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# Numerical analysis of transient magnetic field in a capacitor-discharge impulse magnetizer

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# NUMERICAL ANALYSIS OF TRANSIENT MAGNETIC FIELD IN A CAPACITOR-DISCHARGE IMPULSE MAGNETIZER

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## ABSTRACT

A method for analyzing the magnetic field in a capacitor-discharge impulse magnetizer is established by modifying the finite element method. The effects of charging voltage, capacitance and resistance on the behaviour of the localized fluxes in the impulse magnetizer are analyzed quantitatively. As the detailed distribution of the flux density can be obtained, the optimum design of the magnetizer which produce desired magnet will be possible using our new method.


## 1. INTRODUCTION

In a capacitor-discharge impulse magnetizer, a magnet is magnetized by the discharging current of capacitors. The conventional design of the magnetizer has been based on many year's experience. The behaviour of flux in the magnetizer should be calculated in order to produce the desired magnets. The analysis of the flux distribution is quite difficult. This is because both the magnetizing current and the applied voltage to the magnetizer are unknown.

In this paper, a method for analyzing the magnetic field in the impulse magnetizer is established by the modified finite element method[1]. The behaviour of the localized fluxes in the impulse magnetizer can be analyzed quantitatively using our method. The validity of the method is verified by comparing the calculated results with results measured.

## 2. MAGNETIZER

Figure 1 shows the impulse magnetizer being analyzed with its exciting circuit. Figure 2 shows the analyzed two-dimensional region. The capacitance C is 4400(μF), and the number of turns of the coil is 4 turns per pole. The pole pieces and the yoke are made of steel with conductivity $\sigma$ of $0.7\times10^7$(S/m). The permanent magnet is made of polymer-bonded material (Br=0.264(T), Hc=$1.93\times10^5$(A/m)).

## 3. METHOD OF ANALYSIS[1]

3.1 Outline of the method

Magnetic fields with eddy currents and permanent magnets are denoted by the following equation[2]:

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=-J+\nu_0\left(\frac{\partial M_y}{\partial x}-\frac{\partial M_x}{\partial y}\right)+\sigma\left(\frac{\partial A}{\partial t}+\frac{\partial \phi}{\partial z}\right) \quad (1)$$

where, A is the magnetic vector potential, $\nu$ is the reluctivity and $\nu_0$ is the reluctivity of vacuum. $\sigma$ and $\phi$ are the conductivity and the electric potential respectively. J is the exciting current density and Mx and My are the x- and y-components of magnetization.

As the exciting current J of impulse magnetizer in Eq.(1) is unknown, it is difficult to analyze magnetic fields in such a device using a conventional finite element method[3].

A new method for calculating currents and flux distributions in impulse magnetizer has been developed. In this method, Eq.(1) in conjunction with Kirchhoff's equation for the exciting circuit is used. Both the vector potentials and the exciting current are treated as unknown values.

Figure 3 shows an equivalent circuit of the device. The finite element region enclosed by the broken line corresponds to the magnetizer shown in Fig.2. Here, C, R and L are the capacitance, the resistance and the leakage inductance outside the finite element region. Furthermore, Rc is the resistance of the winding in the finite element region.

The equation obtained from the Kirchhoff's law in the exciting circuit is as follows:

$$\frac{\partial \Phi}{\partial t}+(R+R_c)I+L\frac{\partial I}{\partial t}-\frac{1}{C}\left(Q_0-\int I\,dt\right)=0 \quad (2)$$

where, $\Phi$ is the interlinkage flux to the winding and $Q_0$ is the initial charge of the capacitor.

