * *)}$

In the first place the conditions (i) and (ii) require a scalar boson, $s$, and/or a vector boson with tensor coupling interaction to the nucleon, $v(t)$. Taking only $v(t)$, we have the following potentials:

$$
\begin{aligned}
& { }^{\mathrm{i}} V_{\dot{C}}^{+}=-2 m\left(f_{v}^{2} / 4 \pi\right) Y(m x), \\
& V_{\bar{L} s}=-(3 / 2) m(m / M)^{2}\left(f_{v}^{2} / 4 \pi\right) \xi(m x),
\end{aligned}
$$

[^4]\[

$$
\begin{aligned}
& V_{\bar{r}}=-(1 / 3) m\left(f_{v}^{2} / 4 \pi\right) r(m x), \\
& { }^{3} V_{\bar{c}}=(2 / 3) m\left(f_{v}^{2} / 4 \pi\right) Y(m x)
\end{aligned}
$$
\]

where $m$ is the boson mass and

$$
Y(x)=e^{-x} / x, \gamma(x)=\left(1+3 / x+3 / x^{2}\right) e^{-x} / x, \xi(x)=(1+1 / x) e^{-x} / x^{2}
$$

Thus $v(t)$ satisfies the conditions (i), (ii) and (iii), but not (iv) since ${ }^{3} V_{\bar{c}}$ is repulsive. The case of pure $v(t)$ is therefore excluded. On the other hand $s$ gives attractive ${ }^{3} V_{\bar{c}}$. Since $s$ does not satisfy condition (iii), the possibility of coexistence of $v(t)$ and $s$ is to be investigated.

The case of $v(t)$ and $s$, however, should be corrected with a small addition of bosons of other type in order to get reasonable ratio of $V_{L S}$ to ${ }^{1} V_{\dot{G}}^{+}$. For this purpose we here take $v(v)$ from the standpoint of minimizing the number of boson fields.

While we have thus introduced both $s$ and $v$, their charge state cannot be determined from the data in the $T=1$ states alone. Both $I=0$ and $I=1$ bosons will contribute to the potential in the $T=1$ states in general. Details will be discussed in 2-3 and 2-4.

2-2. Estimate of coupling constants from the data in the $T=1$ states
In order to get the short-range spin-orbit potential, mass of, the bosons must be $m \geq 3$. On the other hand we obtain several bosons of strangeness zero and $I=0$ or 1 having the mass $m=3 \sim 6$ applying the mass formula by Sawada and one of the authors (M.Y.) ${ }^{14}$ to the states of the full symmetry theory. In the following we put $m=4$, because the discussions do not depend seriously on the detailed value of $m$ as discussed in $\S 1$.

Taking Hamada's potential ${ }^{4}$ for the standard phenomenological one we set $\sum$ (OBEP) equal to (Hamada's potential)-(OPEP) at the middle point, $x=0.7$, of the region considered. Then we have

$$
\begin{array}{ll}
{ }^{1} V_{\sigma_{1}}^{+}: & \left(g_{1}-2 f_{1}-h_{1}\right) Y(m x)=-0.63, \\
{ }^{3} V_{\bar{o}}: & \left(g_{1}+(2 / 3) f_{1}-h_{1}\right) Y(m x)=-0.08, \\
V_{\bar{T}}^{-}: & -(1 / 3) f_{1} \gamma(m x)=-0.15, \tag{4}
\end{array}
$$

and

$$
\begin{equation*}
V_{\bar{L} s}: \quad-\left\{(3 / 2)\left(m^{2} / M^{2}\right)\left(f_{1}+g_{1}\right)+\left(m^{2} / 2 M^{2}\right) h_{1}+4(m / M) G_{1}\right\} \xi(m x)=-0.37 \tag{5}
\end{equation*}
$$

where $x=0.7$ and $m=4$. Here

$$
\begin{aligned}
& m\left(g_{v}{ }^{2} / 4 \pi+g_{v}^{\prime 2} / 4 \pi\right) \equiv g_{1}, \text { for } v(v) \\
& m\left(f_{v}^{2} / 4 \pi+f_{v}^{\prime 2} / 4 \pi\right) \equiv f_{1}, \text { for } v(t)
\end{aligned}
$$

$$
m\left(f_{v} g_{v} / 4 \pi+f_{v}^{\prime} g_{v}^{\prime} / 4 \pi\right) \equiv G_{1}
$$

and

$$
m\left(g_{s}^{2} / 4 \pi+g_{s}^{\prime 2} / 4 \pi\right) \equiv h_{1}, \text { for } s(s)
$$

where primed (unprimed) quantities correspond to the coupling constants between nucleon and $I=1(I=0)$ bosons.

From (4) we have

$$
\begin{equation*}
f_{1} \cong 9 \tag{6a}
\end{equation*}
$$

Putting (6a) into (2) or (3), we get $g_{1}-h_{1} \cong-11.0$ or -9.7 , respectively. Let us therefore set

$$
g_{1} \cong h_{1}-10(\geq 0) .
$$

In order to obtain $h_{1}$ and $G_{1}$, data in the $T=0$ state must be taken into account in addition.

## 2-3. Data from the $T=0$ states

The experimental information is still not sufficient to deduce the unique picture for the potential in the inner region $(x<1)$ in the $T=0$ states. For example, the phenomenological potentials by Hamada ${ }^{288 *)}$ and by Gammel and Thaler ${ }^{29)}$ do not agree with each other. The static OPEP alone is also sufficient to explain the experimental data below $100 \mathrm{Mev}{ }^{30}{ }^{30}$

In Table I we show qualitative features of $V_{\text {phen }}-$ (OPEP), taking as $V_{\text {phen }}$ Hamada's and Gammel-Thaler's potentials and the static OPEP. In spite of these ambiguities of the phenomenological nuclear potential, however, we can set the following conditions on $\sum$ (OBEP).

