Memoirs of the Faculty of Engineering, Okayama University, Vol. 44, pp. 7-12, January 2010

Numerical Simulation of Acoustic Waves in a Two-Dimensional Phononic Crystal: Negative Refraction

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(Received December 22, 2009)

The lens effect of acoustic waves in a two-dimensional (2D) phononic crystal is studied by numerical simulation based on the finite-difference time-domain (FDTD) method. We calculate the phonon band structure of 2D phononic crystals, consisting of metal cylinders placed periodically in water. Lens effect is observed by the negative refraction of acoustic waves, which results in refocusing of the waves at the point outside the crystal. To increase the focal intensity, we introduce a 2D phononic crystal shield with a different composition of material, which returns the incident waves back to the lens via the perfect reflection. Also, the dependence on filling fraction of metal in the crystal is studied.

1. INTRODUCTION

The periodic structure consisting of a media that has different permittivity and permeability is called photonic crystal. Having the unique characteristics, such as the photonic band gaps, this artificial crystal can control the electromagnetic wave propagation. Therefore, photonic crystal is expected to play important roles in various applications. Recently, the research focus in this direction has been extended to the study of acoustic waves in periodic composites, called phononic crystals[1]. Phononic crystal is also expected to poses the complete band gaps in which sound and vibration are forbidden to propagate in all directions. Therefore, phononic crystal can be used to design various acoustic devices, such as the acoustic filters and waveguide [2].

Recently, new phenomenon called negative refraction is suggested as another unique feature in photonic crystal [3]. By the multiple scatterings, waves refract at the crystal surface as if crystal has negative refractive index. Analogous to this in the photonic crystal, negative refraction is also investigated in phononic crystal [4,5,6]. Negative refraction results in "lens effect" without the convex shape, in other words, phononic crystal can be a flat acoustic lens. This

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characteristic is promising for novel acoustic devices, acoustic sensor and power supply to heart pacer. There are, however, some issues to resolve before applying the material to these applications, such as the high reflectivity at the surface of phononic crystal. Purpose of the present study is to approach to the issue by increasing the focal intensity. Specifically, we focus two approaches. One of approaches is covering the aoustic source with a phononic crystal shield, and the other is varying the filling fraction of the crystal.

This paper is organized as follow: Chapter 2 explains mechanism of negative refraction. Methodologies used in the present study are described in Chap. 3. Chapter 4 shows and discusses the results of the simulations. Finally, summary is given in Chap.5

2. NEGATIVE REFRACTION

It is called negative refraction that waves refract at crystal surface as if the crystal has negative refractive index. A diagram of negative refraction is shown in Fig.1. Negative refraction can be caused if the phonon band structure has a convex peak at the M point. For example, we suppose that the band structure of certain phononic crystal consisting metal cylinder in liquid and that of liquid look like Fig.2. A purple line represents the band structure of phononic crystal, and a black line is that of liquid base. The yellow region is the frequency range where negative refraction can be observed. The relationship between incident and refracted k-vector on the equivalent frequency surface (EFS) is depicted in Fig.3. The thin and thick arrows represent the wave vector and group velocity, respectively. The group velocity is determined by the frequency gradient at the k point ($v_g = \partial\omega/\partial k$). When the incident waves propagate in liquid base, both the group velocity and the ncident waves are parallel to the wave vector. As the waves enter the region of phononic crystal, the wave vector of refracted wave is changed with conserving the vector component parallel to the interface. On the EFS of phononic crystal, direction of the group velocity is changed to the direction pointing to the M point in the k space, leading to the negative refraction.

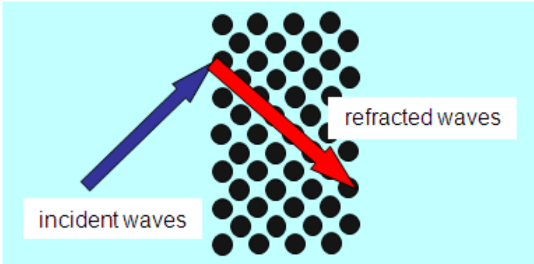

Fig.1 Schematics of negative refraction. Blue and red thick arrow indicate incident and refracted waves, respectively.

