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Numerical Methods for Predicting Melting through the Gap Applied to the One-Dimensional Stefan Problem Using Taylor Series

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Abstract

There is always a gap in melting process in the contact surfaces of two materials. The gas trapped in the gap can inhibit heat transfer and cause the contact resistance. The purpose of the study was to determine the effect of the gap to the melting rate. Numerical methods applied to the one-dimensional Stefan problem using a Taylor series expansion is used to calculate the melting rate and the influence of the material properties of the melt rate. The result is a greater temperature difference between the inlet temperature (To) to the melting temperature (T_m) caused the melting speed difference Δu will be larger, the smaller of the melting time difference Δt. The greater of Stefan number will cause melting speed difference Δu greater. The small value of latent heat (C) and density (ρ) will cause the small energy requirements for melting causing the Δu a large, otherwise the small value of thermal conductivity (k) will lead to small Δu so that the ratio of the value of $k/\rho C$ is the variable values of Δu.

Keywords: melting, numeric, gap, one-dimensional Stefan problems, interface, Taylor series.

1. Introduction

The smelting research on objects through the gap is still very rare. Completion of the smelting process through a gap containing gas approximated by one-dimensional Stefan method and the Taylor series expansion. There needs to be a theoretical solution, which is not difficult, but accurate enough to solve the problem of an one-dimensional melting last equation produces a calculated melting in the melt region only for the solid is not taken into account.

A lot of melting research aims to solve one-dimensional Stefan problem, but does not discuss melt through the gap. The study presents the move interface location and use the grid as the coordinates for the location of the time of the moving interface [13]. Move an adaptive mesh method is developed for the numerical solution of the enthalpy formulation of the heat conduction problem with phase change [5]. The formulation is based on a three-level finite difference methods, stability analysis equations with matrix methods [11]. Heat Balance Integral Methods to solve some heat and phase changes, explains how to calculate the three-base error on the heat problems and shows how to calculate the heat balance formulation [8]. Finite difference method is used to solve the problem of melting, the results are in accordance with the results of calculations using the method of integral nodes [9]. Numerical methods to solve the problem Stefan equations are used for the process of melting with boundaries using Cartesian coordinates, and by applying a front-fixing technique with finite differences [1]. Solving the problems with the line method is applied to the location of melting on the many variations of Stefan numbers, the results can accurately determine the temperature distribution and movement of the liquid-solid interface [10]. Iteration method is used to solve problems that arise during the motion boundary melting or freezing of the area half the limit when the physical properties (thermal conductivity and specific heat) of the two regions that depend on temperature [6]. Research about comparison of numerical methods conformity with established models, the mesh move in the field, suggesting that the numerical and analytical methods can be used in solid-solid phase changes and solid-liquid boundary changes [3].

The practical application in melting, the most important thing that should be identified before the melting process is due to the material properties because it for determine how the process will be completed. It is, therefore necessary to develop a simple formula for predicting melting on contact area based on the properties of materials.

The outcome of the paper is new simple formula to solve the problem of heat transfer in the complex melting through the gap. The equation derived from the combination of the heat transfer equation and thermodynamic equation then calculated by numerical methods with the Taylor series expansion. Calculation of heat transfer in the contact melting is only determined by the properties of the material without taking into account external influences.

The contact surface is considered perfect so narrow gap in the flat contact surface. Grashof number is very small, under 2500, that the convection heat transfer can be neglected [12]. Thus, the heat transferred in the gas trapped in the gap is considered conduction only

Nomenclature

A	area (m^2)	x	horizontal axis
C	latent heat ($J\ kg^{-1}$)		
Cp	specific heat ($J\ kg^{-1}K^{-1}$)		*Greak symbols*
Gr	Grashof number	α	thermal diffusivity (m^2/s)
k	thermal conductivity ($W\ m^{-1}\ K^{-1}$)	β	melt rate per temperature
L	gap width or gas layer (m)	γ	barrier melt rate per temp.
Q	heat flux (W)	δ	stating ratio L/dx
s(t)	position of interface	ρ	density ($kg\ m^{-3}$)
t	time (s)		*Subscripts*
To	inlet temperature	g	gas
T	temperature (K)	l	liquid
u	interface speed ($m\ s^{-1}$)	m	melting
v	heat propagation speed (ms^{-1})	s	solid

2. Mathematical Model

The physical model of this study considered the basis of figure 1. Figure 1a conduction heat transfer in the solid phase through the gap, in the gas-filled gap. Figure 1b shows an illustration molten material starting from left to right through the gap.