Table I.

|  | (Hamada) ${ }^{28)}-(\mathrm{OPEP})^{*)}$ | $(\text { Static })^{30}-(\mathrm{OPEP})$ | $(\mathrm{GT})^{29}-(\mathrm{OPEP})$ |
| :---: | :---: | :---: | :---: |
| triplet even, |  |  |  |
| central, ${ }^{3} V^{+}{ }^{+}$ | repulsive | $\approx 0$ | attractive |
| tensor, $V_{T}^{+}$ | (weak) repulsive | $\approx 0$ | repulsive |
| LS, $V_{L_{S}^{+}}^{+}$ | repulsive | $\approx 0$ | attractive |
| singlet odd, ${ }^{1} V_{G}^{-}$ | repulsive | repulsive | repulsive |

v) The tensor part $\left(V_{T}^{+}\right)$is not attractive. This condition is required in order to reproduce the experimental value of the quadrupole moment of the deuteron. It is noted that $V_{T}^{+}$must be definitely repulsive, if there exists a repulsive spin-

[^5]orbit coupling potential in the triplet even states.*)
vi) Although an attractive $V_{L S}^{+}$does not seem to be plausible in that it gives the negative contribution to the magnetic moment of the deuteron, ${ }^{3132)}$ let us here put the following rather weak restriction on $V_{t s}^{+}$: ${ }^{32)}$
\[

$$
\begin{equation*}
1 / 4 \geq V_{L S}^{+} / V_{\bar{L} s}>-1 \tag{7}
\end{equation*}
$$

\]

vii) An extremely deep ${ }^{3} V_{d}^{+}$would result in too large a cross section for $n-p$ scattering. A strong repulsive ${ }^{3} V_{\sigma}^{+}$, on the other hand, is also unlikely, because the bound state of deuteron could not be obtained with such a potential. Thus we put

$$
10 \times\left({ }^{3} V_{C}^{+} \text {in OPEP }\right) \lesssim^{3} V_{C}^{+} \leqq-5 \times\left({ }^{3} V_{C}^{+} \text {in OPEP }\right)
$$

viii) The singlet odd potential, ${ }^{1} V \bar{\sigma}$, is not attractive.

2-4. Determination of coupling constants from the data in the $T=0$ states
We have for $\sum$ (OBEP) in the $T=0$ states

$$
\begin{aligned}
& { }^{3} V_{o}^{+}=\left(g_{0}+(2 / 3) f_{0}-h_{0}\right) Y(m x), \\
& V_{T}^{+}=-(1 / 3) f_{0} \gamma(m x), \\
& V_{Z S}^{+}=-\left\{(3 / 2)\left(m^{2} / M^{2}\right)\left(f_{0}+g_{0}\right)+\left(m^{2} / 2 M^{2}\right) h_{0}+4(m / M) G_{0}\right\} \xi(m x), \\
& { }^{1} V_{\bar{c}}=\left(g_{0}-2 f_{0}-h_{0}\right) Y(m x) .
\end{aligned}
$$

Here $g_{0}, f_{0}$ and $g_{0}$ are defined as

$$
g_{0} \equiv m\left(g_{v}{ }^{2} / 4 \pi-3 g_{v}^{\prime 2} / 4 \pi\right), f_{0} \equiv m\left(f_{v}^{2} / 4 \pi-3 f_{v}^{\prime 2} / 4 \pi\right), \cdots,
$$

which satisfy the inequalities

$$
\begin{equation*}
g_{1} \geq g_{0} \geq-3 g_{1}, f_{1} \geq f_{0} \geq-3 f_{1}, \cdots \tag{8}
\end{equation*}
$$

In the first place the condition (v) gives $f_{0} \leq 0$, while (6a) and (8) give $-27 \leq f_{0} \leq 9$, so that we have

$$
\begin{equation*}
-27 \leq f_{0} \leq 0 \tag{9}
\end{equation*}
$$

Thus we arrive at the conclusion that $v(t)$ is not a pure $I=0$ but a pure $I=1$ one or a combination of $I=0$ and $I=1$ bosons.

Next we can obtain the following inequalities from the conditions (vii) and (viii) respectively:

$$
\begin{align*}
& -25<(2 / 3) f_{0}+g_{0}-h_{0}<10  \tag{10}\\
& -2 f_{0}+g_{0}-h_{0}>0 \tag{11}
\end{align*}
$$

[^6]The region of $g_{0} / g_{1}$ and $h_{0} / h_{1}$ restricted by (5) $\sim(11)$ is shown in Fig. 1. In the calculation the coupling constants of $v(v)$ and $v(t)$ are assumed to be in phase, i.e. $f_{v} g_{v} / 4 \pi>0$ and $f_{v}^{\prime} g_{v}^{\prime} / 4 \pi>0$. Other cases may be ruled out to be improbable, as shown below :

|  |  | $\begin{aligned} & \text { ign of } \\ & g_{v} / 4 \pi \end{aligned}$ | $\begin{gathered} \text { sign of } \\ f_{v}^{\prime} g_{v}^{\prime} / 4 \pi \end{gathered}$ | values of coupling constants | ratio $V_{L S S}^{+} / V_{\bar{L} s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| case | 1 | + | + | present | calculation |
| case | 2 | + | - | $\simeq 2 \times($ case 1) | $V_{L S}^{+} / V_{L s}^{-}>1$, inconsistent with (7). |
| case | 3 | - | $+$ | $(3 \sim 5) \times($ case 1) | $V_{\bar{L} s}^{+} / V_{\bar{L} s}^{-}<-1$, also inconsistent with (7). |
| case | 4 | - | - | $>7 \times$ (case 1) | reasonable but unstable*) |

From Fig. 1 we can get the second conclusion that both the $v(v)$ - and s-bosons with $I=0$ must be present.