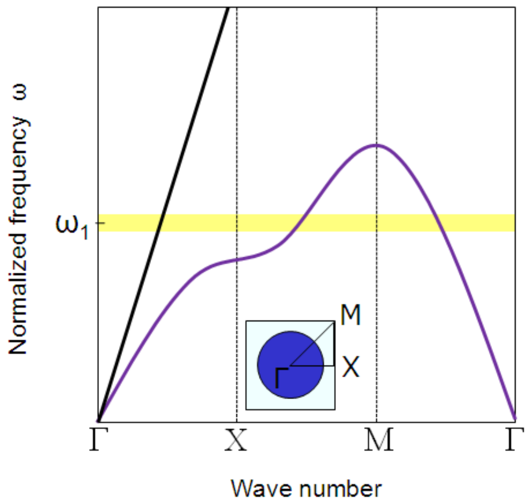

Fig.2 The band structure of the phononic crystal (purple line) and liquid base (black line). The yellow region is the frequency range where negative refraction can be observed. Inset shows a square lattice and its Brillouin zone.

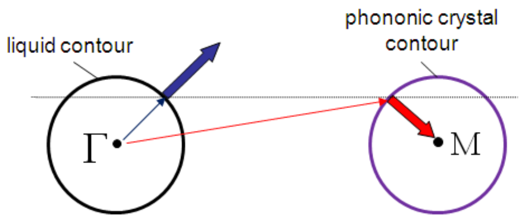

Fig.3 EFS in k space of liquid (black contour) and phononic crystal (purple contour) in ω_1. The thin and thick arrows indicate the wave vector and the group velocity, respectively.

Appling this principle of negative refraction, we demonstrate "lens effect" illustrated in Fig.4. The phononic crystal slab is placed in liquid with the surface normal to the ΓM direction, and a line source is placed at the left side of the slab. Acoustic waves are emitted from the line source and propagate into the phononic crystal slab, which has negative refractive index. The transmitted waves are then refocused at the right side of slab.

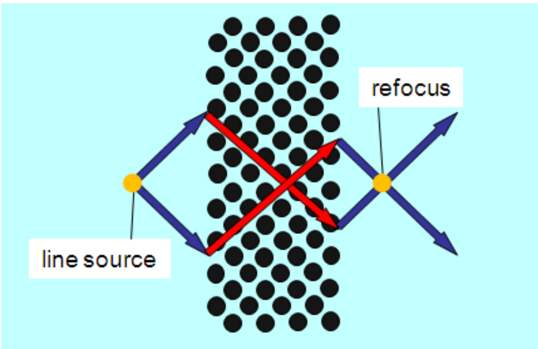

Fig.4 Schematics for "lens effect" by negative refraction.

3. METHODOLOGY

3.1 Basic Equation

We consider a 2D system consisting of infinitely long cylinders parallel to the z axis and the material parameters are independent on the coordinate z. The propagation of elastic wave is assumed to be only in x-y plane. The elastic wave equation in such a system is written as

$$\rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \qquad (1)$$

$$\rho \frac{\partial^2 u_y}{\partial t^2} = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \qquad (2)$$

where u_x, u_y are the x- or y- component of the displacement and ρ is the mass density. The stress tensor $\tau_{i,j}(i,j = x,y)$is represented as

$$\tau_{xx} = C_{11}\frac{\partial u_x}{\partial x} + C_{12}\frac{\partial u_y}{\partial y} \qquad (3)$$

$$\tau_{xy} = C_{44}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) \qquad (4)$$

$$\tau_{yy} = C_{12}\frac{\partial u_x}{\partial x} + C_{11}\frac{\partial u_y}{\partial y} \qquad (5)$$

where C_{11}, C_{12} and C_{44} are elastic constants which depend on the position. In an isotropic system, these are related to the longitudinal and transverse speeds of wave c_l and c_t as $C_{11} = \rho c_l^2$, $C_{44} = \rho c_t^2$ and $C_{12} = C_{11} - 2C_{44}$.

3.2 FDTD Method

The FDTD method, which is based on a discrete algorithm for solving a differential equation in spatial and time domain, is a powerful tool to analyze the wave transmission. Using this method, we can observe displacement field as time advances.

