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Nucleon Electromagnetic Form Factors at High Momentum Transfers in an Extended Particle Model Based on the Quark Model

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Taking account of the Lorentz contraction effect of the extended nucleon core as a nucleon but not as a quark, it is shown that the Gaussian inner orbital wave function can produce the form factor very close to the dipole formula.

Recent experiments show that the nucleon electromagnetic form factor are empirically described by the "scaling law" $e^{-1}G_E{}^P=\mu_P{}^{-1}G_M{}^P=\mu_n{}^{-1}G_M{}^n$ $(\equiv F)$ and $G_E{}^n=0$ and by the "dipole formula" $F=(1+K^{-2}|t|)^{-2}$, where we have followed the usual notations and $K^2=0.71$ $(\mathrm{GeV}/c)^2$. The scaling law was already discussed on the theoretical basis of the nonrelativistic urbaryon (quark) model.[1),2)] Ishida et al.[2)] and Drell et al.[3)] attempted to extract information about the inner orbital wave function at short distances from the $|t|$-dependence of F in a wide region of $|t|$ over M^2 (M being the nucleon mass), using nonrelativistic formulas. In this note we show that if possible relativistic effects as a nucleon (not as a quark), especially the Lorentz contraction of the nucleon core, are taken into account in a proper way, their conclusions become never true but the simple Gaussian inner orbital wave function can produce the form factor very close to the dipole formula.

Those who are working with the nonrelativistic quark model have believed that if $|t|\ll M_q{}^2$ (M_q being the quark mass), nonrelativistic formulas can be used for everything. As for the form factor, therefore, they have used

$$F=W_{\mathrm{NR}} \quad \text{(Drell et al. and others)}, \tag{1a}$$

$$F=(1+|t|/m_{\mathrm{V}}^2)^{-1}W_{\mathrm{NR}} \quad \text{(Ishida et al.)}, \tag{1b}$$

with the nonrelativistic formula

$$W_{\mathrm{NR}}=\iint e^{iqx}|\phi(x,\ \cdots)|^2dxd(\cdots), \tag{2}$$

where m_{V}^2 is the mean square mass of ρ and ω mesons and q the momentum transfer. $\phi(x,\ \cdots)$ stands for the inner orbital wave function, where independent inner coordinates are denoted by x and $\cdots$. Assuming the simple Gaussian function for ϕ, we have got

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$$W_0 = \exp(-\tfrac{1}{6}\langle r^2\rangle_c |t|), \tag{3}$$

where $\langle r^2\rangle_c$ is the mean square radius of the nucleon core. It is evident that the simple Gaussian function never gives us the form factor consistent with the dipole formula for $|t| \gtrsim M^2$. This is the reason why Ishida et al. introduced a singular wave function and Drell et al. discussed singular potentials among constituent particles. It is, however, to be noted that Eq. (2) is a nonrelativistic formula to be verified not only for $|t| \ll M_q^2$ but also for $|t| \ll M^2$. Here we want to emphasize that relativistic effects as a nucleon (but not as a quark) become very important for $|t| \gtrsim M^2$. Indeed, we can see that the Lorents contraction effect as a nucleon for $|t| > M^2$ should reduce $\langle r^2\rangle_c$ in Eq. (3) by the Lorentz factor, Γ^{-1}, approximately proportional to $M^2|t|^{-1}$. Hence Eq. (3) must be modified essentially in its $|t|$-dependence in the following way:

$$\exp\ (-\tfrac{1}{6}\Gamma^{-1}\langle r^2\rangle_c |t|)\, O_{\mathrm{FI}}(q^2), \tag{4}$$

where $O_{\mathrm{FI}}(q^2)$ is the overlap integral defined by $(\phi_{\mathrm{F}}, \phi_{\mathrm{I}})$.

Hence we cannot exclude the Gaussian inner wave function. Furthermore note that the region $M_q^2 \gg |t| \gtrsim M^2$ covers a wide range from one $(\mathrm{GeV}/c)^2$ to several ten $(\mathrm{GeV}/c)^2$ if $M_q \simeq (5 \text{ to } 10) \times M$.