In Eq.(2), the unknown exciting current I must be integrated when the capacitor voltage drop is calculated. The integration of I with respect to t is not easy. If the charge Q on the capacitor is treated as an unknown value instead of I, the capacitor voltage can be easily calculated. Then, a method using the

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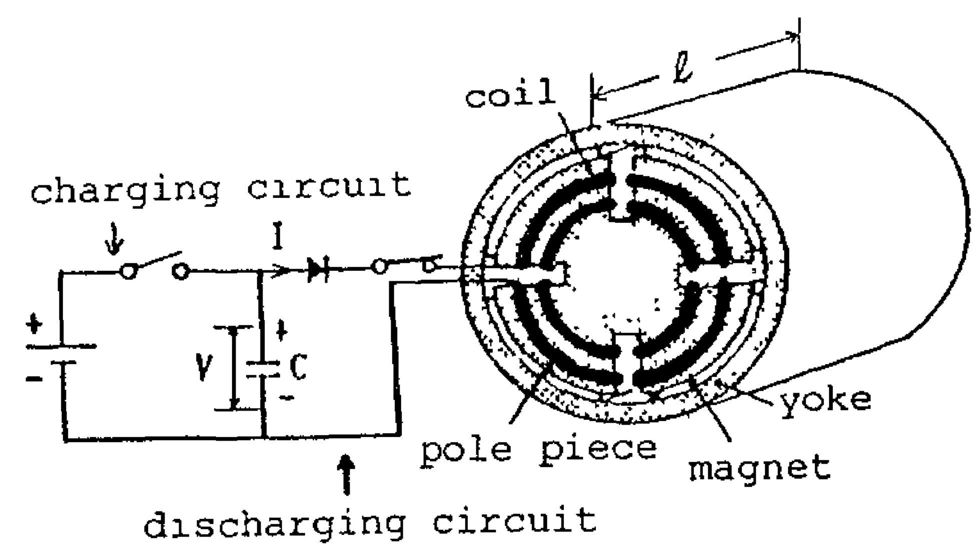


Fig.1 Impulse magnetizer.

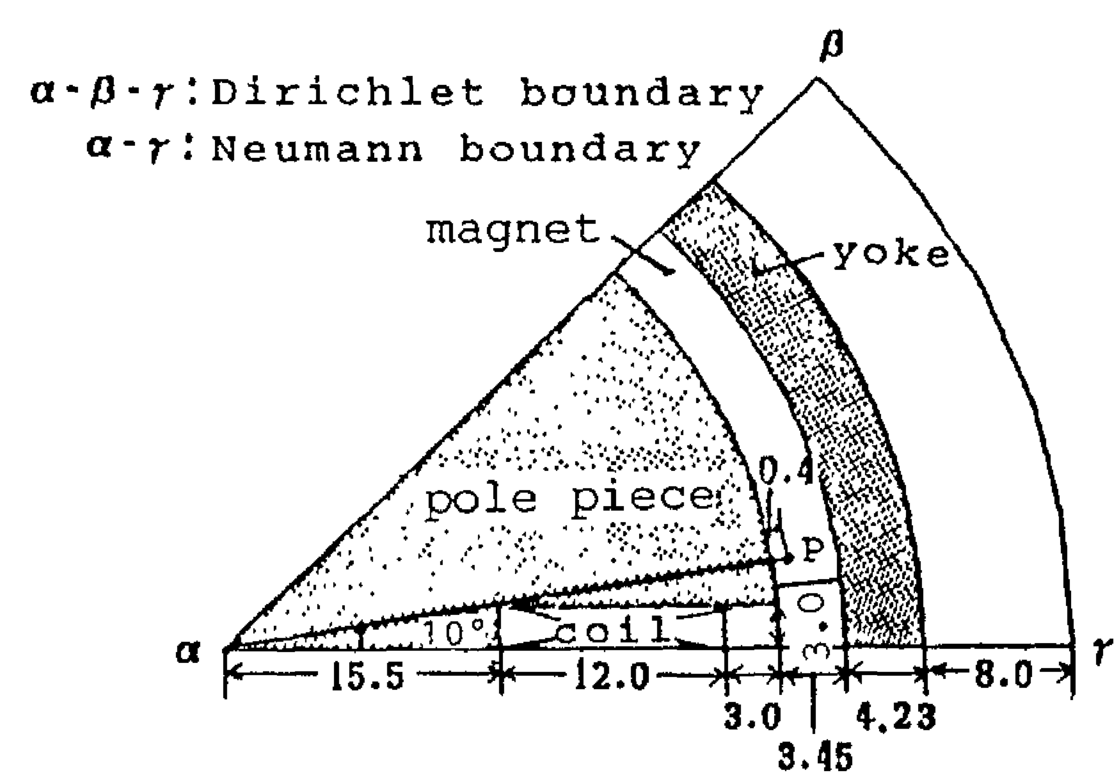


Fig.2 Analyzed model of a magnetizer.