The FDTD method is also able to calculate the dispersion relations of phonons [7,8]. Based on the Bloch theorem, owing to the periodicity, the displacement and the stress are written as

$$u(\boldsymbol{r},t) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}U(\boldsymbol{r},t) \qquad (6)$$

$$\tau(\boldsymbol{r},t) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}T(\boldsymbol{r},t) \qquad (7)$$

where $\boldsymbol{r} = (x,y)$ is a position in x-y plane, $\boldsymbol{k} = (k_x, k_y)$ is a Bloch wave vector, and $U(\boldsymbol{r},t)$ and $T(\boldsymbol{r},t)$ are periodic functions which satisfy $U(\boldsymbol{r}+\boldsymbol{a},t) = U(\boldsymbol{r},t)$ and $T(\boldsymbol{r}+\boldsymbol{a},t) = T(\boldsymbol{r},t)$ with $\boldsymbol{a}$ being a lattice translation vector. After the stationary state is reached, temporal spectra of the displacements are Fourier-transformed to frequency domain. We then identify the eigenfrequency at a given wave vector by finding the peaks in the frequency spectra.

4. RESULTS

4.1 Band Structure

The band structure calculated with the FDTD method is shown in Fig.5. The material parameters, the mass density and sound speed, are as follow: $\rho = 19.3\ \mathrm{g/cm^3}$, $c_l = 5.09\ \mathrm{km/s}$, $c_t = 2.8\ \mathrm{km/s}$ for tungsten; $\rho = 10.5\ \mathrm{g/cm^3}$, $c_l = 3.70\ \mathrm{km/s}$, $c_t = 1.7\ \mathrm{km/s}$ for silver; $\rho = 1.0\ \mathrm{g/cm^3}$, $c_l = 1.49\ \mathrm{km/s}$ for water, respectively.

Figure 5 (a) plots the band structure of phononic crystals consisting of silver and tungsten cylinders in water, which have different filling fraction f. (silver: f=0.7, tungsten: f=0.5). The colored central region is the frequency range which can reflect incident waves. Frequency values are in units of $2\pi c/a$, c and a are longitudinal sound speed of tungsten and lattice constant, respectively. Inset shows a square lattice and its Brillouin zone.

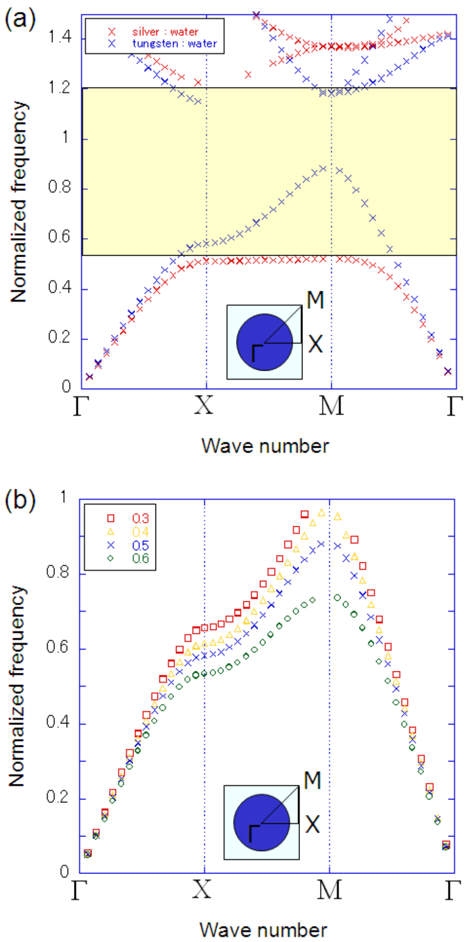

Fig.5 Band structures of 2D phononic crystals; (a) phononic crystals consisting of silver (red cross) and tungsten (blue cross) cylinders in water, respectively. The colored central region is the frequency range in which incident waves are reflected; (b) phononic crystals with different filling fractions. Inset shows a square lattice and its Brillouin zone.

