Numerical Modeling of Suprathermal Electron Transport in Laser-Produced Plasmas

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Report No. 36
August 1976

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by

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ACKNOWLEDGEMENT

The research reported on herein was supported by the following principal sponsors:

Exxon Research & Engineering Company
General Electric Company
Northeast Utilities Service Company
University of Rochester

Others contributing to the efforts of the work in the Laboratory for Laser Energetics include:

Empire State Electric Energy Research Company
U.S. National Science Foundation
New York State Energy Research Development Administration
New York State Science & Technology Foundation
U.S. Air Force Office of Scientific Research

Such support does not imply endorsement of the content by any of the parties.

AUGUST 1976

1.1 INTRODUCTION

When intense laser light is focused onto a target during a laser plasma interaction experiment, energetic electrons are observed either directly with an electron mass spectrograph,[1] or film stack,[2] or indirectly through k-alpha radiation from moderate-Z filled microballoons[3] or their hard x-ray signature.[4] These electrons, variously referred to as non-Maxwellian, non-thermal or supra-thermal, can be created by resonant absorption[5] or parametric or stimulated processes[6] at the plasma critical (plasma frequency equals laser frequency) or quarter critical surfaces. Whatever the exact mechanism for creating these supra-thermal electrons, it is clear that a substantial fraction are too energetic ($\gtrsim$10keV) to be thermalized on any time scale of interest in the hydrodynamic computational models used to simulate laser plasma interaction.

Supra-thermal electrons can radically alter the dynamics of imploding laser fusion targets by preheating the target material, by transforming energy to a rapid ion blowoff or, due to mean free path effects, decoupling the core from the corona. In order to correctly simulate the dynamics of a laser driven target, it is necessary to develop an accurate physical and computational model of these electrons. The difficulties inherent in this problem can be appreciated by noting that the supra-thermal velocities can vary by over an order of magnitude while the density of the background thermal electron fluid through which they propagate can vary by over four orders of magnitude. Since the supra-thermal mean free-path is proportional to v^4/n_e, there can be an eight order of magnitude variation in this scaling parameter.

Most models of particle transport in a thermal background start with the Boltzmann equation. Both S_n[7] and Monte-Carlo[8] methods have been used extensively in neutron transport problems, however the latter are inefficient when used in conjunction with a one-dimensional Lagrangian hydrodynamic code as is our intention here. In the limit of small angle scattering, the Fokker Plank equation replaces the Boltzman equation and this equation is tractable to solution by several approaches. Since the transport of supra-thermal electrons must be followed in both the diffusive and free streaming (i.e., very short and very long mean-free-path) limits, neither flux limited diffusion models[9] nor truncated moment methods[10] are adequate.

We require a model which simultaneously includes the two mean free path limits, allows bidirectional non-isotropy in velocity space but retains one-dimensionality in real and velocity space. This last requirement is again dictated by computational restraints. There are several reasons for the existence of a strong bidirectional (but parallel to a radius vector) asymmetry in the distribution function when the coupling of supra-thermal and thermal electrons is weak. These include strongly non-isotropic flow away from source regions, preferential acceleration by electric fields and collisional depletion of electrons returning toward the source region. Note that the non-isotropy of the source is explicitly restricted to be parallel to a radial vector (i.e., the suprathermals must be created either isotropically or preferentially in the radial direction).

A three component model of a transport of supra-thermal electrons is developed here to satisfy the requirements stated above. In this model, a nearly isotropic distribution function is used in the diffusive regime when the radial anisotropy is weak, while two half-isotropic distributions, peaked respectively in the positive and negative radial directions, are

used in the free streaming regine when the radial anisotropy is strong. Motivation for this particular form of a multigroup transport model follows from both S_n methods[7] and methods useful in radiation transport.[11] Each velocity group in each Lagrangian computational cell of the hydrodynamic computational model[12] is identified as being in either the short or long mean free path regime and treated accordingly. The division into the two regimes is based on comparing the 90^o deflection mean free path at the mean group velocity and local plasma conditions with the suprathermal density scale length.

Section 1.2 reviews the derivation of the Fokker-Planck equation for the transport of supra-thermal electrons through a less energetic thermal background. The supra-thermals are assumed to couple collisionally only to the thermal background. Equations for the diffusive, nearly isotropic regime are obtained in section 1.3. The full three-component model is developed in section 1.4 and the equations describing the self-consistent thermal return current and electric field are developed in section 1.5. Finally, the source function is discussed in section 1.6.

1.2 Fokker-Planck Equation

The Fokker-Planck equation is derived from the Boltzmann equation in the limit of dominant small angle collisions. The Boltzmann equation is given by

$$\frac{\partial\phi(\bar{r},\bar{v},t)}{\partial r} + \bar{v}\cdot\frac{\partial\phi}{\partial\bar{r}} + \frac{1}{m_e}\frac{\partial}{\partial v}(\bar{F}\phi) = \left(\frac{\partial\phi}{\partial t}\right)_c + \left(\frac{\partial\phi}{\partial t}\right)_s \qquad (1)$$

where $\bar{F}$ includes forces othern than collisions. The Fokker-Plank collision operator is obtained by assuming that a particle has the probability $\psi(\bar{v},\bar{\Delta})$, independent of the particle's history, that the velocity $\bar{v}$ acquires an increment $\Delta\bar{v}$ in a time $\Delta\bar{v}$ in a time Δt.[13] This collision operator can then be expressed as

$$\left(\frac{\partial\phi}{\partial t}\right)_c = -\frac{\partial}{\partial\bar{v}}\cdot\left(\phi\langle\Delta\bar{v}\rangle\right) + \frac{1}{2}\frac{\partial^2}{\Delta\bar{v}\Delta\bar{v}} : \left(\phi\langle\Delta\bar{v}\Delta\bar{v}\rangle\right) \qquad (2)$$

Where

$$\begin{Bmatrix}\langle\Delta\bar{v}\rangle\\ \langle\Delta\bar{v}\Delta\bar{v}\rangle\end{Bmatrix} = \frac{1}{\Delta t}\int\psi\ (\bar{v},\Delta\bar{v})\begin{Bmatrix}\Delta\bar{v}\\ \Delta\bar{v}\Delta\bar{v}\end{Bmatrix}\ d(\Delta\bar{v}) \qquad (3)$$

The first term is the coefficient of dynamical friction and the second term is the coefficient of diffusion whose effect is to spread a stream of initially uni-directional particles.[14]

To define the probability ψ, multiple collisions are treated as sequences of binary collisions.[14] Collisions among supra-thermals can be ignored due to their large velocities and relatively low densities. Using the Coulomb scattering cross-section, the collision operator can be written

$$\left(\frac{\partial \phi}{\partial t}\right)_c = \Gamma\left\{-\frac{\partial}{\partial \bar{v}} \cdot \left[\phi(v)\frac{\partial H(\bar{v})}{\partial \bar{v}}\right] + \frac{1}{2}\frac{\partial^2}{\partial \bar{v}\partial \bar{v}} : \left[\phi(\bar{v})\frac{\partial^2 G(\bar{v})}{\partial \bar{v}\partial \bar{v}}\right]\right\} \tag{4}$$

where

$$\Gamma = \frac{4\pi e^4}{m_e^2}$$

$$H = \sum_j \frac{m_e+m_j}{m_e} Z_j^2 \int \frac{\phi_j(\bar{v}_2)\ell n\Lambda_{sj}}{|\bar{v}-\bar{v}_2|} d\bar{v}_2$$

$$G = \sum_j Z_j^2 \int \phi_j(\bar{v}_2)|\bar{v}-\bar{v}_2|\ell n\Lambda_{sj}\, d\bar{v}_2$$

$$\ln\Lambda_{sj} = \min\left[1, \ln\left[\left(\frac{kT}{4\pi n_e e^2}\right)^{\frac{1}{2}} \frac{m_e v}{h}\right]\right]$$

and where the summation over j represents the contribution of all thermal species, $\phi_j(\bar{v}_2)$ is the distribution function of each thermal species and $\ell n\Lambda_{sj}$ is the Coulomb logarithm for collisions between suprathermal and thermal species.