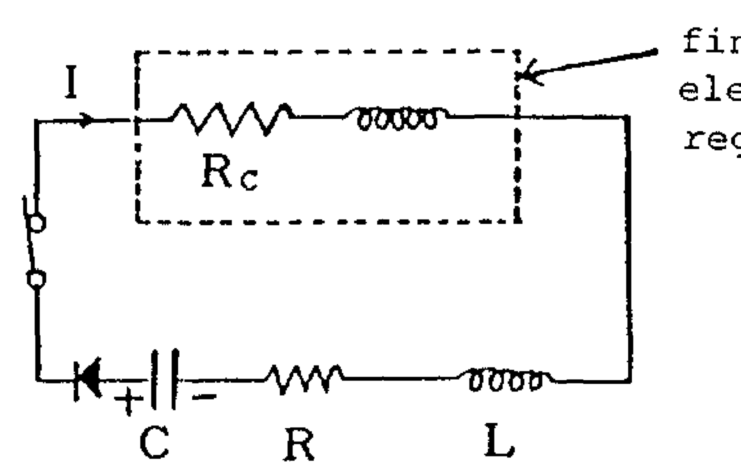


Fig.3 Equivalent circuit.

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charge Q on the capacitor is developed. The current I can be expressed using the charge Q as:

$$I=\frac{\partial Q}{\partial t} \tag{3}$$

Substituting I in Eq.(2) by Eq.(3) and representing $\phi$ in Eq.(2) by the vector potential A, the following equation can be obtained:

$$\oint_s \frac{\partial A}{\partial t} ds + (R+Rc)\frac{\partial Q}{\partial t} + L\frac{\partial^2 Q}{\partial t^2} - \frac{Q}{C} = 0 \tag{4}$$

where, s is a path along the coil.

Substituting I in Eq.(3) into J in Eq.(1) and solving Eqs.(1) and (4) simultaneously while treating vector potential A and charge Q as unknown values, the transient magnetic field during the discharge of the capacitor can be analyzed[1].

Mx and My in Eq.(1) are determined by the magnetization curve or the demagnetization curve shown in Fig.4. When the magnetizing current is raised, the flux density B in the magnet is increased along the magnetization curve. When the magnetizing current is reduced, the flux density is decreased along the demagnetization curve[4].