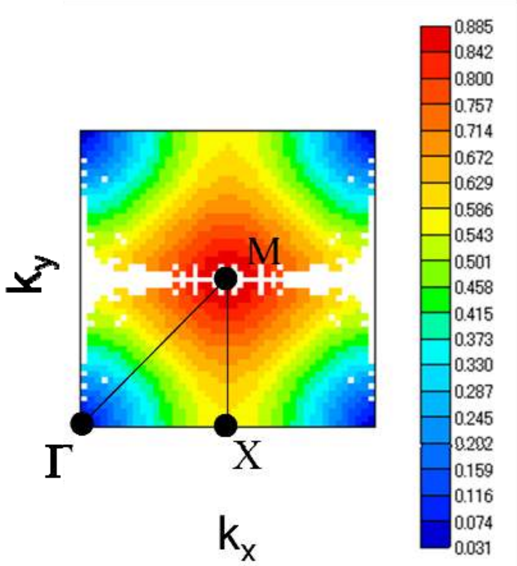

Fig.6 EFS of phononic crystal consisting of tungsten cylinders in water. (filling fraction f=0.5)

Figure 5 (b) shows the band structure for different filling fraction. As the filling fraction is lowered, the first band increases.

Next, we analyze EFS of the phononic crystal consisting of tungsten cylinders in water with f=0.5, as shown in Fig.6.

4.2 Effect of Shielding

We show the result of propagating acoustic waves via the FDTD simulations. Figure 7 depicts time average of normalized intensity distributions of displacement field without shield. We can see that the acoustic waves are focused at the right side of the phononic crystal slab in water. The normalized incident acoustic wave frequency is ω=0.66, at which we expected the negative refraction from the band-structure analysis.

Next, we show similarly the result with a shield in Fig.8. We can see the refocused point at the right side with higher intensity than the result without shield. The normalized intensity of the refocused point increases 2.2 times compared with the normalized intensity without the shield.

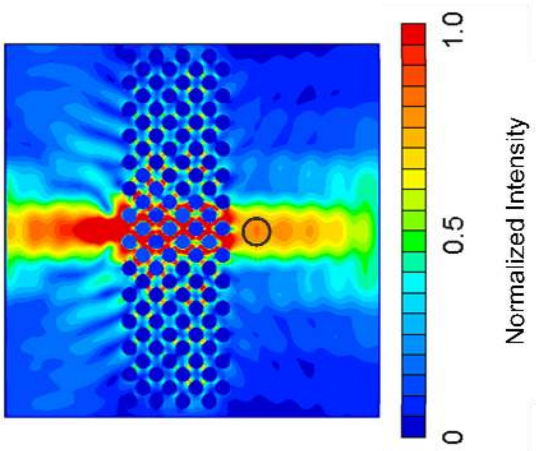

Fig.7 Time average of normalized intensity distributions of displacement field without shield. Normalized incident frequency is ω=0.66.

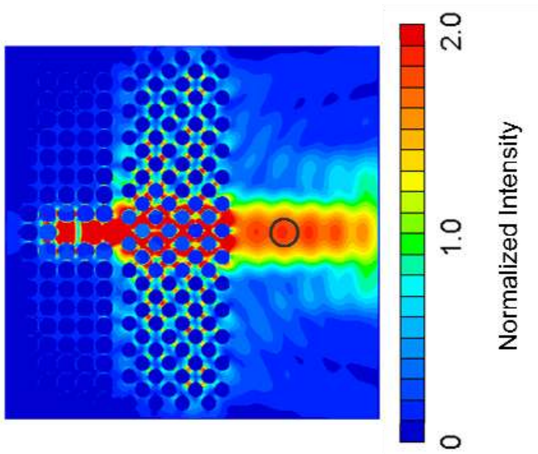

Fig.8 Time average of normalized intensity distributions of displacement field with shield. Normalized incident frequency is ω=0.66.

4.3 Dependence on Filling Fraction

By varying filling fraction, the band structure of phononic crystal can be changed significantly, as indicated in Fig.5 (b). Setting filling fraction to several values between 0.3 and 0.6, we perform the FDTD calculation on a similar model to that in Fig.7. The results are shown in Fig.9. These figures clearly reveal that the focal intensity is dependent strongly on the filling fraction, which has been expected from the EFS contour analysis.

