Nuclear Physics from lattice QCD at strong coupling

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PhD thesis of Michael Fromm (ETH)

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and in progress

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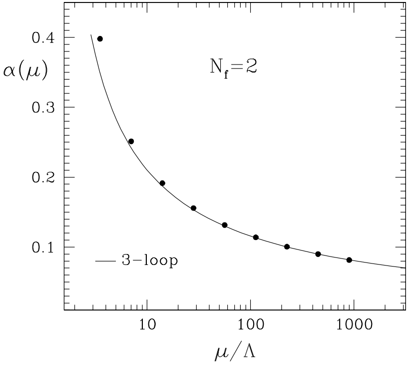
Nuclear physics from lattice QCD ?

- QCD is accepted theory of strong interactions among gluons & quarks:

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i (\not{D} + m_i) \psi_i$$

Asymptotic freedom:

lattice simulations

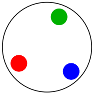


perturbation theory

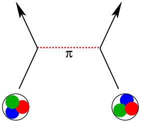
Successes of lattice QCD simulations

Physics of **color singlets**

- "One-body" physics: **confinement**
  hadron masses
  form factors, etc..

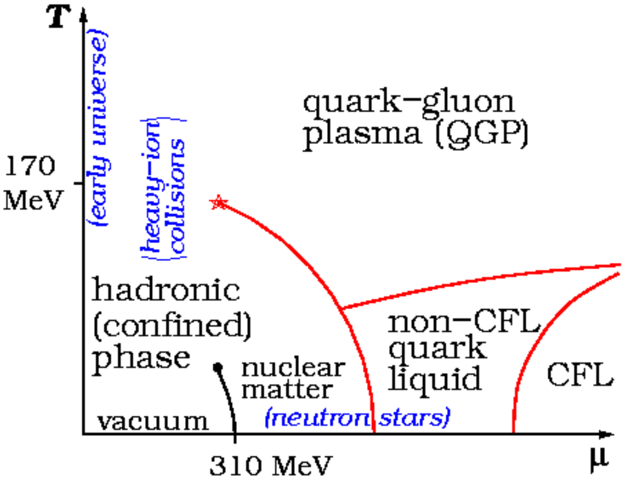


- Two-body physics: **nuclear interactions**
  just starting   Hatsuda et al, Savage et al



- Many-body physics: **nuclear matter**
  (μ, T) **phase diagram**

QCD phase diagram according to Wikipedia



This talk is about: **hadron $\leftrightarrow$ nuclear matter** transition and $T = 0$ **nuclear interactions**

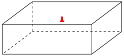
The difficulty with many-body physics

Non-zero baryon density $\Longrightarrow$ "sign problem"

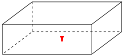
$$Z = \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{N_f}\bar{\psi}_i(\not{D} + m_i)\psi_i\right)$$

Integrate out fermions $\rightarrow$ $\det(\not{D} + m + \mu\gamma_0)$, *complex* when $\mu \neq 0$ (or $i\mu_I$)

Integration measure $d\varpi$ must be complex for correct physics:



$$\langle \mathrm{Tr\ Polyakov}\ \rangle = e^{-\frac{1}{T}F_q} = \int\ \mathrm{Re(Pol)} \times \mathrm{Re}(d\varpi) - \mathrm{Im(Pol)} \times \mathrm{Im}(d\varpi)$$



$$\langle \mathrm{Tr\ Polyakov}^\dagger \rangle = e^{-\frac{1}{T}F_{\bar{q}}} = \int\ \mathrm{Re(Pol)} \times \mathrm{Re}(d\varpi) + \mathrm{Im(Pol)} \times \mathrm{Im}(d\varpi)$$

$$F_q \neq F_{\bar{q}} \ \Rightarrow\ \mathrm{Im}(d\varpi) \neq 0$$

Complex integrand $\Longrightarrow$ cancellations $\Longrightarrow$ CPU resources $\propto$ $\exp(\mathrm{Volume})$

No way out?

$$Z = \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{N_f}\bar{\psi}_i(\not{D} + m_i)\psi_i\right)$$

Try integrating over the gauge field first!