The expressions for H and G in eq.(4) can be simplified by treating the suprathermal electrons as test particles interacting with field particles which are in thermal equilibrium.[15] When a Maxwellian distribution is substituted for $\phi(\bar{v}_2)$, the functions H and G become

$$H(\bar{v}) = \sum_j \left(\frac{m_e+m_j}{m_e}\right) Z_j^2 \frac{n_j}{\bar{v}} \Phi(a_j v) \ell n \Lambda_{sj} \tag{5}$$

$$G(\bar{v}) = \sum_j Z_j^2 n_j \left[\left(v + \frac{1}{2a_j^2 v}\right) \Phi(a_j v) + \frac{e^{-a_j^2 v^2}}{a_j \sqrt{\pi}}\right] \ell n \Lambda_{sj} \tag{8}$$

where n_j denotes thermal electrons or ions.

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy, \quad a_j^2 = \frac{m_j}{2 k T_j}$$

The supra-thermal electrons are selected by the criterion $a_e^2 v^2 \geq 4$, (i.e., $v \geq 2v_{e_{th}}$). This requires that electrons with velocity less than twice the electron thermal velocity be thermalized on a time short compared to the other hydrodynamic times of interest in laser plasma interaction (e.g., electron-ion thermal equilibration time and acoustic or thermal propagation times[12]). It will be shown later that this is a reasonable requirement. Under these conditions, the error function becomes

$$\Phi(x) \simeq 1 \text{ for } x \geq 2$$

and the functions H and G simplify to

$$H(\bar{v}) = \sum_j \left(\frac{m_e+m_j}{m_j}\right) \frac{n_j}{v} Z_j^2 \ell n \Lambda_{sj} \tag{7}$$

$$G(\bar{v}) = \sum_j n_j v Z_j^2 \ell n \Lambda_{sj} \tag{8}$$

Equation 4 can now be rewritten as

$$\left(\frac{\partial\phi}{\partial t}\right)_c = \Gamma \left\{ \frac{\partial}{\partial \bar{v}} \cdot \left[\phi(\bar{v}) \ \frac{\bar{v}}{v^3}\right] \sum_j \left(\frac{m_e+m_j}{m_j}\right) \ Z_j^2 \ n_j \ \ln\Lambda_{sj} \right.$$
$$\left. + \frac{1}{2} \ \frac{\partial^2}{\partial \bar{v}\partial \bar{v}} : \left[\phi(v) \ \frac{(v^2\bar{\bar{I}} - \bar{v}\ \bar{v})}{v^3}\right] \sum_j Z_j^2 \ n_j \ \ln\Lambda_{sj} \right\} \qquad (9)$$

where ϕ denotes the supra-thermal electron distribution function. The diffusion term in eq.(9) can be expanded to become

$$\frac{1}{2} \frac{\partial}{\partial \bar{v}} \cdot \left[-2\phi(\bar{v}) \ \frac{\bar{v}}{v^3} + \left(\frac{v^2\bar{\bar{I}} - \bar{v}\ \bar{v}}{v^3}\right) \cdot \frac{\partial\phi\ (\bar{v})}{\partial \bar{v}}\right] \sum_j Z_j^2 \ n_j \ \ln\Lambda_{sj} \qquad (10)$$

The first term in the bracket is a dynamical friction term and the second term is a diffusion term which vanishes when the velocity is either uni-directional or isotropic. The two friction terms in eq.(9) and (10) combine to yield

$$\frac{\partial}{\partial \bar{v}} \left[\phi \frac{\bar{v}}{v^3}\right] \sum_j \ Z_j^2 \left(\frac{m_e}{m_j}\right) n_j \ \ln\Lambda_{sj} \qquad (11)$$

Since the friction term is proportional to the mass ratio, electron-ion collisions can be neglected. On the other hand, the diffusion term is inversely proportional to the electron mass so that both thermal species must be included.

The Fokker-Planck equation now reduces to

$$\frac{\partial\phi}{\partial t}(\bar{r},\bar{v},t) + \bar{v} \cdot \frac{\partial\phi}{\partial \bar{r}} + \frac{\partial}{\partial \bar{v}} \cdot (\bar{F}\phi) = \left(\frac{\partial\phi}{\partial t}\right)_f + \left(\frac{\partial\phi}{\partial t}\right)_d + \left(\frac{\partial\phi}{\partial t}\right)_S \qquad (12)$$

where

$$\left(\frac{\partial \phi}{\partial t}\right)_f = \Gamma\, n_e \frac{\partial}{\partial \bar{v}} \cdot \left[\ln\Lambda_{se} \frac{\phi \bar{v}}{v^3}\right]$$

$$\left(\frac{\partial \phi}{\partial t}\right)_d = \frac{\Gamma}{2}\left(n_e + Z^2 n_i\right) \frac{\partial}{\partial \bar{v}} \cdot \left[\ln\Lambda_{se} \frac{\bar{v}^2 \bar{I} - \bar{v}\,\bar{v}}{\bar{v}^3} \cdot \frac{\partial \phi(\bar{v})}{\partial \bar{v}}\right] \tag{14}$$

and $\left(\frac{\partial \phi}{\partial t}\right)_S$ is the source term.

There are two other effects, apart from collisions, which must be accounted for: the self-consistent electric field and Bremsstrahlung radiation. The force due to the electric field is given by

$$\bar{F}_E = - \frac{e\,\bar{E}}{M_e} \tag{15}$$

The Bremsstrahlung radiation can be treated as a force and can thus be written as[16]

$$\bar{F}_B = -\bar{e}_v\, n_i \int_0^{\omega_{max}} \chi(\omega)\, d\omega \tag{16}$$

where

$$\chi(\omega) = \frac{16}{3} \frac{Z^2 e^6}{M_e^2 c^3} \frac{1}{v^2} \ln\left[\frac{\left(E^{1/2} + \sqrt{E-\hbar\omega}\right)^2}{\hbar\omega}\right]$$

$$E = \frac{1}{2} m v^2$$

$$\omega_{max} \simeq \frac{m v^2}{\hbar}$$

The integral can be shown to reduce to unity and the force becomes

$$\bar{F} = \frac{-16\, n_i Z_i^2 e^6}{3 m_e c^3 \hbar} \bar{e}_v \tag{17}$$

A more useful form of the Fokker-Planck equation can be obtained by substituting

$$\bar{\Omega} = \frac{\bar{v}}{v} = \cos\theta\ \bar{e}_r \tag{18}$$

into equations 12, 13, 14, 15, and 17 and the double partial in the diffusion term becomes

$$\frac{\partial}{\partial \bar{v}} \cdot \left[\frac{\bar{\bar{I}} - \bar{\Omega}\,\bar{\Omega}}{v} \cdot \frac{\partial \phi}{\partial \bar{v}} \right] = \frac{1}{v^3 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\phi \frac{\partial \phi}{\partial \theta} \right)$$

