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## **Numerical and experimental investigation on multibody systems with revolute clearance joints**

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### **Abstract**

A comprehensive combined numerical and experimental study on the dynamic response of a slider-crank mechanism with revolute clearance joints is presented and discussed in this paper to provide an experimental verification and validation of the predictive capabilities of the multibody clearance joint models. This study is supported in an experimental work in a test rig, which consists of a slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod. The motion of the slider is measured with a linear transducer and an accelerometer. Dynamic tests at different operating crank speeds and with several clearance sizes are performed. The maximum slider acceleration, associated with the impact acceleration, is used as a measure of the impact severity. The obtained results demonstrate the dynamical behavior of a multibody mechanical system with a clearance joint. Finally, the correlation between the numerical and experimental results is presented and discussed leading to validated models of clearance revolute joints.

*Keywords:* Multibody Dynamics, Clearance Joints, Contact Forces, Dry Contact, Experimental Test Rig

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## 1. Introduction

A mechanical system is made of several components, which can be divided in two major groups, namely, links, that is, bodies with a convenient geometry, and joints, which introduce some restrictions on the relative motion of the various bodies of the system. Usually, the bodies are modeled as rigid and/or deformable bodies, while the joints are modeled through a set of kinematic constraints. That is, the joints are not modeled as contact pairs in the strict sense of the word contact, but as algebraic constraints to which implicit forces are associated. The functionality of a joint relies upon the relative motion allowed between the connected components. In most cases, this implies the existence of a clearance between the mating parts, and thus surface contact, shock transmission and the development of different regimes of friction and wear. On the other hand, no matter how small that clearance is, it can lead to vibration and fatigue phenomena, lack of precision or, even random overall behavior.

Over the last few decades, a number of researchers have proposed various methodologies for modeling mechanisms with clearance joints [1-15]. However, most of these studies only deal with numerical models. The literature reporting on experimental studies on mechanical systems with clearance joints is confined to a few publications. Dubowsky and Moening [16] studied experimentally the interactions between clearance joints and the system elasticity, using a Scotch-Yoke mechanism. Accelerometers were used in that study to measure the impact accelerations. A failure occurred at the Scotch-Yoke mechanism due to fatigue, caused by large impact forces developed at the clearance joint. Grant and Fawcett [17] investigated the effects of clearance size, lubrication and material properties on the contact loss in a four bar linkage with one clearance joint. Their experimental results confirmed the validity of the theoretical approach proposed, but only for a limited class of systems. Dubowsky et al. [18] studied analytically and experimentally a simple system, called the Impact Ring Model, to predict the impacts in planar mechanical systems with clearance joints. Haines [19] carried out an experimental investigation on the dynamic behavior of a simple journal-bearing with varying degrees of freedom. In that study, the contact loss between the journal and bearing was predicted using proximity transducers. Bengisu et al. [20] studied the contact loss in revolute clearance joints of a four bar mechanism, being the relative motion between the journal and bearing measured by employing optical methods. Soong and Thompson [21] presented a theoretical and experimental investigation of the dynamic response of a slider-crank mechanism with a revolute clearance joint where the slider and the connecting rod accelerations were quantified by using accelerometers. More recently, Khemili and

Romdhane [22] studied the dynamic behavior of a planar flexible slider-crank mechanism with clearance joints using simulation and experimental tests. Erkaya and Uzmay [23] presented an interesting work, in which an extensive experimental data of a slider-crank mechanism with clearance joints were discussed. These two last works consider a similar test rig, where the system describes the motion in the vertical plane.

The main goal of this work is to provide an experimental verification and validation of the predictive capabilities of the clearance joint models revised in this work. The experimental procedure complements the numerical studies in the literature and it provides a proposal for the coherent combination of numerical and experimental work which needs to be undertaken, in order to establish validated models and to identify directions for subsequent studies allowing for the improvement of the models. This study is supported in an experimental test rig that consists of a slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod. Dynamic tests at different operating crank speeds and with several clearance sizes are performed. The maximum slider acceleration, associated with the impact acceleration, is used as a measure of the impact severity. The obtained results demonstrate the dynamical behavior of a clearance joint and they provide qualitative measures that can be associated with fatigue and wear phenomena, when the system components have to operate with real joints. The correlation between the numerical and experimental results is presented and discussed allowing to define the range validity of the models. The motivation to develop a new experimental test rig comes from the necessity to have a flexible apparatus that allows for the study of the effect of the clearance in joints on the dynamic response of the system. In addition, the new experimental test rig allows studying of different test conditions such as, number and size of clearance joints, different contacting materials, lubrication actions, and several input conditions. Furthermore, the approach presented in this work can also be expanded to include spatial clearance joints with only minor modifications.

## 2. Modeling revolute joints with clearance

In the classical analysis of a revolute joint, the journal and bearing centers coincide, that is, the revolute joint is considered ideal or perfect, but the inclusion of the clearance allows for the separation of these two centers. Consequently, two extra degrees-of-freedom are added to the system. Figure 1(a) depicts a revolute clearance joint, the so-called journal-bearing, in which the radial clearance,  $c$ , is measured by the difference between the bearing radius and the journal radius,  $R_B$  and  $R_J$ , respectively. A

revolute clearance joint does not impose any kinematic constraints on the system, but limits the journal orbit to stay inside the bearing's boundaries [24].

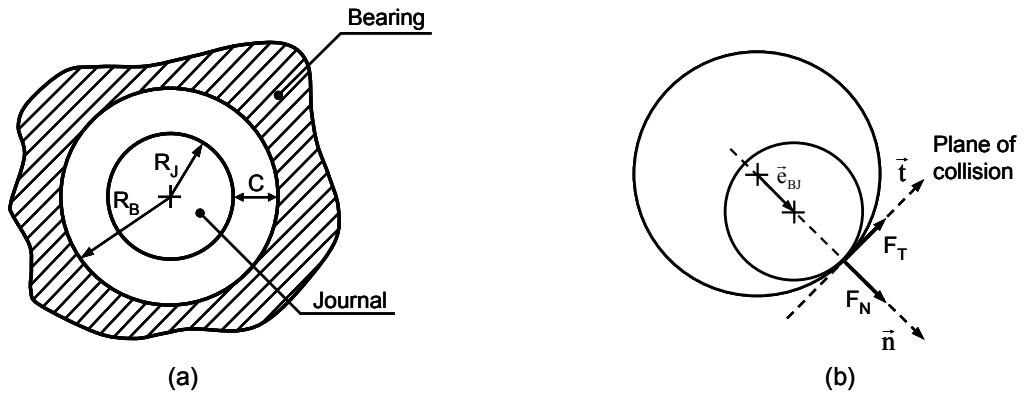


Figure 1. (a) Revolute joint with clearance; (b) Normal and tangential forces due to the impact between the journal and bearing surfaces.

In a dry contact situation, the journal can move freely within the bearing until contact between the two bodies takes place. When the journal impacts the bearing wall, a normal contact force together with a friction force are evaluated to obtain the dynamics of the journal-bearing. These forces are of a complex nature, and their corresponding impulse is transmitted throughout the mechanical system [25]. Figure 1(b) illustrates the normal and tangential force components due to the impact between the journal and the bearing. The impact, which has both normal and tangential relative velocities, is treated as an eccentric oblique collision between two bodies.

From the dynamic configuration of the system, the relative penetration depth between the journal and the bearing can be defined as

$$\delta = e_{BJ} - c \quad (1)$$

where  $e_{BJ}$  is the magnitude of the eccentricity vector defined between the bearing and journal center, as seen in Fig. 1b, and  $c$  is the radial clearance, which is a specified parameter. A more detailed description on the clearance joint modeling can be found in references [26-28].