In order to take properly the relativistic effect into account we must inevitably use the four-dimensional inner orbital wave function. In the quark model, the nucleon is assumed to be a composite particle of three quarks. Suppose that the three quarks have, respectively, four-position coordinates x_1, x_2 aed x_3. After separating the center-of-mass coordinate $X = (x_1 + x_2 + x_3)/3$, we keep two independent relative coordinates $r = (x_2 - x_3)/\sqrt{6}$ and $s = (-2x_1 + x_2 + x_3)/3\sqrt{2}$. Now, as the simplest example, we can choose the four-dimensional Gaussian function

$$\phi(r, s; P) = (\alpha/\pi)^2 \exp\left[\frac{\alpha}{2}\left\{r^2 + s^2 - \frac{2}{M^2}(P\cdot r)^2 - \frac{2}{M^2}(P\cdot s)^2\right\}\right] \tag{5}$$

for the inner orbital wave function,*) where P stands for the center-of-mass momentum of the composite system, i.e. the nucleon momentum, and $(\alpha/\pi)^2$ is the normalization costant determined by $\int\int |\phi|^2 d^4r d^4s = 1$. The constant α is related to the mean square radius of the nucleon core through $\alpha^{-1} = \langle r^2\rangle_c/3$. It may be worth while to emphasize another reason why Eq. (5) is used here: Equation (5) represents the ground state eigenfunction of the Hamiltonian of a four-dimensional harmonic oscillator consistent with the famous linearly raising trajectory in an extended particle model.[4] Our procedure should be regarded as one theoretical attempt in an extended particle model represented by a trilocal field based on the quark model, rather than one in the naive relativistic quark model.

*) Note that $r^2 = r_0^2 - \boldsymbol{r}^2$.

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The relativistic form factor is given by the formula*)

$$W_{\mathrm{R}} = \iint \psi^*(r,\ s;\ P_{\mathrm{F}})\, e^{iq\cdot(ar+bs)} \psi(r,\ s;\ P_{\mathrm{I}})\, d^4r d^4s\,, \tag{6}$$

or symbolically

$$W_{\mathrm{R}} = (\psi_{\mathrm{F}},\ e^{iqx}\psi_{\mathrm{I}})\,, \tag{6'}$$

where P_{I} and P_{F} are, respectively, the initial and final momenta of the nucleon and $q = P_{\mathrm{F}} - P_{\mathrm{I}}$. a and b is one of the following pairs; $(0,\ -\sqrt{2})$, $(\sqrt{3/2},\ 1/\sqrt{2})$ and $(-\sqrt{3/2},\ 1/\sqrt{2})$. Note here that $a^2 + b^2 = 2$ for every pair. Inserting Eq. (5) into Eq. (6), one obtains

$$W_{\mathrm{R}} = \left(\frac{1}{\sqrt{\Gamma}}\right)^B \exp\left(-\frac{1}{6}\langle r^2\rangle_c \Gamma^{-1}|t|\right). \tag{7}$$

Here the first factor is not other than the overlap integral

$$O_{\mathrm{FI}}(q^2) = (\psi_{\mathrm{F}},\ \psi_{\mathrm{I}}) = \left(\frac{1}{\sqrt{\Gamma}}\right)^B$$

with (8)

$$\Gamma = 1 + \frac{|t|}{2M^2} \quad \text{and} \quad B = 4\,,$$

where $B = 4$ means the number of dimensions giving the Lorentz contraction, namely, the longitudinal space-like inner coordinates and two time-like inner coordinates. As mentioned above we can see in Eq. (7) that $\sqrt{\Gamma}$ behaves just like the effective Lorentz contraction factor and the exponential function in Eq. (7) goes to a constant as $|t|$ increases over M^2. The $|t|$-dependence of the form factor for $M_q^2 \gg |t| \gtrsim M^2$ is, therefore, governed mainly by the overlapping-effect factor $(1/\sqrt{\Gamma})^B$ which goes to $(|t|/2M^2)^{-B/2}$. We can remark that power B is nothing other than the number of independent relative coordinates. Thus the dipole-like behavior of W_{R} can be obtained from $B = 4$, namely, the inner freedom of motion equivalent to four indepencent relative coordinates. Thus the dipole-like behavior of W_{R} can be obtained from $B = 4$, namely, the inner freedom of motion equivalent to four independent relative coordinates.