The final form of the Fokker-Planck equation is now,

$$\frac{\partial \phi}{\partial t}(\bar{r},\bar{\Omega},v,t) + \bar{\Omega} \cdot \frac{\partial}{\partial \bar{r}}(\phi v) - \frac{e}{m_e} \frac{\partial}{\partial \bar{v}}(\phi E \cdot \bar{\Omega}\,\bar{\Omega}) - K_B \frac{\partial}{\partial \bar{v}} \cdot (\phi \bar{\Omega})$$

$$- K_f \frac{\partial}{\partial \bar{v}} \cdot \left(\frac{\phi}{v^3} \ln\Lambda_{se}\, \bar{\Omega} \right) - \frac{K_d}{v^3 \sin\theta} \ln\Lambda_{se} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial \phi}{\partial \theta} \right) = \left(\frac{\partial \phi}{\partial t} \right)_S \qquad (19)$$

where

$$K_B = \frac{16\, n_i Z_i^2\, e^6}{3 m_e^2 c^3 \hbar};$$

$$K_f = \Gamma n_e;$$

$$K_d = \frac{\Gamma}{2}(n_e + Z^2 n_i);$$

$$\Gamma = \frac{4\pi e^4}{m_e^2}$$

The Fokker-Planck equation is two dimensional in velocity space. The angular dependence is generated by the diffusion term and possibly by the source term. If the electrons undergo few collisions with the thermal particles, the collision term can be neglected and eq.(19) becomes one-dimensional in velocity space - the electrons are then transported by a streaming equation. On the other hand, if collisions are dominant, the collision term must be retained - a one-dimensional description can be recovered through the derivation of a diffusion equation in which the distribution function is assumed nearly-isotropic.

1.3 THE DIFFUSION EQUATION

The diffusion equation is obtained in the limit of a nearly isotropic distribution of particles in velocity space. There are several ways of deriving this equation; here we use a spherical harmonic expansion of the distribution function in velocity space which will allow us to retain some mathematical simplicity while still being able to understand the nature of the physical approximations inherent in this approach. Any distribution function can be expanded in the complete set of spherical harmonics.[17] It is sufficient, for the derivation of the diffusion equation to keep only the first two terms

$$\phi(\bar{r},\ v,\ \bar{\Omega},\ t) = A + \bar{\Omega}\cdot\bar{B} \tag{20}$$

where $A/|\bar{B}|$ is not a function of $\bar{\Omega}$. Since the set of equations developed below are one-dimensional (spherically symmetric) in the velocity space, the magnitude and angular dependence of the velocity vector have been separated in the argument of ϕ. The near isotropy of the distribution function for the diffusive mode requires that the terms omitted in eq.(20) be appropriately small.

For convenience in the following work, we note here several useful integrals. If $\bar{A}$ and $\bar{B}$ are any vectors independent of $\bar{\Omega}$, which may include the gradient operator, then integration over the entire solid angle yields

$$\int \bar{\Omega}\ \bar{A}\ d\bar{\Omega} = 0$$

$$\int \bar{\Omega}\cdot\bar{A}\ d\bar{\Omega} = 0$$

$$\int (\bar{\Omega}\cdot\bar{A})(\bar{\Omega}\cdot\bar{B})\ d\bar{\Omega} = \frac{4\pi}{3}\ (\bar{A}\cdot\bar{B})$$

$$\int \bar{\Omega}(\bar{A}\cdot\bar{\Omega})\ d\bar{\Omega} = \frac{4\pi}{3}\ \bar{A}$$

$$\int \bar{\Omega}(\bar{A}\cdot\bar{\Omega})(\bar{B}\cdot\bar{\Omega})\ d\bar{\Omega} = 0$$

Following the usual definitions of particle kinetic theory, the number density and flux are related to the distribution by

$$n(\bar{r},\, v,\, t) = \int \phi d\bar{\Omega}$$
$$\bar{J}(\bar{r},\, v,\, t) = \bar{e}_r \int (\bar{\Omega}\phi v)\cdot d\bar{\Omega} \qquad (21)$$

where $\bar{e}_r$ is a unit vector in real space. The zero and first moments of eq.(20) then yield

$$A = (4\pi)^{-1}\, n(\bar{r},\, v,\, t), \quad \bar{B} = 3(4\pi v)^{-1}\bar{J}(\bar{r},\, v,\, t)$$

and the distribution function can be rewritten as

$$\phi(\bar{r},\bar{v},\bar{\Omega},t) = \frac{1}{4\pi}\left[n(\bar{r},v,t) + \frac{3}{v}\,\bar{\Omega}\cdot\bar{J}(\bar{r},v,t)\right] \qquad (22)$$

We now suppress the $(\bar{r},v,t)$ arguments of n and $\bar{J}$.

Equations for the number density and flux are obtained by taking the corresponding moments of the Fokker-Planck equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \bar{r}}\cdot J - \frac{e}{m_e v^2}\,\frac{\partial}{\partial v}\,(v\bar{E}\cdot\bar{J}) - \frac{K_B}{v^2}\,\frac{\partial}{\partial v}\,(v^2 n) -$$
$$\frac{K_f}{v^2}\,\frac{\partial}{\partial v}\,(n \ln\Lambda_{se}) = \left(\frac{\partial n}{\partial t}\right)_S \qquad (23)$$

$$\frac{1}{v}\,\frac{\partial \bar{J}}{\partial t} + \frac{v}{3}\,\frac{\partial n}{\partial \bar{r}} - \frac{e}{3m_e}\,\frac{1}{v^2}\,\frac{\partial}{\partial v}\,(\bar{E} n v^2) - \frac{K_B}{v^2}\,\frac{\partial}{\partial v}\left(v^2\,\frac{\bar{J}}{v}\right) -$$
$$\frac{K_f}{v^2}\,\frac{\partial}{\partial v}\,\left(\ln\Lambda_{se}\,\frac{\bar{J}}{v}\right) + 2\,\frac{K_d}{v^3}\,\ln\Lambda_{se}\,\frac{\bar{J}}{v} = \left(\frac{\partial \bar{J}}{\partial t}\right)_S \qquad (24)$$

This set of coupled equations has been closed by the truncation of the spherical harmonic expansion explicit in eq.(20). Equations (23) and (24) will be applied to the description of the transport of supra-thermal electrons in the diffusive mode. The source terms on the right-hand sides

represent the creation of suprathermal particles and their associated flux by promotion out of the thermal distribution.