- Problem: $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \;\rightarrow\; \beta_{\rm gauge}{\rm Tr}U_{\rm Plaquette}$, ie. 4-link interaction
- Solution: set $\beta_{\rm gauge} = \frac{2N_c}{g^2}$ to zero, ie. $g = \infty$, strong coupling limit
- Then integral over gauge links *factorizes*: $\sim \int \prod dU \exp(\bar{\psi}_x U_{x,\hat{\mu}}\psi_{x+\hat{\mu}})$

- analytic 1-link integral $\rightarrow$ only color singlets survive

- perform Grassmann integration last $\rightarrow$ hopping of color singlets
$\rightarrow$ hadron worldlines

- sample gas of worldlines by Monte Carlo

Note: when $\beta_{\rm gauge} = 0$, quarks are *always* confined $\forall(\mu, T)$, ie. **nuclear matter**

The price to pay: not continuum QCD

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\rm gauge}) \propto \exp(-\frac{\beta_{\rm gauge}}{4N_c b_0})$

ie. $a \to 0$ when $\beta_{\rm gauge} \equiv \frac{2N_c}{g^2} \to +\infty$. Here $\boxed{\beta_{\rm gauge} = 0}$!!

- Lattice “infinitely coarse”
- Physics not universal

Nevertheless:

- Properties similar to QCD: confinement and χSB
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\rm gauge} > 0$

When $\beta_{\rm gauge} = 0$, sign problem is **manageable** $\to$ **complete solution**

Valuable insight?

Further motivation

- 25^+ years of analytic predictions:

  80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto

  $$\mu_c(T=0) = 0.66, \quad T_c(\mu=0) = 5/3$$

  90's: Petersson et al., $1/g^2$ corrections

  00's: detailed (μ, T) phase diagram: Nishida, Kawamoto,...

  now: Ohnishi et al. $\mathcal{O}(\beta)$ & $\mathcal{O}(\beta^2)$, Münster & Philipsen,...

  How accurate is mean-field ($1/d$) approximation?

- Almost no Monte Carlo crosschecks:

  89: Karsch-Mütter $\to$ MDP formalism $\to$ $\mu_c(T=0) \sim 0.63$

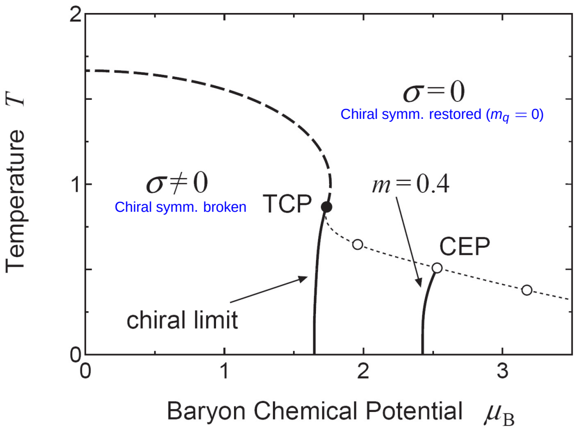
  92: Karsch et al. $T_c(\mu=0) \approx 1.40$

  99: Azcoiti et al., MDP ergodicity ??

  06: PdF-Kim, HMC $\to$ hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram from Nishida (2004, mean field, cf. Fukushima)



- Very similar to conjectured phase diagram of $N_f = 2$ **QCD**
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass $= M_{\text{proton}}$ $\Rightarrow$ lattice spacing $a \sim 0.6$ fm not universal

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp(-\bar{\psi}(\not{D}(U)+m)\psi)$, no plaquette term ($\beta_{\rm gauge} = 0$)

- One complex colored fermion field per site (no Dirac indices, spinless)
- $\not{D}(U) = \frac{1}{2}\sum_{x,\nu}\eta_\nu(x)(U_\nu(x) - U_\nu^\dagger(x-\hat{\nu}))$, $\eta_\nu(x) = (-)^{x_1+..+x_{\nu-1}}$

