

NUMERICAL SIMULATION OF SAND WAVE EVOLUTION IN SHALLOW SHELF SEAS

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Abstract: Sand waves form a prominent regular pattern in the offshore seabed of sandy shallow seas. A 2DV morphological numerical model describing the behaviour of these sand waves is under development. The first goal is to make the new numerical model reproduce the results obtained with a linear stability analysis. We start with a constant current after which the behaviour of sand waves is investigated in a tidal environment. Furthermore, calculations using Delft3D have been made to test the general setup of the new numerical model.

INTRODUCTION

Large parts of shallow seas, such as the North Sea (Fig. 1), are covered with bed features having various spatial scales. Sand waves form a prominent bed pattern with a crest spacing of about 500 m. Usually sand waves are observed at a water depth in the order of 30 m and their heights can reach up to several metres. The crests are often assumed to be oriented perpendicular to the principal current (Johnson *et al.*, 1981; Langhorne, 1981). Based on a theoretical analysis, Hulscher [1996] showed that sand wave crests may deviate up to 10° anti-clockwise perpendicular from the principal direction of the current.

Observations indicate that these sand waves are dynamic (Allen, 1980; Lanckneus *et al.*, 1991; Maren, 1998) and can migrate with speeds of up to several metres per year. Knaapen *et al.* [2000] showed that if we dredge the top of a sand wave, the bed form is

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able to grow back in only a matter of years. Insight into the behaviour of these sand waves is therefore crucial to enable cost-effective management practices.

An analysis of the sand market [Peters, 2000] showed that a deficit of sand is expected in the future in the Netherlands. The crests of sand waves are assumed to have the preferable composition for use in concrete. Furthermore the behaviour of sand waves plays an important role during the selection of areas for large-scale sand extraction pits and their design. It is necessary to know whether or not sand waves can grow back after excavation. The presence of sand waves changes the hydrodynamics and therefore the recolonization possibilities of benthic fauna. Furthermore, it needs to be known whether or not the behaviour of sand waves changes in the area surrounding a pit or if new sand waves can appear develop [Stolk, 2000].

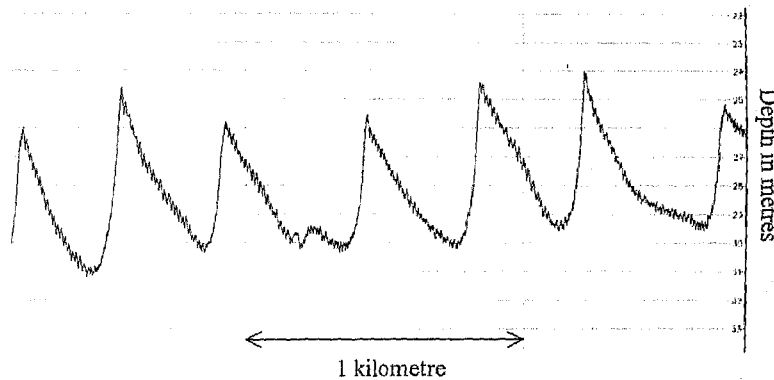


Fig. 1. Cross-section of sand waves based on bathymetry measurements made in the North Sea (Courtesy Clyde Petroleum Exploratie; Holland Offshore Consulting; State Supervision of Mines, 2000)

Huthnance [1982] was the first to look at a system consisting of depth-averaged tidal flow and an erodible seabed. Within this framework, one can investigate whether certain regular patterns develop as free instabilities of the system. Unstable modes comparable to tidal sandbanks were found, whereas smaller modes corresponding to sand waves were not initiated. This work was extended by Hulscher [1996] using a model allowing for vertical circulations and found formation of sand waves based on a horizontally averaged symmetrical tidal motion (see also Gerkema [2000] and Komarova & Hulscher [2000]). The work showed that net convergence of sand can occur at the top of the sand waves over an entire tidal cycle (see Fig. 2). Németh *et al.* [2000] extended the previous work by including a non-symmetric basic flow inducing sand wave migration.

