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NUMERICAL ANALYSIS AT THE NATIONAL BUREAU OF STANDARDS*

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Abstract. In 1946/47, Dr. E. U. Condon (1902–1974), the fourth director of the National Bureau of Standards, arranged for the organization of a Division of Applied Mathematics, under the leadership of Dr. John H. Curtiss. A principal objective of this division was the development of the mathematics appropriate for the automatic digital computing machines then being constructed. An account is given here of some of the activities in numerical analysis carried out by that division in its first 25 years.

I am very glad to speak on this subject. Mrs. Todd and I spent about 10 years, 1947–57, at the NBS—she as a resident consultant and I in various capacities. On the whole it was a happy and exciting time, but there were several heartaches. More recently I have been a member of the NASNRC Advisory Panel.

It is difficult to summarize 25 years' work of an organization in the time and space allotted and so I must restrict my coverage severely and apologize for omissions, deliberate or inadvertent. I shall confine myself to more theoretical numerical analysis, omitting discussions of (1) the solutions of special computational problems, which are one of the major contributions of the NBS group, (2) the various tables prepared to aid in statistical analyses [45] and the evaluation of computer programs (e.g., least squares by R. H. Wampler [36], [37]), (3) the more recent work on combinatorial algorithms [38], (4) collaboration in the development of Omnitab [32].

Detailed accounts of the work of the Applied Mathematics Division are given in the various progress reports which are available. Some of the technical material appeared in the Applied Mathematics Series of which over 60 volumes have been published to date and in the Journal of Research of the National Bureau of Standards (since 1947 and especially since 1968 when separate issues were devoted to Mathematical Sciences).

It is almost exactly 25 years since the start of modern numerical mathematics *at NBS*. Personally, I consider the beginning of modern numerical mathematics took place about 1936. This was the date of Comrie's paper [1] showing how to use business machines for scientific purposes—a thing which we are still doing—and of Turing's first papers [2]; about this time also Ostrowski introduced the "Horner" as a unit for scientific computation [3], and studied the numerical stability of the Newton process [4]. I am, however, far from objecting to the choice of the appearance of the von Neumann–Goldstine paper [5] as the zero date, for it and the companion Turing paper [6] influenced my scientific development insofar as I wrote half a dozen papers on condition numbers [7].

The actual prospectus for the National Applied Mathematics Laboratories was dated February 1947. [This name was abandoned in the early 1950s so I shall speak of the Applied Mathematics Division (AMD).] I actually arrived late

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in September 1947 to join the staff of the Institute for Numerical Analysis (INA), which was one of several sections into which the AMD was divided. Mrs. Todd and I traveled in a troop ship from London with 100 war brides in the officers' cabins; Mrs. Todd was in the sergeants' quarters and I was down in the hold. It was a Liberty ship and, having spent some time earlier on the analysis of the losses of welded ships, I was not always too happy among the creaking bulkheads.

Actually it was only in April 1948 that Prof. O. Szász, Mrs. Todd and I joined Dr. G. Blanch at INA which was located on the campus of the University of California at Los Angeles (UCLA) having spent the time between at the NBS in Washington and in Princeton with von Neumann at the Institute for Advanced Study Computer Project.

The contribution of Professor John H. Curtiss to the development of modern numerical mathematics cannot be overestimated. Like many of us he was first (and still is) a pure mathematician. He got involved in statistics as a Naval officer in World War II and then was a focal point for the development of a central federal mathematical establishment, a project supported by various agencies, notably the Office of Naval Research. The idea for this activity apparently originated in 1965 in the Office of Planning and Invention of the Navy Department. Part of his administrative contributions should be clear from this survey; for his technical contributions we note his work on Monte Carlo [34], his work on the conjugate direction method [33] and a long series of substantial contributions to the theory of approximation from which we cite only two [35].

[On the other side of the Atlantic, I had organized an Admiralty Computing Service (ACS) and later, with my colleagues in it (especially A. Erdélyi and D. H. Sadler), worked toward the foundation of a British National Mathematical Laboratory—this ultimately was incorporated as a division of the British National Physical Laboratory and is the organization with which J. H. Wilkinson has long been associated and of which A. M. Turing was a member. [For some of these activities see [8].]