The dynamics of a dry journal-bearing is characterized by two different situations. First, when the journal and the bearing are not in contact, with each other, there is no contact force associated with the journal-bearing. Second, once the contact between the two bodies takes place, the contact-impact forces are evaluated based on a nonlinear Hertzian-type contact force law for normal force and on the Coulomb friction law for the tangential forces. The contact conditions can be expressed for penalty forces models as

$$\begin{aligned} F &= 0 & \text{if } \delta < 0 \\ F &= F_N + F_T & \text{if } \delta > 0 \end{aligned} \quad (2)$$

in which,  $F_N$  and  $F_T$  are normal and tangential forces, respectively. In short, when the journal reaches the bearing wall an impact takes place. This impact is treated as a continuous event, that is, the local deformations and the contact forces are continuous functions of time. The impact analysis of the system is performed simply by including the normal and tangential contact forces into the system equations of motion [29-34].

For any contact in multibody systems, such as the ones owing in revolute clearance joint, the contact between the journal and bearing can be modeled in the most fundamental form by using the Lankarani and Nikravesh force model given by [35]

$$F_N = K \delta^n \left[ 1 + \frac{3(1-e^2)}{4} \frac{\dot{\delta}}{\dot{\delta}^{(-)}} \right] \quad (3)$$

where  $K$  is the contact stiffness,  $\delta$  is the relative penetration depth,  $e$  is the restitution coefficient,  $\dot{\delta}$  is the relative penetration velocity and  $\dot{\delta}^{(-)}$  is the initial impact velocity. The exponent  $n$  is set to 1.5 for most metal contacts. The parameter  $K$  depends on the material and geometric properties of the contacting surfaces. For two spherical surfaces in contact, one inside the other, the generalized stiffness parameter is given by [36]

$$K = \frac{4}{3(h_B + h_J)} \left[ \frac{R_B R_J}{R_B - R_J} \right]^{\frac{1}{2}} \quad (4)$$

where the parameters  $h_B$  and  $h_J$  are given by

$$h_k = \frac{1 - \nu_k^2}{E_k}, \quad (k=B, J) \quad (5)$$

$R_B$  and  $R_J$  are the radius of the bearing and the journal, respectively,  $\nu_k$  is the Poisson's ratio and  $E_k$  is the Young's modulus.

In a revolute clearance joint, three different modes of motion between the journal and the bearing can be considered, namely: continuous contact mode, free-flight mode, and impact mode. These three types of the journal motion are illustrated in Fig. 2 [24].

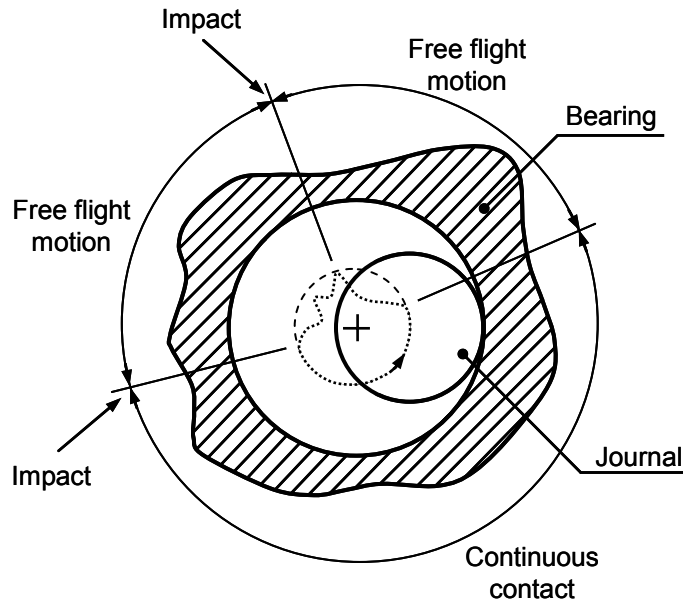


Figure 2. Different types of journal motion inside the bearing.

In the continuous contact mode, the journal and the bearing are in contact and a sliding motion related to each other is assumed to exist. In this mode, the penetration depth varies along the circumference of the journal. This mode is ended at the instant when the journal and bearing separate and the journal enters the free flight mode. In the free flight mode, the journal can move freely inside the bearing boundaries; i.e., the journal and the bearing joint are not in contact; and hence there is no reaction force between these two elements. After the end of the free flight mode, the journal enters the impact mode, in which impact forces are applied and removed. This mode is characterized by a discontinuity in the kinematic and dynamic characteristics, and a significant exchange of momentum occurs between the two impacting bodies. At the termination of the impact mode, the journal can enter either free flight or following mode. During the dynamic simulation of a revolute joint with clearance, if the path of the journal center is plotted for discrete instant of time, the different modes of motion of the journal inside the bearing can easily be observed.

### 3. Experimental test rig

In order to investigate the dynamic response of mechanical systems with clearance joints, an experimental test rig was designed and constructed. The experimental equipment was designed with the intent of providing data that can support the identification of different numerical models and their validation in the framework of the dynamic analysis of mechanisms with clearance joints. The experimental test rig has been constructed and operated at the Computational Mechanics Laboratory at the National Institute for Aviation Research, Wichita State University, Kansas, USA [2].



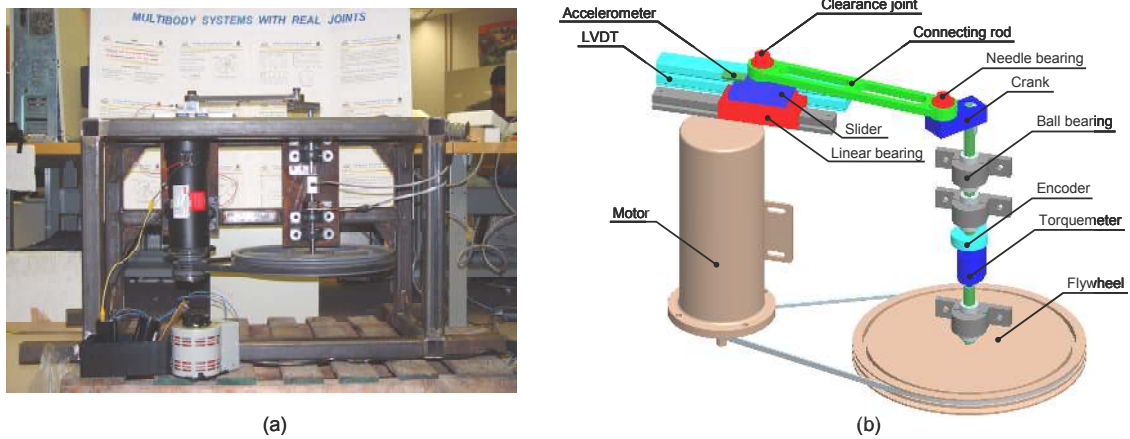


Figure 3. (a) Photograph of the experimental test rig; (b) Schematic drawing of the test rig.