In Table II, an example of numerical values of the coupling constants is given and the resulting $\sum$ (OBEP) + (OPEP) is compared with Hamada's potential, taking $g_{0} / g_{1}=0.48$ and $h_{0} / h_{1}=1$ for a time. In this case the coupling constants of bosons are considerably larger than that of the pionnucleon coupling constant, though they are consistent with those of Sakurai ${ }^{24)}$ and of Hara. ${ }^{33}$ ) This might seem to cause a trouble that the potential with such a large coupling constant would affect even the OPEP-tail. However, this is not the case by two reasons. Firstly each OBEP in question is of short range ( $m=4$ ).


Fig. 1. Values of $g_{0} / g_{1}$ and $h_{0} / h_{1}$ deduced from the identification $\left(V_{\text {pher }}-\right.$ OPEP $)=\Sigma($ OBEP $)$. The conditions (5) $\sim(11)$ restrict $g_{0} / g_{1}$ and $h_{0} / h_{1}$ to the values in the shaded region. $f_{0}$ falls in the full interval given in (9) only for rather exceptional cases because of (7) and (11). In fact it is only for the case $g_{0} / g_{1} \approx 1, h_{0} / h_{1} \sim 0$ that $f_{0}$ takes the lower limit, $-27 . \quad V_{L S}{ }^{+} / V_{\overline{L S}}$ is positive only for $g_{0} / g_{1}>0$ and $h_{0} / h_{1}>0$. The cross marks labeled with GT, Hamada and Table II correspond to the potentials by Gammel and Thaler, ${ }^{29)}$ Hamada, ${ }^{28)}$ and the example in Table II, respectively.

[^7]Table II.

$\Sigma$ (OBEP) involves OPEP. The figures in the row specified as Hamada are values of Hamada's potential ${ }^{4}$ ) and given for comparison.

Secondly there occur considerable cancellations between OBEP's due to $s$ and $v$ bosons, so that the resulting total potential has the desired asymptotic form. The numerical values of $\sum(\mathrm{OBEP})$ at $x=1.0$ in Table II will clarify the situation.

## § 3. One-boson-exchange-potentials to be added to pion-theoretical potential

We now turn to approach II mentioned in § 1. Although the process of analysis is the same as in $\S 2$, the underlying idea is in contrast.

While we have as yet no quantitatively reliable two-pion-exchange-potential, Tamagaki and two of the authors (S.O. and W.W.) ${ }^{6)}$ have found that the nonstatic TPEP except for the spin-orbit part is not inconsistent with experimental data, if the ( $3 / 2,3 / 2$ )-resonance effect of pion-nucleon system is properly taken into account. For the spin-orbit part, the TPEP considerably differs from the phenomenological one in the $T=1$ states. There are various possibilities to fill up this difference.
$3-1$. In this subsection let us consider $\sum$ (OBEP) as the origin of $V_{\bar{L} s}$. In the first place we note the following features of OBEP:
i) $v(v)$ and $v(t)$ give the central and tensor OBEP which are just opposite in sign to those due to $s$ and $p v(p v)$ respectively. On the other hand, the spinorbit parts due to these bosons are equally negative.
ii) $p s$ and $p v(p t)$ also lead to the same OBEP's of opposite sign. No spin-orbit OBEP results from these two bosons.
iii) Mixing of ( $p v$ )- and ( $p t$ )-couplings for a $p v$ boson is not allowed.

These characteristics lead us to conclude that any of the following three sets of boson fields or the combination of these sets yield only $V_{\bar{L} s}$ :

$$
\begin{equation*}
[s, v(v)] \quad[p v(p v), v(t)] \tag{12}
\end{equation*}
$$

and

$$
[p v(p t), p s]
$$

where in each set the coupling constants must be taken equal for both bosons and the masses must also be equal to each other. The last set can be shown, however, to give only zero-contribution. Taking a combination of these sets, each of which can have the different values of the boson mass and coupling constant, we can get an attractive $V_{\bar{L} s}$ of fairly arbitrary form. (In this case $p v(p v)$ and $p v(p t)$ do not mean the same boson of different coupling, of course.)

As far as we are concerned with the minimum set of bosons to be needed, we have to take either of the former two sets in (12). In this case the coupling constant and the boson mass are uniquely determined from the requirement that $V_{\bar{z} s}$ should take the same value with that of the assumed phenomenological one at two points of $x$. If we set $m=4$ for the sake of comparison with the results in $\S 2$, then $V_{\bar{z} s}$ of Hamada's potential, can be fairly reproduced, for example, by the pair $[s, v(v)]$ with $g_{s}{ }^{2} / 4 \pi=g_{v}{ }^{2} / 4 \pi \cong 12$.*

We do not discuss here the charge states of bosons in question, since the knowledge of $V_{L S}^{+}$in the $T=0$ states is still uncertain.
3-2. The "true" two-pion-exchange-potential might be different from the phenomenological one also in the other parts than the spin-orbit part, $V_{\bar{L} s}$. Assuming that it is really the case, we shall make a similar analysis as in $\S 2$ in what follows, taking, for convenience, the $\mathrm{TMO}^{34}$ or $\mathrm{KMO}^{233}$ potential for the "true " pion-theoretical one. Thus

$$
\begin{equation*}
V_{p h e n}-(\text { the TMO or KMO potential }) \tag{13}
\end{equation*}
$$

