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Numerical optimisation of sandwich and laminated composite structures

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Abstract

The main object of present investigations is numerical optimisation of damping the vibrations and weight of sandwich and laminated composite structures in harmonic oscillations at frequencies close to the natural frequencies of structure. The method of planning of experiments is used for this purpose. Computations are carried out by using a finite element method and two methods of dynamic analysis: the method of complex eigenvalues and method of the potential energy of eigenmodes. The frequency response analysis is also applied. For numerical analysis of examined structures the finite element models of sandwich and laminated composite beams and plates were developed. Energy of dissipation in the viscoelastic layers is taking into account using complex modules of elasticity. As examples the optimal design of sandwich and laminated composite beams and plates for a maximum damping and minimum weight is presented.

1 Introduction

Fibre composite materials and damping coatings are widely used in structural applications requiring high stiffness-to-weight and strength-to-weight ratios, and a high damping. The significance of damping to the dynamic performance of structures is broadly recognised. Passive damping is an essential dynamic parameter for vibration and sound control, fatigue endurance and impact resistance. On a practice, the fibre composite materials are used more often as a laminated structures and the damping coatings comprise of one or two layers, and can be considered as a sandwich structures. Damping in the viscoelastic layers is represented by using the complex stiffness approach which derives from the elastic-viscoelastic correspondence principle. Because of the increased need for high damped and light-weight structures, significant progress has been recently achieved in the optimisation of damping the vibrations and weight of the laminated composite structures [1,2], and sandwich structures [3,4]. However, the direct optimisation methods used now in a practice is ineffective, as they require a large volume of calculations and expenditures of machine time for computation of the dynamic characteristics of structures with damping. Therefore the present investigation is devoted to the optimal design procedure based on the planning of experiments [5]. Some numerical examples for sandwich and laminated composite beams and plates are also examined.

2 Method of the planning of experiments

This engineering approach of optimisation consists of three fundamental stages: the carrying out of numerical calculations by the informative plans of experiments; the approximation of obtained results; the solution of optimisation problem by methods of non-linear programming. In each of these stages it is possible to solve a problem by various methods.

2.1 Numerical calculations by the informative plans of experiments

Let us consider a criterion for elaboration of the plans of experiments which is independent on a mathematical model of the designing object or process. The initial information for elaboration of the plan is number of factors n and number of experiments k. The points of experiments in the domain of factors are distributed as regular as possible. For this reason the following criterion is used

$$\Phi = \sum_{i=1}^{k} \sum_{j=i+1}^{k} \frac{1}{l_{ij}^2} \Rightarrow \min ,$$

where l_{ij} is the distance between the points having numbers i and j ($i \neq j$). Physically it is equal to the minimum of potential energy of repulsive forces for the points with unity mass if the magnitude of these repulsive forces is inversely proportional to the distance between the points. This problem is solved by the program PLANEX. For each number of factors n=*2...15* and number of experiments k=*2...25* it is possible to elaborate a plan of experiments. But it needs much computer time, therefore each plan of experiment is elaborated only once and it can be used for various designing cases. The plan of experiments is characterised by the matrix of plan B_{ij}. When the domain of factors is determined as $x_j \in [x_j^{\min}, x_j^{\max}]$, the points of experiments are calculated by the following expression

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1}\left(x_j^{\max} - x_j^{\min}\right)\left(B_{ij} - 1\right), \quad i = 1,2,\ldots,k, \quad j = 1,2,\ldots n .$$

Then the numerical calculations or physical experiments are carried out in these points.

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2.2 Approximation of results

In this approach, the form of the equation of regression is unknown previously. There are two requirements for the equation of regression: accuracy and reliability. Accuracy is characterised as minimum of standard deviation of the table data from the values given by equation of regression. Increasing the number of terms in the equation of regression it is possible to obtain a complete agreement between the table data and the values given by equation of regression. However, it is necessary to note that prediction at the intervals between the table points can be not so good. For improvement of prediction, it is necessary to decrease of distance between the points of experiments by increasing of the number of experiments or by decreasing of the domain of factors. Reliability of the equation of regression can be characterised by affirmation that standard deviations for the table points and for any other points are approximately the same. Obviously the reliability is greater for a smaller number of terms of the equation of regression.

