Prog. Theor. Phys. Vol. 94, No. 3, September 1995, Letters

Nucleon Superfluidity in Kaon-Condensed Neutron Stars

Tatsuyuki TAKATSUKA and Ryozo TAMAGAKI*,*)

Faculty of Humanities and Social Sciences, Iwate University, Morioka 020
**Kamitakano Maeda-Cho 26-5, Kyoto 606*

(Received May 15, 1995)

Nucleon Superfluidity under the occurrence of kaon condensation in neutron star cores is investigated. Due to a large proton mixing characteristic of the condensate, both of neutrons and protons can utilize the $^3P_2$-pairing interaction, but the realization of these $^3P_2$ superfluids is unlikely unless the effective nucleon-mass parameter $m^*(\equiv m_N^*/m_N)$ is exceptionally larger than ~ 0.7 at high densities.

Superfluidity in neutron stars is of great interest not only from the viewpoint of quantum many-body problem in nucleon matter but also from neutron star phenomena such as the cooling and the glitches. In 1970's, we have shown by a realistic approach that neutrons and admixed protons in the core region of neutron stars are in the $^3P_2$-superfluid of a new type and the $^1S_0$-superfluid of a usual type, respectively, and these "core superfluids" are coexistent at the densities $\rho \simeq (0.7-3)\rho_0$ with $\rho_0 = 0.17$ nucleons $/\mathrm{fm}^3 \simeq 2.8\times 10^{14}\mathrm{g}/cc$ being the standard nuclear density.[1]

In recent years, however, the possibility of core superfluids at higher densities $\rho \gtrsim 3\rho_0$ has been attracting much attention in relation to the cooling problem of neutron stars; to be compatible with observations, we need the efficient cooling mechanism such as the pion- or the kaon-coolings expected to operate at $\rho \gtrsim 3\rho_0$ and at the same time the role of nucleon superfluidity to suppress their too rapid cooling.[2] This poses a challenging problem whether the existence of core superfluids can extend to $\rho \gtrsim 3\rho_0$ under the situation that pion- or kaon-condensate occurs at these high densities. The aim of this paper is to investigate the case with kaon condensate under current interest.

Characteristics of kaon-condensed phase

In neutron stars, kaon condensation is expected to occur for $\rho \gtrsim (3-4)\rho_0$ through the weak interaction process not to conserve strangeness, $n \rightleftarrows p + K^-$, and by the energy gain coming from the strong interaction, i.e., the so-called KN sigma term.[3)~6)] In this new phase, nucleons are described by the quasiparticle states, η and ζ, which are the superposition of neutron (n) and proton (p) states, i.e., $|\eta\rangle = u|n\rangle + v|p\rangle$, $|\zeta\rangle = u^*|p\rangle - v^*|n\rangle$ with $|u|^2 + |v|^2 = 1$. Since $u = 1 + O(G_W{}^2)$ and $v = O(G_W)$ with the weak coupling constant G_W, they can safely be treated as pure neutron and proton states constituting two independent Fermi seas.[7] In this sense, nucleon system under kaon condensate is very similar to that in a normal phase of usual neutron star matter.

A distinguished point of this new phase is in a remarkabley large proton-mixing ratio $Y_p(\equiv \rho_p/\rho,\ \rho = \rho_n + \rho_p)$. This is due to that the charge neutrality condition is assured by K^-, not by e^- which causes a large energy increase. As an example, Y_p

*) Professor Emeritus, Kyoto University.