is equated to $\Sigma$ (OBEP).
Fig. 2. Values of $g_{0} / g_{1}$ and $h_{0} / h_{1}$ deduced from the identification $\left(V_{\text {phen }}-\mathrm{TMO}\right)=\Sigma$ (OBEP). This identification gives the values of $g_{0} / g_{1}$ and $h_{0} / h_{1}$ on the solid line. The numerals attached to the four points on the line are the values of $V_{L S}^{ \pm} / V_{\bar{L}}$ (unparenthesized) and $h_{1}$ (parenthesized). In the TMO-case, $s$ with $I=0,1$ and $v(v)$ with $I=0,1$ give the main contribution. $v(t)$ with $I=1$ and $p s$ with $I=1$ are also needed as a small correction. For example, the point with -1 (34) corresponds to the following solution:
$g_{s}{ }^{2} / 4 \pi \approx 6, g_{s}^{\prime 2} / 4 \pi \approx 2 ; g_{v}{ }^{2} / 4 \pi \approx 6, g_{v}{ }^{2} / 4 \pi \approx 2.3, f_{v}{ }^{\prime 2} / 4 \pi \approx 0.5$; $f_{p s}^{\prime} / 4 \pi \approx 0.25$.


[^8]

Fig. 3. Values of $g_{0} / g_{1}$ and $f_{0} / f_{1}$ deduced from the identification $\left(V_{\text {phen }}-\mathrm{KMO}\right)=\Sigma(\mathrm{OBEP})$. The data in the $T=0$ states except for ${ }^{3} V_{L S}{ }^{+}$(conditions corresponding to (v), (vii) and (viii) in $\S 2$ ) confine $g_{0} / g_{1}$ and $f_{0} / f_{1}$ to the values in the shaded region, where $g_{1}$ and $f_{1}$ take $g_{1} \cong 32, f_{1} \cong 10$ from the data in the central and tensor parts in $T=1$ states. $V_{L S^{-}}$of Hamada's potential restricts $g_{0} / g_{1}$ and $f_{0} / f_{1}$ further to the values on the solid curve. For $g_{0} / g_{1}$ and $f_{0} / f_{1}$ at the ends of the solid curve, $V_{L S^{+}} / V_{L S^{-}}$ takes the values -1.27 and 0.46 . The charge state of the $p s$-boson is not determined, while the order of the coupling constant is $m\left(f_{p s}{ }^{2} / 4 \pi+f_{p s}{ }^{\prime 2} / 4 \pi\right) \sim 12$.

In the TMO potential, the repulsive ${ }^{3} V_{C}^{+}$is added to the one-pion-exchange part. Also there are strong attractive TPEP ${ }^{1} V_{c}^{+}$and ${ }^{3} V_{\bar{c}}$ in addition to OPEP. Comparing these properties with the results in §2, we can see at once that the $v(v)$ plays more important role than in $\S 2$ as the origin of repulsive ${ }^{1} V_{\square}^{+}$ and ${ }^{3} V_{C}^{-}$and attractive ${ }^{3} V_{C}^{+}$. The coupling constant $g_{v}{ }^{2} / 4 \pi$ in the TMO-case takes a comparable value with $g_{s}^{2} / 4 \pi$, and the solution in this case is quite similar to the one in $3-1$.

The results of the anaylysis are shown in Fig. 2.
The KMO potential is the TMO potential corrected by the ( $3 / 2,3 / 2$ )-resonance effect in pion-nucleon system. This resonance effect yields a strong attractive central force equally in all states. Thus in this case the neutral $s$ boson is not necessary, and the $p s$-boson will be required in order to weaken the strong attractive central potential to the appropriate one. The results are shown in Fig. 3.

## §4. Experimental evidences of bosons other than nuclear forces

It is emphasized that the bosons to be needed are of the following three (four) types regardless of the different standpoints of the analysis:
$I=0$ vector boson mainly interacting with nucleon by vector coupling,
$I=1$ vector boson mainly interacting with nucleon by tensor coupling,
$I=0$ (and $I=1$ ) scalar (pseudoscalar for the KMO case) boson.
In this section let us discuss some other experimental evidences of these bosons than the nuclear forces. At present we have rather scanty information on bosons having the strangeness zero except for a few cases, consequently the
following discussions can not avoid some ambiguity.
$I=1$ vector
Among the experimental evidences for bosons with $S=0$, the most definite one is the data of a two-pion resonance state with $I=1$ vector and having the mass $\sim 5$, observed in the single pion production by pion-nucleon collision. ${ }^{35)(88)}$ The analyses of experimental results of $\pi^{-}+p \rightarrow \pi+\pi+N$ reaction at $1.78 \mathrm{Bev} / c$ and 1.25 Bev by Goebel's and Chew-Low's method ${ }^{377}$ have shown that the $\pi-\pi$ system has a $P$-wave resonance state. ${ }^{36}$ ) This is often called $\rho$-meson.

Another important suggestion for the existence of the $I=1$ vector boson has been obtained from the analysis of the electromagnetic form factor of the nucleon. If we apply the similar consideration to the electromagnetic interaction of nucleon with that for "OBEP" for the nucleon forces as discussed in $\S 1,{ }^{38}$ ) then the iso-vector part of the form factor is expected to take the following form for low momentum transfer,

$$
\begin{equation*}
F_{1,2 V}\left(q^{2}\right) \simeq a_{1,2 V}+b_{1,2 V} /\left(m^{2}+q^{2}\right), \tag{14}
\end{equation*}
$$

where $m$ is the mass of the lowest $I=1$ vector boson, and $a$ involves the contributions of core interaction, higher mass states, etc. In fact the recent data can be expressed as ${ }^{\text {s9) }}$

$$
\begin{equation*}
F_{1,2 V}\left(q^{2}\right) \simeq-0.20+1.20 /\left(1+0.10 q^{2}\right), \quad\left(q^{2} \text { in unit of } 10^{+28} \mathrm{~cm}^{-2}\right) \tag{15}
\end{equation*}
$$

From this the mass of the boson is estimated as $\sim 4.5$. This value is nearly equal to the results from the single pion production experiments.