Equations (1a) and (1b) should, respectively, be replaced with

$$F = W_{\mathrm{R}}\,, \tag{9a}$$

$$F = (1 + \tfrac{1}{6}\langle r^2\rangle_{\mathrm{V}} \Gamma^{-1}|t|)^{-1} W_{\mathrm{R}}\,, \tag{9b}$$

where $\langle r^2\rangle_{\mathrm{V}} = 6m_{\mathrm{V}}^{-2}$ is the mean square radius of the vector meson cloud. Both formulas (9a) and (9b) together with Eq. (7) behave like the dipole formula

*) It is to be noted that most of the infinite component field theories have identified the form factor with the overlap function $(\psi_F,\ \psi_I)$ but never with $(\psi_F,\ e^{iqx}\psi_I)$ itself. In fact, some authors have derived our later result, $(\psi_F,\ \psi_I) = (1 + |t|/2M^2)^{-2}$ as the form factor, using the infinite component field theory. See A. O. Barut's lecture given at the Colorado Summer School in 1967. We must emphasize here that the form factor should not be given by $(\psi_F,\ \psi_I)$.

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modified by a constant factor for $M_q^2 \gg |t| \gtrsim M^2$. Note that they contain only one free parameter $\langle r^2 \rangle_c$ to be adjusted. Let us first compare the theoretical form factor given by Eq. (9a) together with Eq. (7)——call it Case (i)——with experiment[5] in Fig. 1, in which we have used $\langle r^2 \rangle_c = 7.50$ (GeV/c)$^{-2}$. From them one can see that the theoretical curve given by Eqs. (9a) and (7) is not inconsistent with the experimental plot but quite different from the nonrelativistic Gaussian form factor W_O. Next we examine Case (ii) in which Eq. (9b) is combined with Eq. (7), namely, each quark has the vector meson cloud. Choosing $\langle r^2 \rangle_c = 1.82$ (GeV/c)$^{-2}$, we see in Fig. 2 that the theoretical curve can reproduce the experimental plot in a wide range of $|t|$ from zero to about 25 (GeV/c)^2. It is repeatedly noted that this fit has been obtained by adjusting only one parameter $\langle r^2 \rangle_c$, and that the nonrelativistic Gaussian form factor is strongly modified in its essence.