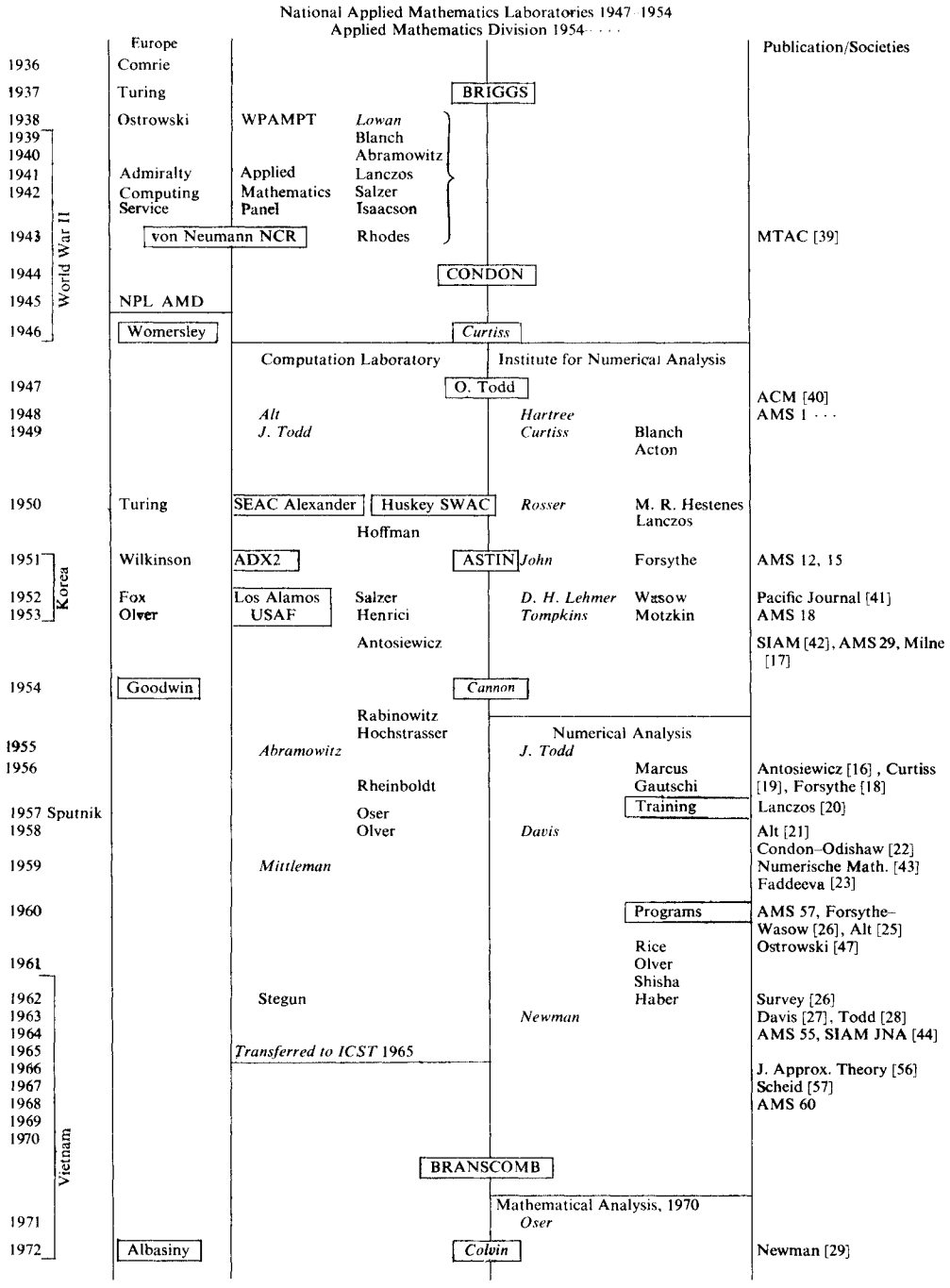
I digress briefly here to call attention to the fact that one of the first contacts von Neumann had with programming computers occurred during his visit to the ACS in 1943. [For details see [9].]

Dr. Curtiss realized that any experienced pure mathematician could find, if he chose, attractive and challenging problems in numerical mathematics and he recruited established mathematicians as administrators and staff for AMD. Naturally, there were a few recruits who did not respond to the challenges.

In the accompanying diagram I have listed the leaders of groups and given related information indicating the part played by the AMD in the formation of professional societies, the establishment of journals and some of the contributions of its members to the literature.

L. J. Briggs, E. U. Condon, A. V. Astin and L. W. Branscomb were the directors of the NBS in the periods 1933–45, 1945–51, 1951–69, 1969–72, respectively. J. H. Curtiss, E. W. Cannon and B. H. Colvin were the chiefs of the AMD in the periods 1946–53, 1955–71, 1972 respectively.