It has also been shown by Bowcock et al. that the existence of such boson is favorable for explaining the small phase shift of the pion-nucleon scattering. ${ }^{40)}$
$I=0$ vector
This boson will have the dominant decay mode $\pi^{0}+\gamma$ if its mass is smaller than 3 , and the mode $3 \pi$ if larger than 3 . In the latter case, it will contribute to the reactions such as $\pi+N \rightarrow 3 \pi+N$ and $N+\bar{N} \rightarrow n \pi$.

Recently from the analysis of experiments of $p+\bar{p} \rightarrow 5 \pi$ and $7 \pi$, the existence of a three-pion resonance state having mass $\sim 5.5(780 \mathrm{Mev})$ has been reported. ${ }^{41)}$ The iso-spin of the observed three-pion resonance state is zero and the vector seems to be most probable for its spin and parity. This boson has been named as $\omega$-meson. As in the case of $I=1$ vector boson, it is considered that $I=0$ vector boson will contribute to the iso-scalar part of the nucleon electromagnetic form factor. With such expectation, the experimental results have been analysed by assuming a similar form to (14) and it has been obtained ${ }^{39)}$

$$
\begin{align*}
& F_{1 S}\left(q^{2}\right) \simeq 0.44+0.56 /\left(1+0.214 q^{2}\right) \\
& F_{2 S}\left(q^{2}\right) \simeq 4.0-3.0 /\left(1+0.214 q^{2}\right) \tag{16}
\end{align*}
$$

The estimated mass is $\sim 3(420 \mathrm{Mev})$ in țhis c̣ase, Evidenc̣e of the boson isobar
that can be identified with that suggested from the iso-scalar form factor has very recently been reported by Nusbaum et al. from the experiment of $\pi^{+}+d \rightarrow$ $2 p+\pi^{+}+\pi^{-}+\pi^{0} .{ }^{42)}$

Another evidence might be the $I=0$ boson with mass $\sim 2(305 \mathrm{Mev})$ inferred from the $\mathrm{He}^{3}$ momentum spectrum in $d+p \rightarrow \mathrm{He}^{3}+?^{0}+?^{0}$ by Abashian et al. ${ }^{43)}$ However, it seems possible to give another interpretation of this phenomena such as an $I=0 S$-wave resonance by Troung, ${ }^{44)}$ etc. It has also been suggested that the existence of neutral vector boson (other possibility is $p s$ or $p v$ ) of mass $\sim 2.5$ improves the fit of the electron spectrum in $K_{e 3}^{+}$decay by $V$ - $A$ theory. ${ }^{45)}$

Here we note on the coupling types of these vector bosons. If we assign the vector bosons required from the nuclear forces to the poles appeared in the electromagnetic form factors, then we can discuss about their coupling types.

For example, we shall have the following relation for the $I=1$ vector boson from (15)

$$
f^{\prime}{ }_{V} / g^{\prime}{ }_{V} \simeq 1.20 \times 3.70 \times\left(m^{\prime} / M\right) / 1.20 \simeq 2.5,
$$

where $m^{\prime}$ is the mass of the vector boson and $M$ the nucleon mass. Similarly for the coupling constants of the $I=0$ boson, we have from (16)

$$
f_{V} / g_{V} \simeq-3.0 \times(-0.12) \times(m / M) / 0.56 \simeq 0.27
$$

Although the experimental data seem to be inaccurate, we could obtain the conclusion that the $I=1$ vector boson mainly interacts with nucleon through tensor coupling, while the $I=0$ vector boson does mainly through vector coupling. This is consistent with the results obtained from the nuclear forces.
$I=1$ scalar
At present we have neither experimental information nor theoretical suggestion about the necessity of $I=1$ scalar boson. It also requires further investigation of the $T=0$ states to determine whether we really need this boson or not for the explanation of the nucleon-nucleon scattering.
$I=0$ scalar
Only few suggestions have been made so far for the existence of such boson. One is the resonance-like behaviour of the $\pi-\pi$ scattering cross section reduced from the data of $\pi^{-}+p \rightarrow \pi^{-}+\pi^{0}+p$ and $\rightarrow \pi^{-}+\pi^{+}+n$ reactions. Walker et al. have indicated a possibility for that the $\pi-\pi$ system has the low energy $S$-wave resonance, although the statistics of their experiment are rather poor. ${ }^{38)}$ This $I=0$ resonance of $\pi-\pi$ system is also more directly suggested from the following fact concerning the similar experiments of the single pion production. In $\pi^{-}+p \rightarrow \pi^{-}+\pi^{0}+p$ reaction the evidence of $I=1$ boson is clearly found in the proton momentum spectrum or the $Q$-value distribution of $\pi^{-}-\pi^{0}$ system, while such spectra of $\pi^{-}+p \rightarrow \pi^{+}+\pi^{-}+n$ reaction do not clearly show the evidence of $I=1$ boson isobar, whose contribution is naturally expected. ${ }^{35}$ ) This fact could
be consistently interpreted if we introduce the $I=0$ boson having $2 \pi$ decay mode and mass smaller than that of the $I=1$ boson. ${ }^{48)}$

An explanation of the experiment by Abashian et al. by $S$-wave $\pi-\pi$ resonance has also been proposed. ${ }^{44}$

## $I=0$ pseudoscalar

There have been several speculations on the $I=0$ pseudoscalar boson of relatively small mass. ${ }^{47,21)}$ But we have no clear evidence of such boson as far as the present experimental data are concerned.

## Relation between the bosons and the states of the full symmetry theory in the Sakata model

In the following we shall examine the correspondence of the bosons so far discussed with the states of the full symmetry theory of Ikeda, Ogawa and Ohnuki. ${ }^{\text {21) }}$

We now have no reliable theory for the spin and parity of the states of full symmetry theory. Accordingly our discussion is limited only to examining whether the mass level scheme of the Sakata model, which is obtained by applying the mass formula ${ }^{14)}$ to the full symmetry theory, can consistently explain the bosons so far discussed.