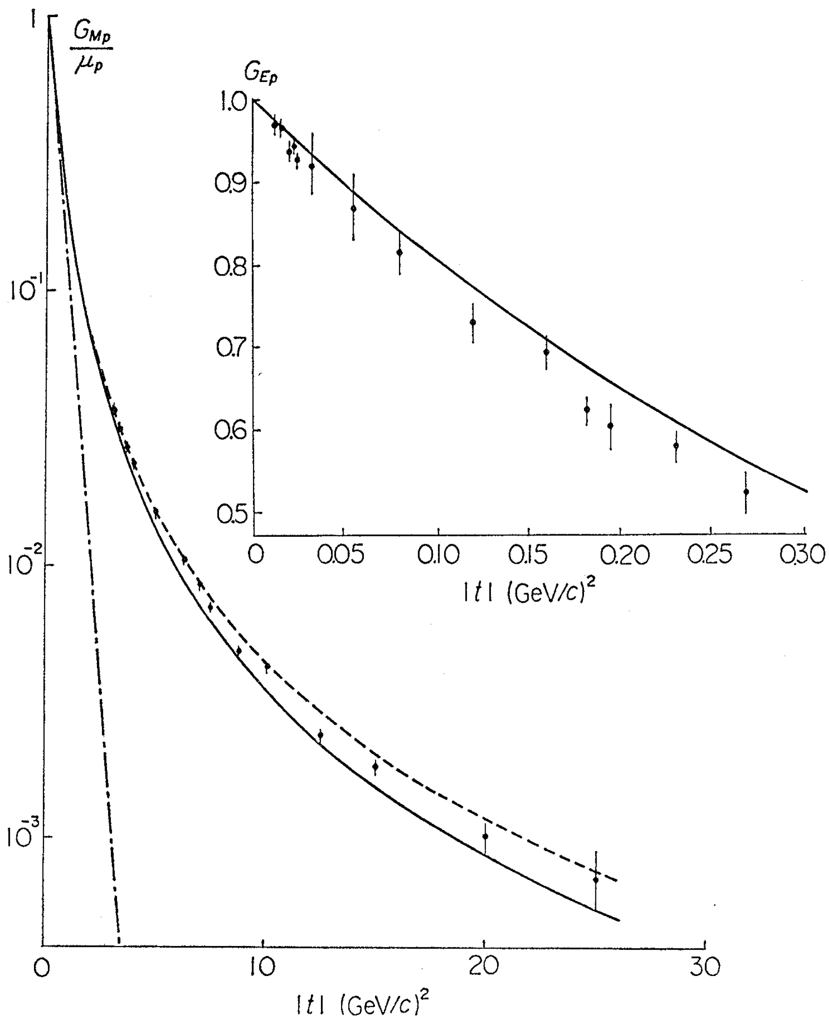

Fig. 1. Comparison of the theoretical form factors with experiments in Case (i) with $\langle r^2 \rangle_c = 7.50$ (GeV/c)$^{-2}$. The dipole formula and the nonrelativistic Gaussian form factor are, respectively, shown by the broken and chain lines.

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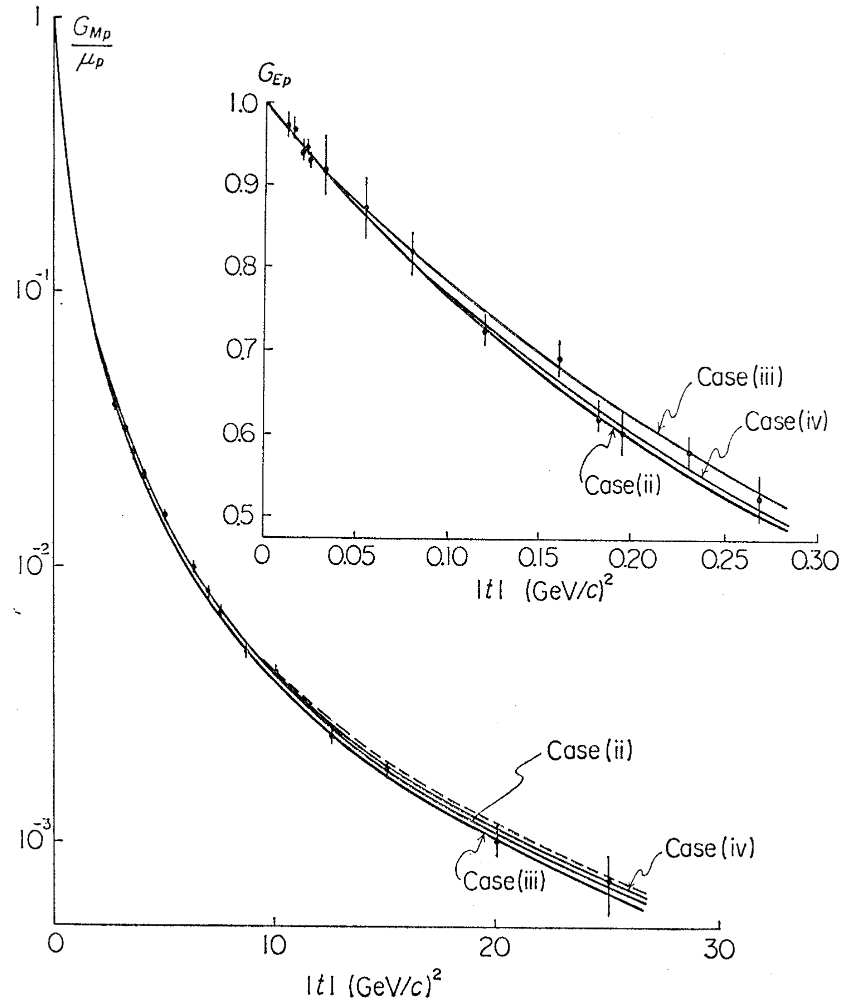

Fig. 2. Comparison of the theoretical form factors with experiments in Case (ii) with $\langle r^2\rangle_c=1.82$ (GeV/c)$^{-2}$, in Case (iii) with $\langle r^2\rangle_c=8.81$ (GeV/c)$^{-2}$ and $\lambda=1.4$, and in Case (iv) with $\langle r^2\rangle_c=1.20$ (GeV/c)$^{-2}$ and $\lambda=0.9$. The broken line shows the dipole formula.