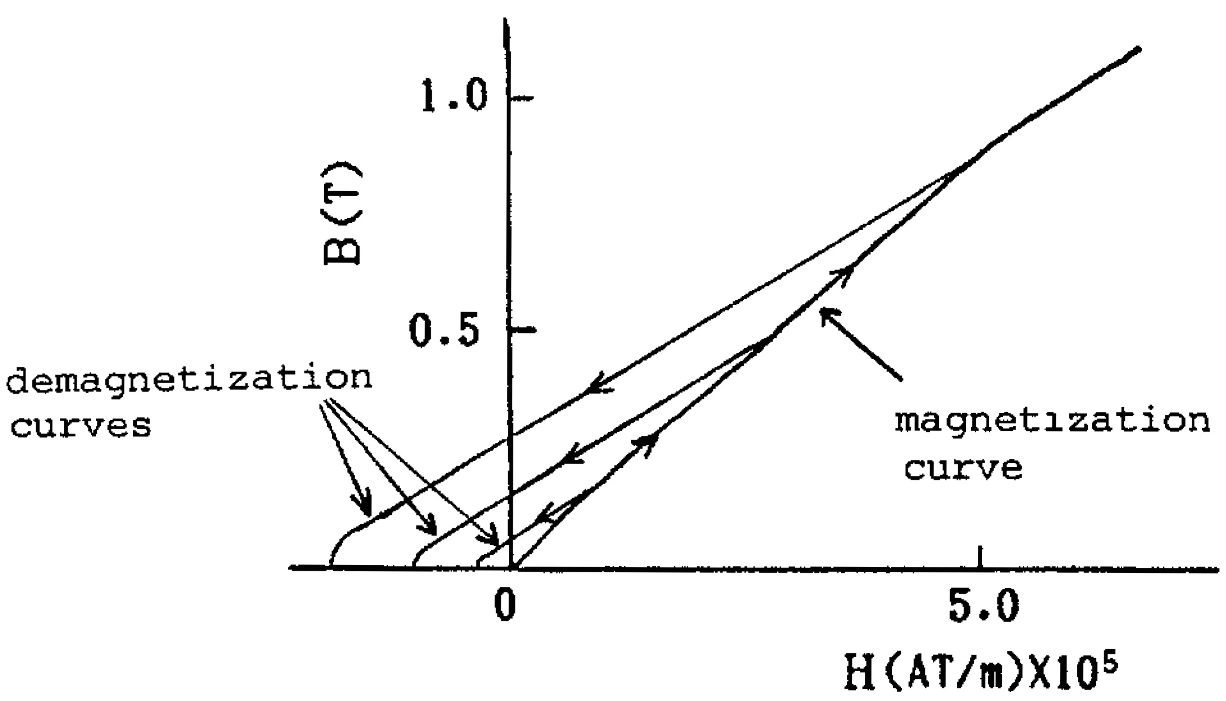



Fig.4 Magnetizing process of a polymer-bonded magnet.

3.2 Finite Element formulation

The following equation can be obtained by Galerkin's method from Eq.(1) [1].

$$G_i^t=\sum_{e=1}^{ne}\{\nu^{(e)t}\sum_{k=1}^{3}S_{ik e}A_{ke}^{t}+\sigma\sum_{k=1}^{3}\frac{\Delta^{(e)}}{12}(1+\delta_{ike})\frac{A_{ke}^{t}-A_{ke}^{t-\Delta t}}{\Delta t} - \frac{n\Delta^{(e)}}{3S\Delta t}(Q^t-Q^{t-\Delta t}) \tag{5}$$

Where ne is the number of element, and $A_{ke}$ is the vector potential at a node ke. The superscript t denotes the instant of the calculation and $\Delta t$ is the time interval. $\Delta^{(e)}$ is the area of the element e. $\delta_{ike}$ is the Kronecker's delta. $S_{ike}$ is defined by

$$S_{ike}=\frac{1}{4\Delta^{(e)}}(c_i^{(e)}c_{ke}+d_i^{(e)}d_{ke}) \tag{6}$$

$c_{ie}$ and $d_{ie}$ are denoted by

$$\left.\begin{aligned} c_{ie}&=y_{je}-y_{ke}\\ d_{ie}&=x_{ke}-x_{je}\end{aligned}\right\} \tag{7}$$

with the other coefficients obtained by a cyclic permutation of subscripts in the order i, j, k.

Let us define $\eta^t$ as follows:

$$\eta^t=\oint_s\frac{\partial A^t}{\partial t}ds+(R+Rc)\frac{\partial Q^t}{\partial t}+L\frac{\partial^2 Q^t}{\partial t^2}-\frac{Q^t}{C} \tag{8}$$

Equation (8) can be discretized as follows:

$$\eta^t=\ell\sum_{e=1}^{nc}\frac{n\Delta^{(e)}}{3S\Delta t}\sum_{k=1}^{3}(A_{ke}^{t}-A_{ke}^{t-\Delta t})+\frac{Q^t}{C}+(R+R_c)\frac{Q^t-Q^{t-\Delta t}}{\Delta t}+L\frac{Q^t-2Q^{t-\Delta t}+Q^{t-2\Delta t}}{(\Delta t)^2} \tag{9}$$

where, n is the number of turn of the exciting coil, nc is the number of element in the cross section of the coil, $\ell$ is the thickness of the pole piece and yoke shown in Fig.1.