Since the masses of the bosons in question are $\sim 4$, it may be enough to consider the states composed of one baryon and one antibaryon (2-body configuration) and two baryons and two antibaryons (4-body configuration). In fact, applying the semi-empirical mass formula, mass levels higher than 1 Bev are obtained for the states of $S=0 I=0,1$ of 6 -body system and higher configuration.

Then, besides the pion ( $B_{2}{ }^{1}(0,1)$ ), the following four (five) states appear as the states of $S=0 I=0$ and $I=1$ of 2 - and 4 -body configuration :

$$
\begin{aligned}
& I=0: B_{2}^{1}(0,0), B_{4}^{1}(0,0),\left(B_{2}^{2}(0,0)\right), \\
& I=1: B_{4}^{1}(0,1), B_{4}^{4,6}(0,1)
\end{aligned}
$$

Strictly speaking, $B_{4}^{4}(0,1)$ and $B_{4}{ }^{6}(0,1)$ are mutually independent states. But they are in the particle-to-antiparticle relation to each other so that it may be allowed to treat them together.*)

The reason why we put $B_{2}{ }^{2}(0,0)$ in brackets is that it is an "excited" state of vacuum, while the other particles are ground state. It is an open question for the theory whether $B_{2}^{2}(0,0)$ should exist as a particle state or not, but the assumption of its existence might have some profit in explaining the bosons in terms of the scheme of the full symmetry theory, as will be discussed later.

We see that the full symmetry theory seems to give the necessary and sufficient number of bosons which are needed from the phenomenological analysis

[^9]of the nuclear forces. The mass level scheme of the boson states of the full symmetry theory is given in Fig. 4. In the following we give a possible assignment of the bosons.

We have two possibilities in the choice of the assignment of $I=1$ vector boson, $B_{4}^{1}(0,1)$ and $B_{4}^{4,6}(0,1)$. Here we tentatively assign $I=1$ vector boson to $B_{4}^{4,6}(0,1)$ whose theoretical mass is nearer to the observed one.

There might be two $I=0$, vector bosons. One is the iso-singlet $3 \pi$ resonance observed in $p+\bar{p} \rightarrow 5 \pi$ and $7 \pi$ reactions and has relatively higher mass $\sim 780 \mathrm{Mev}^{41}$ ) and the other suggested from the pole of isoscalar part of nucleon electromagnetic form factor and also from the experiment of double pion production by the pion-deu-


Fig. 4. The mass level of bosons with $S=0, I=0$ and 1 obtained from the full symmetry theory by applying the mass formula. ${ }^{14)}$ teron collision has relatively small mass ( $\sim 420 \mathrm{Mev}$ from form factor and $\sim 550$ Mev from pion-deuteron collision) ${ }^{39, \text {, 42) }}$ In view of these observed masses, we could assign the higher mass to $B_{4}{ }^{1}(0,0)$ and the lower one to $B_{2}{ }^{2}(0,0)$, assuming the existence of $B_{2}{ }^{2}(0,0)$ state. Of course, if the existence of lower mass vector boson is not the case, we need not introduce $B_{2}{ }^{2}(0,0)$ into our level scheme of boson.

The $I=1$ scalar boson should be assigned to $B_{4}^{1}(0,1)$, if its existence is required.

Here we note on the assignment of $I=1$ bosons. In the above we have tentatively assigned $B_{4}^{4,6}(0,1)$ to the observed vector boson of mass $\sim 5$. If we conversely assign the higher one $B_{4}{ }^{1}(0,1)$ to the vector boson (this possibility must be taken into account, if the recent $\zeta$-meson ${ }^{48)}$ is real) then only scalar or vector would be allowed for $B_{1}^{4,6}(0,1)$. If it has other spin and parity, we need some additional boson to suppress the effect of $B_{i}^{4,6}(0,1)$ to the nuclear forces. But in the full symmetry theory there is no space for boson that can play such a role.

The $I=0$ scalar boson should be assigned to the remaining one, $B_{2}{ }^{1}(0,0)$. The present experimental evidence of $S$-wave $\pi-\pi$ resonance seems to occur at very low energy. If the $I=0$ scalar boson would have small mass as $\sim 2$, then it might be appropriate to interchange the assignment for $B_{2}{ }^{1}(0,0)$ and $B_{2}{ }^{2}(0,0)$.

For the case of the pseudoscalar bosons, we could borrow the arguments for the scalar bosons.

The above arguments are only a possible guess that can be inferred from the rather poor experimental data at present and should be changed according
to the progress of experiments in future.
Finally we should like to stress the importance of determining the parity of the supposed $I=0$ spinless boson. Not only it gives the definite answer to the standpoints taken in $\S 2$ and $\S 3$ of this paper, but also gives some key to the relationship of the full symmetry theory to the physical reality.

