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Numerical simulation of dispersive shallow water waves with Rosenau-KdV equation

Research Article

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Abstract: In this paper, the Rosenau-KdV equation that is one of the significant equations in physics was discussed. The collocation finite element method is implemented to find the numerical simulation of the dispersive shallow water waves with Rosenau-KdV equation using the quintic B-spline basis functions. A linear stability analysis based on von Neumann approximation theory of the numerical scheme is investigated. To demonstrate the precise and efficiency of the proposed method, the motion of solitary wave is studied by calculating the error norms L_2 and L_∞. The invariants I_1, I_2 and their relative changes have been computed to define the conservation properties of the simulation. As a result, the obtained results are found better than some recent results.

MSC: 35Q51 • 35Q53 • 35Q58

Keywords: Rosenau-KdV equation • Solitons • Shallow water • Collocation

1. Introduction

Many physical phenomena can be defined by the Korteweg–de Vries (KdV) equation which was discovered by Korteweg and de Vries as the following [1–8]:

$$U_t + UU_x + U_{xxx} = 0. \tag{1}$$

Eq. (1) plays a important role in the study of nonlinear dispersive shallow water waves. These types of equations have been significant class of nonlinear equations with numerical simulations in physical phenomena. The case of wave-wave and wave-wall interactions can not be defined using the well-known KdV equation in the study of the dynamics of dense discrete systems,. To accomplish this deficiency of the KdV equation, Rosenau equation was derived [9, 10]:

$$U_t + U_{xxxxt} + U_x + UU_x = 0. \tag{2}$$

The theoretical results on existence, uniqueness and regularity of the solution for Eq. (2) was proved [11]. On the numerical solutions of the Eq. (2), many studies have been performed by the scholars [12–17]. Then, for the further consideration of the nonlinear wave, Jin-Ming Zuo developed the Rosenau-KdV equation and discussed the solitary wave solutions and its periodic solutions in Ref.[18]

$$U_t + U_{xxxxt} + U_x + UU_x + U_{xxx} = 0, \tag{3}$$

where U_{xxx} is the viscous term and the independent variables x and t indicate the spatial and temporal variables, respectively. Recently, the solitary solutions for the generalized Rosenau-KdV equation with consuetudinary power law

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nonlinearity were examined [19–21]. Two invariants were given for the equation in [19, 20]. Especially, in [20] both the singular single soliton solution was derived with the ansatz method and the perturbation theory was used. The ansatz method is implemented to obtain the topological soliton (shock) solution of the generalized Rosenau-KdV equation [21]. The G'/G expansion, ansatz and the exp-function methods are implemented to achieve several solutions to the equation [22]. Singular solitons, solitary waves and shock waves with conservation laws of the Rosenau-KdV-RLW equation are obtained [23]. The conservation laws of the Rosenau-KdV-RLW equation are computed with power law nonlinearity by the aid of multiplier approach in Lie symmetry analysis [24]. Solutions of the perturbated Rosenau-KdV-RLW equation are obtained [25]. A conservative three-level linear finite difference scheme for the numerical solution of the initial-boundary value problem of the Rosenau-KdV equatin is suggested [26]. This is the only method applied to obtain the numerical solution of the Rosenau-KdV equation in the literature.

In this paper, Rosenau-KdV equation is solved numerically by using collocation method with the quintic B-spline basis functions.

2. Numerical scheme using quintic B-Spline basis functions

Let us take into consideration the Rosenau-KdV Eq. (3) by the following boundary conditions:

$$\begin{aligned} &U(a,t)=0, \quad U(b,t)=0, \\ &U_x(a,t)=0, \quad U_x(b,t)=0, \quad t>0. \end{aligned} \tag{4}$$

and the initial condition.

$$U(x,0)=f(x) \quad a\le x\le b, \tag{5}$$

To able to apply the numerical method, the solution region of the problem is restricted over an interval $a \le x \le b$. The interval is partitioned into uniformly sized finite elements of length h by the knots x_m such that $a = x_0 < x_1 < \ldots < x_N = b$ and $h = \frac{b-a}{N}$. The set of quintic B-spline functions $\{\phi_{-2}(x), \phi_{-1}(x), \ldots, \phi_{N+1}(x), \phi_{N+2}(x)\}$ forms a basis over the solution interval $[a,b]$. The numerical solution $U_N(x,t)$ is represented in terms of the quintic B-spline basis functions

$$U_N(x,t)=\sum_{j=-2}^{N+2}\phi_j(x)\delta_j(t) \tag{6}$$

where $\delta_j(t)$ are time dependent parameters to be defined from the boundary and collocation conditions.