The equation of regression can be written in the following form

$$y = \sum_{i=1}^{p} A_i f_i(x_j) \, ,$$

where A_i are the coefficients of the equation of regression, $f_i(x_j)$ are the functions from the bank of simple functions $\varphi_1, \varphi_2, \ldots, \varphi_m$ which are assumed as,

$$\varphi_m(x_j) = \prod_{i=1}^{s} x_j^{\alpha_{mi}} \, ,$$

where α_{mi} is a positive or negative integer including zero. Synthesis of the equation from the bank of simple functions is carried out in two stages: selection of perspective functions from the bank and then step by step elimination of the selected functions. The program RESINT is used for elaboration of this mathematical model.

2.3 Solution of optimisation problem

Constrained non-linear optimisation problem can be written in the following form

$$\min F(x); \; H_i(x) \geq 0; \; G_j(x) = 0;$$
$$i = 1,2,\ldots,I; \; j = 1,2,\ldots,J \, ,$$

where I and J are the numbers of inequality and equality constraints. This problem is replaced to the unconstrained minimisation problem in which the constraints are taken into account with the penalty functions. New version of random search method (the program SUPEX) is used for solving of the optimisation problem. The algorithm of program SUPEX is presented on the Figure 1.

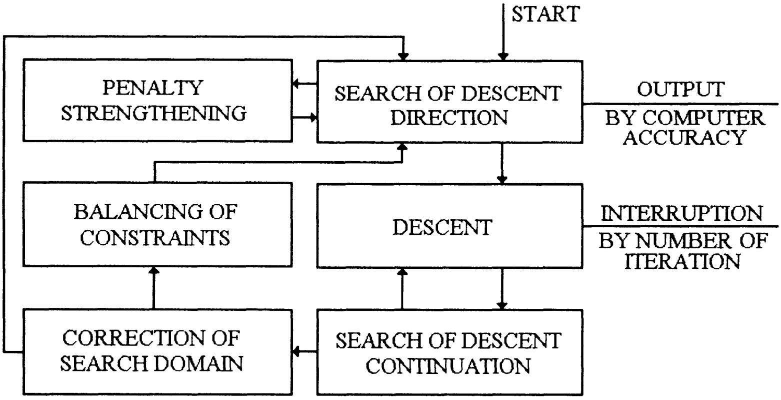

Figure 1: Algorithm of the program SUPEX.

3 Numerical results

Using in a practice of sandwich and laminated composite structures with damping is connected first of all with desire of designers to construct the structures with maximum damping and minimum weight. Attached to that, the structure must be answer to a number of operating requirements on the rigidity, strength, frequency, dimensions, etc. Moreover, designer has a possibility to choose a material of separate layers. Below some numerical examples are examined.

3.1 Damping optimisation of a sandwich beam

For testing, the results of paper [4] were used where the optimisation problem is solved by Vardi algorithm for the equality constrained minimisation. The material properties and geometrical parameters of sandwich cantilever are

$$h_1 = 5\ \mathrm{mm}\ ;\ E_1 = 71000\ \frac{\mathrm{N}}{\mathrm{mm}^2}\ ;\ \upsilon_1 = 0.32\ ;\ \rho_1 = 2.7*10^{-9}\ \frac{\mathrm{N\,s}^2}{mm^4}\ ;\ \eta_1 = 0.01\ ;$$

$$\upsilon_2 = 0.49\ ;\ \rho_2 = 1.3*10^{-9}\ \frac{\mathrm{N\,s}^2}{mm^4}\ ;$$

$$E_3 = 71000\ \frac{\mathrm{N}}{\mathrm{mm}^2}\ ;\ \upsilon_3 = 0.32\ ;\ \rho_1 = 2.7*10^{-9}\ \frac{\mathrm{N\,s}^2}{mm^4}\ ;\ \eta_1 = 0.01\ ;$$

$$b = 12\ \mathrm{mm}\ ;\ L = 180\ \mathrm{mm}\ .$$

Shear module and loss factor of the viscoelastic material are given by following expressions