$U(1)_V \times U(1)_A$ symmetry:

$$\left.\begin{array}{l}\psi(x) \to e^{i\theta}\psi(x)\\ \bar{\psi}(x) \to e^{-i\theta}\bar{\psi}(x)\end{array}\right\} \text{unbroken} \Rightarrow \text{quark number} \Rightarrow \text{chem. pot.}$$

$$\left.\begin{array}{l}\psi(x) \to e^{i\varepsilon(x)\theta}\psi(x)\\ \bar{\psi}(x) \to e^{i\varepsilon(x)\theta}\bar{\psi}(x)\\ \varepsilon(x) = (-)^{x_1+x_2+x_3+x_4}\end{array}\right\} \text{spont. broken } (m=0) \Rightarrow \text{quark condensate}$$

($N_f = 1 \longrightarrow U(1)$ chiral symmetry)

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- Chemical potential $\mu \quad \to \exp(\pm a\mu)U_{\pm 4}$
- Alternative 1: integrate over fermions
  $Z = \int \mathcal{D}U \, \det(\not{D}(U)+m) \quad \to \quad$ HMC, severe sign pb. for $\mu \neq 0$

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- Alternative 2: $\mathcal{D}U = \prod dU$ *factorizes* $\rightarrow$ integrate over links Rossi & Wolff

  $\rightarrow$ **Color singlet** degrees of freedom:

- **Monomer** (meson $\bar{\psi}\psi$) $M(x) \in \{0,1,2,3\}$
- **Dimer** (meson hopping), non-oriented $n_\nu(x) \in \{0,1,2,3\}$
- **Baryon** hopping, oriented $\bar{B}B_\nu(x) \in \{0,1\}$ $\rightarrow$ *self-avoiding loops C*

Point-like, hard-core baryons in pion bath

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$$Z(m,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{m^{M(x)}}{M(x)!} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{n_\nu(x)!} \prod_{\text{loops}\,C} \rho(C)$$

with **constraint** $(M + \sum_{\pm\nu} n_\nu)(x) = 3\ \forall x \notin \{C\}$

MDP Monte Carlo

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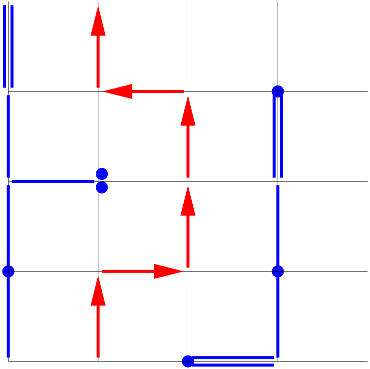

Constraint: 3 blue symbols or a baryon loop at every site

MDP Monte Carlo

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- sign of $\prod_C \rho(C)$: geometric factor $\varepsilon(C) = \pm 1$ for each baryon loop C

Karsch & Mütter: Resum analytically $\to$ sign pb. eliminated at $\mu = 0$

$\to$ "MDP ensemble"

MDP Monte Carlo

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Karsch & Mütter: Resum analytically $\rightarrow$ sign pb. eliminated at $\mu = 0$

$\rightarrow$ "MDP ensemble"

Further, algorithmic difficulties:

- changing monomer number difficult: weight $\sim m^{\sum_x M(x)}$
  monomer-changing update (Karsch & Mütter) restricted to $m \sim \mathcal{O}(1)$
- tight-packing constraint $\rightarrow$ local update inefficient, esp. as $m \rightarrow 0$

Solved with **worm algorithm** (Prokof'ev & Svistunov 1998)

Worm algorithm for MDP

- Sample $G(x,y)$ rather than Z, ie. add source and sink
- Monte Carlo: guided random walk of sink, ie. *local* steps ("**worm**")
- When $y=x$, contribution to Z $\rightarrow$ *global* change
- cf. "directed path" (Adams & Chandrasekharan) for $U(N)$