In the nonlinear analysis using Newton-Raphson iteration technique, the increments $\{\delta A_j^t\}$ and $\delta Q^t$ at the instant t are obtained from the following equation:

$$\begin{bmatrix} [\frac{\partial G_i^t}{\partial A_j^t}] & \{\frac{\partial G_i^t}{\partial Q^t}\} \\ \{\frac{\partial \eta^t}{\partial A_j^t}\} & \frac{\partial \eta^t}{\partial Q^t}\end{bmatrix}\begin{Bmatrix}\{\delta A_j^t\}\\ \delta Q^t\end{Bmatrix}=\begin{Bmatrix}\{-G_i^t\}\\ -\eta^t\end{Bmatrix} \tag{10}$$

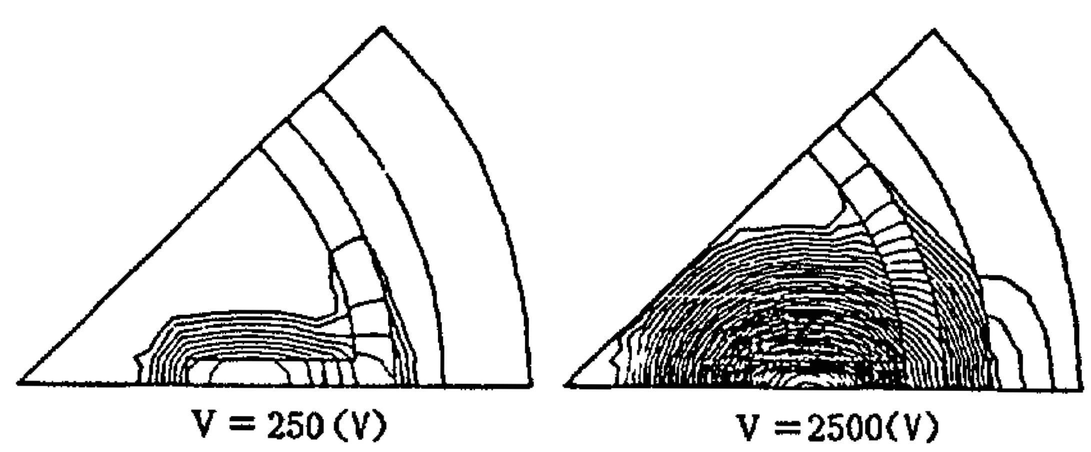
where, $\partial G_i^t/\partial A_j^t$ etc. are derived from Eqs.(5) and (9).

The transient magnetic fields and exciting currents can be calculated by the so-called "step-by-step method[5]". For example, $\{A_j^t\}$ and $Q^t$ at the instant t can be calculated by carrying out the nonlinear iteration of Eq.(10). In the calculation of Eq.(10), the already obtained values $\{A_j^{t-\Delta t}\}$, $Q^{t-\Delta t}$ and $Q^{t-2\Delta t}$ are used as the initial values for $[G_i^t]$ and $\eta^t$ in Eqs.(5) and (9). $\{A_j^{t+\Delta t}\}$ and $Q^{t+\Delta t}$ at the instant $(t+\Delta t)$ can be calculated in the same way.