A consequence of linear stability analyses is that their validity is limited to small-amplitude sand waves. But this is far from the final aim, i.e. understanding the entire evolutionary process of sand wave formation. Komarova and Newell [2000] extended the linear analysis by Komarova & Hulscher [2000] into the weakly non-linear regime

for investigation of the behaviour of finite-amplitude idealised sand waves. Migration is not described here. Fredsøe and Deigaard [1992] describe the behaviour of finite-amplitude dunes under a steady current. They assume the time-dependence of the flow to be negligible when modelling sand waves in a tidal environment. Johns *et al.* [1990] discussed the finite amplitude behaviour of sand wave like bedforms under a constant current.

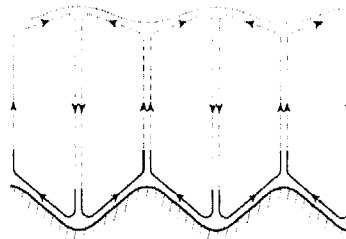


Fig. 2. Residual current directed upwards towards the crest over a tidal cycle [Hulscher, 1996]

Numerical methods can form a tool to overcome these limitations and enable the study of the non-linear behaviour of these bed forms. This would enable the description of the *entire* evolutionary process of sand waves. It gives a clue on the most important mechanisms, determining (the growth and) stabilisation of sand waves. Furthermore, migration and the impact of different shapes than sinusoidal on sand wave behaviour can be investigated.

The first important question to be answered here is whether the formation process of small amplitude sand waves can be reproduced or cannot and verified with a numerical model. The results of a linear stability analysis [Németh *et al.*, 2000], are used for validation of the numerical model. Subsequently the amplitudes will be allowed to become finite. The general goal of this study is therefore to create a numerical model describing the intermediate term behaviour of fully developed sand waves. This fully non-linear model will be validated against sand wave data from in the North Sea and the Bisanseto Sea [Katoh *et al.*, 1998].

Within this paper we will focus on the first step, which is to reproduce the results obtained with a linear stability analysis for small amplitude sand waves with a numerical model. Firstly, we present the mathematical formulation of the sand wave model. It is based on the two-dimensional vertical shallow water equations combined with a simple sediment transport equation, describing bed load transport. The scaling for studying sand waves is discussed in detail in Németh *et al.* [2000]. Next, the linear stability analysis is summarised and the main results for comparison are presented. Subsequently the setup for the numerical model is tested with the help of the software package Delft3D. Afterwards the numerical approach pursued in this work is discussed shortly. In the final section we present the discussion and conclusions.

PHYSICAL MODEL

In the present study we start with simulating a non-tidal situation, in which a constant current dominates. Herein we focus on simulating the residual vertical circulation cells which make sand waves evolve as free instabilities of the system (See Fig. 2). In this model the constant current can have two different physical origins: generated by a wind stress applied at the sea surface or forced by a pressure gradient (See Eq. 10 & 11). The vertical flow structure in these cases differs, resulting in differences in bed shear stress and sediment transport. Subsequently we investigate the simulation of the initial behaviour of sand waves in a tidal regime under the influence of symmetrical tidal movement (M_2).

The hydrodynamic equations

The Coriolis force only slightly affects sand waves. The behaviour of sand waves can therefore be described with the help of the two-dimensional vertical (2DV) shallow water equations, after making the shallow water approximation. Furthermore, neglecting the horizontal viscosity gives us (See also Fig. 3):

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -g \frac{\partial \zeta}{\partial x} + \frac{\partial}{\partial z} \left(A_v \frac{\partial u}{\partial z} \right), \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0. \quad (2)$$

The symbols g and A_v indicate respectively the acceleration due to gravity and the vertical eddy viscosity. Time is represented by t . The velocities in the x - and z - directions are u , respectively w . The water level is denoted by ζ and H represents the mean water depth. The level of the seabed is represented by h .