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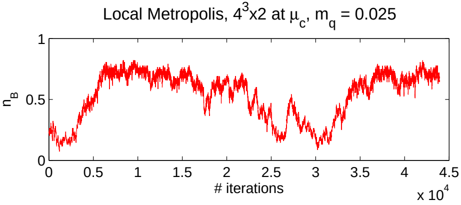
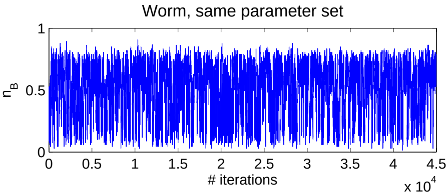
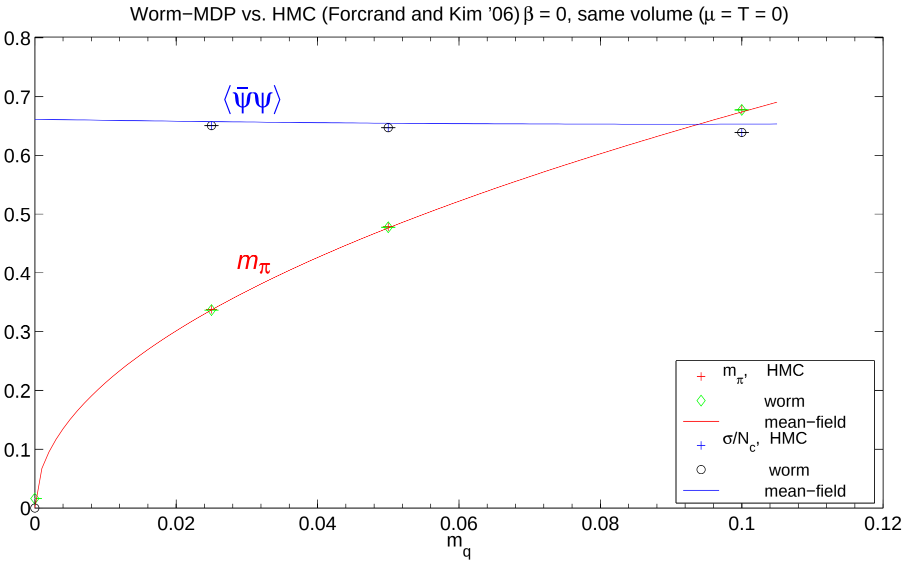
- **Efficient even when $m_q = 0$**

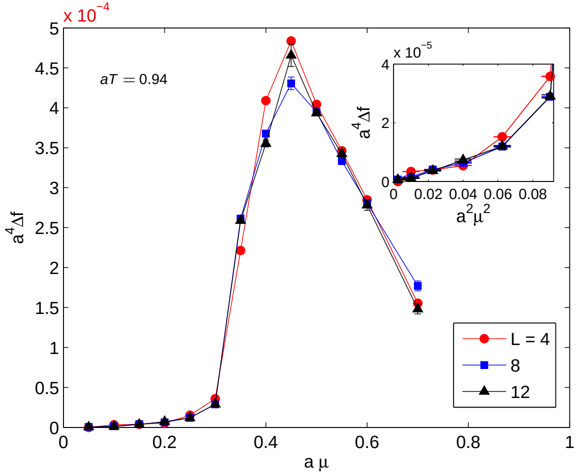




[Non-trivial] consistency check with HMC

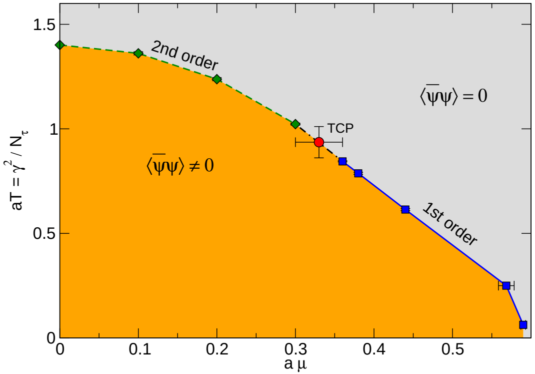


Sign problem? Monitor $-\frac{1}{V}\log\langle \mathrm{sign}\rangle$



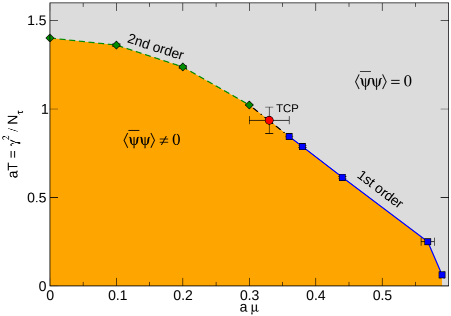
- $\langle \mathrm{sign}\rangle = \frac{Z}{Z_{||}} \sim \exp(-\frac{V}{T}\Delta f(\mu^2))$ as expected; $\Delta f \sim \mu^2 + \mathcal{O}(\mu^4)$
- Can reach $\sim$ **$16^3 \times 4$** $\forall\mu$, ie. adequate