A slider-crank mechanism, in which the revolute joint that connects the slider and the connecting rod has a variable and controlled radial clearance, was chosen due to its simplicity and importance within all possible candidate machines and mechanisms. An overall view of the experimental apparatus built is shown in a photograph in Fig. 3(a), and in a schematic drawing in Fig. 3(b). The main sub-assembly of the experimental test rig consists of a slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod, shown in Figure 4. This joint is designed as a dry journal-bearing, as illustrated in Figure 4(b). The remaining kinematic joints were constructed as close to the ideal joints as possible, that is, with minimum clearance and friction in order to minimize any contamination of the data that are intended to be measured. Moreover, these joints are lightly oiled to minimize the friction in their connections. A standard sleeve element is press-fitted to the extremity of the connecting rod, working as bearing, with its diameter fixed to a very tight tolerance. The journal is rigidly connected to the sliding block and incorporates a standard pin with a variable diameter, as pictured in Figure 4(b). Thus, the clearance at the test journal-bearing can be altered by simply changing the pin. A particular journal-bearing set is also manufactured in order to simulate an ideal or zero-clearance joint, which is used to obtain the reference data associated with an ideal mechanism. Different sets of journal diameters are also built to allow the analysis of the influence of the clearance on the system's dynamic response. The crankshaft is keyed to the crank and it is supported by ball bearings. A needle bearing, with minimal radial clearance and high rigidity, connects the crank to the connecting rod. The sliding block component is screwed onto a linear translational bearing, which has a precision preloaded system with zero-clearances. Table 1 shows the type of joints used in the

experimental slider-crank mechanism and their nominal or operating clearances. Thus, for numerical purposes, they can be considered as ideal or zero-clearance joints [2].

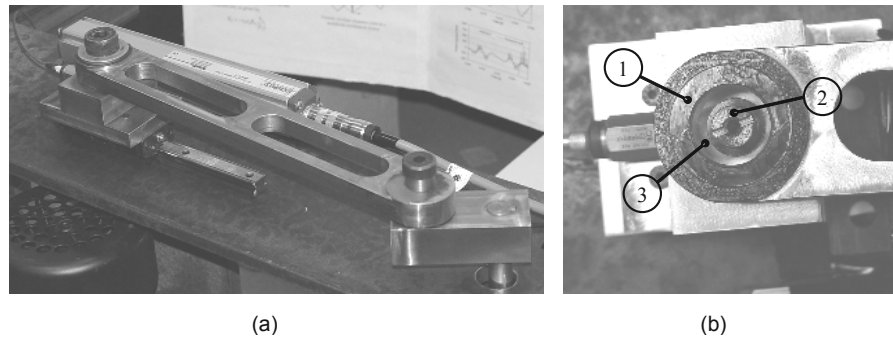


Figure 4. (a) Experimental slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod; (b) Test revolute joint in which the clearance is altered by simply changing the journal diameter: 1-bearing/sleeve, 2-journal/pin; 3-clearance. Clearance is exaggerated for illustration.

Connection	Joint type	Diameter [mm]	Clearance [mm]
Ground – Crank	Ball bearing	17.0	0.009
Crank – Connecting rod	Needle bearing	10.0	0.005
Connecting rod – Slider	Journal-bearing	22.2	0.002
Ground – Slider	Translational bearing	–	0.001

Table 1. Type of joints used in the slider-crank mechanism and the corresponding nominal clearances.

Between the driven pulley and the crankshaft, an encoder and a torque sensor are incorporated. The encoder is used to measure the crank angular position and velocity, whereas the torque sensor allows the measurement of the reaction moment that acts on the crank. Furthermore, an accelerometer and a linear voltage differential transducer (LVDT) are used to monitor the slider acceleration and displacement, respectively. The slider velocity is obtained either by performing the numerical integration of the acceleration value, or the numerical differentiation of the displacement data. The impact force between the journal and bearing is measured indirectly, that is, the impact accelerations are directly related to the impact forces.

The slider-crank mechanism works on the horizontal plane and, due to its rigidity and alignment, the gravitational effects on the system's dynamic responses can be neglected. The mechanism components are built entirely from steel and, hence for practical purposes, are assumed to be perfectly rigid. The connecting rod is built with a hollow cross-section in order to reduce the mass, while maintaining a high stiffness and, thus, reducing the flexible effects. The slider-crank mechanism and all other mechanical components are mounted on a heavy stiff frame. The mechanical arrangement of the experimental test rig is schematically illustrated in Figure 3(b). A summary of the physical properties of the experimental slider-crank model is given in Table 2, where

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the crank inertia properties include the shaft, encoder, torque sensor and flywheel.

Similarly, the slider-block inertia properties take into account the linear bearing and the accelerometer characteristics. These values are used in the numerical simulations. The overall mass of the experimental equipment, including frame and moving parts, is about 130 kg.

Body	Length [m]	Mass [kg]	Moment of inertia [kgm <sup>2</sup> ]
Crank	0.05	17.900	0.460327
Connecting rod	0.30	1.130	0.015300
Sliding block	-	1.013	0.000772

Table 2. Physical properties of the experimental slider-crank mechanism.

#### 4. Test scenarios and results

A comprehensive combined study on the dynamic response of the experimental slider-crank mechanism, with different clearances in one of the revolute joints, is used here to validate the numerical approach. The experimental test rig allows adjusting two parameters: the journal diameter, i.e., the radial clearance, and the driving frequency, i.e., the crank speed. Table 3 shows the test matrix performed with the experimental slider-crank mechanism. The radial clearance is obtained by changing the journal diameter and maintaining constant the diameter of the sleeve/bearing.

In the experimental setup, some difficulties have been experienced with the data acquisition by the torque sensor. These difficulties are related to a high residual torque that is always present in the output signal of the torquemeter, which contaminates the its outcome. Thus in what follows, only the dynamic characteristics of the slider motion are used. The dynamic behavior of the experimental slider-crank mechanism is monitored using the LVDT and accelerometer, and related to the crank angle. The crank speed is constant and equal to 200 rpm and the measured radial clearance is 0.25 mm, which is a large value for the journal-bearing used. Before carrying out the experimental tests, the joint elements that constitute the clearance joint are washed out using an alcohol to remove all traces of oil due to the manufacturing process, ensuring that the contact is dry.

Fixed parameters	Variable parameters		
Sleeve or bearing diameter [mm]	Journal diameter [mm]	Radial Clearance [mm]	Crank speed [rpm]
22.25	22.05	0.10	100
-	21.75	0.25	150
-	21.25	0.50	200
-	20.25	1.00	250

Table 3. Test matrix performed with the experimental slider-crank mechanism.

Figure 5 presents the experimental response of the slider-crank for ideal and clearance joints during one crank revolution. The dynamic response obtained with the experimental slider-crank setup is compared with that of the numerical models in which all the joints were considered to be ideal or perfect. Figures 5(a) and 5(c) show that experimental and numerical results for slider-crank mechanism with ideal joints are similar. In fact, the position data is almost identical and only a small deviation between the numerical and experimental acceleration response is observed. The position of the slider is not affected visibly by the clearance. This result is expected because the radial clearance is very small when compared to the maximum amplitude of the slider displacement. In sharp contrast to the slider position, the slider acceleration presents significant differences between the dynamic response histories of the system with and without clearance. Figure 5(d) clearly shows that the journal and bearing come out of contact once in every crank revolution. This separation occurs at approximately 43 degrees of the crank angle rotation and the contact is restored again at about 63 degrees. After this impact, the journal and bearing remain in an impact-rebound mode during a angular period of approximately 60 degrees. Between 230 and 330 degrees of the crank angle, the relative motion between journal and bearing is characterized by impacts and rebounds of small magnitude. Some periods of permanent contact between the journal and bearing are also noticed. It must be highlighted that the maximum slider acceleration for the system with the clearance joint with 0.25 mm is approximately 300% larger than that of the case without clearance.