Equations (23) and (24) are too difficult to solve in their present form. To make these equations tractable to our analysis, we introduce the standard diffusion assumptions: that the flux source and scattering are isotropic and that the diffusion time is short compared with any other time scale of interest. Only the second and last terms on the left-hand side of (24) are retained and the resulting simplification yields

$$\bar{J} = -\frac{v}{6(K_d \ln\Lambda_{se}/v^4)} \frac{\partial n}{\partial \bar{r}} = -\frac{v\lambda}{3} \frac{\partial n}{\partial r} = -D \frac{\partial n}{\partial \bar{r}} \qquad (25)$$

where a 90^o deflection mean free path

$$\lambda = 2 \tau_{90^o} v = \frac{v^4}{2K_d \ln\Lambda_{se}}$$

has been defined.

The assumption on time scales which was used to eliminate the time derivative in eq.(24) does not hold very well in the problem of charged particle slowing down in plasma. When the diffusion equation is used to describe particles with large energy, the diffusion coefficient must be corrected because, as the speed of the particles increases, the mean-free-path, which varies as v^4, will exceed the free-streaming "distance" given by $v\Delta t$ (where Δt is a computational time step[12]). For large velocities the diffusion term must go over to free-streaming. To correct this, a flux limiter can be introduced and the diffusion coefficient written as[9]

$$D = \frac{v}{\frac{3}{\lambda} + \frac{2\partial n}{n\partial r}} \qquad (26)$$

However, diffusive and streaming transport are basically different and incompatible. In diffusive transport the "driving force" for the flux of particles is the density gradient; if the density is uniform there is no flux. On the other hand, in streaming transport the "driving force" is the initial impulse on the particles and their flux is not affected by their density gradient; thus a local flux can exist with a uniform density. In addition, the flux-limited transport assumes that near-isotropy, which exists in the diffusive regime because of collisions, will carry over in the streaming formulation; this is rather doubtful in situations where the source is localized in space and the particles move directly away. Because of these problems, we avoid the ad hoc flux limiter formulation and develop the three component model in the next section.

1.4 THE THREE-COMPONENT MODEL

Two equations have been derived which can describe the transport of the non-thermal electrons over two velocity regimes: a streaming Fokker-Planck equation for the high energy, near-collisional regime and a diffusion equation for the low energy collisional regime. For computational purposes it is necessary to sharply divide these two regimes although a smooth transition exists in reality. This transition point in velocity space can be found by considering the limits of the validity of the diffusion equation and, in particular, the point at which the diffusion coefficient would have to be flux-limited to retain physical meaning.

We delimit the boundary between the diffusive and streaming formulations by reference to the density scale length

$$\frac{\lambda}{3} < \frac{n}{\partial n/\partial r} \equiv L_s \qquad \text{diffusion regime}$$

$$\frac{\lambda}{3} > L_s \qquad \text{streaming regime.} \tag{27}$$

$$\lambda = \frac{v^4}{2K_d \ln\Lambda_{se}}$$

To see that this is consistent with the formulation of the diffusion equation, we note that a diffusion time can be obtained from eq.(24)

$$\tau_D \simeq \frac{v^3}{2K_d \ln\Lambda_{se}} = \frac{\lambda}{2v}$$

The diffusion time scale follows from (27)

$$\frac{1}{\tau_D} \sim \frac{2v}{\lambda} > \frac{2v}{3n}\frac{\partial n}{\partial r} \sim \frac{1}{\tau_t}$$

where τ_t is associated with the convective term in (24). This condition is

only weakly satisfied at the boundary where $\tau_d \simeq \tau_t$. Note that streaming non-thermals undergo very few collisions and, on the average, are deflected only through very small angles.

The simplest streaming model which can accommodate the major sources of anisotropy, a source region localized to a thin spherical region and strong radial electric fields (the source function itself is discussed in section 1.6), is a semi-isotropic distribution which treats separately electrons with positive and negative radial velocities (see figures 1). This semi-isotropic distribution function has the two components

$$g_s(\bar{r},v,t) \equiv \int \phi(\bar{r},v,\bar{\Omega},t)\, d\bar{\Omega} = 2\pi\phi \quad \text{for} \quad 0 \leq \theta \leq \pi/2$$

$$h_s(\bar{r},v,t) \equiv \int \phi(\bar{r},v,\bar{\Omega},t)\, d\bar{\Omega} = 2\pi\phi \quad \text{for} \quad \frac{\pi}{2} \leq \theta \leq \pi$$

Substitution of this distribution function into eq.(19) and integration over the angles yields

$$\frac{\partial g_s}{\partial t}(\bar{r},v,t) + \frac{\partial}{\partial \bar{r}} \cdot \bar{J}_{s+} - \frac{e}{2mv^2} \frac{\partial}{\partial v}(v\bar{E}\cdot\bar{J}_{s+}) - \frac{K_B}{v^2}\frac{\partial}{\partial v}(g_s v^2) -$$

$$\frac{K_f}{v^2}\frac{\partial}{\partial v}(g_s \ln\Lambda_{es}) = \left(\frac{\partial g_s}{\partial t}\right)_S \qquad (28)$$

$$\frac{\partial h_s}{\partial t}(\bar{r},v,t) + \frac{\partial}{\partial \bar{r}} \cdot \bar{j}_{s-} - \frac{e}{2mv^2} \frac{\partial}{\partial v}(v\bar{E}\cdot\bar{j}_{s-}) - \frac{K_B}{v^2} \frac{\partial}{\partial v}(h_s v^2) -$$

$$\frac{K_f}{v^2} \frac{\partial}{\partial v}(h_s \ln\Lambda_{es}) = \left(\frac{\partial h_s}{\partial t}\right)_s \tag{29}$$

where

$$\bar{j}_{s+} = \int_0^{\pi/2} \bar{\Omega}\phi v d\bar{\Omega} = \frac{v}{2} g_s \tag{30}$$

$$\bar{j}_{s-} = \int_0^{\pi/2} \phi v \bar{\Omega} d\bar{\Omega} = -\frac{v}{2} h_s \tag{31}$$

It will be necessary to retain the general expression for the flux rather than its integrated form.

These two equations are coupled by the boundary conditions at the center of the target where all electrons with negative velocity are reversed and at the outer boundary where some of the electrons with positive velocity are reversed and part could escape through a potential sheath. Because of the difficulty of the sheath problem, we have this to future work and here assume that all outgoing electrons are reversed at the outer boundary.

Equations (28) and (29) are coupled in another more subtle way at each point in the plasma because of the use of the semi-isotropic two component distribution function. In traversing any spherical annulus of radius r and width dr, part of the electron distribution with negative velocity will move tangentially in such a way as to re-emerge at the outer rather than the inner boundary. Such electrons must then be transferred from the negative to the positive velocity distribution via source-sink terms in the equations. Failure to do this means that

ingoing electrons penetrate to the center of a sphere giving rise to anomalously large preheat. These source-sink terms are treated explicitly in the chapter on finite difference methods.

The transport equations are to be solved in the Lagrangian frame[12] of a hydrodynamics code. The transformation from Eulerian to Lagrangian frames leads to a term of the form

$$(\overline{v}-\overline{U})\nabla\phi$$

for the convective term where $\overline{U}$ is the macroscopic thermal velocity. It will be seen a posteriori that since $\overline{v}$ is always required to be larger than the thermal velocity of the thermal electrons, it is always at least an order of magnitude larger than $\overline{U}$. This small correction can be then neglected and equations 23, 28, and 29, together with the definitions of the fluxes, can be used to describe the transport with respect to the thermal fluid.