Let me note that there was already a solid basis for computation at NBS—the Lowan group, whose activities have been described by Lowan [31]. This became a nucleus for the Computation Laboratory (CL), another section of the AMD.


National Applied Mathematics Laboratories 1947–1954
Applied Mathematics Division 1954 · · ·
Europe
Publication/Societies
1936 Comrie
1937 Turing
BRIGGS
1938 Ostrowski
WPAMPT
Lowan
1939 Blanch
1940 Abramowitz
1941 Admiralty
Applied
Lanczos
1942 Computing Service
Mathematics Panel
Salzer
Isaacson
World War II
1943 von Neumann NCR
Rhodes
MTAC [39]
1944
CONDON
1945 NPL AMD
1946 Womersley
Curtiss
Computation Laboratory
Institute for Numerical Analysis
1947
O. Todd
ACM [40]
1948 Alt
Hartree
AMS 1 · · ·
1949 J. Todd
Curtiss
Blanch
Acton
1950 Turing
SEAC Alexander
Huskey SWAC
Rosser
M. R. Hestenes
Lanczos
Hoffman
1951 Wilkinson
ADX2
ASTIN
John
Forsythe
AMS 12, 15
Korea
1952 Fox
Los Alamos
Salzer
D. H. Lehmer
Wasow
Pacific Journal [41]
1953 Olver
USAF
Henrici
Tompkins
Motzkin
AMS 18
Antosiewicz
SIAM [42], AMS 29, Milne [17]
1954 Goodwin
Cannon
Rabinowitz
Hochstrasser
Numerical Analysis
1955
Abramowitz
J. Todd
1956
Marcus
Gautschi
Antosiewicz [16], Curtiss [19], Forsythe [18]
Rheinboldt
Training
1957 Sputnik
Oser
Lanczos [20]
1958
Olver
Davis
Alt [21]
Condon–Odishaw [22]
1959
Mittleman
Numerische Math. [43]
Faddeeva [23]
1960
Programs
AMS 57, Forsythe–Wasow [26], Alt [25]
Rice
Ostrowski [47]
1961
Olver
Shisha
1962
Stegun
Haber
Survey [26]
1963
Newman
Davis [27], Todd [28]
1964
AMS 55, SIAM JNA [44]
1965 Transferred to ICST 1965
1966
J. Approx. Theory [56]
1967
Scheid [57]
1968
AMS 60
1969
1970
Vietnam
BRANSCOMB
Mathematical Analysis, 1970
Oser
1971
1972 Albasiny
Colvin
Newman [29]

Governments, and other institutions, have their ups and downs. The INA operation at UCLA was unfortunately terminated in 1954; at this time a Numerical Analysis Section (NA) was set up in Washington. The CL was transferred to what is now the Institute for Computer Sciences and Technology in 1965 and the NA was absorbed into a Mathematical Analysis section in 1970. Due to these events and to personnel changes and deaths (e.g., of Abramowitz, Tompkins and Motzkin) the residual operations in numerical mathematics at UCLA and Washington are now a small shadow of what they were in the early 1950s although other areas have flourished and been developed according to the national needs. However the alumni of NBS are now scattered all over the world continuing to develop numerical mathematics and all have a lasting affection for Dr. Curtiss and the NBS and its directors, Dr. Condon and Dr. Astin, who supported their work.

Before discussing in more detail the activities in numerical analysis let me acknowledge the contributions of F. L. Alt and, more recently, of A. J. Goldman in various capacities in the AMD, of S. N. Alexander and H. D. Huskey in providing machines, and for the basic training on them which many had from I. Rhodes.

INA 1947–54. After a short stay by Hartree the first efforts of INA were led by Dr. Curtiss in the directions of Monte Carlo and conformal mapping. Symposia on these topics were organized—see AMS 12, AMS 18 and its sequel, AMS 42. Professor Wasow and Dr. Curtiss were active in Monte Carlo and work in this area is continuing in Washington under Dr. Haber.

Rosser led an effort on matrix problems. This period included the 50th anniversary of the NBS which was marked by "Gatlinburg—1", a symposium largely planned by Mrs. Todd; she, with Paige, edited the Proceedings, AMS 29; sequels to this, AMS 39, 49 appeared later. Highlights of this have been mentioned by Prof. Householder, notably the so-called conjugate direction method. A brief sketch of this method follows. (This is based in part on an account included in [0(e)] due to G. E. Forsythe.)