The mathematical model in this paper builds on the figure 1b. Gap contains gas will disturbing of heat transfer from left side to right side, melt motion when the contact surface will be disturbed. Gap containing gas is considered a source of negative energy, when the temperature of the objects up the gas in the gap will absorb the heat, consequently the gas temperature in the gap will up. This is consistent with the first law of thermodynamics, gas will reduce heat energy. The heat of the gas will be forwarded to a solid object whose temperature is lower than the gas temperature, consequently a solid temperature rises to reach the melting temperature.

The gas trapped in the gap (L) is very thin. Heat convection in the gas can be neglected because the Grashof number is a function of L^3 is very small, so that heat transferred through the gas by conduction only.

The gas in the gaps will absorb the heat of Q [2]

$$Q = \rho_g \, Cp_g.A.dT_g \, \frac{dx_g}{dt_g} \qquad (1)$$

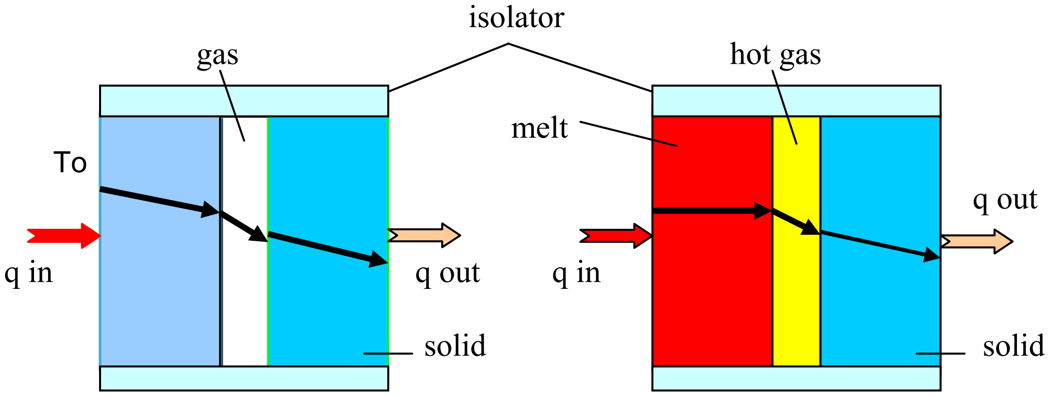


Figure 1 Mechanism of melting

Figure 2 shows the melting begins with the heating or inlet temperature (To) which is at the left of the object, the melt will move to the right. At the melt reach the left side of the gap the interface will stop for a moment because it was blocked by the gas and at the beginning of the energy used to raise the temperature of the gas.

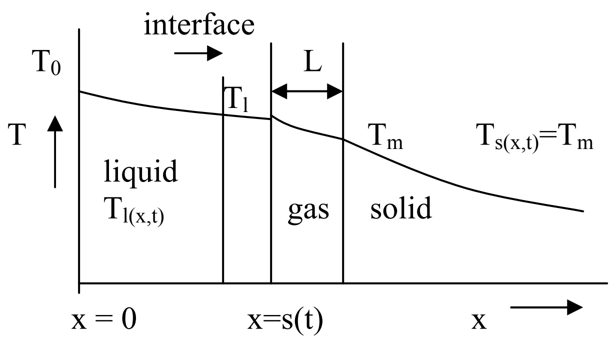


Figure 2 Melting process of half face

Movement of the interface with speed u from left to right, in the picture shows that there are equations with the following restrictions:

$$T_{l(x,t)} = To \qquad \text{at} \qquad x = 0, \; t > 0 \qquad (2)$$

Interface,

$$T_{l(x,t)} > T_m \qquad \text{at} \qquad x = s(t),\ t > 0 \tag{3}$$

$$-k_l \frac{\partial T_l}{\partial x} = \rho_l C \frac{ds}{dt} \qquad \text{at} \qquad x = s(t),\ t > 0 \tag{4}$$

From figure 2 is taken a small portion adjacent to the gap as shown in figure 3.