The non-thermal electrons are now described by a three-component distribution function and their transport in r-v space is treated by three equations which have been obtained from the Fokker-Planck equation under the assumptions of the three-component model. Still to be specified in these equations are the electric field and the thermal return current which are considered in the next section.

1.5 SELF-CONSISTENT ELECTRIC FIELDS AND RETURN CURRENTS

A complete description of a plasma with two thermal components and suprathermal electrons requires two continuity, momentum and energy equations, Poisson's equation for the electric field and the transport equations of the last section. Electric fields are created by the transport of supra-thermal electrons away from the source region, by the excess of electrons in regions of the fluid where supra-thermals have been thermalized, and by the electron thermal pressure gradient. Return currents will in turn be created which tend to cancel the field. The effectiveness of this cancellation depends on the electron-ion collision frequency,

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{3} \frac{n_e Z^2 e^4 \ln\Lambda}{m_e^{1/2}(kT_e)^{3/2}} \tag{32}$$

High temperature, low density plasmas carry larger return currents while low temperature, high density plasma will impede the cancellation of the electric fields. The return current can also be impeded by such phenomena as ion acoustic turbulence which is equivalent to higher collision frequencies.

The complete set of equations for the plasma are listed in Table I. These equations must be manipulated to yield a description of the motion of the thermal plasma and a description of the return current relative to the hydrodynamic motion. All of the species conservation equations in Table I contain mass, momentum and energy source terms due to thermal-suprathermal coupling as well as momentum and energy exchange terms between the two thermal components (we emphasize that although many different ionic states may be present in the plasma, all ions are assigned the same temperature).

The hydrodynamic equations are obtained by addition of the species equation,

$$\frac{d\rho}{dt} = \frac{\partial m_e n_e}{\partial t}\bigg|_s \tag{33a}$$

$$\frac{d\rho\underline{U}}{dt} + \underline{\nabla}p - e(zn_i - n_e)\underline{E} = -\underline{K}_{en} - \underline{K}_{in} \tag{33b}$$

$$\frac{d\rho_e e_e}{dt} - p_e \frac{d}{dt}\frac{1}{\rho} - \dot{Q}_e - \dot{W} - \Delta\dot{E} = \dot{S}_{ne} \tag{33c}$$

$$\frac{d\rho_i e_i}{dt} - p_i \frac{d}{dt}\frac{1}{\rho} - \dot{Q}_i + \Delta\dot{E} = \dot{S}_{ni} \tag{33d}$$

where ρ, ρU, P are the total mass, momentum, and pressure of the plasma, ρ_i, ρ_e, n_i, n_e, e_i, and e_e are the ion and electron mass and number densities and energies, Z the local average ionization state and $\dot{Q}$ and $\dot{W}$ the thermal flux and rate of laser energy deposition, $\Delta\dot{E}$ the electron-ion energy transfer and $\frac{d}{dt}$ the Lagrangian operator. The mass, momentum, and energy source terms have been isolated on the right-hand sides of (33) where $(\ )_n$ is used to denote supra-thermal components. The mass and momentum sources in these equations are $O(m_e/m_i)$ and can be neglected. The energy source terms are quite important and are the (negative) sum of the dissipation and slowing down terms in the supra-thermal transport equations. An explicit formulation for these terms will be given in the next chapter. Equations (33), in the form given, emphasize the one velocity, two temperature nature of the thermal fluid.

The return current equations are obtained by subtracting the species continuity and momentum equations to obtain

$$\frac{\partial \delta}{\partial t} + \nabla \cdot \bar{f} = -\left(\frac{\partial n_e}{\partial t}\right)_S \tag{34a}$$

$$\frac{\partial \bar{f}}{\partial t} + \nabla \cdot \left(\frac{Z_I p_I}{m_I} - \frac{p_e}{m_e}\right) - e\bar{E}\left(\frac{Zn_I}{m_I} + \frac{n_e}{m_e}\right) = \bar{K}_{ie}\left(\frac{Z_I}{m_I} + \frac{1}{m_e}\right) - \frac{\bar{K}_{in}}{m_I} + \frac{\bar{K}_{en}}{m_e} \tag{34b}$$

where

$$\delta = Zn_I - n_e \tag{35}$$
$$f = Zn_I\bar{u}_I - n_e\bar{u}_e$$

are the local net charge and flux. In our notation "j" and "J" are also used for flux and it is convenient to work with these quantities rather than go explicitly to currents.

Equation (34b) was obtained by neglecting terms quadradic in ($\underline{U}_i$-$\underline{U}_e$) and $\underline{f}$ which is consistent with the Chapman-Enskog formulation of a multi-species diffusion equation. Also consistent with this formulation is the neglect of $\partial\bar{f}/\partial t$. Replacing $\underline{K}_{ie}$ with $-m_e\nu_{ei}\underline{f}$ and noting that $\delta/n_e \ll 1$ (as required by the one velocity formulation above), then

$$\rho \simeq m_i n_i$$

$$n_e \simeq z\rho/m_i - \delta$$

and (34b) becomes

$$\bar{f} = +\frac{1}{m_e\nu_{ei}}\left(\nabla p_e + e\bar{E}n_e + m_e\bar{K}_{en}\right) \tag{36}$$

Equations (34a) and (36) are directly transformed (following some elementary manipulation) into Lagrangian form

$$\frac{\partial}{\partial\tau}\delta + \nabla\cdot\bar{f}' + \delta\nabla\cdot\bar{U} = -\left(\frac{\partial n_e}{\partial\tau}\right)_S \qquad (37a)$$

$$\bar{f}' = \frac{1}{m_e\nu_{ei}}\left(\nabla p_e + e\bar{E}n_e + m_e\bar{K}'_{en}\right) \qquad (37b)$$

where $\bar{f}' = n_e(\bar{u}_e - \bar{U})$ is the "flux" in the moving frame, and the source terms are calculated from the transport of non-thermals in the Lagrangian frame. The sources will be discussed later.

Equations (37) can be combined to give

$$\frac{\partial\delta}{\partial\tau} + \nabla\cdot\left[\frac{1}{\nu_{ei}m_e}\left(\nabla p_e + e\bar{E}n_e + m_e\bar{K}_{en}\right)\right] + \delta\nabla\cdot\bar{U} = -\left(\frac{\partial n_e}{\partial\tau}\right)_S \qquad (38)$$

The electric field is to be obtained from Poisson's equation

$$\nabla\cdot\bar{E} = 4\pi e(\delta - n_n)$$

where n_n is the local density of suprathermals. Taking advantage of the spherical symmetry of the system and the fact that $|U|/c<<1$, we can use this equation in the computations in an integral form

$$\underline{E} = \underline{e}_r E_r = \frac{\underline{e}_r e}{r^2}\int_o^r (\delta - n_n)dV \qquad (39)$$

The time scale for the relaxation of the charge separation is estimated from eq.(38) and (39) in the form

$$\frac{\partial \delta}{\partial \tau} \simeq \frac{e^2 n_e \delta}{m_e \nu_{ei}} \tag{40}$$

which yields

$$\tau_\delta \sim \frac{m_e \nu_{ei}}{e^2 n_e} = \frac{4\sqrt{2\pi}}{3} \frac{m_e^{1/2} Z^2 e^2 \ln\Lambda}{(kT_e)^{3/2}}$$

This relaxation time depends only weakly on n_e through the coulomb logarithm and is shown in Fig. 2 as a function of electron temperature for $Z^2=25$ and $\ln\Lambda=2$. It is anticipated that the largest electric fields and return currents occur near the source regions at the critical surface in a laser plasma. As can be seen from Fig. 2, except for very early times when the plasma is cold, the charge relation time is short compared with other possible time scales of interest (which are all $0(10^{-14})$sec or longer) and a steady state calculation for δ would follow from (38). Fortunately the time dependent calculation is not difficult, and no further approximation is made in this equation.