## § 5. Upper bound of coupling constant for possibly present neutral boson

The possible existence of neutral boson is interesting not only from the fact that its experimental discovery is very difficult but also from the fact that a possible interpretation of the 2 -body configuration of the full symmetry theory implies the existence of a neutral pseudoscalar boson with the mass of about $2 \sim 3 .{ }^{21) *)}$ Here we will first discuss the effects of pseudoscalar boson on nuclear forces rather in detail and only summarize those for the other types of neutral boson. In the following discussion we consider only one type of neutral boson of mass $m$ for simplicity and assume that OPEP + TPEP+one-neutral-boson-exchange-potential could explain the experiments. The analysis will be made about static potentials.
$I=0$ pseudoscalar
(i) The deuteron parameters except for the binding energy are mainly determined by $V_{T}^{+}$of OPEP in the region $x>1.5$, from which the pion coupling constant $g_{\pi}{ }^{2} / 4 \pi$ is determined as ${ }^{49}$

$$
\begin{equation*}
0.09>g_{\pi}^{2} / 4 \pi>0.06 \tag{17}
\end{equation*}
$$

In the presence of the neutral pseudoscalar boson whose coupling constant is $f_{p s}{ }^{2} / 4 \pi$, the OPEP tensor potential

$$
V_{T}^{+}=-\left(g_{\pi}{ }^{2} / 4 \pi\right) \gamma(x)
$$

changes into

$$
\begin{equation*}
V_{T}^{+}=-\left(g_{\pi}^{2} / 4 \pi\right) \gamma(x)\left\{1-(m / 3)\left(f_{p z}^{2} / g_{\pi}^{2}\right) \gamma(m x) / \gamma(x)\right\} . \tag{18}
\end{equation*}
$$

Considering that the uncertainty of $g_{\pi}^{2} / 4 \pi$ in Eq. (17) is about $20 \%$ and $\gamma(m x) / \gamma(x)$ in Eq. (18) rapidly decrease as $x$ increases, it may be difficult to see the effect of the boson from the deuteron parameters when the factor in parentheses of Eq. (18), which we call the neutral boson factor, satisfies the following condition:

$$
\begin{equation*}
1-(m / 3)\left(f_{p s}^{2} / g_{\pi}^{2}\right) \gamma(m x) / \gamma(x) \mid=0.8 . \tag{19}
\end{equation*}
$$

[^10]Eq. (19) gives

$$
\begin{equation*}
f_{p s}^{2} / g_{\pi}^{2} \leq 5.0 \quad \text { when } \quad m=2 \tag{20}
\end{equation*}
$$

Owing to the rapidly decreasing properties of $\gamma(m x) / \gamma(x), V_{T}^{+}$becomes weaker at most by 0.046 at $x=2.0$ by the neutral boson factor.
(ii) However, an effect of the neutral boson is clearly revealed in the singlet even state. ${ }^{1} V_{c}^{+}$changes into

$$
{ }^{1} V_{\sigma}^{+}=-\left(g_{\pi}^{2} / 4 \pi\right) Y(x)\left\{1+m\left(f_{p s}^{2} / g_{\pi}^{2}\right) Y(m x) / Y(x)\right\}
$$

The neutral boson factor is as large as 2.14 and 2.89 at $x=1.5$ and 1.0 respectively, when the equal sign in Eq. (20) is assumed. Then there is only a small room for TPEP.
(iii) The boson largely contributes also in the triplet odd state. The neutral boson factor for $V_{\bar{F}}$ is 1.60 at $x=1.5$ and 1.14 at $x=2.0$, which gives rise to too large $p-p$ total cross section, ${ }^{26)}$ and requires an extraordinary strong attractive tensor TPEP for compensation.
(iv) The unnecessity of TPEP in ${ }^{1} V_{C}^{+}$and the severe necessity in $V_{\vec{r}}$ are to be in contradiction, and a compromise can be obtained by assuming a small value for $f_{p s}^{2} / g_{\pi}{ }^{2}$. For example, assume that

$$
f_{p p}^{2} / g_{\pi}{ }^{2} \leq 2 \quad \text { when } \quad m=2 .
$$

Then the largest values of the neutral boson factor are:

$$
\begin{array}{llllllll}
\text { for } & { }^{1} V_{C}^{+} & 1.46 & \text { at } & x=1.5, & 1.76 & \text { at } & x=1.0, \\
\text { for } & V_{\bar{T}} & 1.05 & \text { at } & x=2.0, & 1.25 & \text { at } & x=1.5 .
\end{array}
$$

The left rooms for TPEP are for ${ }^{1} V_{C}^{+}$and $V_{\bar{r}}$ about one-half and comparable of the TMO TPEP respectively, which is reasonable in view of the uncertainty of a factor about two of TPEP.
(v) It is difficult to find effects of the one-neutral-boson-exchange-potential experimentally for the other states, since they are
a) relatively small for $V_{C}^{+}$, while $V_{C}^{+}$itself plays only a minor role for the deuteron parameters. ${ }^{49}$
b) repulsive for $V_{\bar{c}}$, being not inconsistent with the experimental requirement, ${ }^{277}$
c) attractive for ${ }^{1} V_{\bar{c}}$ but do not overwhelm the strong repulsive OPEP even if $f_{p k}^{2} / g_{\pi}^{2} \sim 5$ is assumed.
(vi) When $m=3$ is assumed,

$$
f_{p s}^{2} / g_{\pi}{ }^{2} \leq 29
$$

is obtained instead of (20) but the similar compromise discussed in (iv) leads to

$$
f_{p_{3}}^{2} / g_{\pi}{ }^{2} \leq 10 \quad \text { when } m=3
$$

$I=0$ scalar
The room left for the neutral scalar boson is rather small since its contribution must not destroy the weak repulsive central hamp due to OPEP in the triplet odd state. Namely the requirement that

$$
\begin{gathered}
V_{\bar{c}}=(1 / 3)\left(g_{\pi}{ }^{2} / 4 \pi\right) Y(x)\left\{1-3 m\left(g_{s}{ }^{2} / g_{\pi}{ }^{2}\right) Y(m x) / Y(x)\right\}>0 \\
\text { for } x>1
\end{gathered}
$$