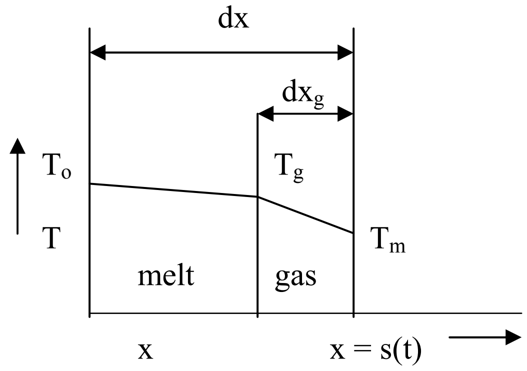


Figure 3. Conduction heat transfer in the melt and gas

Figure 3 shows the temperature distribution while considered linear because of the short distance. Melting through the gap will be disrupted so that the equation (4) will be changed to equation (5),

$$-k_l \frac{\partial T_l}{\partial x} + Q/A = \rho_l C \frac{ds}{dt} \qquad \text{at} \qquad x = s(t),\ t > 0 \tag{5}$$

Substitution of equation (1) into equation (5)

$$-k_l \frac{\partial T_l}{\partial x} + \rho_g\, Cp_g.dT_g \frac{dx_g}{dt_g} = \rho_l C \frac{ds}{dt} \tag{6}$$

To solve the equation (6) using a Taylor series expansion equation,

$$T_{(x)} = T_{(o)} + (x - x_o)\, T' + \frac{(x - x_o)^2}{2} T'' + \ldots \tag{7}$$

was taken only two levels and is set in such a that it becomes,

$$T' = \frac{(T_{(x)} - T_{(o)})}{(x - x_o)} - \frac{(x - x_o)}{2} T''$$

or

$$\frac{\partial T}{\partial x} = \frac{(T_{(x)} - T_{(o)})}{(x - x_o)} - \frac{(x - x_o)}{2} \frac{\partial^2 T}{\partial x^2} \tag{8}$$

3. Numerical methods

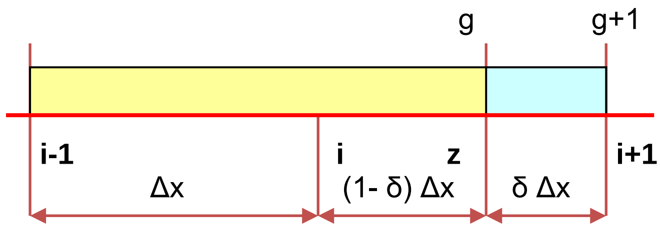


Figure 4. Discretization scheme

Here δ is a dimensionless scale which is the ratio between L and dx or $\delta = L/dx$.

From figure 4 it could be demonstrated that when:

$\delta = 0$	means no gap
$0 < \delta < 1$	means the gap is less than Δx
$\delta = 1$	means the gap equals to Δx
$\delta > 1$	means the gap is greater than Δx

The temperature in the nodal is

$$T_g = T_z$$
$$T_{i+1} = T_z + \delta\,(T_{i+1} - T_i)$$

Equation (8) is converted into a numerical equation that result is equation (9)

where: $T_{(x)} = T_{i+1}$, $T_{(o)} = T_{i-1}$, $(x - x_o) = 2\Delta x$

$$\frac{\partial T}{\partial x} = \frac{(T_{(i+1)} - T_{(i-1)})}{2\Delta x} - \frac{2\Delta x}{2}\,\frac{T_{i-1} + T_{i+1} - 2T_i}{\Delta x^2}$$

$$\frac{\partial T}{\partial x} = \frac{(T_{(i+1)} - T_{(i-1)})}{2\Delta x} - \frac{T_{i-1} + T_{i+1} - 2T_i}{\Delta x} \tag{9}$$

Equation (9) multiplied by k_l the result is equation (10):

$$k_l\frac{\partial T}{\partial x} = k_l\frac{(T_{(i+1)} - T_{(i-1)})}{2\Delta x} - k_l\frac{T_{i-1} + T_{i+1} - 2T_i}{\Delta x} \tag{10}$$

Substitution of equation (10) into equation (6) it becomes equation (11)

$$\rho_l C\,\frac{ds}{dt} = -\,k_l\frac{(T_{(i+1)} - T_{(i-1)})}{2\Delta x} + k_l\frac{(T_{i-1} + T_{i+1} - 2T_i)}{\Delta x} + \rho_g\,Cp_g.dT_g\,\frac{dx_g}{dt_g} \tag{11}$$

if $dT_g = \delta\,(T_{i+1} - T_i)$, equation (11) it becomes equation (12)

$$\rho_l C\,\frac{ds}{dt} = k_l\frac{(T_{(i-1)} - T_{(i+1)})}{2\Delta x} + k_l\frac{(T_{i-1} + T_{i+1} - 2T_i)}{\Delta x} - \rho_g\,Cp_g\,\frac{dx_g}{dt_g}\,\delta\,(T_i - T_{i+1}) \tag{12}$$

where: $u = \dfrac{ds}{dt}$ and $v = \dfrac{dx_g}{dt_g}$

then equation (12) turns into equation (13)

$$u = \frac{3k_l}{2\rho_l C\Delta x} T_{i-1} + \left(\frac{k_l}{2\rho_l C\Delta x} + \frac{\rho_g Cp_g v.\delta}{\rho_l C}\right) T_{i+1} - \left(\frac{2k_l}{\rho_l C\Delta x} + \frac{\rho_g Cp_g v.\delta}{\rho_l C}\right) T_i \quad (13)$$

if : $\beta = \frac{k_l}{2\rho_l C\Delta x}$ and $\gamma = \frac{\rho_g Cp_g v.\delta}{\rho_l C}$