In order to obtain expressions for the source terms, it is necessary to return to the Fokker-Planck equation (19). It will be necessary to take various velocity moments of this equation and it must be recalled that at each space-time point a suprathermal electron is either in the diffusive region $V_{min} \leq V \leq V^*$ or in the streaming region, $V>V^*$. V_{min} is the speed at which the suprathermals are considered thermalized and are demoted into the thermal electron fluid and V^* follows from condition (27). As $V \to \infty$ all of the distribution functions must vanish. Since electrons

in both regions exist at the same point, we define the three quantities

$$g(\bar{r},t) \equiv \int_{V_{min}}^{\infty} g_s v^2 dv \qquad \text{for } 0 \leq \Theta \leq \pi/2$$

$$h(\bar{r},t) \equiv \int_{V_{min}}^{\infty} h_s v^2 dv \qquad \text{for } \frac{\pi}{2} \leq \Theta < \pi \qquad (41)$$

$$n_d(\bar{r},t) \equiv \int_{V_{min}}^{\infty} n_s v^2 dv \qquad \text{for } 0 \leq \Theta \leq \pi$$

The integrations over angle of eq.(19) has already yielded eqs. (23), (28), and (29). Integrating now the zero (mass) order moment of these equations over the velocity space for $V \geq V_{min}$ and using (27) and (41) yields,

$$\frac{\partial g}{\partial t} + \int \nabla \cdot \bar{j}_{s+} v^2 dv - \frac{e\bar{E}}{m_e} \cdot \left[v\bar{j}_{s+} \right]_{v^*} - \left[g_s \ln\Lambda_{es} (K_B + \frac{K_f}{v^2}) \right]_{v^*} = \left(\frac{\partial g}{\partial t} \right)_S$$

$$\frac{\partial h}{\partial t} + \int \nabla \cdot j_{s-} v^2 dv - \frac{e\bar{E}}{m_e} \cdot \left[v j_{s-} \right]_{v^*} - \left[h_s \ln\Lambda_{es} (K_B + \frac{K_f}{v^2}) \right]_{v^*} = \left(\frac{\partial h}{\partial t} \right)_S \qquad (42)$$

$$\frac{\partial n_d}{\partial t} + \int \nabla \cdot \bar{j}_s \, v^2 dv + \frac{e\bar{E}}{m_e} \cdot \left[v j_s \right]_{v^*} - \frac{e\bar{E}}{mc} \cdot \left[v j_s \right] V_{min} + \left[n_s \ln\Lambda_{es} \right.$$

$$\left. (K_B + \frac{K_f}{v^2}) \right]_{v^*} - \left[n_s \ln\Lambda_{es} \, (K_B + \frac{K_f}{v^2}) \right]_{V_{min}} = \left(\frac{\partial n_d}{\partial t} \right)_S$$

We have explicitly assumed that any particle demoted from the streaming to the thermal regime must pass through the diffusive regime. Effectively, the suprathermal electrons pass through the velocity distribution and drop out the bottom at V_{min} so the source term in (38) becomes

$$\left(\frac{\partial n_e}{\partial t} \right)_S = \left[(K_B + \frac{K_f}{v^2}) \, n_s \ln\Lambda_{es} \right]_{v_{min}} + \frac{e\bar{E}}{m_e} \cdot \left[v\bar{j}_s \right]_{v_{min}} - \left(\frac{\partial g}{\partial t} \right)_S - \left(\frac{\partial h}{\partial t} \right)_S - \left(\frac{\partial n_d}{\partial t} \right)_S \qquad (43)$$

Note that the electric field term is effective only when $\underline{E}\cdot\underline{j}>0$; when $\bar{E}\cdot\bar{j}<0$, thermal particles cannot be promoted to suprathermal. The first term on the right-hand side represents the loss of supra-thermal electrons due to Bremsstrahlung radiation and collisions with thermals and the last term is the number of thermal electrons raised into the supra-thermal component because of various absorption mechanisms.

To obtain the momentum contributions, it is necessary to redo the angular integrations of the Fokker-Planck regime for the first moment, $m_e V\bar{\Omega}$. For all three components of the model, the angle integrations yield,

$$\frac{\partial}{\partial t}\bar{j}_s + \frac{v^2}{3}\frac{\partial}{\partial \bar{r}}\, g_s - \frac{ev}{3m_e v^2}\frac{\partial}{\partial v}\,(v^2\bar{E}g_s) - \frac{K_B v}{v^2}\,\frac{\partial}{\partial v}\left(v^2\,\frac{\bar{j}_{s+}}{v}\right)$$
$$- \frac{K_f v}{v^2}\frac{\partial}{\partial v}\,(\bar{j}_{s+}\ln\Lambda_{es}) = \left(\frac{\partial \bar{j}_{s+}}{\partial t}\right)_S$$

$$\frac{\partial}{\partial t}\bar{j}_{s+} + \frac{v^2}{3}\,\frac{\partial h_s}{\partial \bar{r}} - \frac{ev}{3m_e v^2}\,\frac{\partial}{\partial v}\,(v^2\bar{E}h_s) - \frac{K_B v}{v^2}\,\frac{\partial}{\partial v}\left(v^2\,\frac{\bar{j}_{s-}}{v}\right)$$
$$- \frac{K_f v}{v^2}\frac{\partial}{\partial v}\,(\bar{j}_s\ln\Lambda_{es}) = \left(\frac{\partial \bar{j}_{s-}}{\partial t}\right)_S \qquad (44)$$

$$\frac{\partial}{\partial t}\bar{j}_s + \frac{v^2}{3}\,\frac{\partial n_s}{\partial \bar{r}} - \frac{ev}{3m_e v^2}\,\frac{\partial}{\partial v}\,(\bar{E}n_s v^2) - \frac{K_B v}{v^2}\frac{\partial}{\partial v}\left(v^2\,\frac{\bar{j}_s}{v}\right)$$
$$- \frac{K_f v}{v^2}\frac{\partial}{\partial v}\,(\bar{j}_s\ln\Lambda_{es}) + \frac{2K_d v}{v^3}\ln\Lambda_{es}\,\frac{\bar{j}_s}{v} = \left(\frac{\partial \bar{j}_s}{\partial t}\right)_S$$