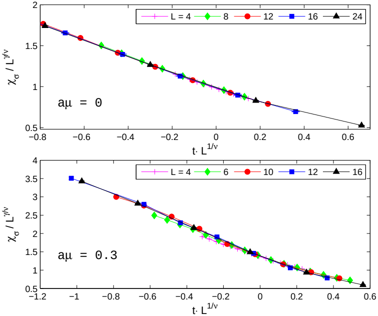
Phase diagram in the chiral limit $m_q = 0$



- Phase boundary for breaking/restoration of $U(1)$ chiral symmetry
- Mean field analysis: 2nd order at $\mu = 0$, $T_c = 5/3$
- If 2nd order, then expect 3d O(2) universality class
- Monte Carlo: 2nd order at $\mu = 0$ (Karsch et al, 1992)
- 1rst order at $T = 0$: ρ_B jumps from 0 to 1 baryon per site $\Longrightarrow$ tricrit. pt. TCP

Phase diagram in the chiral limit



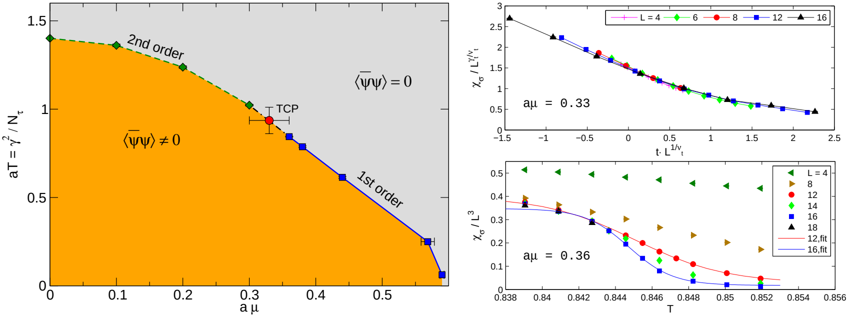


- Chiral susc. $\chi_\sigma = \frac{1}{V}\frac{\partial^2}{\partial m_q^2}\log Z = \langle \sum_x \bar\psi\psi(x)\bar\psi\psi(0)\rangle \sim L^{\gamma/\nu}\tilde\chi(tL^{1/\nu})$

  $\rightarrow$ $\frac{\chi_\sigma}{L^{\gamma/\nu}}$ is *universal* function of $tL^{1/\nu}$

- Data collapse using 3d O(2) exponents for $a\mu = 0$ and 0.30

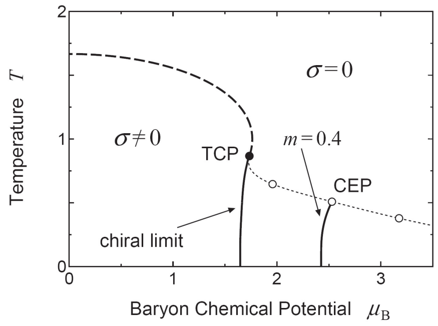
Phase diagram in the chiral limit

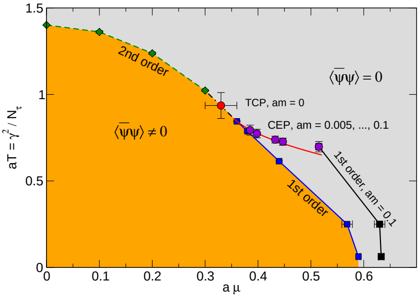


- Data collapse using mean-field exponents (d=3 is TCP upper crit. dim.) for $a\mu = 0.33$