The equation (13) it becomes equation (14)

$$u = 3\beta T_{i-1} + (\beta + \gamma) T_{i+1} - (4\beta + \gamma) T_i \quad (14)$$

When the value of $\delta = 0$ or there is without gap then the equation (14) will be the equation (15) which is the equation of melting speed or interfaces speed in melting state without a gap.

$$u = \beta (3 T_{i-1} + T_{i+1} - 4 T_i) \quad (15)$$

To calculate the temperature distribution on the object using equation (16) of the journal [2]

$$\frac{dT}{dt} = \lambda_l (T_{i-1} + T_{i+1} - 2T_i) \quad (16)$$

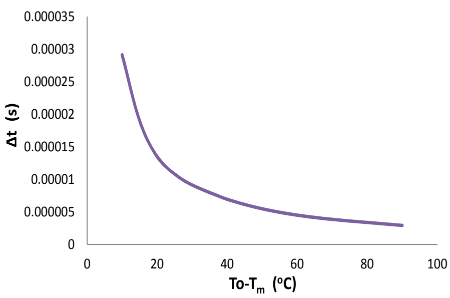
where: $\lambda_l = \frac{\alpha_l}{\Delta x^2}$

4. Results and Discussion

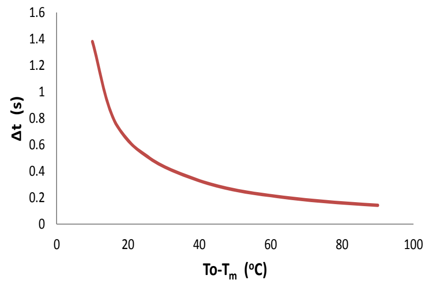
To determine the effect of the contact surface of the gap during the melting process using equation (14) and (15) are applied to various metals and non-metals materials such as steel, copper, polypropylene and limestone. The properties of these materials are taken from the results of a preliminary study as well as from [4], [7] as shown in Table 1. Other variables are $\delta = 0.02$, the heat propagation velocity v = 0.1 m/s and $\Delta x = 0.002$ m.

Table 1 Properties of materials

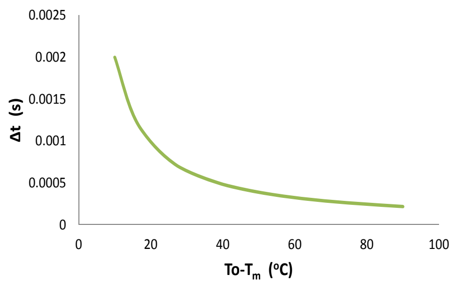
Material	Polypropylene	Air	Copper	Steel	Limestone
Conductivity, k (W/m^2 K)	0,16	0.03; 0.0837; 0.111	345	35	1.1
Density, ρ (kg/m^3)	739	1.14; 0,2707; 0.200	8900	7600	2450
Heat Capacitance, Cp (J/kg.K)	1450	1009 ; 1197; 1267	370	430	700
latent heat C (J/kg.)	88000	-	205000	250000	400000
Melting point T_m (^oC)	140	-	1084	1500	1000



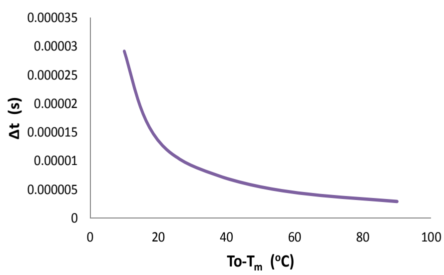
a) Polypropylene



b) Limestone



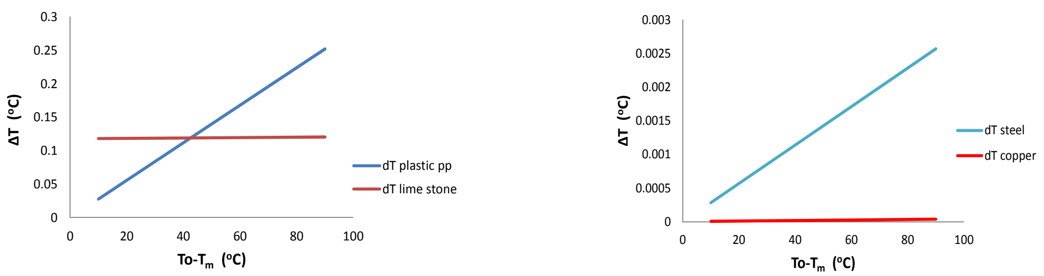
c) Steel



d) Copper

Figure 5 Smelting time difference (Δt)