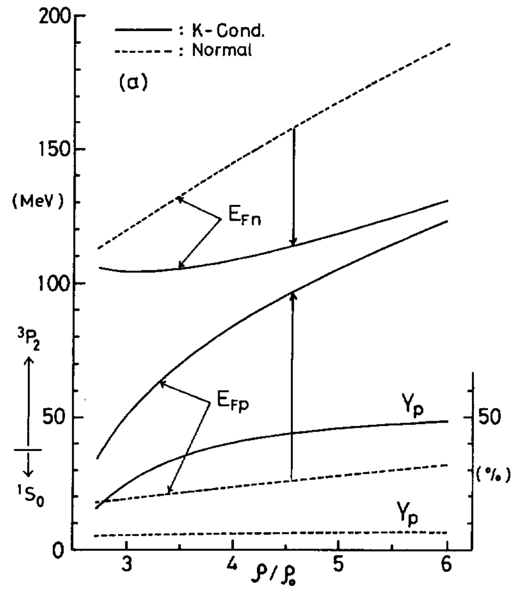

Fig. 1. Proton mixing ratio Y_p and Fermi energy of neutrons E_{F_n} and protons E_{F_p} in the kaon-condensed phase (solid lines) as compared with those in normal phase (dotted lines). ρ denotes the total density of nucleons and ρ_0 the standard nuclear density. On the left of the ordinate, the appearance range of $^3P_2$ or $^1S_0$ superfluid.

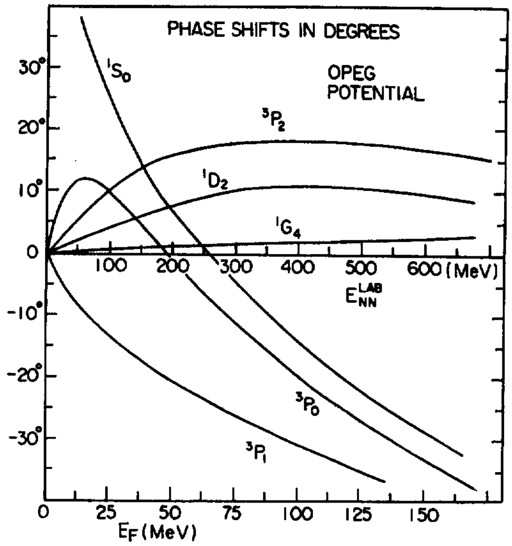

Fig. 2. Phase shifts of two-nucleon scattering by use of the OPEG potential (^1E-1 and ^3O-$2M$ in the isospin triplet state).[10),11)] $E_{NN}^{LAB}(E_F)$ is the scattering energy in laboratory system (Fermi energy).

in the kaon-condensed phase calculated by Maruyama et al.[8)] (the case denoted PM3 there, the condensed phase sets in at $\rho_c \simeq 2.6\,\rho_0$) are illustrated in Fig. 1. For reference, the Y_p in normal phase (usual neutron star matter) obtained by use of the effective two-nucleon interaction based on the Reid-soft-core potential[9)] are also shown as a standard case. We remark a large difference in Y_p; the $Y_p=(10-50)\%$ ($(5-6)\%$) for the kaon-condensed (normal) phase. Therefore it is of interest how the difference in Y_p affects the occurrence of nucleon superfluids.

The pairing interaction responsible for nucleon superfluidity depends on the density of neutrons (ρ_n) and protons (ρ_p). In usual neutron star matter, neutrons are a dominant component ($\rho_n \sim \rho$), having high Fermi energy ($E_{F_n}=\hbar^2(3\pi^2\rho_n)^{3/2}/2m_N \gtrsim 50$ MeV, m_N being the nucleon mass) and hence the Cooper pair of neutrons is formed primarily in the $^3P_2$ state which is most attractive at high scattering energies ($E_{NN}^{LAB}=4E_{F_n} \gtrsim 200$ MeV), as seen in Fig. 2. By contrast, protons admixed are a small component ($\rho_p/\rho \lesssim 0.06$) and so the Cooper pair utilizes the $^1S_0$-interaction most attractive at low scattering energies ($E_{NN}^{LAB}=4E_{F_p} \lesssim 80$ MeV). On the other hand, in the case with kaon condensate, the situation is different. Due to the large Y_p, both of neutrons and protons are able to utilize the $^3P_2$-interaction, which is understood from Fig. 1 showing the ρ-dependence of E_{F_n} and E_{F_p}, together with Fig. 2. Here, we study the $^3P_2$-superfluid of neutrons and protons in this kaon-condensed phase.