Integrating the continuity equation over the depth and using a kinematic condition at the water surface leads to the free surface equation. With this equation the vertical velocity can be determined:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} \left(\int_h^{\zeta+H} u dz \right) = 0. \quad (3)$$

Initial and boundary conditions

The boundary conditions and the free surface ($z = 0$) are prescribed by:

$$\frac{\partial u}{\partial z} = \hat{\tau}_w; \quad w = 0, \quad (4)$$

in which τ_w describes the wind induced shear stress at the sea surface. The horizontal flow components at the bottom are described with the help of a partial slip condition (S is the resistance parameter controlling the resistance at the seabed). This description is

equal to [Németh *et al.*, 2000] making it possible to compare the results directly. The vertical velocity component at the bed ($z = -H+h$) is described by the kinematic condition:

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = w \text{ and } A_v \frac{\partial u}{\partial z} = Su. \quad (5)$$

Inflow/Outflow

At the inflow boundary a velocity profile in the vertical is prescribed. The outflow boundary is physically an open boundary condition. The numerical model however needs a prescription of the boundary condition. Therefore an estimate of the water level is supplied to the model at the outflow boundary. In the case of tidal movement the in- and outflow boundaries change during the tidal cycle (See Fig.3).

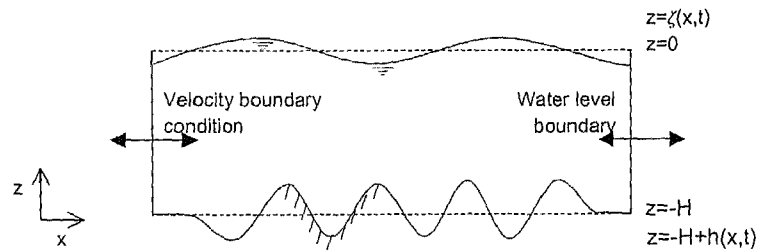


Fig. 3. Illustration framework model

Sediment transport

The sediment transport model only describes bed load transport. This mode of transport is assumed to be dominant in offshore tidal regimes. As the flow velocities in the vertical direction are being calculated explicitly, bed load transport can be modelled here as a direct function of the bottom shear stress. The following general bed load formula is used [Komarova and Hulscher, 2000]:

$$S_b = \alpha |\tau_b|^b \left[\tau_b - \lambda \frac{\partial h}{\partial x} \right], \quad (6)$$

with τ_b the bottom shear stress defined by:

$$\tau_b = A_v \left(\frac{\partial u}{\partial z} \right) \Big|_{z=\text{bottom}}. \quad (7)$$

S_b is the sediment transport. The power of transport, represented by b is set at 1/2. The proportionality constant α , set at a value of 0.3 s m^{-2} , is a parameter and can be computed from Van Rijn. The scale factor for the bed slope mechanism is λ . It takes into account that sand is transported more easily downward than upward and incorporates the effects

of the critical shear stress on the slope. The default value is set at 8 in this study conform the analytical analysis [Németh *et al.*, 2000].

The net inflow of sediment is assumed to be zero. This results in the following sediment balance, which couples the flow model (Eq. 1,2,3) with the sediment transport model (Eq. 6):

$$\frac{\partial h}{\partial t} + \frac{\partial S_b}{\partial x} = 0. \quad (8)$$

RESULTS LINEAR STABILITY ANALYSIS

The results of the numerical model are validated with the results of a linear stability analysis [Németh *et al.*, 2000]. This analysis gave insight in the initial evolution of sand waves. It furthermore showed that a steady current inducing an asymmetry in the basic state causes migration of sand waves (See Eq. 9). The order of magnitude for the migration rates and wavelengths found are 5-10 m/a and 600 m respectively. These values are in agreement with values reported in the literature.