leads to

$$
g_{s}^{2} / g_{\pi}^{2}<0.5 \quad \text { for } \quad m=2 .
$$

and

$$
g_{s}^{2} / g_{\pi}^{2}<1.5 \quad \text { for } \quad m=3
$$

With such values of $g_{s}{ }^{2}$, effects of the one-neutral scalar-boson-exchange-potential for $V_{G}^{+},{ }^{1} V_{\mathscr{C}}^{+},{ }^{1} V_{\bar{\sigma}}^{-}$are found negligible.
$I=0$ vector
A severe restriction for the coupling constant of the neutral vector boson comes from the fact that its contribution in the singlet even state must not be repulsive. That is,

$$
\begin{equation*}
2\left(f_{v}^{2} / 4 \pi\right)-g_{v}^{2} / 4 \pi>0 \tag{21}
\end{equation*}
$$

with which the effects of the one-neutral vector-boson-exchange-potential are

| repulsive | for $V_{\bar{o}}$, |
| :--- | :--- |
| attractive | for $V_{\bar{F}}$, |
| relatively small | for $V_{\bar{F}}^{+}$, |

being not inconsistent with the experimental requirements. Thus the coupling constants $f_{v}{ }^{2} / 4 \pi$ and $g_{v}{ }^{2} / 4 \pi$ can be fairly large as long as they satisfy Eq. (21). For example, from a modification of $V_{T}^{+}$corresponding to Eq. (18) we obtain

$$
f_{v}{ }^{2} / g_{\pi}{ }^{2} \leq 5.0 \quad \text { when } m=2
$$

No serious contradictions could be found when we further put $g_{v}{ }^{2} / 4 \pi=0$ and assume that TPEP is weak.
$I=0$ pseudovector
The presence of the boson is very improbable since it gives a repulsion for ${ }^{1} V_{c}^{+}$.

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[^0]:    *) Some part of the present paper was published in Soryushiron Kenkyu. ${ }^{1)}$
    **) We express the isospin of two-nucleon system by $T$ and that of boson by $I$.
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[^1]:    *) $x$ is the inter-nucleon distance in the unit of the Compton wavelength,
    **) See the footnote on p. 18,

[^2]:    *) In this paper, the pion mass is taken as the unit of mass.
    **) The relation that the force range is inversely proportional to the boson mass is not practically useful for a heavy boson and a small change of the boson mass scarcely affect the potential shape when the coupling constant is adjusted.

[^3]:    *) Minami has proposed the idea that the $\omega$ meson through vector coupling is mainly responsible for the hard cores in nuclear forces and $A-N$ forces. ${ }^{25)}$. We think, however, that the hardcore like repulsions may originate from the properties of forces between fundamental baryons, as we discussed in $\S 1$.
    ${ }^{* *}$ ) In this paper pseudovector boson with pseudotensor coupling to the nucleon is expressed as $p v(p t)$, and $p s$ means pseudoscalar boson, and so on.

[^4]:    *) The spin-orbit potential $V_{L S}(r) \boldsymbol{L} \cdot \boldsymbol{S}$ or the tensor potential $V_{T}(r) S_{12}$ is called attractive when $V_{L S}$ or $V_{T}$ is negative.
    **) In reference 22), only bosons of spin zero and one are considered and effects arising from the finite lifetime of the boson are neglected. In the present paper, nonstatic effects except spinorbit force are not taken into account in addition. In approach I, the non-static effects for the singlet even states mentioned in $\S 1$ should be attributed to the non-static effects for the OBEP. We can easily show that this is the case for the results of this section.

[^5]:    *) Hamada's potential involves the term proportional to

    $$
    Q_{12}=(1 / 2)\left\{\left(\boldsymbol{L} \cdot \boldsymbol{\sigma}_{1}\right)\left(\boldsymbol{L} \cdot \boldsymbol{\sigma}_{2}\right)+\left(\boldsymbol{L} \cdot \boldsymbol{\sigma}_{2}\right)\left(\boldsymbol{L} \cdot \boldsymbol{\sigma}_{1}\right)\right\}
    $$

[^6]:    *) A repulsive spin-orbit force is practically equivalent to an attractive tensor potential in the ${ }^{3} S_{1}+{ }^{3} D_{1}$ state. Therefore, in order to reproduce the deuteron parameters, the repulsive $V_{L S}^{+}$ must be compensated by a.repulsive $V_{T}^{+}$. Such compensation was found by Hamada and Tamagaki (private communication).

[^7]:    *) The large coupling constants in case 4 show that there exists very "delicate" cancellation between OBEP's due to $s, v(v)$ and $v(t)$. Even a slight change of each value of the masses of $s, v(v)$ and $v(t)$ would change the resulting potential drastically. In this sense the solution is quite unstable, and may also be excluded.

[^8]:    *) Such values larger than that in $\S 2$ come from the fact that the interference term between $v(v)$ and $v(t)$, which gives an important contribution to the spin-orbit force, is absent here.

[^9]:    ${ }^{*)}$ However, it is noted that there might be two different mass levels present corresponding to the states $(1 / \sqrt{ } 2)\left(B_{4}{ }^{4}(0,1) \pm B_{4}{ }^{6}(0,1)\right)$ as in the case of $K_{1}{ }^{0}$ and $K_{2}{ }^{0}$ particles.

[^10]:    *) In Ohnuki's interpretation there are two classes of bosons, each class corresponding to the 2 -body configuration of the full symmetry theory $B_{2}{ }^{1}(0,1), B_{2}{ }^{1}(0,0), B_{2}{ }^{1}\left(1, \frac{1}{2}\right), B_{2}{ }^{1}\left(-1, \frac{1}{2}\right)$. One is pseudoscalar mesons $\pi, \pi_{0}{ }^{\prime}, K, \bar{K}$ and the other vector ones $\rho, \omega, K^{*}, \bar{K}^{*}$. Between these two classes, agreement of isospin and strangeness and correspondence of mass hold. See also discussions in §1. Such assignments differ from these in §4 in that a state of the full symmetry theory corresponds to only one boson in the latter, while to two bosons in the former.

