

Numerical Data Fitting in Dynamical Systems

A Practical Introduction with Applications and Software

by

Klaus Schittkowski

*Department of Mathematics,
University of Bayreuth,
Bayreuth, Germany*



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