In the stability analysis the velocity in the horizontal direction consists of a periodic part representing tidal motion (M_2) and a steady current ($u_r(z)$). The tidal motion has a depth-averaged amplitude of 1 m s⁻¹ [Hulscher, 1996]. The basic state is formulated as follows:

$$u_0 = \beta u_r(z) + (1 - \beta) \{ u_s(z) \sin t + u_c(z) \cos t \}, \quad (9)$$

in which β enables us to vary the ratio of the steady and periodic part, in accordance with the used scaling method. Two possible types of steady flow components were investigated. These are (I) a wind driven current and (II) a current induced by a pressure gradient. Their vertical structures are deduced from the basic state [Németh *et al.*, 2000]:

$$\text{I: } u_r = \bar{\tau}_w \left(1 + \frac{E_v}{\hat{S}} + z \right), \quad (10)$$

$$\text{II: } u_r = P \left(\frac{1}{2} z^2 - \frac{E_v}{\hat{S}} - \frac{1}{2} \right) \text{ with } P = \frac{L}{10\delta E_v} \zeta, \quad (11)$$

in which the length of the tidal wave is described by L . The dimensionless resistance parameter is defined by \hat{S} , E_v can be seen as a measure for the influence of the viscosity on the water movement in the water column and δ is the Stokes layer thickness:

$$\hat{S} = \frac{S}{\sigma\delta}, \quad E_v = \frac{A_v}{\sigma\delta^2} \text{ and } \delta = \sqrt{\frac{2A_v}{\sigma}}, \quad (12)$$

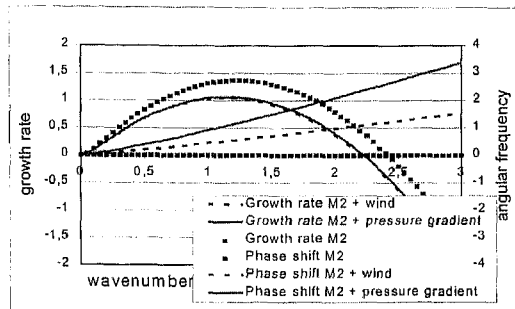


Fig. 4. Growth rate (curved lines corresponding to the left axis) and angular frequencies (almost straight lines corresponding to the right axis) as a function of the wave number ($\hat{S} = 0.005$).

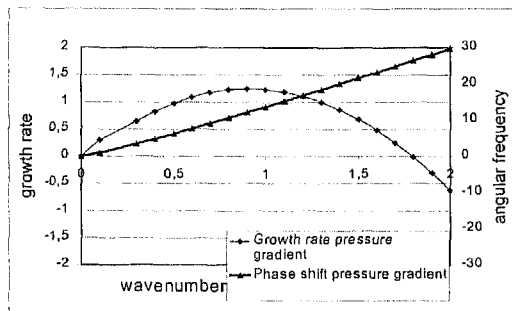


Fig. 5. Growth rate and angular frequencies as a function of the wave number ($\hat{S} = 0.01$).

In Fig. 4 the results are shown for a tidal current with a depth averaged amplitude of 1 m s^{-1} ($M_2, \beta = 0$). The morphological time scale (T_m) is about 6 years. As was found from previous research (Gerkema, 2000; Hulscher, 1996; Komarova & Hulscher, 2000; Blondeaux *et al.*, 1999) positive growth rates appear for a range of wave numbers k (for the used Fourier transformation see Németh *et al.* [2000]). The dimensional wavelength follows from:

$$L_{\text{sandwave}} = \frac{20\pi}{k} \sqrt{\frac{2A_v}{\sigma}}, \quad (13)$$

with A_v the eddy viscosity of $0.01 \text{ m}^2 \text{ s}^{-1}$ and σ the tidal frequency with a value of $1.4 \cdot 10^{-4} \text{ s}^{-1}$. The wavelength having the largest growth rate is the mode we expect to find in nature. In this case the fastest growing mode has a wave number $k = 1.25$ (See Fig. 4) corresponding to a wavelength of about 600 m. In this case, no migration is found.

Sand waves are found to be migrating due to an asymmetry in the water motion. The magnitude of this migration depends on the nature of the steady part (I or II) and on the

magnitude of the asymmetry in the water movement. Fig. 4 also shows the results for a depth-averaged residual current of 0.1 m s^{-1} (M_0) superimposed on a tidal current of 0.9 m s^{-1} (M_2) ($\beta = 0.1$). The fastest growing modes have in both cases wavelengths in the order of 700 m. For the fastest growing modes the migration rates for case I become 6 and for case II 15 m/a. This dimensional form of the migration rate can be found as follows:

$$V_{\text{sandwave}} = \frac{10\omega_i}{2\pi T_m} \sqrt{\frac{2A_v}{\sigma}}, \quad (14)$$

with ω_i the angular frequency of the bed forms. If we only apply a constant current with a magnitude of 1 m s^{-1} ($M_0, \beta = 1$), we find, if we increase the resistance parameter, a similar curve for the growth rate (Fig. 5). But the migration rates are a factor 10 larger than those found in Fig. 4.