- 1st-order Borgs-Kotecky: $Z(T) = \exp(-\frac{V}{T}f_1(T)) + c\,\exp(-\frac{V}{T}f_2(T))$ for $a\mu = 0.36$

Compare with Nishida (2004)





- TCP: $(\mu, T) = (0.33(3), 0.94(7))$ (Monte Carlo) vs $(0.577, 0.866)$ (mean-field)
  Beware of quantitative mean-field predictions for phase diagram
- No reentrant phase diagram (caused by *decreasing* entropy in dense phase)
  cf. Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta S} \to$ vertical at $T = 0$
- $m_q \neq 0$: trajectory of CEP obeys tricritical scaling

Transition to nuclear matter: $T = 0, \mu = \mu_c$

Puzzle:

- Mean-field baryon mass is ≈ 3 $\Rightarrow$ expect $\mu_c = \frac{1}{3} F_B(T = 0) \approx 1$
- Mean-field estimate $\mu_c \sim 0.55 - 0.66$ much smaller, ie. $\mu_c^B \sim 600$ MeV !

Mean field gives wrong M_B? wrong μ_c?

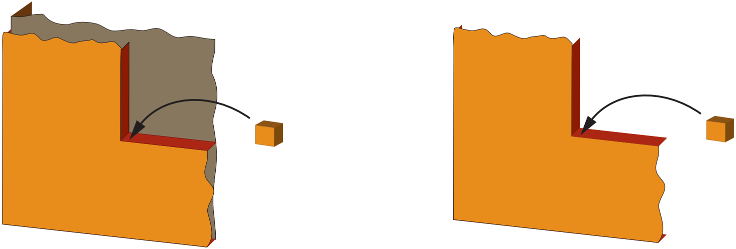
- Check M_B and μ_c by Monte Carlo $\longrightarrow$ ok (next slide)

Remaining explanation: nuclear attraction $\sim$ **1/3** baryon mass !!

Why so large ? Nuclear potential ? Nuclear spectroscopy ??

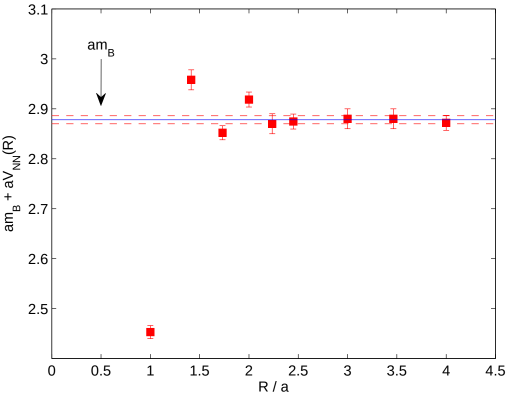
$\mu_c(T=0)$

- $T=0$ dense phase is baryon crystal (1 baryon per site)
- μ_c is free energy necessary to add 1 baryon to dense phase



- Monte Carlo: $a\mu_c^B = 1.78(1)$ compared with $am_B = 2.88(1)$
- Each baryon binds to 3 nearest-neighbours $\rightarrow$ attraction
  $V_{NN}(r=a) \sim \frac{2.88-1.78}{3} a^{-1} \sim 120$ MeV !!
- Surface tension: first layer of dense phase $\rightarrow$ 2 nearest-neighbours only
  $$\sigma \approx \frac{1}{2}|V_{NN}(r=a)|a^{-2} \quad \sim 200 \text{ MeV/fm}^2$$
- Complete nuclear potential ?

Nuclear potential

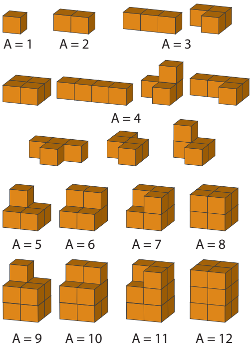


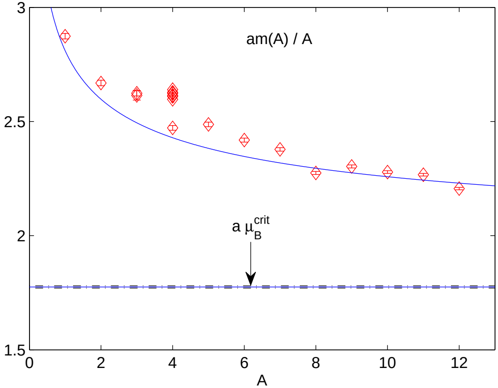
- Nucleons are point-like $\longrightarrow$ no ambiguity with definition of static potential
- Nearest-neighbour attraction $\sim$ 120 MeV at distance $\sim$ 0.5 fm: cf. real world