The convective and Bremsstrahlung terms will not contribute to the momentum transfer to the thermal fluid. After integration and summation, the momentum equation becomes

$$\frac{\partial}{\partial t}\int_{v_{min}}^{\infty}(\bar{j}_{s+}+\bar{j}_{s-}+\bar{j}_{s})v^2dv+\frac{1}{3}\frac{\partial}{\partial r}\int_{v_{min}}^{\infty}v^2(g_s+h_s+n_s)v^2dv-$$

$$K_B\int v\frac{\partial}{\partial v}\,v^2(\bar{j}_{s+}+\bar{j}_{s-}+\bar{j}_{s})\,dv=-\frac{\bar{K}_{ne}}{m_e}-\frac{\bar{K}_{ni}}{m_e} \qquad (45)$$

where the thermal-suprathermal interaction is separated into ion and electron components,

$$\frac{\bar{K}_{ne}}{m_e}=-\frac{e\bar{E}}{3m_e}\int_{v_{min}}^{\infty}v\frac{\partial}{\partial v}\left[v^2(g_s+h_s+n_s)\right]dv-K_f\int v\frac{\partial}{\partial v}\left[\ln\Lambda_{es}(\bar{j}_{s+}+j_{s-}+j_s)\right]dv$$

$$+K_f\int\frac{\ln\Lambda_{es}}{v^3}\,\bar{j}_s v^2dv-\int\left[\left(\frac{\partial j_{s+}}{\partial t}\right)_s+\left(\frac{\partial j_{s-}}{\partial t}\right)_s+\left(\frac{\partial j_s}{\partial t}\right)_s\right]v^2dv \qquad (46)$$

$$\frac{K_{ni}}{m_e}=\Gamma Z^2n_I\int\frac{\ln\Lambda_{es}}{v^3}\,\bar{j}_s\,v^2dv$$

As before, it must be remembered that n_d, g_s and h_s are non-zero only for certain ranges of v and the same for the corresponding j's. However now, any change in suprathermal momentum is transferred to the thermal fluid (this will also be true of energy below) not just at the bottom of the distribution where $v=v_{min}$. These integrals could be simplified by expanding by parts, but the difference scheme used in the transport equation (see chapter 2) requires that they be in this form before differencing.

The energy contribution of the suprathermal electrons to the thermal fluid is found in exactly the same manner using the energy moment $\frac{1}{2}m_e v^2$. Omitting the lengthy manipulation, the thermal electron source is

$$\dot{S}_e = -\int \frac{e\bar{E}}{4} \cdot v^2 \left[\frac{\partial}{\partial v} v(\bar{J}_{s+}+\bar{J}_{s-}+\bar{J}_s)\right] dv$$

$$- \frac{m_e}{2} K_f \int v^2 \frac{\partial}{\partial v} \left[\ln\Lambda_{es} \ (g_s+h_s+n_s)\right] dv - \left(\frac{\partial W_s}{\partial t}\right)_S \qquad (47)$$

and the ion source is $O(m_e/m_i)$ and is neglected. The ohmic heating term has already been explicitly represented in eqs.(33) and hence is not repeated here. $(\frac{\partial W_s}{\partial t})_S$ represents not only the direct energy absorption from the laser field, but also the energy already contained in the thermal electrons when they are promoted to the suprathermal distribution.

1.6 THE SUPRATHERMAL SOURCE

The source function in this transport model is characterized by the energy distribution of the suprathermal electrons and the fraction of the incident energy absorbed. This description must be obtained from plasma physics and, in particular, from the numerical simulation of the plasma instabilities of various absorption processes. Resonant absorption has been investigated numerically and despite extensive numerical simulations under diverse plasma conditions, it is difficult to extract absorption histories and energy distribution functions.

When a sphere is irradiated by a polarized focused laser beam, absorption conditions are complex. The propagation vector is non-radial over most of the surface, the rays continuously diffracted, and the absorption will be a mixture of collisional, resonant and parametric processes. The situation becomes even more complex for multi-beam illumination. The absorption processes are also dependent on the intensity of the laser light, the self-consistent density profile, the quality of the beam or target surface and the temperature of the plasma; there are uncertainties about the effect of cavities and filamentation on the production of high energy electrons.

Because of the difficulties in describing the absorption physics, a simplified model is used in the transport code. An approximate description of the distribution function of the hot electrons can be obtained from simulation codes[18]; unfortunately this distribution function is usually given for only one set of conditions. One approach is to describe the source of hot electrons by a second isotropic Maxwellian distribution with temperature αT_e where T_e is the thermal electron temperature at the critical density and α varies between 4 and 12 [9,18];

the lower number applies to resonant absorption with very steep profiles and the higher to parametric processes. An isotropic distribution, while not compatible with the directed motion of the electrons calculated in simulation code, approximates the complex behavior of the absorption on the pellet surface.

The number of electrons contained in the high temperature Maxwellian is obtained from the energy absorbed. Fractional absorption usually published for a given case and over the short time of the simulation can vary from 15% to 55% depending on the angle of incidence.[18] In view of the complexity of the illumination pattern on spherical pellets, a realistic absorption fraction would be impossible to obtain. The simplest way to obtain the energy absorbed into the suprathermal fluid is to assume that the part of the total specified absorbed energy which has not already been absorbed through inverse Bremsstrahlung is absorbed anomalously; i.e., the energy "dump" usually employed in hydrodynamic codes is transferred from the thermal to the non-thermal component. Let E_c be the remaining energy to be absorbed at the critical surface and let N_e be the number of electrons raised into the non-thermal distribution; then the energy "absorbed" in that distribution is

$$E_T = E_c + N_e \, C_v \, T_e$$

This energy can be expressed in terms of the distribution function as

$$E_T = \frac{m_e}{2} \int_{-\infty}^{\infty} v^2 f_n(v) dv$$

where

$$f_n(v) = \frac{4v^2 N_e}{\pi^{\frac{1}{2}} (\alpha v_e^2)^{3/2}} e^{-\frac{v^2}{\alpha v_e^2}} \tag{48}$$

The integration yields

$$E_T = \frac{3}{4} m_e N_e \alpha v_e^2$$

and the number of electrons is given by

$$N_e = \frac{4 E_c}{3 m_e \alpha v_e^2 - 4 C_v T_e} \tag{49}$$

Equations 48 and 49 are used as the source function with α left as a free parameter.

Extensive testing and usage of this supathermal package for the SUPER code is being carried out. A complete documentation of the suprathermal code will be available as an internal theory technical note.[19] Theoretical interpretation[20] and design of experiments[21] have been completed through simulations including the effects of suprathermals. By a comparison of theory and experiment, it will be possible to work backwards through the numerical simulations to pin down the free parameters in equations such as (48 and (49). Ultimately this should be a direct guide to an understanding of the suprathermal electron generations processes.

TABLE 1

Two-Fluid Equation - Lab Frame

Continuity Equation

$$\frac{\partial n_e}{\partial t} + \nabla\cdot (n_e u_e) = \left(\frac{\partial n_e}{\partial t}\right)_s$$

$$\frac{\partial n_I}{\partial t} + \nabla\cdot (n_I u_I) = 0$$

MOMENTUM EQUATION

$$\frac{\partial}{\partial t} (m_e n_e \bar{u}_e) + \nabla\cdot m_e n_e \bar{u}_e \bar{u}_e + \nabla\cdot p_e + n_e e\bar{F} = -\bar{K}_{ei} - \bar{K}_{en}$$

$$\frac{\partial}{\partial t} (m_I n_I \bar{u}_I) + \nabla\cdot m_I n_I \bar{u}_i \bar{u}_i + \nabla\cdot p_I - Z n_I e\bar{E} = -\bar{K}_{ie} - \bar{K}_{in}$$

where $\bar{U} = \dfrac{m_e n_e \bar{u}_e + m_I n_I \bar{u}_I}{m_e n_e + m_I n_I}$

Poisson's Equations

$$\nabla\cdot\bar{E} = 4\pi e(Z n_i - n_e - n_n)$$

FIGURE CAPTIONS

FIGURE 1: Velocity distribution function in streaming model

FIGURE 2: Charge separation equilibriation time as a function of thermal electron temperature.