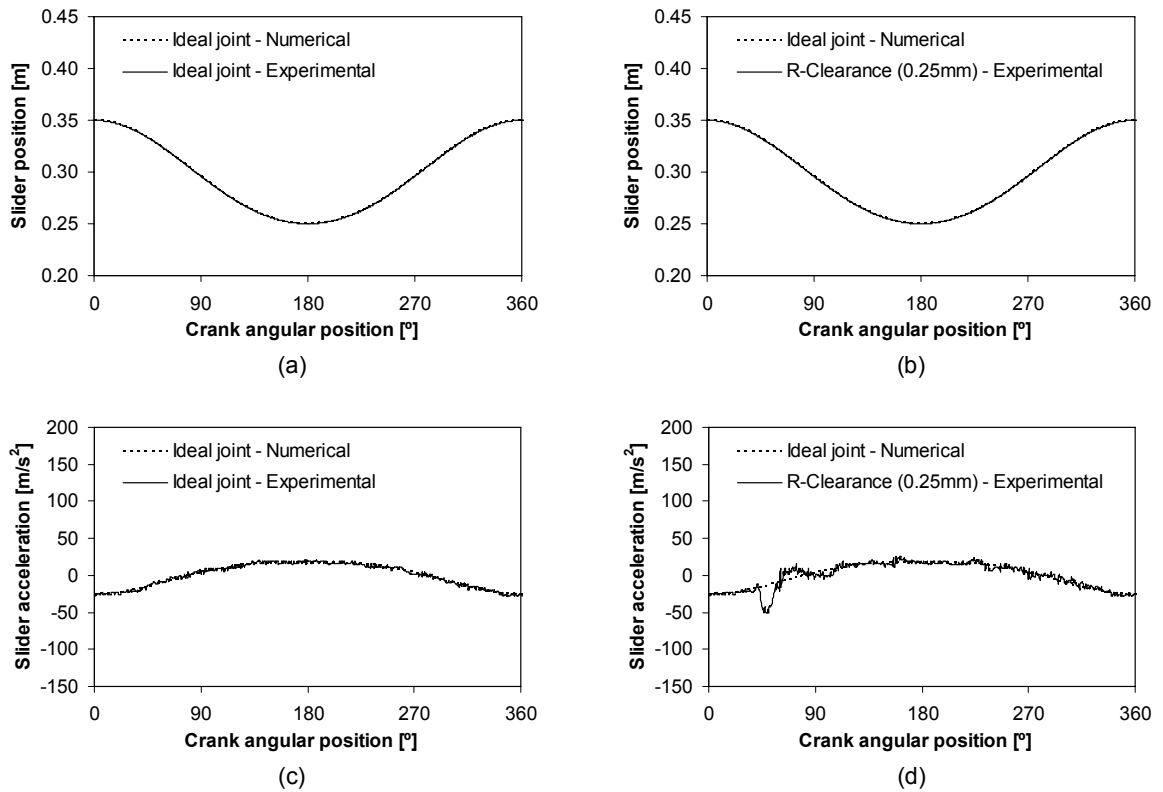


Figure 5. Dynamic response of the experimental slider-crank mechanism for a crank speed of 200 rpm with reference to the numerical ideal joint response: (a) slider position for ideal joint; (b) slider position for radial clearance equal to 0.25 mm; (c) slider acceleration for ideal joint; (d) slider acceleration for radial clearance equal to 0.25 mm.

Figure 6 presents slider acceleration results of the experimental slider-crank model operating at 200 rpm, for radial clearances of 0.10, 0.25, 0.50 and 1.00 mm. In Figure 6 it is observed that the maximum slider impact acceleration increases with the radial clearance. Furthermore, when the radial clearance is increased, the number of impacts increases as well. For the lower radial clearance of 0.10 mm, the maximum slider acceleration is about 3.5 times larger than that the maximum slider acceleration for the ideal joint, whereas for a larger radial clearance of 1.00 mm, the maximum slider acceleration is about 20 times larger than for the ideal joint. When the radial clearance is reduced, the peaks on the slider acceleration curve are also lower; i.e., the response is smoother, and the slider-crank behavior tends to be closer to the ideal mechanism as the journal and bearing are in contact with each other for longer periods of time. Figure 7 presents a Fast Fourier Transformation analysis (FFT) of the slider acceleration data plotted in Figure 6. It can be observed that not only the number of dominant frequencies in the system increases with clearance on the joint, but also that the magnitude of the contribution of each frequency increases.

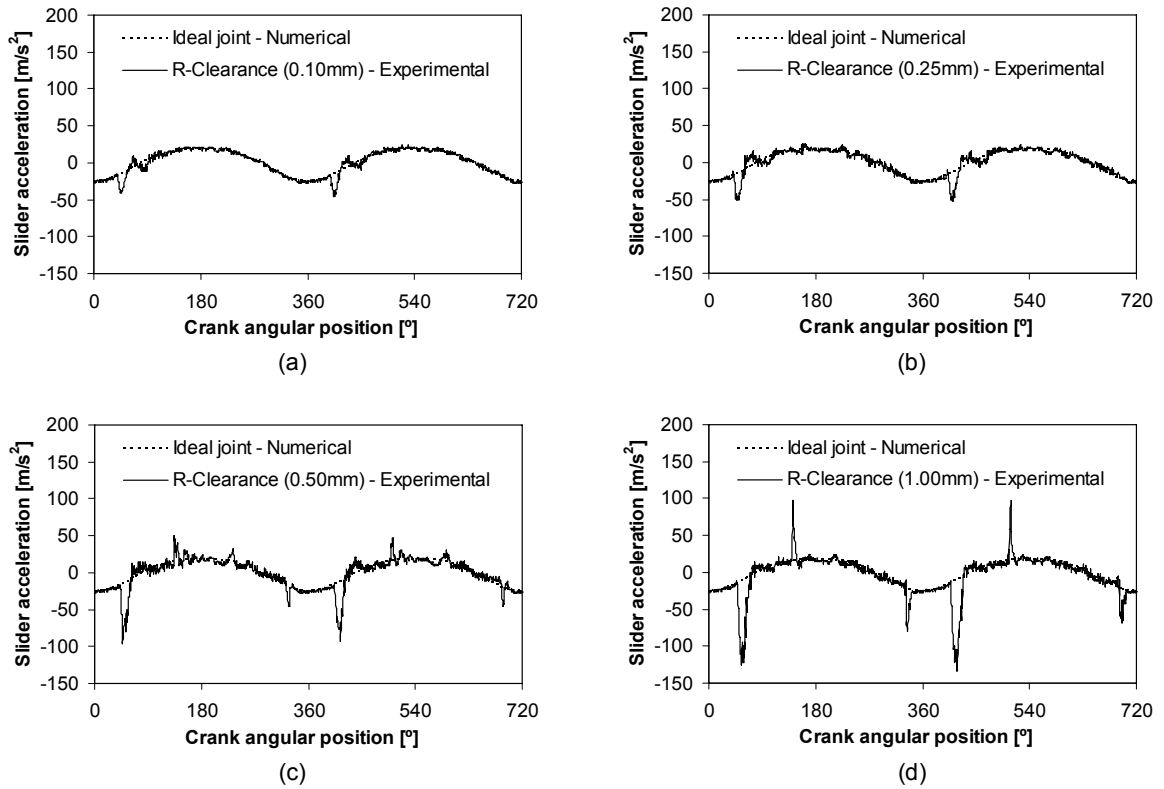


Figure 6. Slider acceleration for crank speed of 200 rpm and for different clearance sizes: (a)  $c=0.10$  mm; (b)  $c=0.25$  mm; (c)  $c=0.50$  mm; (d)  $c=1.00$  mm.