The solution of an $n \times n$ linear system $Ax = b$ is equivalent to the minimization of $f(x) = \|Ax - b\|_2$, and so, essentially, the determination of the vertex V of an elliptic paraboloid π. A method of steepest descent consists in moving from a point P_0 on π in the direction of steepest descent at P_0 to a point P_1, which is the local minimum. From P_1 we move similarly in the direction of steepest descent at P_1. It can be shown that $P_n \to V$. This is, in general, an *infinite* process. Using the geometrical fact that the midpoint of parallel chords of an ellipse lie on the conjugate diameter, an iterative scheme can be set up which terminates in at most n steps. This scheme was devised in 1951, simultaneously by E. Stiefel in Zürich and M. R. Hestenes at INA. There was considerable preliminary work at INA in which Forsythe, Karush, Motzkin and Rosser also participated. At the same time Lanczos, also at INA, adapted his (1950) method of minimized iterations for the determination of the characteristic polynomial of A to one for the solution of $Ax = b$ and arrived at the same conjugate direction method. There was much further work at INA by Paige, M. Stein, Hayes, Hochstrasser, L. Wilson and Curtiss. The definitive report is [30].

It is probably fair to say now that although the conjugate direction method has not been as successful as originally thought for the linear equation problem,

it has proved very satisfactory in connection with certain optimization problems and for certain sparse systems of equations.

The next period was under the influence of Fritz John when attacks were made on problems of partial differential equations, with contributions by Blanch, Forsythe and Wasow. Milne worked on his book on ordinary differential equations [17]. Representative publications are cited under [53].

Next the Lehmers came to INA and there were some changes in the direction of number theory. Representative publications are cited under [52].

Lehmer was followed by Tompkins whose interests included linear programming, and the calculus of variations. Representative publications are cited under [54].

Throughout this period, M. R. Hestenes and Forsythe [55] were always active and, for part of it, Acton, Lanczos, Motzkin and Karush were involved.

Various training programs took place in the summers.

CL 1947–54. The research activities during this period were largely concerned with mathematical tables under Abramowitz and Stegun, interpolation under H. E. Salzer and with linear programming. The latter subject was created and promoted by G. B. Dantzig: the work at NBS was developed by A. J. Hoffman, in part in collaboration with Motzkin at INA. Hoffman and Antosiewicz [16] ran a symposium on this topic in Washington in 1955.

In addition, Mrs. Todd started a project on the estimation of eigenvalues of matrices and functions of matrices and one in number theory in which both the young postdoctoral staff and the senior visitors participated. Representative publications are cited in [46].

As regards actual computation: this was on a rather small scale due to the "occupation" of SEAC by USAF and the AEC: The USAF actually paid for the machine and the AEC donated a building so that we could be near it. Among the work accomplished was work of a number theoretical character started by Mrs. Todd (Monte Carlo, consecutive power residues mod p (with J. C. P. Miller), Wilson primes and the Baker–Campbell–Hausdorff formula (K. Goldberg)) and some controlled experiments on conformal mapping [48], and on linear programming [49].

Towards the end of this period there was the ADX2 incident and a change of administration which led to a heavy reduction in force. There seems to be no connected account of the happenings available—this is an opportunity for a science historian with illustrations available from Herblock. It will be recalled that government scientists were reprimanded for not taking account of "the play of the marketplace" and that the director, Dr. Astin, was asked to resign, but was reinstated, and the NBS decisions were fully supported by a special committee appointed by the National Academy of Science. One outcome of this was an investigation of the NBS as a whole by a select committee under the chairmanship of Dr. M. J. Kelly. In the report of this committee (submitted 15 October 1953) the Applied Mathematics Division was one of two which were specially commended. However this committee deferred to the Applied Mathematics Advisory Council for recommendations on the future of the INA which was then, with the rest of the AMD, in a precarious financial situation due in part to a decision of the Department of Defense not to permit transfer of funds to the NBS.

1954–70. The numerical analysis activities were concentrated in Washington under my direction and at this time Abramowitz took over the CL. In my time the main emphasis was on controlled computational experiments, e.g., the evaluation of matrix inversion programs with M. Newman [50], who had prepared one of the first matrix packages. This was continued after I left NBS in 1957 by my successor P. Davis, who, for example, with Rabinowitz studied orthonormalization methods [51]. Davis also exploited functional analytic methods (cf. [27]). Davis left in 1963 and his successor, Newman, was largely occupied by number theory, e.g., exact solution of linear systems, until the Numerical Analysis section was merged into a Mathematical Analysis section whose chief is Oser in 1970. However, during this period there was outstanding work by J. R. Rice, a postdoctoral research associate working on approximation theory. I organized a Training Program for Numerical Analysts in 1957—this resulted in the "Survey" [26]. A repetition of this, again with NSF support, was organized by Davis in 1960.