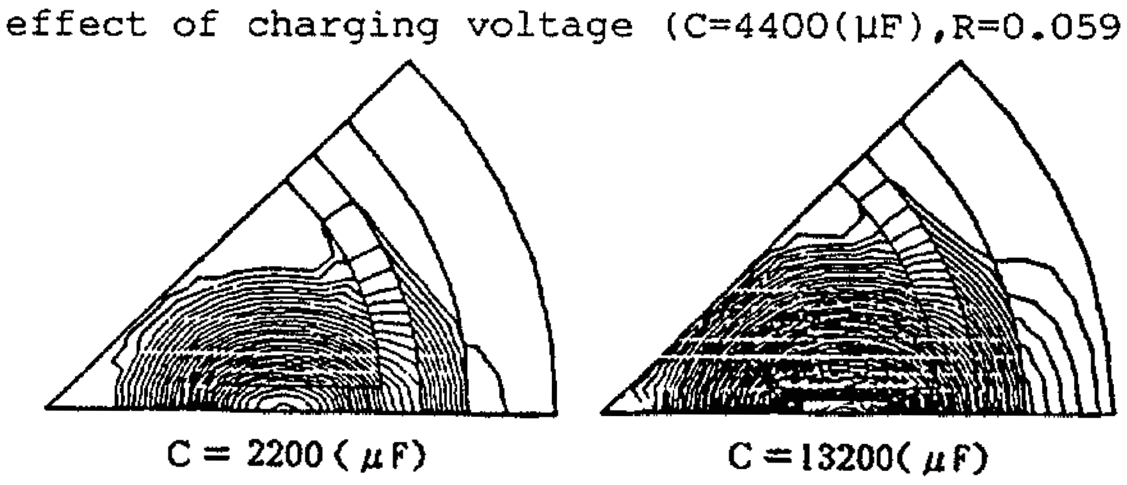
## 4. RESULTS AND DISCUSSIONS

Figure 5 shows the effects of the charging voltage V, capacitance C and resistance R on the flux distributions. As the flux density becomes a maximum at t=0.3(msec), the flux distribution at this instant is investigated. The time interval $\Delta t$ is chosen as 0.1(ms).

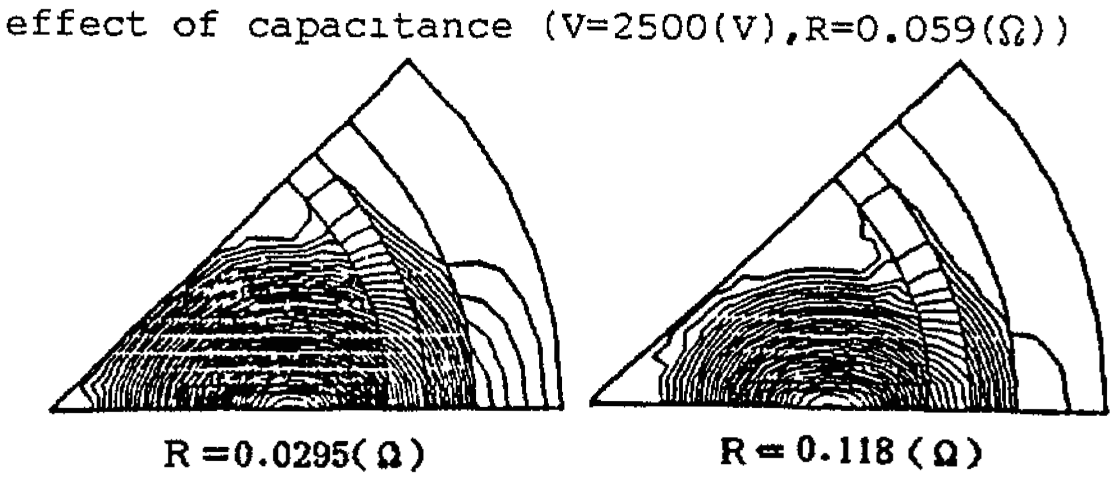
Figure 6 denotes the distributions of magnetization obtained by the magnetizer.