Here we want to introduce a new parameter, say λ, into the inner orbital wave function as follows:

$$\psi_\lambda(r,\ s;\ P)=\left(\frac{\alpha}{\pi}\right)^2\sqrt{2\lambda-1}\ \exp\left[\frac{\alpha}{2}\left\{r^2+s^2-\frac{2\lambda}{M^2}(P\cdot r)^2-\frac{2\lambda}{M^2}(P\cdot s)^2\right\}\right]. \tag{10}$$

It is easy to see that the parameter λ distinguishes the time-like extension from the space-like one of the inner orbital motion, and that $\lambda=1$ gives us the original one Eq. (5). Using ψ_λ, we have got

$$W_{\mathrm{R}}^{(\lambda)}=\Gamma_\lambda^{-1}\left(1+\frac{\lambda}{2\lambda-1}\frac{|t|}{2M^2}\right)^{-1}\exp\left[-\frac{1}{6}\langle r^2\rangle_c\Gamma_\lambda^{-1}|t|\right], \tag{11}$$

where

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$$\Gamma_\lambda = 1 + \lambda \frac{|t|}{2M^2}. \tag{12}$$

The form factor F_λ is obtained by Eq. (11) together with the modified formulas

$$F_\lambda = W_{\mathrm{E}}^{(\lambda)}, \tag{13a}$$

$$F_\lambda = (1 + \tfrac{1}{6}\langle r^2\rangle_{\mathrm{V}} \Gamma_\lambda^{-1}|t|)^{-1} W_{\mathrm{E}}^{(\lambda)}. \tag{13b}$$

In case (iii) Eq. (13a) is combined with Eq. (11), and Case (iv) is given by Eq. (13b) together with Eq. (11). In both cases the form factor goes to one proportional to $(2M^2|t|^{-1})^2$ like the dipole formula. Figure 2 shows us that the experimental plot can be fitted by the theoretical curves with $\langle r^2\rangle_c = 8.81$ $(\mathrm{GeV}/c)^{-2}$ and $\lambda = 1.4$ in Case (iii) and with $\langle r^2\rangle_c = 1.20$ $(\mathrm{GeV}/c)^{-2}$ and $\lambda = 0.9$ in Case (iv). The theoretical curves are in good agreement with experiment. Needless to say, Cases (iii) and (iv) include Cases (i) and (ii), respectively, as their special cases with $\lambda = 1$.

From the above arguments we have inferred that the Lorentz contraction of the extended nucleon core can be a possible origin of the "dipole formula". The same effects will appear also in inelastic electron proton collisions leading to the isobar excitation. In the nonrelativistic quark model we have got the differential cross section for the inelastic collision in the following form:[6)]

$$\frac{d^2\sigma}{d\Omega dE} \propto (\langle r^2\rangle_c q^2)^L \exp[-A\langle r^2\rangle_c q^2] \tag{14}$$

using the simple Gaussian wave function, where L is an integral number determined by the type of transition, and A a numerical factor. If we take the Lorentz contraction factor into account, then we can infer that Eq. (14) should be replaced with

$$(\Gamma^{-1}\langle r^2\rangle_c q^2)^L \exp[-A\Gamma^{-1}\langle r^2\rangle_c q^2]\,|O_{\mathrm{FI}}(q^2)|^2, \tag{15}$$

where $\sqrt{\Gamma}$ and $O_{\mathrm{FI}}(q^2)$ are, respectively, the effective Lorentz contraction factor and the overlap integral in the inelastic collision. The similar structure of the wave function suggests us that $\Gamma \to q^2$ and $O_{\mathrm{FI}}(q^2) \to (q^2)^{-2}$ as q^2 goes over M^2, and then that the inelastic cross section would behave like the dipole formula squared for $q^2 \gtrsim M^2$. Indeed, it seems to us that recent experiments indicate such a behavior for the cross section.[7)] Detailed discussions will be given in a forthcoming paper in which the full nucleon and isobar wave functions and the quark current to be valid for $M_q^2 \gg |t| \gtrsim M^2$ will be formulated.

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