${}^3P_2+{}^3F_2$ coupled gap equation

In order to treat realistically the ${}^3P_2$-pairing, it is necessary to include the ${}^3P_2$-${}^3F_2$ tensor coupling effect bringing about an important attraction to enhance the main ${}^3P_2$ component of energy gap. Then the ${}^3P_2$ gap equation is coupled with the ${}^3F_2$ one through this tensor coupling (${}^3P_2+{}^3F_2$ coupled gap equation) and is given explicitly (the case with $m_j=0$, see Ref. 1) for details):

$$\Delta_1(k)=-\frac{1}{\pi}\int k'^2dk'\langle k'|V^{11}|k\rangle\int d\hat{k}'\{\Delta_1(k')f(\theta)+\Delta_3(k')g(\theta)\}/E_{k}'$$

$$+\frac{1}{\pi}\int k'^2dk'\langle k'|V^{31}|k\rangle\int d\hat{k}'\{\Delta_1(k')g(\theta)+\Delta_3(k')h(\theta)\}/E_{k}'\,, \tag{1}$$

$$\Delta_3(k)=+\frac{1}{\pi}\int k'^2dk'\langle k'|V^{13}|k\rangle\int d\hat{k}'\{\Delta_1(k')f(\theta)+\Delta_3(k')g(\theta)\}/E_{k}'$$

$$-\frac{1}{\pi}\int k'^2dk'\langle k'|V^{33}|k\rangle\int d\hat{k}'\{\Delta_1(k')g(\theta)+\Delta_3(k')h(\theta)\}/E_{k}'\,, \tag{2}$$

where

$$f(\theta)=\frac{1}{8\pi}(1+3\cos^2\theta)\,, \tag{3}$$

$$g(\theta)=-\frac{\sqrt{6}}{64\pi}(1-7\cos^2\theta+5\sin\theta\sin3\theta-10\cos\theta\cos3\theta)\,, \tag{4}$$

$$h(\theta)=\frac{3}{128\pi}(13+4\cos^2\theta+5\sin\theta\sin3\theta+15\cos\theta\cos\theta3\theta)\,, \tag{5}$$

$$E_{k'}=\sqrt{\tilde{\epsilon}_{k'}^2+D^2(\boldsymbol{k}')}\,, \tag{6}$$

$$D^2(\boldsymbol{k}')=\frac{1}{2}f(\theta)\Delta_1^2(k')+\frac{1}{2}h(\theta)\Delta_3^2(k')+g(\theta)\Delta_1(k')\Delta_3(k')\,, \tag{7}$$

$$\tilde{\epsilon}_{k'}=\epsilon(k')-\epsilon(k_F)=\hbar^2(k'^2-k_F^2)/2m_N^*\,, \tag{8}$$

$$\langle k'|V^{l'l}|k\rangle=\int r^2drj_{l'}(k'r)V^{l'l}(r)j_l(kr)\,, \tag{9}$$

$$V^{l'l}(r)=\sum_{\text{spin}}\int d\hat{r}\,\mathcal{Y}^*_{sl'jm_j}(1,2)\,V(1,2)\mathcal{Y}_{sljm_j}(1,2)\,, \tag{10}$$

$$\mathcal{Y}_{sljm_j}(1,2)=\sum_{m_s+m_l=m_j}(slm_sm_l|jm_j)\,Y_{lm_l}(\hat{r})\chi_{sm_s}(1,2)\,. \tag{11}$$

In these expressions, $\Delta_1(k)$ and $\Delta_3(k)$ are the energy gap functions of the ${}^3P_2$- and the ${}^3F_2$-component, respectively, θ denotes the polar angle of the momentum $\boldsymbol{k}$, $\tilde{\epsilon}_k$ is a single-particle energy measured from the Fermi surface with k_F the Fermi momentum. Also, $V(1,2)$ is a two-nucleon potential and $\mathcal{Y}_{sljm_j}(1,2)$ is a spin-angular part for the pair wave function with angular momenta, s, l, j, m_j, in usual notations. The energy gap for the ${}^3P_2+{}^3F_2$ pairing is given by $D(k_F,\hat{k})$ in Eq. (6). Since $D(k_F,\hat{k})$ is angle-dependent, it is convenient to introduce the angle-averaged energy gap $\bar{D}(k_F)$ defined by

$$\bar{D}(k_F) \equiv \left[\frac{1}{4\pi}\int d\hat{k} D^2(k_F, \hat{k})\right]^{1/2} = \frac{1}{\sqrt{8\pi}}\sqrt{\Delta_1^2(k_F)+\Delta_3^2(k_F)} \simeq \Delta_1(k_F)/\sqrt{8\pi}\,. \qquad (12)$$