We show the relationship between the refocused intensity and the filling fraction in Fig.10. The results shown in Fig.10 shows that refocused intensity has maximum value around f=0.36. Vertical axis is focal intensity normalized by the intensity at f=0.36.

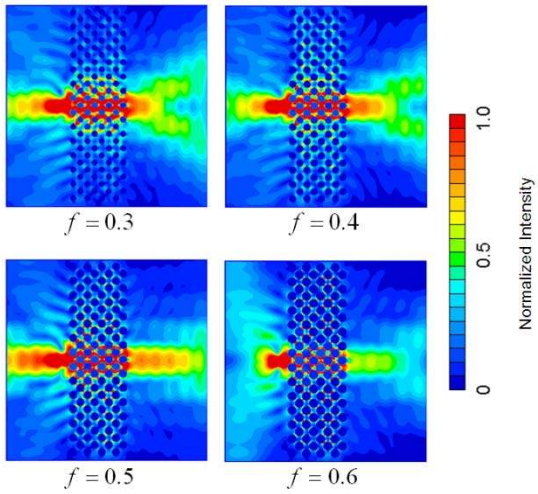

Fig.9 Time average of normalized intensity distributions of displacement field for different filling fractions, 0.3, 0.4, 0.5, 0.6.

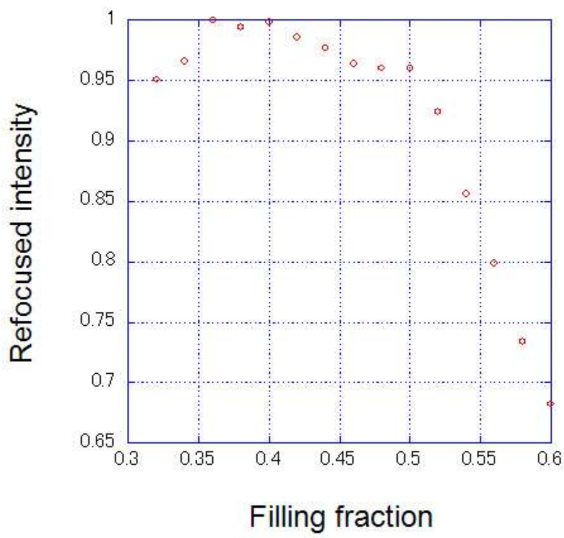

Fig.10 Relationship between refocused intensity and filling fraction. Vertical axis is the focal intensity normalized by the intensity at f=0.36.

5. SUMMARY

We have presented that highly efficient "lens effect" of 2D phononic crystal can be obtained by optimizing its transmission properties by using the FDTD method. We proposed two approaches. The first approach is to cover the acoustic source with the phononic crystal shield, which achieves strong focal intensity. The second approach is to change the filling fraction of metallic cylinder in the phononic crystal. We found that an appropriate value of the filling fraction to maximize the refocused intensity is around 36%.

ACKNOWLEDGEMENTS

This work was supported by Grant-in-Aid for Scientific Research on Priority Areas "Nano Materials Science for Atomic Scale Modification 474" from MEXT of Japan.

REFERENCES

[1] M. S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. Lett. **71** (1993) 2022-2025

[2] Jia-Hong Sun and Tsung-Tsong Wu, Phys. Rev. B **76** (2007) 104304-104311

[3] Chiyan Luo, Steven G. Johnson, and J. D. Joannopoulos, Phys. Rev. B **65** (2002) 201104-201107

[4] Liang Feng, Xiao-Ping Liu, Yan-Bin Chen, Zhi-Peng Huang, Yi-Wei Mao, Yan-Feng Chen, Jian Zi, and Yong-Yuan Zhu, Phys. Rev. B **72** (2005) 033108-033111

[5] Jing Li, Zhengyou Liu, and Chunyin Qiu, Phys. Rev. B **71** (2006) 054302-054306

[6] Liang-Yu Wu, Wen-Pei Yang, and Lien-Wen Chen, Phys. Lett. A **372** (2008) 2701-2705

[7] Y. Tanaka, Y. Tomoyasu, and S. Tamura, Phys. Rev. B **62** (2000) 7387-7392

[8] Yongjun Cao, Zhilin Hou, and Youyan Liu, Solid State Communication **132** (2004) 539-543