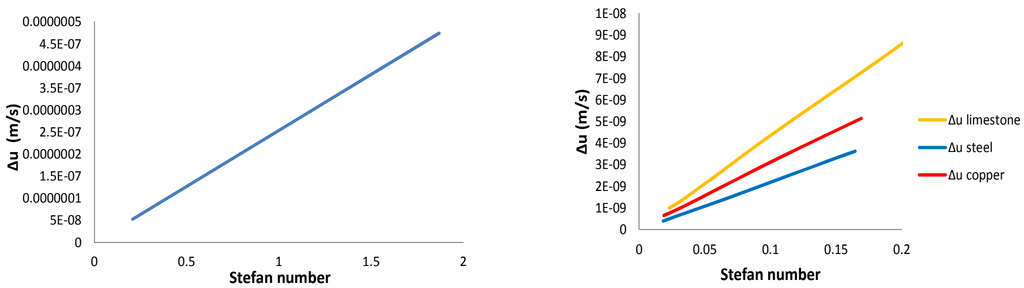
Figure 5 shows the effect of the temperature difference between the inlet temperature (To) with a melting temperature (T_m) or (To-T_m) against the melting time difference Δt, where Δt is calculated from the thick object divided by the speed of melt there is a gap reduced by thick object divided by the speed of melt without a gap. Graph in the figure shows the corresponding hyperbolic equation, Δt value will be infinite if the (To-T_m) is zero and otherwise if the value of the (To-T_m) immense then the values of Δt close to zero. The greater of the (To-T_m) the Δt will be smaller, this is because heat energy given higher to the object and more rapid melting. The value Δt of plastic material is much greater than the limestone material and other materials as well as steel materials higher than copper, this is due to the heat conductivity of plastic pp materials are smaller than other materials and less steel than copper. Thermal conductivity (k) higher will cause the speeds of material melting fast so Δt will be smaller. Other properties that inhibit speed is the latent heat of fusion (C) and density (ρ), the higher (C) heat required for smelting getting bigger and so is the greater density of objects will be more energy for melting.



a) Polypropylene and limestone b) Steel and copper

Figure 6 Temperature difference (ΔT) in the gap

Figure 6 shows the effect of the (To-T_m) to the temperature difference ΔT in the air-filled gap. The greater of the (To-T_m) the ΔT will be greater, this is because the temperature range at equal distances greater then ΔT will be greater. Values of ΔT plastic pp > limestone > steel > copper because the value of (k) plastic pp < limestone < steel < copper, the greater value of (k) the heat will quickly rise. The value (To-Tm) are not so influential on the limestone due to one reason is the value of β is very small, the small value β melting speed is slower so that dT is not so changed.



a) Polypropylene b) Limestone, steel and copper

Figure 7 The effect of Stefan number against speed difference of smelting (Δu)

Figure 7 shows the effect of Stefan number on speed difference melting of material Δu, where Δu is calculated from speed of melt without a gap reduced by the speed of melt there is a gap. The larger of the Stefan number the melt rate will greater because Stefan number is proportional to the temperature difference heating, as well as the Δu. Plastic PP materials have Δu highest due to the influence of the values of (C), ρ and (k) is small. Value (C) and ρ small will cause the energy requirements for melting is small causing the Δu a large, otherwise the value of (k) small will lead to Δu is small so that the ratio of the value of k/ ρC is the variable values of Δu.

5. Conclusion

1. The greater temperature difference between the inlet temperature (To) to the melting temperature (T_m) caused the melting speed difference (Δu) will be larger and the smaller of the melting time difference (Δt)
2. The greater of Stefan number will cause melting speed difference (Δu) greater.
3. The small value of latent heat (C) and density (ρ) will cause the small energy requirements for melting causing the Δu a large, otherwise the small value of thermal conductivity (k) will lead to small Δu so that the ratio of the value of k/ρC is the variable values of Δu.

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