In the following, we discuss results in terms of the critical temperature given by

$$T_C(^3P_2) \simeq 0.60\bar{D}(k_F) \simeq 0.139\Delta_1(k_F) \times 10^{10}\,\mathrm{K} \qquad (13)$$

with $\Delta_1(k_F)$ in MeV.

We solve this $^3P_2+{}^3F_2$ coupled gap equation numerically by an iterative technique. As for $V(1, 2)$, we adopt the OPEG ^3O-$2M$ potential which is used in our previous work[1] and reproduces well the scattering phase shift for the $^3P_2$ two-nucleon state. The OPEG ^3O-$2M$ is a slightly modified version of the OPEG ^3O-2 in Ref. 10) by changing only the depth of the spin-orbit core part ($^{C}V_{LS}^{0}=-100$ MeV $\rightarrow -150$ MeV) in order to improve the fit to $^3P_2$ phase shift at high energies ($E_{NN}^{\mathrm{LAB}} \gtrsim 300$ MeV). In the calculation, we use the effective mass approximation for the single-particle energy $\epsilon(k)$ given by the second equality in Eq. (8) with m_N^* being the effective mass of a nucleon. We use the Y_p of the kaon-condensed phase in Fig. 1 as a typical case (other cases in Ref. 8), PM1 and PM2 cases, give similar results).

Results and discussion

We show results for neutron (solid lines) and proton (dash-dotted lines) $^3P_2$-superfluids in Fig. 3. The following points are remarked: (i) When the effective-mass parameter $m^*(\equiv m_N^*/m_N)=1$, both neutron and proton $^3P_2$-superfluids exist safely at $\rho \gtrsim 3\rho_0$ (even for $\rho_0 \gtrsim 6\rho_0$), since T_c is by far larger than the internal temperature $\sim 10^8$ K in neutron stars. As m^* gets smaller, T_c decreases significantly. (ii) For smaller $m^*(=0.75)$, both superfluids persist, but the existence of proton $^3P_2$-superfluid is pushed to higher densities, $\rho \gtrsim 3.5\rho_0$. For still smaller $m^*(=0.7)$, both superfluids disappear ($T_c < 10^8$ K is not figured). (iii) By comparing the T_c for $m^*=0.75$ in Fig. 3, between the case with (a solid line) and without kaon condensation (a dotted line), we see that the kaon-condesed phase is more preferable for the neutron $^3P_2$-superfluid to occur at high densities. This comes from a smaller E_{Fn} caused by a larger Y_p in the kaon condensate. (iv) At lower densities ($\rho_c \lesssim \rho \lesssim 2.9\rho_0$), proton superfluid is due to

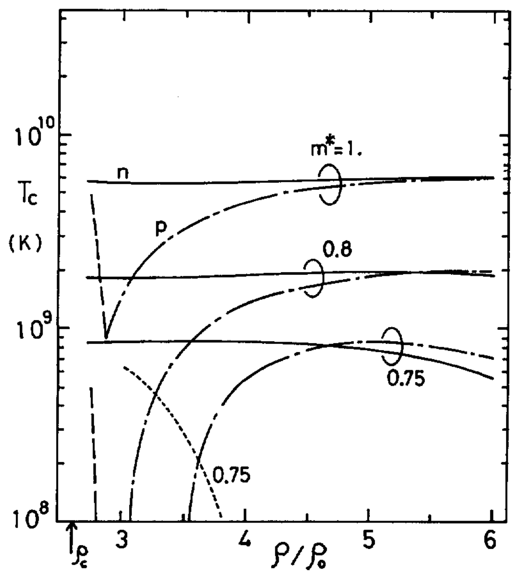

Fig. 3. Critical temperature T_c for the $^3P_2$-superfluid of neutrons (solid lines) and protons (dash-dotted lines) for several effective nucleon-mass parameter m^*, calculated with the OPEG ^3O-$2M$ potential. For $m^* \lesssim 0.70$, T_c is by far lower than the internal temperature of neutron stars $T_i \sim 10^8$ K and not figured. The dotted line illustrates the T_c of neutron $^3P_2$-superfluid in normal phase for reference and the dashed lines show the T_c of proton $^1S_0$ superfluid existing at the vicinity of the onset density ρ_c indicated by an arrow.