$$G_2(f) = 1.007*10^{-3} f + 1.386\ \mathrm{(MPa)}\ ;$$

$$\eta_2(f) = 1.608*10^{-4} f + 0.2564\ ,$$

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where f is the frequency in Hz. The design problem $\eta^{(1)}(\mathbf{X}) \Rightarrow \max$ is solved with vector of project $\mathbf{X} = \{X_1, X_2\}$ $(X_1 = h_2; X_2 = h_3)$ and following constraints

$$0.6 \le h_2 / h_1 \; ; \; 0.2 \le h_3 / h_1 \; ; \; 1.3 \le (h_1 + h_2 + h_3) / h_1 \le 2.1 \; ;$$
$$1.05 \le (W_1 + W_2 + W_3) / W_1 \le 1.5 \, ,$$

where W is the weight of layer. On a first stage, the plan with 20 points and two variables is chosen. Then loss factors of sandwich beam [6] are determined in these points by method of the direct calculation of frequency characteristics (on the basis of complex eigenvalues method) [6]. By carrying out calculations, the polynomial equation of the results approximation are constructed

$$\eta^{(1)} = 0.04159 + 0.005386X_1 + 0.02036X_2 + 0.0006246 / X_1 -$$
$$- 0.002445X_1X_2 - 0.005660X_2^2 \, ,$$

where $X_1 = -1.5000 + 0.6667h_2$ (mm) ; $X_2 = -0.1667 + 0.6667h_3$ (mm). Here the coefficient of correlation is 98.6%. Then this mathematical model is used as objective function in the optimal design problem. The results of optimisation and verification in optimal point are given in Table 1. A good agreement is observed between the present optimisation results and results of verification (0.7%), and paper [4] (3.8%).

Two problems connected with obtaining the maximum loss factor of sandwich structure spring up more often in design:

1.) add a constrained viscoelastic layer to the structure;
2.) replace a given homogeneous structure by sandwich or build a sandwich structure.

These two problems of optimisation are examined on the example of sandwich beam in the paper [7].

3.2 Damping optimisation of a laminated composite beam

The design problem $\eta^{(1)}(\mathbf{X}) \Rightarrow \max$ is solved for a cantilever laminated composite beam with the following material properties of layer

$$E_L = 144.90 \text{ GPa} \; ; \; E_T = 9.66 \text{ GPa} \; ; \; G_A = 4.14 \text{ GPa} \; ; \; G_T = 3.45 \text{ GPa} \; ;$$
$$\upsilon_A = 0.3 \; ; \; \rho = 3.598 \text{ kg/m}^3 \; ; \; \eta_{E_L} = 0.1 \; ; \; \eta_{E_T} = \eta_{G_T} = \eta_{G_A} = 0.5 \; ;$$
$$h_i = 0.0005 \text{ m} \; ; \; b = 0.0254 \text{ m} \; ; \; L = 0.381 \text{ m} \, ,$$

and the following configuration $(+\alpha / -\alpha)_{\frac{N}{2}S}$. The optimisation problem is examined with a vector of project $\mathbf{X} = \{X_1, X_2\}$ $(X_1 = N; X_2 = \alpha)$ and the following constraints

$$4 \le N \le 64 \; ; \; 0^0 \le \alpha \le 90^0 \; ; \; \omega^{(1)} \ge 500 \text{ (rad/s)}.$$

Table 1. Optimisation results of the sandwich beam.

Method	h_2, mm	h_3, mm	$\eta^{(1)}$
[4]	3.000	1.055	0.0523
Optimisation	3.000	1.065	0.0543
Verification	3.000	1.065	0.0539