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STREAMING MODEL

UR LLE

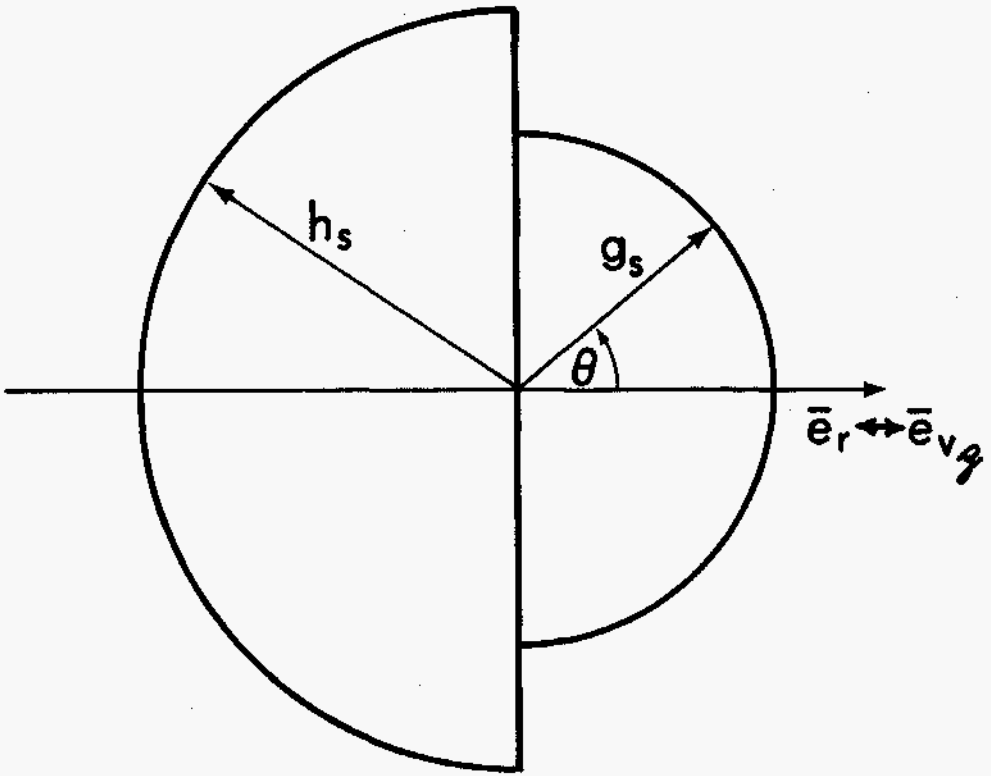

Figure 1

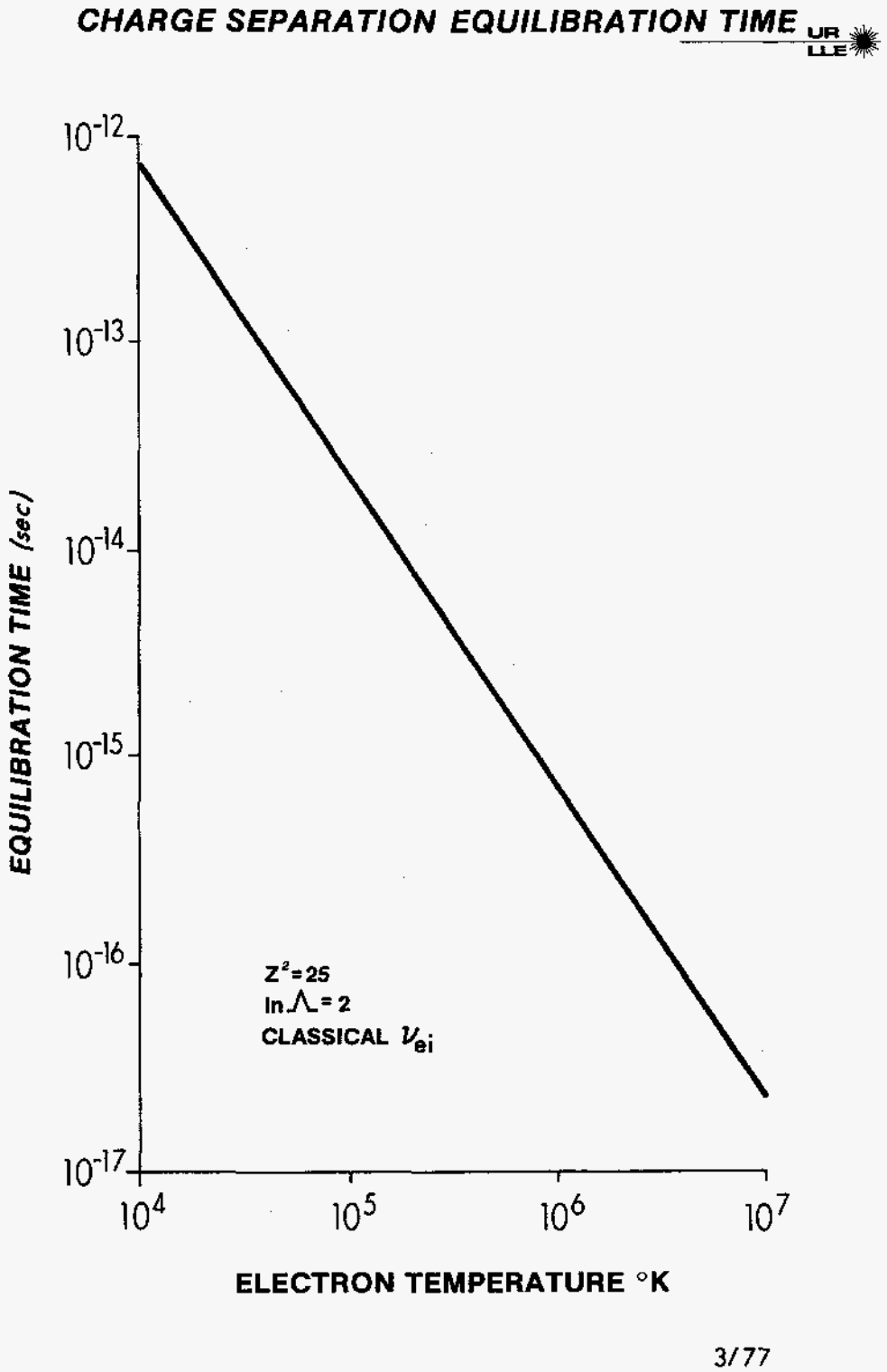
CHARGE SEPARATION EQUILIBRATION TIME
UR
LLE
EQUILIBRATION TIME (sec)
10-12
10-13
10-14
10-15
10-16
10-17
Z²=25
ln Λ = 2
CLASSICAL νei
10⁴
10⁵
10⁶
10⁷
ELECTRON TEMPERATURE °K
3/77

Figure 2