the $^1S_0$-pairing (dashed lines in Fig. 3) as in normal phase, but very soon ($\rho \gtrsim 2.9\rho_0$) is caused by the $^3P_2$-pairing. This is because the most attractive pairing interaction changes from the $^1S_0$ to the $^3P_2$ as ρ and Y_p increase (see Fig. 1).

Concluding remarks

The existence of superfluid depends strongly on m^*. The G-matrix calculation of dense nucleon matter suggests that $m^* > 0.7$ for $\rho \gtrsim 3\rho_0$ is unlikely in the normal phase, especially for asymmetric nuclear matter with $Y_p \sim (0.2-0.5)$, and $m_p^* \lesssim m_n^* \lesssim (0.5-0.6)$ is probable. Therefore we conclude that both neutron and proton superfluids under the kaon condensation is hard to be expected. This means that we cannot suppress the too rapid cooling if the kaon condensate takes part in neutron stars. Conversely, this might suggest the non-existence of kaon condensate in neutron stars discussed in Ref. 2), that is, the onset density of kaon condensation would exceed the central density of these neutron stars. Then, there arises a question; is there another possibility to give an efficient cooling mechanism together with nucleon superfluids ? We remark the following new phases realizable at high densities: One is a case with pion condensation leading to a rapid cooling. In the case of combined neutral and charged pion condensate ($\pi^0\pi^c$ condensate), we have larger m^*, $m^* \sim 0.9$ for $\rho = (3-5)\rho_0$,[11),12)] and so can expect the nucleon superfluids. The other is a case with hyperon matter generating an equally efficient cooling mechanism.[13)] In this case, hyperon superfluid has to be examined. Study on these possibilities is our forthcoming subject and will be reported elsewhere.

The authors wish to thank Dr. T. Tatsumi and Dr. T. Muto for their discussions. This work is indebted to the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (No. 06640366).

---

1) R. Tamagaki, Prog. Theor. Phys. **44** (1970), 905.
T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. **46** (1971), 114.
T. Takatsuka, Prog. Theor. Phys. **48** (1972), 1517; **50** (1973), 1754, 1755.
As a review article: T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 112 (1993), 27.
2) H. Umeda, K. Nomoto, S. Tsuruta, T. Muto and T. Tatsumi, Astrophys. J. **431** (1994), 309.
3) D. B. Kaplan and A. E. Nelson, Phys. Lett. **B175** (1986), 57.
4) G. E. Brown, K. Kubodera and M. Rho, Phys. Lett. **B192** (1987), 273.
5) T. Tatsumi, Prog. Theor. Phys. **80** (1988), 22.
6) T. Muto and T. Tatsumi, Phys. Lett. **B283** (1992), 165.
7) H. Fujii, T. Muto, T. Tatsumi and R. Tamagaki, Nucl. Phys. **A571** (1994), 758.
8) T. Maruyama, H. Fujii, T. Muto and T. Tatsumi, Phys. Lett. **B337** (1994), 19.
9) S. Nishizaki, T. Takatsuka, N. Yahagi and J. Hiura, Prog. Theor. Phys. **86** (1991), 853.
10) R. Tamagaki, Prog. Theor. Phys. **39** (1968), 91.
11) K. Tamiya and R. Tamagaki, Prog. Theor. Phys. **66** (1981), 948, 1361.
12) T. Takatsuka, Y. Saito and J. Hiura, Prog. Theor. Phys. **67** (1982), 254.
13) M. Prakash, M. Prakash, J. M. Lattimer and C. J. Pethick, Astrophys. J. **390** (1992), L77.