One of the outstanding events of this period was the publication in 1964 of AMS 55 which was prepared by Abramowitz and Stegun and their team. To date about 200,000 copies have been sold and nine printings have been made. (This number includes the sales of a softback Dover edition.)

There were several other significant expository works in this period. The chapters on mathematics in the Condon–Odishaw *Handbook of Physics* [16] were almost all prepared by members of the AMD. In 1960 A. M. Ostrowski's 1952 lecture notes appeared in book form [47] and AMS 57, *Basic theorems in matrix theory*, by M. Marcus was published: these have become standard references in many numerical analysis courses. In 1968, AMS 60, *Matrix representations of groups*, by M. Newman was published.

Finally, let me mention some who participated in the work in Washington or in Los Angeles or both and who contributed to numerical mathematics:

S. Agmon, L. Bers, H. Cohn, Ky Fan, J. M. Hammersley, T. Kato, J. C. P. Miller, A. M. Ostrowski, I. J. Schoenberg, P. Stein, E. Stiefel, H. Wielandt. (I omit those summer visitors at Los Angeles who were involved in a classified project.)

Among the "younger" people who "graduated" from the NBS during this period were F. S. Acton, E. C. Dade, H. P. Edmundson, Walter Gautschi, A. A. Goldstein, R. M. Hayes, P. Henrici, U. W. Hochstrasser, M. Muller, W. C. Rheinboldt, J. R. Rice, F. Scheid, J. L. Selfridge, T. H. Southard (a former SIAM President), M. L. Stein, D. Teichroew and C. Witzgall.

To conclude, let me enumerate some probable firsts in numerical mathematics which were carried out at the NBS during the first 25 years of the AMD:

(a) Automatically computed earth–moon trajectory [10]. (Earlier, rocket/comet orbits were tabulated by Herrick using punched card equipment: See AMS 20 (1953).)
(b) Optimal scaling of matrices [11].
(c) Computational complexity [12], [13].
(d) Studies of the Hilbert matrix [14], [15], [46(a)].

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(d) ———, Some recent trends in applied mathematics, American Scientist, 37 (1969), (#4), pp. 587 ff.
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(f) Applied Mathematics Series 1 (1948). To date over 60 volumes have been published by the U.S. Government Printing Office. We refer specifically to the following:
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AMS 15 (1951), *Problems for the numerical analysis of the future*, Four papers presented at Symposia, July 25–31, 1948. Foreword by J. H. Curtiss.
AMS 18 (1952), *Construction and applications of conformal maps*, Proc. of a Symposium, June 22–25, 1949, E. F. Beckenbach, ed.
AMS 42 (1955), *Experiments in the computation of conformal maps*, John Todd, ed.
AMS 29 (1953), *Simultaneous linear equations and the determination of eigenvalues*, Proc. of a Symposium, Aug. 23–25, 1951, L. J. Paige and Olga Taussky, eds.
AMS 39 (1954), *Contributions to the solution of systems of linear equations and the determination of eigenvalues*, Olga Taussky, ed.
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AMS 65 (1964, 9th printing 1972), *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, M. Abramowitz and I. A. Stegun, eds.

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[40] The Association for Computing Machinery was founded in 1947. The first president was J. H. Curtiss, the third was F. L. Alt (who was given the distinguished Science Award in 1970). Later H. D. Huskey and G. E. Forsythe were presidents of ACM. M. R. Kellington has been editor of ACM since 1959, having served the AMD in a similar capacity since 1948. For further information about ACM see, e.g., Comm. ACM, 15 (1972), no. 7.

[41] Although the Pacific Journal of Mathematics received much support from the NBS in its early days (see e.g., [0(e)]) it never developed into a major one in numerical mathematics.

[42] The Society for Industrial and Applied Mathematics was founded in 1953. One of its early presidents was T. H. Southard, the current president is B. H. Colvin.

[43] Numerische Mathematik was founded in 1959 and former members of the NBS AMD have been active editorially. E. Stiefel and John Todd have been editors-in-chief since the beginning and G. E. Forsythe, W. Gautschi and P. Henrici have been or are associate editors.

[44] The SIAM Journal of Numerical Analysis was founded in 1964. Among those who have served as editors are P. Henrici, F. W. J. Olver and W. C. Rheinboldt.

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Notes added in proof, November 1974.

1. The present [1973–] director of the NBS is R. W. Roberts.
2. For information supplementary to [31] see

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3. Another paper presented at the October 17, 1972 Panel is

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