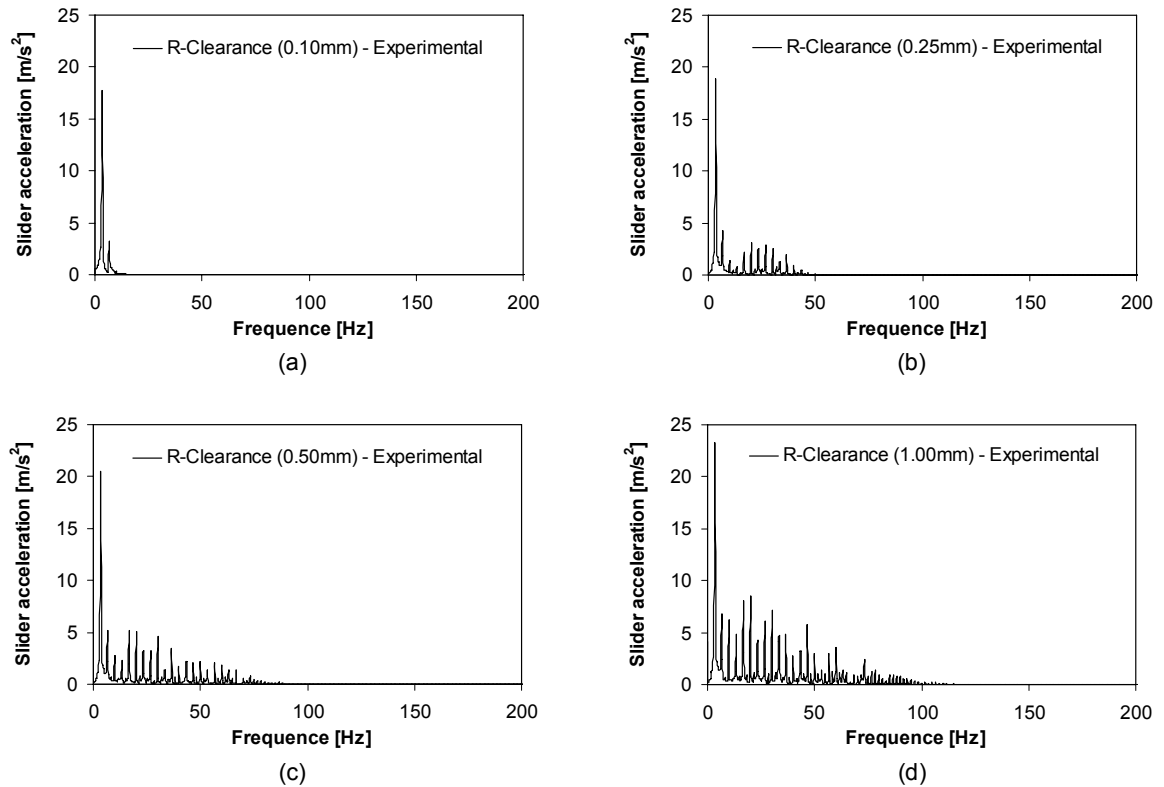


Figure 7. FFT analysis of the slider acceleration for different clearance sizes and for 200 rpm: (a)  $c=0.10$  mm; (b)  $c=0.25$  mm; (c)  $c=0.50$  mm; (d)  $c=1.00$  mm.

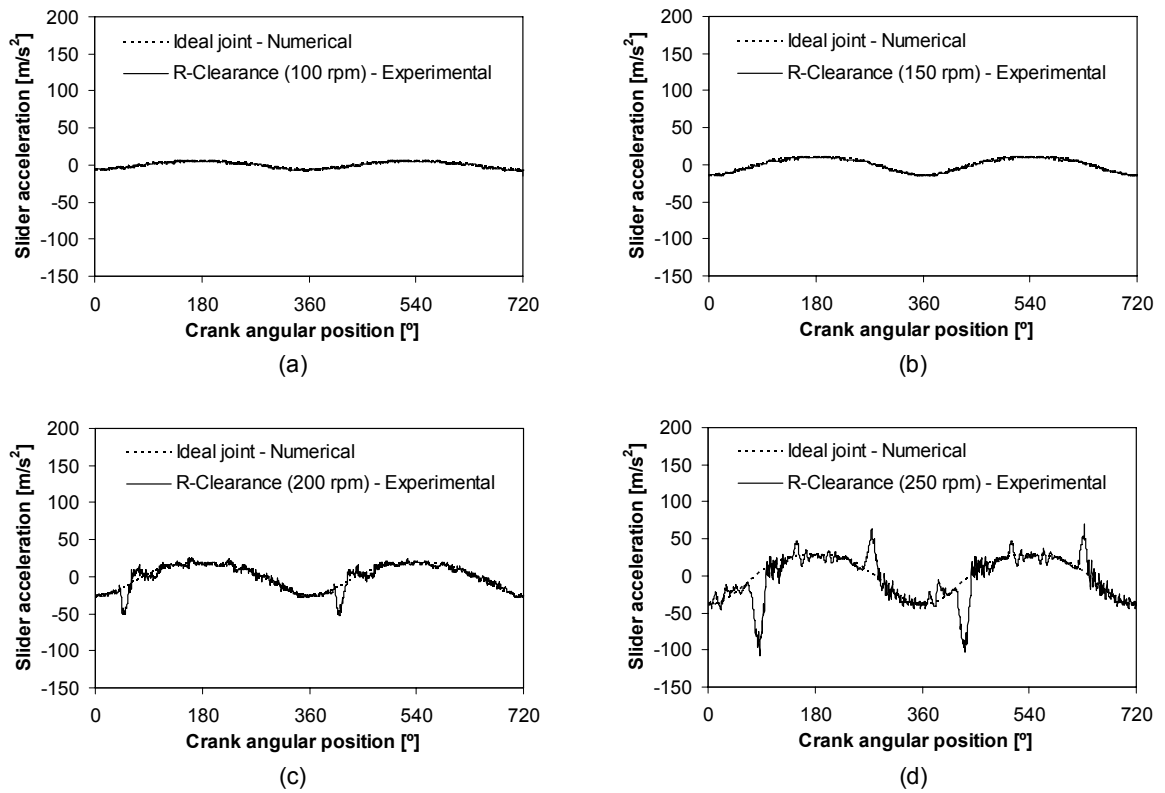


Figure 8. Slider acceleration for different crank angular velocities and clearance size equal to 0.25 mm: (a) 100 rpm; (b) 150 rpm; (c) 200 rpm; (d) 250 rpm.