(a)effect of charging voltage (C=4400(μF),R=0.059(Ω))




(b)effect of capacitance (V=2500(V),R=0.059(Ω))



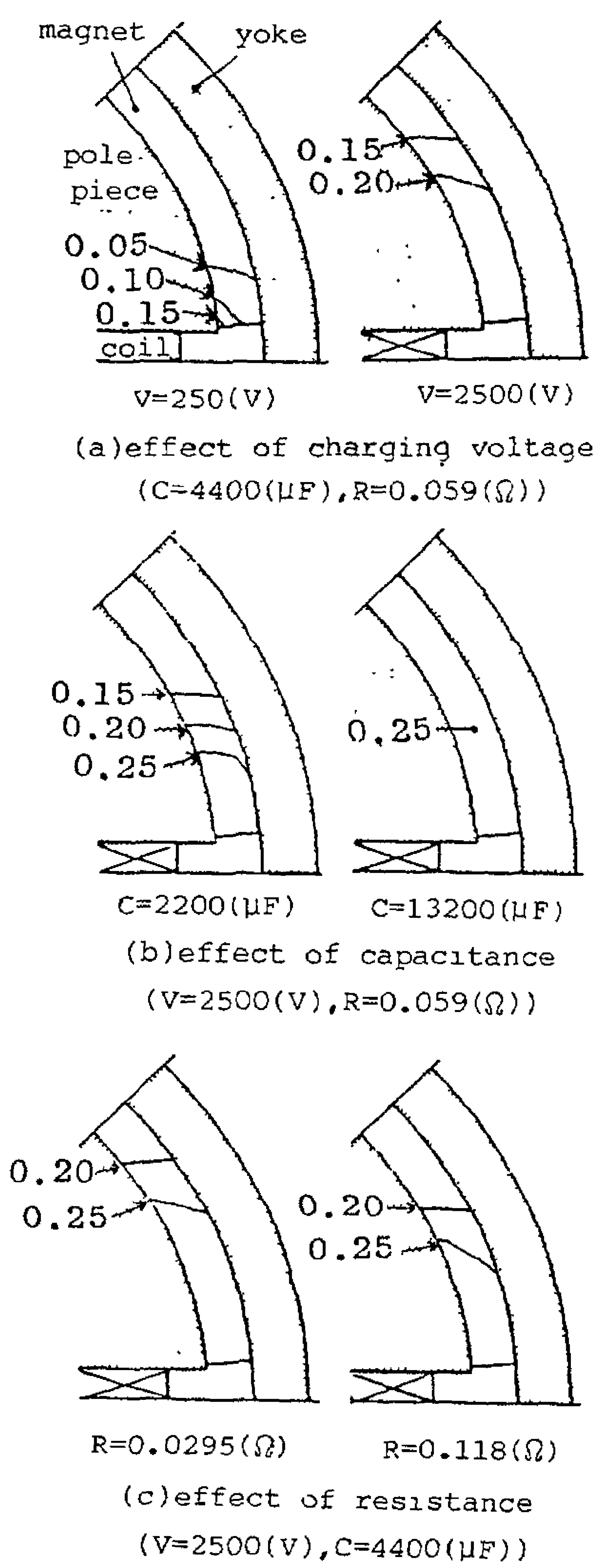

(c)effect of resistance (V=2500(V),C=4400(μF))

Fig.5 Flux distributions(t=0.3(msec)).

Figures 5 and 6 denote that the skin effect is pronounced and, as a result, the magnet is magnetized ununiformly when V or C is small or R is large. The reason is as follows: When R is increased, the permeability μ in the pole piece is increased due to the low flux density of pole piece. As the opposing magnetic field is large in the magnetic material with high permeability, the skin effect becomes remarkable with the increase of R. The effects of charging voltage V and capacitance C on the skin effect can also be explained with the same reason.

Figure 7 shows the calculated and the measured flux density at the point P denoted in Fig.2 during the discharge of the capacitor. The flux density is measured by a Hall probe (0.8mm thick,YEW 3252-02). This figure shows that the transient magnetic field in the magnetizer can be accurately calculated using our method. Table 1 shows the flux densities on the outer surface of the magnet denoted in Fig.8, which is removed from the magnetizer after magnetization. The flux density in Table 1 is calculated from the obtained magnetization. Accordingly, Table 1 shows the reliability of the calculated magnetization.