  Baryon worldlines self-avoiding $\longrightarrow$ no meson exchange here (just hard core)

  Attraction due to bath of neutral pions: cf. Casimir effect (see later)

Nuclear spectroscopy

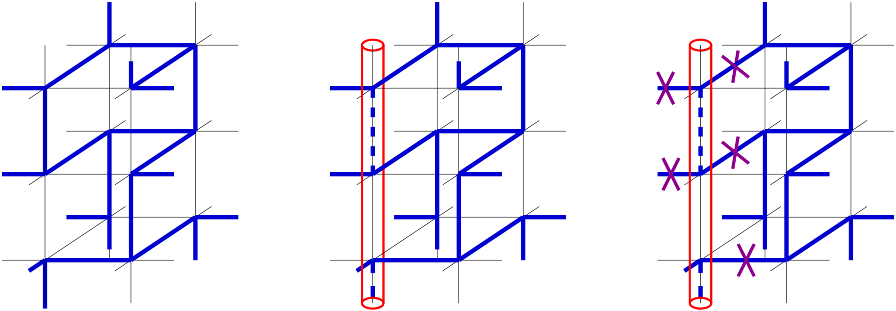




- Can compare masses of differently shaped "isotopes"
- $E(B=2) - 2E(B=1) \sim -0.4$, ie. "deuteron" binding energy ca. 120 MeV
- $am(A) \sim a\mu_B^{crit} A + (36\pi)^{1/3} \sigma a^2 A^{2/3}$, ie. (bulk + surface tension)
  Bethe-Weizsäcker parameter-free (μ_B^{crit} and σ fixed)
- "Magic numbers" with increased stability: $A = 4,\ 8,\ 12$

How the nucleon got its mass

- Point-like nucleon **distorts pion bath** cf. Casimir



- Energy = nb. time-like pion lines

Constraint: 3 pion lines per site ($m_q = 0$) $\rightarrow$ energy density = 3/4 in vacuum

No spatial pion lines connecting to site occupied by nucleon $\rightarrow$ energy increase

Steric effect

- $am_B \approx 2.88 = (3 - 0.75) + \Delta E_\pi$, ie. "valence"(78%) + "**pion cloud**"(22%)

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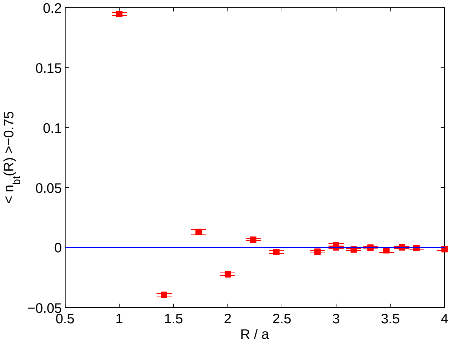

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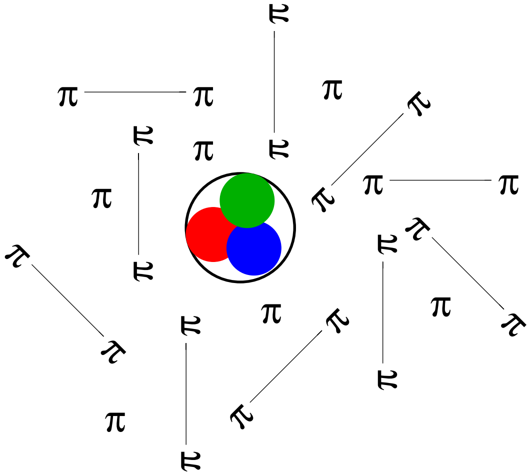
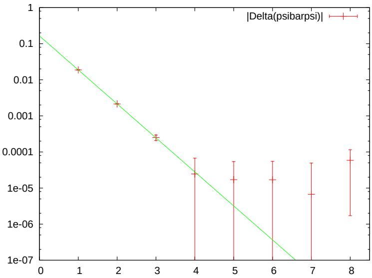
In fact, nucleon is *not* point-like