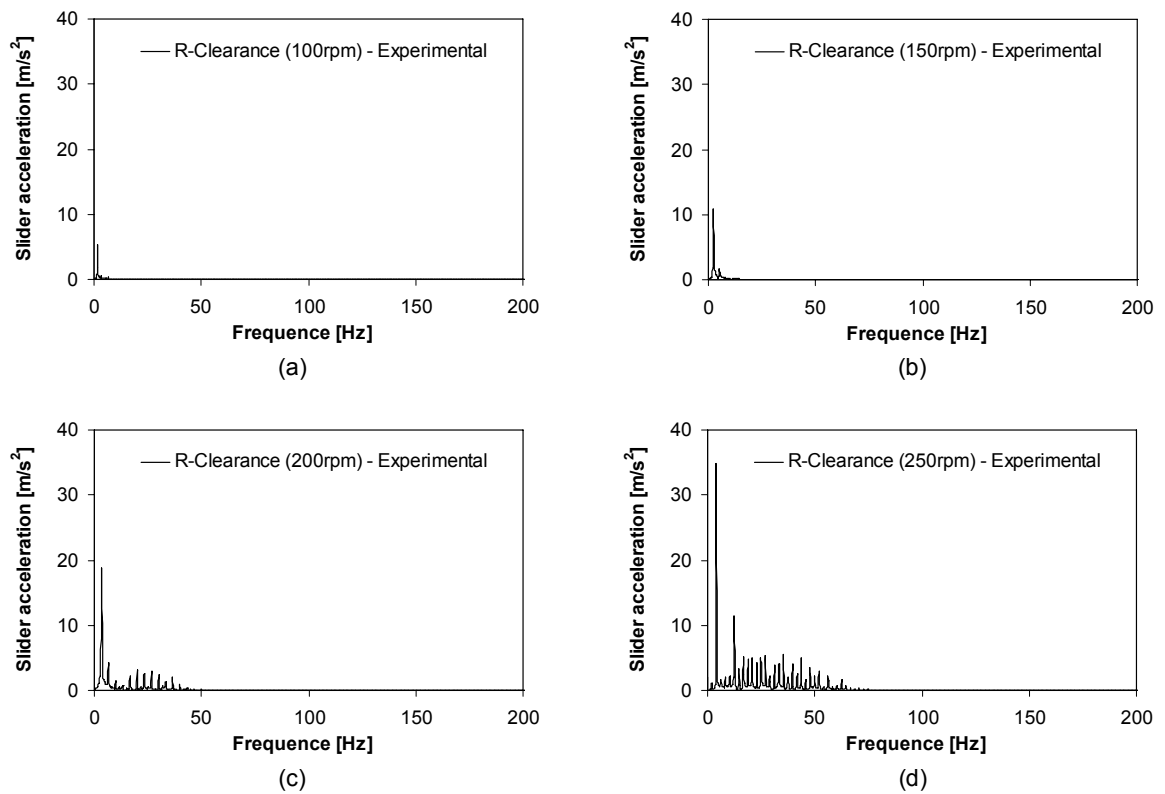


Figure 9. FFT analysis of the slider acceleration for different crank speeds and clearance of 0.25 mm: (a) 100 rpm; (b) 150 rpm; (c) 200 rpm; (d) 250 rpm.

Figure 8 shows the effect of varying the crank angular velocity on the slider acceleration response. The values of the crank velocity are 100, 150, 200 and 250 rpm, and the radial clearance is 0.25 mm. For the lower crank speeds the variation on the gross motion of the slider-crank mechanism is rather small, and the slider acceleration is similar to that observed for the ideal case. However, for higher crank speeds, the number of impacts and the size of the peaks in the slider acceleration increase, as observed in Figures 8(c)-(d) with particular emphasis. In Figure 9, the FFT analysis of the slider acceleration data for different crank speeds is presented. It is observed from this figure that the number and magnitude of the contribution of the dominant frequencies increase with the crank speed.

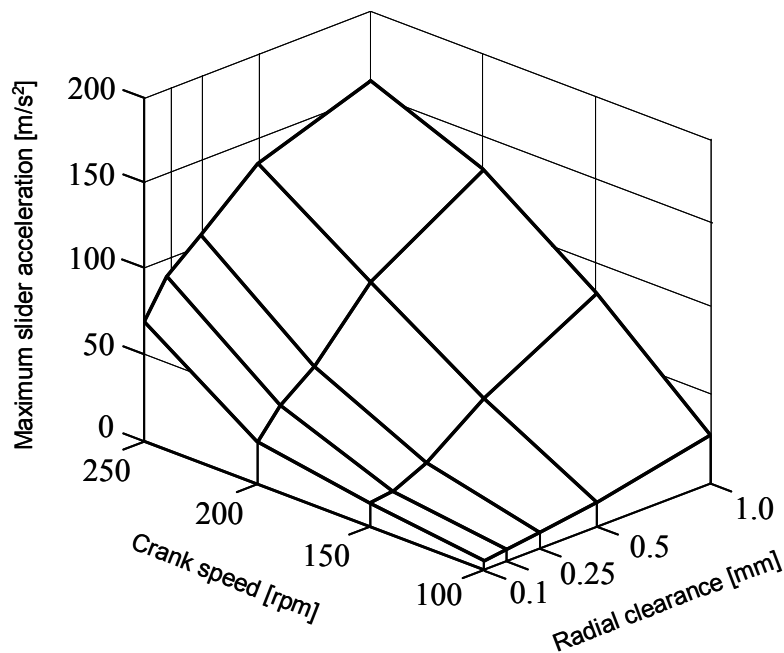


Figure 10. Maximum slider acceleration as function of crank speed and radial clearance.

With the actual construction of the experimental slider-crank setup, the impact force on the clearance joint can not be directly measured. Therefore, the maximum slider acceleration, i.e., the impact acceleration, is used to quantify the severity of the impact at the clearance joint. Figure 10 presents the maximum slider acceleration for the experimental dynamic response histories, as function of the crank speed and radial clearance. Figure 10 shows again that the maximum slider acceleration increases with crank speed and clearance. The three-dimensional design diagram for the maximum slider acceleration, which is distilled from Figures 6 and 8, can be used for analysis and design purposes of mechanical systems with clearance joints. For instance, it is possible to estimate the maximum slider acceleration for a combination of clearance and crank speed by using the design diagram of Figure 10. For low crank speeds, the apparent gross motion characteristics of the slider-crank mechanism remains unchanged, but for



high crank speeds and clearances, the dynamic response is significantly modified. This observation clearly demonstrates the severe variation of the dynamical behavior in the presence of a clearance joint and provides the required data to quantify the fatigue and wear phenomena.

## 5. Correlation between numerical and experimental results

In what follows, the experimental and numerical simulation results are correlated and discussed. The length and inertia properties of the experimental slider-crank mechanism components are listed in Table 2. The parameters used in the dynamic simulations are given in Table 4. Figures 11 and 12 present the numerical and experimental dynamic acceleration response histories of the slider-crank model with a clearance joint.

Restitution coefficient	0.46	Baumgarte - $\alpha$	5
Friction coefficient	0.01	Baumgarte - $\beta$	5
Young's modulus	207 GPa	Integration step size	$10^{-6}$ s
Poisson's ratio	0.3	Integration tolerance	$10^{-7}$

Table 4. Parameters used in the dynamic simulation of the slider-crank mechanism.

In the modeling of the impact phenomenon in multibody systems, the selection of the friction and restitution coefficients is of great importance and influences the outcome of the results. The selection of these parameters is made with the materials and the test conditions in mind, and based on the best published data. Wilson and Fawcett [37, 38] used values of the friction coefficient in the range of 0.007-0.010 and of the restitution coefficient in the range of 0.4-0.6, to investigate the dynamics of the slider-crank mechanism with clearance in the sliding bearing. They showed that the values of friction and restitution coefficients equal to 0.01 and 0.40, respectively, agreed well with experimental data response. Soong and Thompson [21] developed an experimental device to quantify the restitution coefficient in the impact between a journal and a bearing. Based on the kinematic, or Newton restitution coefficient, Soong and Thompson obtained a value of 0.46 for the restitution coefficient as the quotient between the impact velocity of 5.81 cm/s and the rebound velocity of 2.67 cm/s used in this test setup. In addition, Soong used the Coulomb's friction law to obtain the friction coefficient. Thus, in the present work, the values of 0.01 and 0.46 respectively for the friction and the restitution coefficients have been used to model the experimental slider-crank mechanism.

The differences between the experimental and numerical system response profiles, plotted in Figures 11 and 12, must be considered in light of the assumptions made in the formulation for the mathematical model of clearance joints, namely in what concerns to the joints flexibility, which was ignored. The friction in the sliding bearing was also neglected.