PRELIMINARY NUMERICAL TESTS

Before we start the construction of the numerical sand wave model we test the setup for the description of the water movement with the "flow"-module of the software package Delft3D (WL|Delft Hydraulics). For this test an average depth of 20 m was chosen. The total length of the grid is 4400 m consisting of 6 sand waves with a wavelength of 600 m each. The amplitude of the sand waves is gradually increased to 2 metres over one wavelength near the boundaries (See Fig. 3). 20 grid points were used per wavelength. The water movement in the vertical direction is described with a sigma co-ordinate transformation [WL|Delft Hydraulics, 1999] consisting of 15 layers. The density is gradually increased nearer to the seabed.

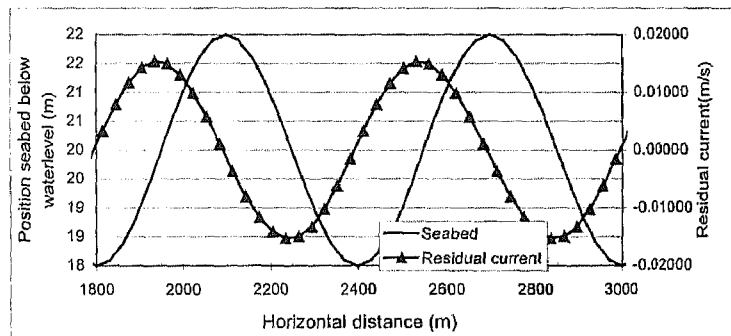


Fig. 6. Residual current near the seabed supplied with sand waves

We compared two runs. One consisting of a flat bed and one with the sand waves imprinted on the seabed. Looking at the averages over a tidal period of these two runs we could determine the net influence of the sand waves on the water movement. Fig. 6 shows the net residual current near the seabed caused by the sand waves. A positive velocity stands for a current oriented in the positive horizontal direction. The residual

current is therefore directed towards the crests of the sand waves. The results of the water movement therefore coincide qualitatively with respect to the linear stability analysis.

NUMERICAL SAND WAVE MODEL

The solutions of the water movement are approximated by a pseudospectral (collocation) method. This method is chosen because of its high accuracy and the fact that it is virtually free of dissipative and dispersive errors. These aspects are very important since we expect that the solution has a very small time-independent flow component on top of the much larger oscillatory tides. The small residual part is crucial for the long-term bed development (see Fig. 2). Furthermore the relative coarse grids allow time and memory efficient calculations [Fornberg, 1996].

The horizontal as well as the vertical direction of the water movement is non-periodic. In both directions a staggered Chebyshev collocation method is used [Canuto *et al.*, 1988; Duncan, 1998]. In the vertical this makes it possible to make an accurate approximation of the vertical flow distribution without using a large number of vertical grid points. The usage of a Gauss-Labatto grid allows us to have efficient grid refinement near the seabed. In the horizontal direction periodic boundary conditions are not an option since the length of the tidal wave is almost three orders of magnitude larger than the wavelength of a sand wave. The description in the horizontal is therefore chosen to be non-periodic. The description used is analogous to the description used in the vertical. A second-order temporal discretisation is used.

DISCUSSION & CONCLUSIONS

The results encompass a numerical model that is able to describe the behaviour of small amplitude sand waves. The influence of a steady current and tidal movement will be investigated. The model will be validated using a linear stability analysis. This work is a necessary step in extending the model for the investigation of the intermediate term (non-linear) behaviour. The fully non-linear model helps to obtain insight in the processes, which are instrumental for sand wave dynamics.

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