As the detailed behaviour of magnetic fields can be verified, the optimum design of the magnetizer to produce the desired magnet will be possible using our new method.




V=250(V) V=2500(V)
(a)effect of charging voltage (C=4400(μF),R=0.059(Ω))

C=2200(μF) C=13200(μF)
(b)effect of capacitance (V=2500(V),R=0.059(Ω))

R=0.0295(Ω) R=0.118(Ω)
(c)effect of resistance (V=2500(V),C=4400(μF))

Fig.6 Distribution of magnetization.

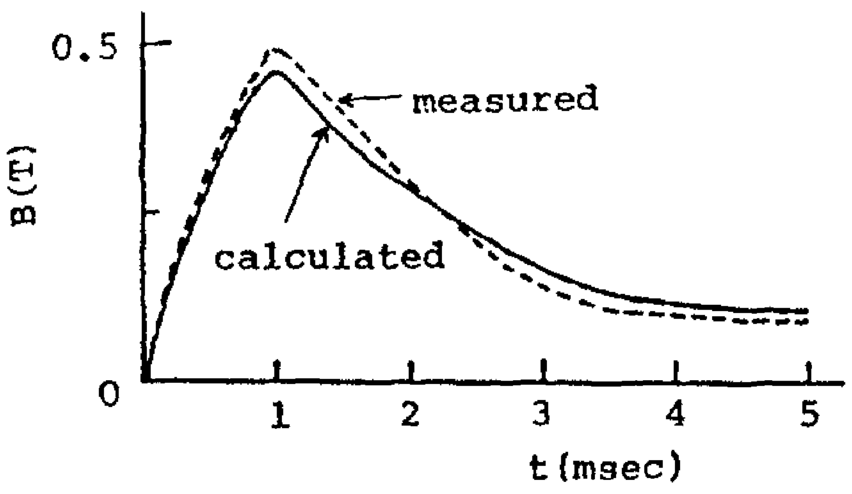



Fig.7 Flux densities at the point P. (V=250(V),C=4400(μF),R=0.059(Ω)).

Table 1 Flux densities on the surface of the magnet (V=250(V),C=4400(μF), R=0.059(Ω)).

| points | | $K_1$ | $K_2$ | $K_3$ | $K_4$ |
|---|---|---|---|---|---|
| flux density (T) | calculated | 0.0020 | 0.0040 | 0.0159 | 0.0341 |
| | measured | 0.0032 | 0.0040 | 0.0144 | 0.0279 |

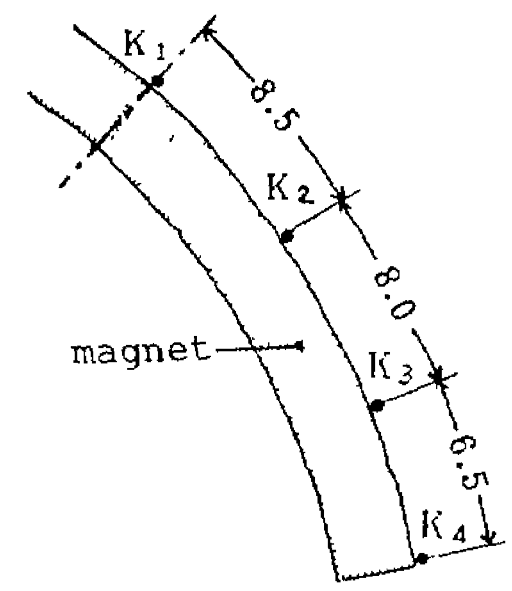



Fig.8 Investigated points.

## 5. CONCLUSIONS

It has become possible to analyze the transient magnetic field in a capacitor-discharge impulse magnetizer by developing a new method. In our method, Maxwell's equation in conjunction with Kirchhoff's equation is used. Both vector potentials and charges are treated as unknown values. The skin effect of the pole piece is considerably affected by the charging voltage, capacitance and resistance.

The results obtained provide information for designing the optimum magnetizer which produce desired magnet.

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