At the first stage, computations are carried out by the method of complex eigenvalues [8] and the finite element of laminated composite beam [9] in 16 points. Then the polynomial equation of the results approximation are constructed for the first eigenfrequency and the corresponding loss factor

$$\omega^{(1)} = -2514 - 975.6X_1 + 1879 / X_2 + 1869 / X_2 X_1 + \\ + 681.4X_2^2 - 104.5 / X_2^4 ,$$

where $X_1 = 0.015625N$; $X_2 = 0.5 + 0.011111\alpha$;

$$\eta^{(1)} = 0.3764 + 0.4413X_2 - 0.1175X_2^2 - 0.2845X_2^3 + \\ + 0.04336X_1X_2^2 + 0.1336X_1X_2^3 - 0.1097X_1^3X_2 ,$$

where $X_1 = -1.1333 + 0.033333N$; $X_2 = -1 + 0.022222\alpha$.
On the next stage, these approximate functions are used as control functions for the optimisation problem. As an initial variant for the optimisation, the composite beam with N=28; α=30° and $\omega^{(1)} = 560.4$ (rad / s); $\eta^{(1)} = 0.204$ has been chosen. The result of the optimisation is

$$N = 62 \; ; \; \alpha = 71^0 \; ; \; \omega^{(1)} = 500.0 \,(\text{rad / s}) \; ; \; \eta^{(1)} = 0.522 .$$

The results of verification in the optimal point are presented in Table 2 by two methods: the method of complex eigenvalues (CEM) [8] and the direct frequency response method (DFRM) [9]. As shown in a literature, the loss factor can be determined by analysing the resonant peaks for a particular mode. It is obtained by the real part of the response spectrum, as explained in Figure 2. The displacements near the first frequency for initial and optimal variants are presented in Figure 3 from which the loss factors for both laminated composite beams can be easily determined. From Table 2 it is clearly seen that the agreement between results obtained by the two methods is very good.

Table 2. Verification of the optimisation problem.

Variant	N	α^o	$\eta^{(1)}$ CEM	$\eta^{(1)}$ DFRM
Initial	28	30	0.204	0.196
Optimal	62	71	0.506	0.508

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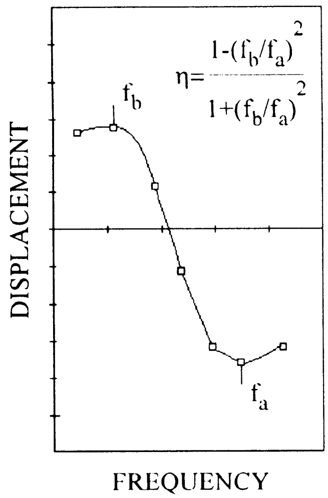

Figure 2: Frequency response.

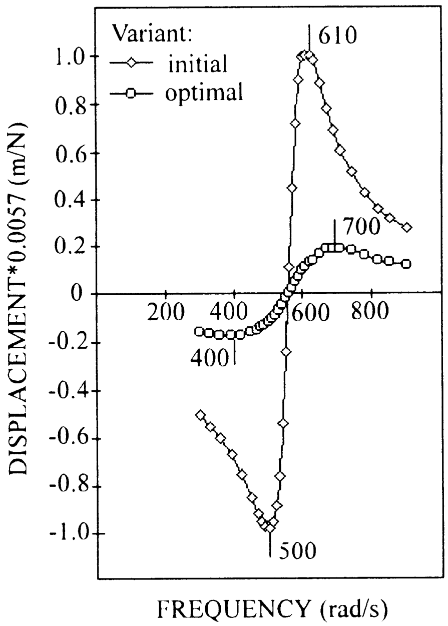

Figure 3: Frequency response of a laminated composite beam near the first eigenfrequency.

3.3 Weight optimisation of a sandwich plate

Numerical example of the sandwich plate optimisation for a minimum mass with the constraints on the thickness of layers, the bending stiffness and the first eigenfrequency is examined in the paper [10].

3.4 Weight optimisation of a laminated composite plate

The problem of mass optimisation on the example of a laminated composite square plate from the glass fibre-reinforced plastic with the constraints on the ply angle, the number of layers, the first eigenfrequency and corresponding specific damping capacity is presented in the paper [7].

4 Conclusion

The carrying out calculations show that proposed approach with sufficient accuracy for a practice can be applied to the optimisation of sandwich and laminated composite structures. Besides, it is necessary to note that more responsible stages are the problem positing and mathematical models composition. The numerical multiobject optimisation in this relation only reflects the errors of previous stages and its role as errors source is insignificant.

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