Point-like “bag” of 3 valence quarks $\rightarrow$ macroscopic disturbance in pion vacuum

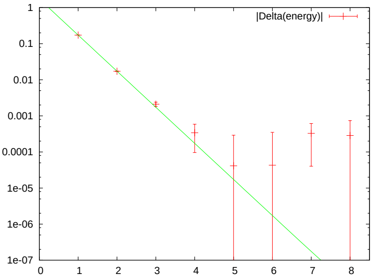


In fact, nucleon is *not* point-like

Point-like “bag” of 3 valence quarks $\rightarrow$ macroscopic disturbance in pion vacuum



$|\Delta(\bar{\psi}\psi)|$

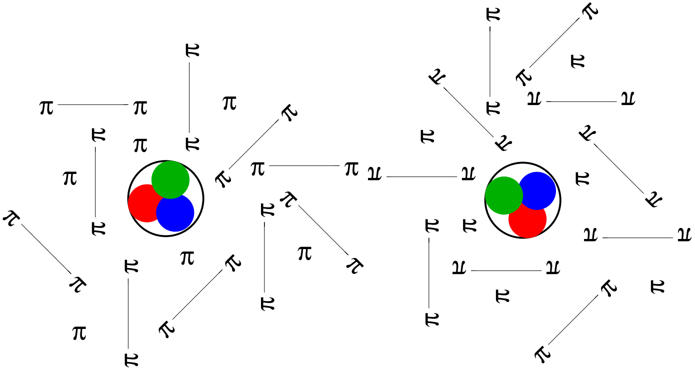


$|\Delta(\text{energy density})|$

Profile $\sim \exp(-\text{meson mass} \times r)$ (in progress)

Nuclear interaction via pion clouds

- Here, baryons make self-avoiding loops $\rightarrow$ no direct meson exchange
- Interaction comes from **overlapping pion clouds**



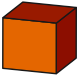
"meson exchange" $\Delta E(r) \sim \exp(-mr)/r$

Nuclear interaction via pion clouds

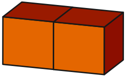
- Here, baryons make self-avoiding loops $\rightarrow$ no direct meson exchange
- Interaction comes from **overlapping pion clouds**

- Example: nearest-neighbour nucleon attraction

- Energy of pion cloud mostly in 6 nearest-neighbours of nucleon



- Nearest-neighbour nucleon pair has only 10 nearest-neighbour sites



$$\Delta E_\pi(2 \text{ nucleons}) \; < 2\Delta E_\pi(1 \text{ nucleon}) \rightarrow \text{attraction}$$

Conclusions

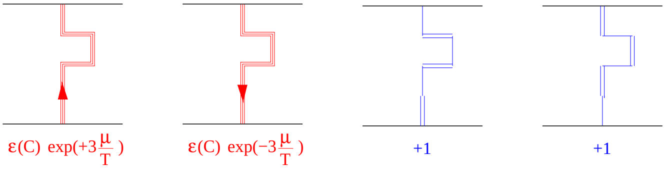
Summary

- Take mean-field results with a grain of salt
- “Clean-up” of phase diagram justified
- [Crude] nuclear matter from QCD
- Nucleon: point-like “bag” + large pion cloud
- “Understand” nuclear interaction as steric effect of valence quarks
- “Meson exchange” from overlapping pion clouds

Outlook

- Non-zero quark mass:
  - Critical end-point as a function of m_q
  - Nuclear potential & spectroscopy as a function of m_q
- Include second quark species $\rightarrow$ isospin (degenerate masses or not)
- Include $\mathcal{O}(\beta)$ effects ?

Backup: Karsch & Mütter's resummation



Karsch & Mütter: Resum into "MDP ensemble" $\longrightarrow$ sign pb. eliminated at $\mu = 0$