Moreover, during the non-contact situation, between the journal and the bearing, in the slider velocity is not constant in the experimental model, but decreases due to the friction in the slider bearing, because a preloaded linear bearing type has been used. For the numerical simulation, when the journal and bearing do not contact each other, the slider velocity is obviously constant and, consequently, the slider acceleration is null. This phenomenon is well visible as horizontal lines in the slider acceleration diagrams of Figures 11(f) and 11(h).

Furthermore, misalignments between journal and bearing elements, always present in actual mechanical systems, are not considered in the numerical simulations. Another important feature in the numerical simulations is the choice of the restitution and friction coefficients, because small variations on these parameters can significantly change the system response. The friction and restitution coefficients depend on the materials and geometric properties of the contacting bodies and can vary during the contact. However, the detailed analysis of these phenomena is considered beyond the scope of the present work being the choice of the restitution and friction coefficients solely based on the best published data. The geometrical tolerances, such as the degree of roundness of the journal and bearing surfaces, also affects the results, whereas for the model, the cross-sections are treated as perfectly round.

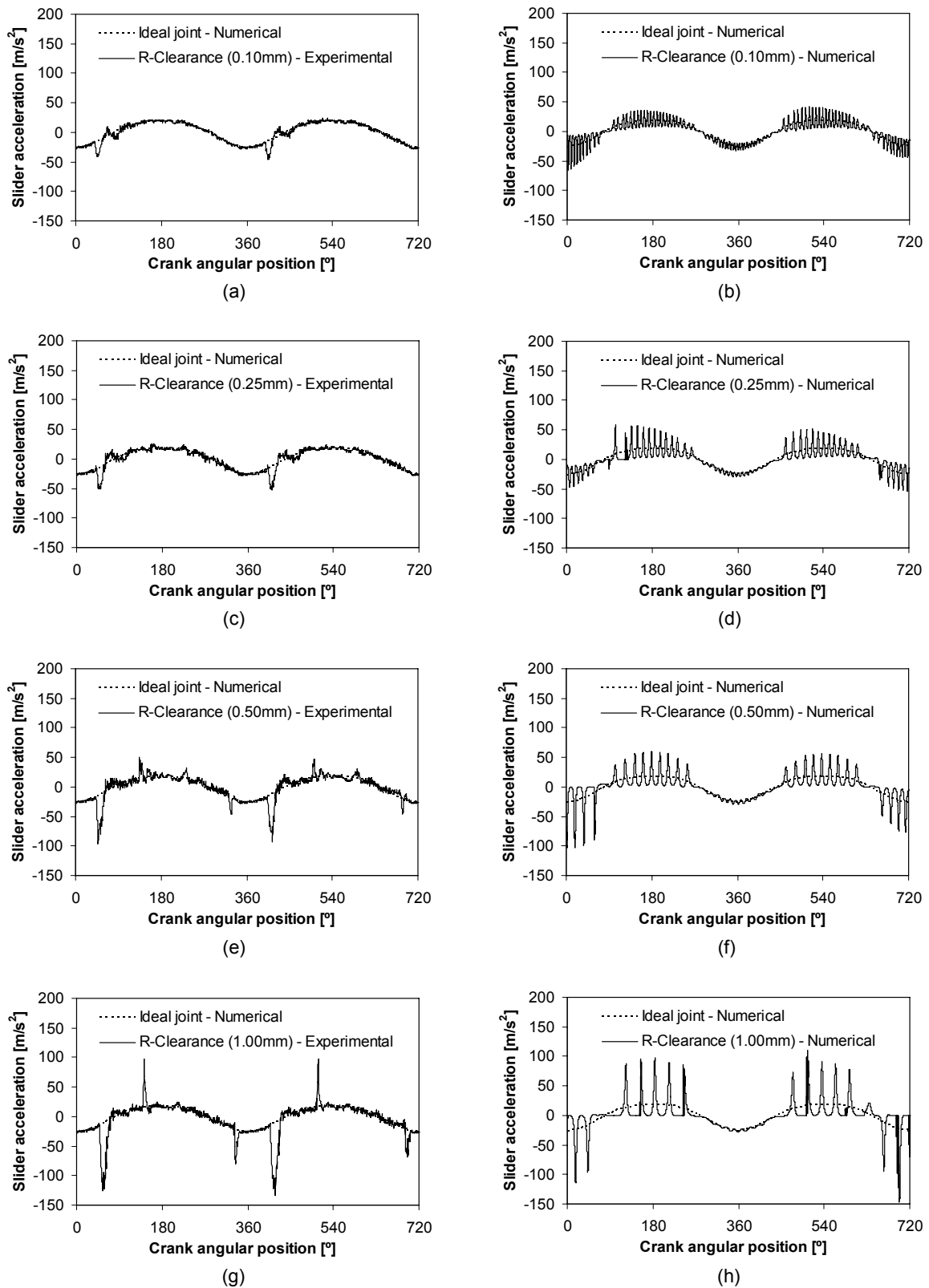


Figure 11. Experimental and numerical slider acceleration for crank speed of 200 rpm and different radial clearance sizes: (a)-(b)  $c=0.10 \text{ mm}$ ; (c)-(d)  $c=0.25 \text{ mm}$ ; (e)-(f)  $c=0.50 \text{ mm}$ ; (g)-(h)  $c=1.00 \text{ mm}$ .

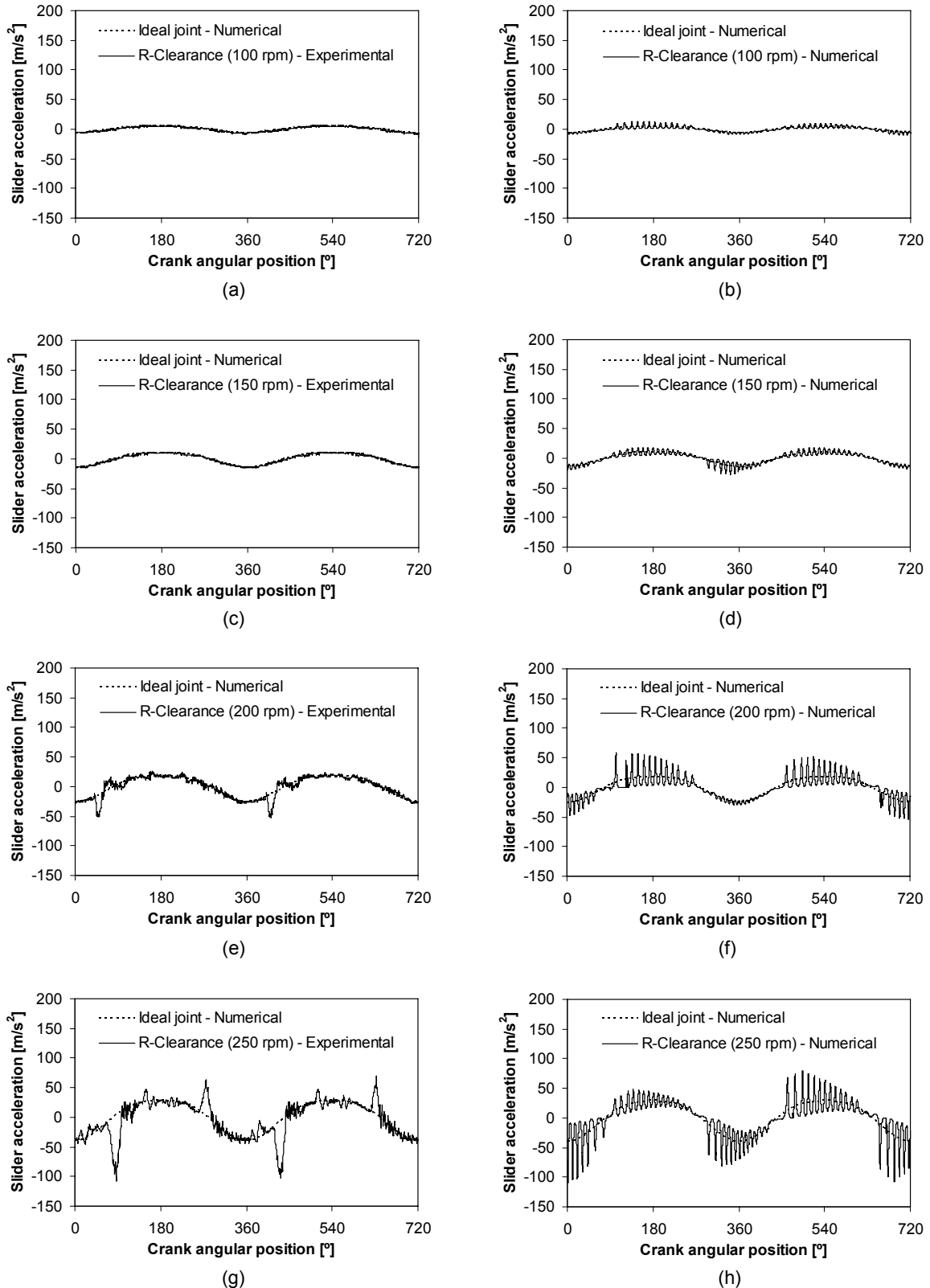


Figure 12. Experimental and numerical slider acceleration for different crank speeds and clearance size equal to 0.25 mm: (a)-(b) 100 rpm; (c)-(d) 150 rpm; (e)-(f) 200 rpm; (g)-(h) 250 rpm.

At this stage it is important to mention that the formulation of the equations of motion for constrained multibody system adopted in the present work follows closely the Nikravesh's work, in which the generalized absolute coordinates are used to

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describe the system configuration [29]. The Newton-Euler approach was employed to obtain the equations of motion of constrained multibody systems that was augmented with the constraint equations, resulting in a set of differential and algebraic equations [30]. A great merit of this formulation is that it is quite straightforward in terms of assemble of the equations of motion and providing all reaction forces. Moreover, the formulation is detailed for the type of coordinates adopted and expressions obtained for the contact-impact models used and presented throughout this work. The issues associated with the contact problem formulation, namely in what concerns the contact detection, the integration algorithm used, the selection of the appropriate time step, the lubrication problem, the influence of the number of clearance joints, and the computational efficiency of the methodology used in this study have been presented in detail in authors' previous works [5, 24-28, 39-43].

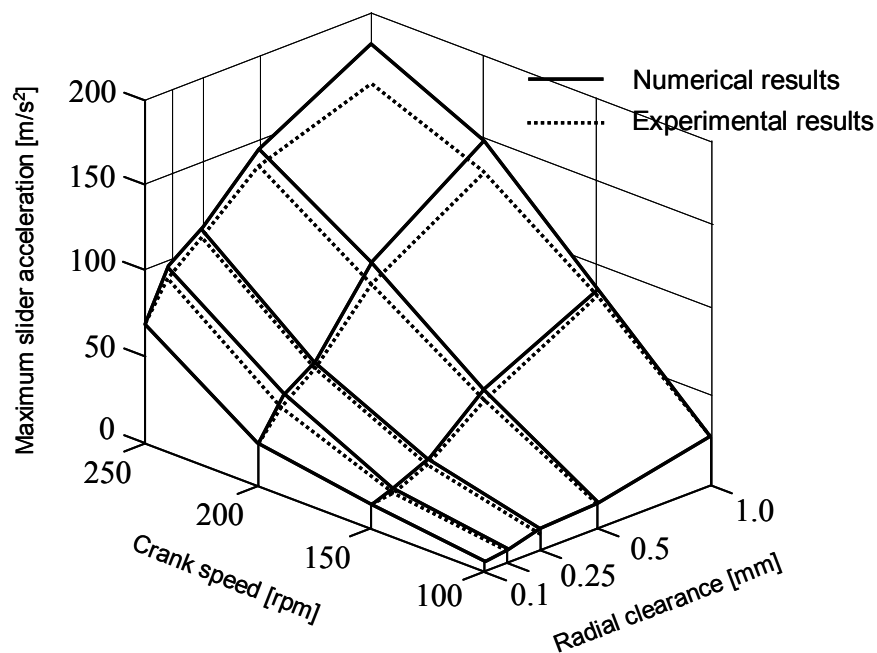


Figure 13. Maximum slider acceleration as function of crank speed and radial clearance: numerical and experimental results.

Figure 13 presents the maximum slider acceleration, for both experimental and numerical dynamic response histories, as function of crank speed for various radial clearances. From Figure 13, it is clear that the maximum slider acceleration increases with crank speed and with clearance, for both experimental and numerical results. Moreover, the maximum acceleration obtained from experimental tests agrees quite well with the numerical simulations, suggesting that the predictive capability of the proposed methodology is a reasonable approach to model multibody systems with impact at clearance joints. This observation confirms other data published on the field

on dynamics of multibody systems with clearance joints [2]. The fact that the existence of the clearance joint has an important effect on the slider acceleration supports the idea that the model of clearance joints must be considered in the analysis and design of the real mechanical systems.

## 6. Conclusions

In this paper, a comprehensive investigation was undertaken for a slider-crank mechanism in which the revolute joint between the slider and the connecting rod has an adjustable clearance. Dynamic response data for different operating frequencies; i.e., various crank speeds, with different radial clearance size were presented. Prior to present the dynamic response histories with clearance joints, the experimental model was used to obtain the global motion characteristics of the slider-crank mechanism with joints as close to ideal joints as possible, i.e., with a very low clearance. The experimental response data and the numerical results for these scenarios are in complete agreement.

The dynamic behavior of the experimental slider-crank setup was monitored with an LVDT and an accelerometer. The response histories were related to the crank angle using an encoder. For comparative purposes, the maximum amplitude of the slider acceleration was taken as a measure of the severity of the impact. This approach provides a reliable form of estimating the impact force that overcomes the deficiency of the actual test rig apparatus to acquire directly the impact force. It is observed that the maximum amplitude of the impact acceleration increases with clearance and crank speed. For low crank speeds, the gross motion characteristics of the slider-crank mechanism remain similar to those of the mechanism with perfect joints. However, for high angular frequencies and clearances, the slider dynamic response is significantly altered, being the maximum impact acceleration observed increased by more than 20 times. This information is important in the design of mechanical systems and allows estimating the tradeoff between journal-bearing life, cost and performance. In addition, the information relative to the maximum impact acceleration can be used to help the control and maintenance of industrial machines with clearance joints.

Some critical parameters and test uncertainties are also identified. The variety of material joint properties, mainly friction and restitution coefficients, should be considered since the selection of these parameters influences the outcome of the numerical results. In absence of better reference, values published in the literature were used. The correct alignment and geometrical precision of the impacting bodies is quite

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difficult to obtain experimentally, thus, the contact force experienced by these may not be distributed over the theoretical contact area. The methodology used in this work is based on some simplifications that include: the contact surfaces are considered as ellipsoidal; the center of the contact area does not change during the contact process; all the components of the system are rigid bodies. The basic fundamental form of Hertzian contact model was used in this study for the development of the numerical model.

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