Quintic B-splines $\phi_m(x)$, ($m= -2(1)N+2$), at the knots x_m are determined over the interval $[a,b]$ by Ref.[27]

$$\phi_m(x)=\frac{1}{h^5}\begin{cases} (x-x_{m-3})^5, & [x_{m-3},x_{m-2}] \\ (x-x_{m-3})^5-6(x-x_{m-2})^5, & [x_{m-2},x_{m-1}] \\ (x-x_{m-3})^5-6(x-x_{m-2})^5+15(x-x_{m-1})^5, & [x_{m-1},x_m] \\ (x-x_{m-3})^5-6(x-x_{m-2})^5+15(x-x_{m-1})^5- 20(x-x_m)^5, & [x_m,x_{m+1}] \\ (x-x_{m-3})^5-6(x-x_{m-2})^5+15(x-x_{m-1})^5- 20(x-x_m)^5+15(x-x_{m+1})^5 & [x_{m+1},x_{m+2}] \\ (x-x_{m-3})^5-6(x-x_{m-2})^5+15(x-x_{m-1})^5- 20(x-x_m)^5+15(x-x_{m+1})^5-6(x-x_{m+2})^5 & [x_{m+2},x_{m+3}] \\ 0, & otherwise. \end{cases} \tag{7}$$

Each quintic B-spline covers six elements so that each element $[x_m, x_{m+1}]$ is covered by six B-splines. A typical finite interval $[x_m, x_{m+1}]$ is mapped to the interval $[0,1]$ by a local coordinate transformation described by $h\xi = x - x_m$, $0 \le \xi \le 1$. Thus, quintic B-splines (7) in terms of ξ over $[0,1]$ can be given as the following:

$$\begin{aligned} \phi_{m-2} &= 1-5\xi+10\xi^2-10\xi^3+5\xi^4-\xi^5, \\ \phi_{m-1} &= 26-50\xi+20\xi^2+20\xi^3-20\xi^4+5\xi^5, \\ \phi_m &= 66-60\xi^2+30\xi^4-10\xi^5, \\ \phi_{m+1} &= 26+50\xi+20\xi^2-20\xi^3-20\xi^4+10\xi^5, \\ \phi_{m+2} &= 1+5\xi+10\xi^2+10\xi^3+5\xi^4-5\xi^5, \\ \phi_{m+3} &= \xi^5. \end{aligned} \tag{8}$$

Using the nodal values of U, U', U'', U''' and $U^{\prime v}$ at the knots x_m are given in terms of the element parameters δ_m by

$$\begin{aligned} U_N(x_m,t) = U_m &= \delta_{m-2}+26\delta_{m-1}+66\delta_m+26\delta_{m+1}+\delta_{m+2} \\ U'_m &= \tfrac{5}{h}(-\delta_{m-2}-10\delta_{m-1}+10\delta_{m+1}+\delta_{m+2}), \\ U''_m &= \tfrac{20}{h^2}(\delta_{m-2}+2\delta_{m-1}-6\delta_m+2\delta_{m+1}+\delta_{m+2}), \\ U'''_m &= \tfrac{60}{h^3}(-\delta_{m-2}+2\delta_{m-1}-2\delta_{m+1}+\delta_{m+2}), \\ U^{\prime v}_m &= \tfrac{120}{h^4}(\delta_{m-2}-4\delta_{m-1}+6\delta_m-4\delta_{m+1}+\delta_{m+2}), \end{aligned} \tag{9}$$

where the symbols $'$,$''$,$'''$and $'^{v}$ symbolize differentiation according to x, respectively. The splines $\phi_m(x)$ and its four principle derivatives vanish outside the interval $[x_{m-3}, x_{m+3}]$. Now we identify the collocation points with the knots and use Eq. (9) to evaluate U_m, its space derivatives and substitute into Eq. (3) to obtain the set of the coupled ordinary differential equations. For the linearization technique we get the following equation:

$$\begin{aligned}
&\dot{\delta}_{m-2} + 26\dot{\delta}_{m-1} + 66\dot{\delta}_m + 26\dot{\delta}_{m+1} + 2\dot{\delta}_{m+2} \\
&+ \tfrac{120}{h^4}(\dot{\delta}_{m-2} - 4\dot{\delta}_{m-1} + 6\dot{\delta}_m - 4\dot{\delta}_{m+1} + \dot{\delta}_{m+2}) \\
&+ \tfrac{5}{h}(-\delta_{m-2} - 10\delta_{m-1} + 10\delta_{m+1} + \delta_{m+2}) \\
&+ \tfrac{5Z_m}{h}(-\delta_{m-2} - 10\delta_{m-1} + 10\delta_{m+1} + \delta_{m+2}) \\
&+ \tfrac{60}{h^3}(-\delta_{m-2} + 2\delta_{m-1} - 2\delta_{m+1} + \delta_{m+2}) = 0
\end{aligned} \tag{10}$$

where

$$Z_m = U_m = \delta_{m-2} + 26\delta_{m-1} + 66\delta_m + 26\delta_{m+1} + \delta_{m+2}$$

and indicates derivative with respect to t. If time parameters δ_i's and its time derivatives $\dot{\delta}_i$'s in Eq. (10) are discretized by the Crank-Nicolson formula and usual finite difference approximation, respectively:

$$\delta_i = \frac{1}{2}(\delta^n + \delta^{n+1}) \quad , \quad \dot{\delta}_i = \frac{\delta^{n+1} - \delta^n}{\Delta t} \tag{11}$$

we obtain a recurrence relationship between two time levels n and $n+1$ relating two unknown parameters δ_i^{n+1}, δ_i^n for $i = m-2, m-1, ..., m+1, m+2$

$$\begin{aligned}
&\gamma_1\delta_{m-2}^{n+1} + \gamma_2\delta_{m-1}^{n+1} + \gamma_3\delta_m^{n+1} + \gamma_4\delta_{m+1}^{n+1} + \gamma_5\delta_{m+2}^{n+1} \\
&= \gamma_5\delta_{m-2}^{n+1} + \gamma_4\delta_{m-1}^{n} + \gamma_3\delta_m^{n} + \gamma_2\delta_{m+1}^{n} + \gamma_1\delta_{m+2}^{n}
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&\gamma_1 = [1 - E(1+Z_m) - M + K], \\
&\gamma_2 = [26 - 10E(1+Z_m) + 2M - 4K], \\
&\gamma_3 = [66 + 6K], \\
&\gamma_4 = [26 + 10E(1+Z_m) - 2M - 4K], \\
&\gamma_5 = [1 + E(1+Z_m) + M + K], \\
&m = 0, 1, \ldots, N, \quad E = \tfrac{5}{2h}\Delta t, \quad M = \tfrac{30}{h^3}\Delta t, \quad K = \tfrac{120}{h^4}.
\end{aligned} \tag{13}$$

For the linearization technique, the term U in non-linear term UU_x is taken as

$$Z_m = U_m = \delta_{m-2} + 26\delta_{m-1} + 66\delta_m + 26\delta_{m+1} + \delta_{m+2}. \tag{14}$$

The system (12) consists of $(N+1)$ linear equations including $(N+5)$ unknown parameters $(\delta_{-2}, \delta_{-1} \ldots, \delta_{N+1}, \delta_{N+2})^T$. To obtain a unique solution to this system, we need four additional constraints. These are got from the boundary conditions and can be used to eliminate δ_{-2}, δ_{-1} and δ_{N+1}, δ_{N+2} from the system (12) which becomes later a matrix equation for the $N+1$ unknowns $d = (\delta_0, \delta_1, \ldots, \delta_N)^T$ of the form

$$A\mathbf{d^{n+1}} = B\mathbf{d^n}. \tag{15}$$

The matrices A and B are pentagonal $(N+1) \times (N+1)$ matrices and this matrix can be solved by usig the pentagonal algorithm. However, two or three inner iterations are implemented to the term $\delta^{n*} = \delta^n + \frac{1}{2}(\delta^n - \delta^{n-1})$ at each time step to cope with the non-linearity caused by Z_m. Before the solution process begins iteratively, the initial vector $\mathbf{d^0} = (\delta_0, \delta_1, ..., \delta_{N-1}, \delta_N)$ must be determined by using the initial condition and the following derivatives at the boundary conditions:

$$\begin{aligned}
&U_N(x,0) = U(x_m,0), && m = 0,1,2,...,N \\
&(U_N)_x(a,0) = 0, && (U_N)_x(b,0) = 0, \\
&(U_N)_{xx}(a,0) = 0, && (U_N)_{xx}(b,0) = 0,
\end{aligned} \tag{16}$$

So we have the following matrix form of the initial vector $\mathbf{d^0}$:

$$W\mathbf{d^0} = B \tag{17}$$

where $W = \begin{bmatrix} 54 & 60 & 6 & & & & & \\ 25.25 & 67.50 & 26.25 & 1 & & & & \\ 1 & 26 & 66 & 26 & 1 & & & \\ & 1 & 26 & 66 & 26 & 1 & & \\ & & & & \ddots & & & \\ & & & 1 & 26 & 66 & 26 & 1 \\ & & & & 1 & 26.25 & 67.50 & 25.25 \\ & & & & & 6 & 60 & 54 \end{bmatrix}$

$\mathbf{d^0} = (\delta_0, \delta_1, ..., \delta_{N-1}, \delta_N)$ and $B = [U(x_0,0), U(x_1,0), ..., U(x_{N-1},0), U(x_N,0)]^T$.

This matrix system can be solved efficiently by using a variant of Thomas algorithm.

3. Stability analysis

In order to apply the von Neumann stability analysis, the Rosenau-KdV equation can be linearized by supposing that the quantity U in the nonlinear term UUx is locally constant. Replacing Fourier mode

$$\delta_j^n = \hat{\delta}^n e^{ijkh} \tag{18}$$

where k is the mode number and h is the element size, into Eq. (12) gives the growth factor g of the form

$$g = \frac{a - ib}{a + ib}, \tag{19}$$

where

$$\begin{aligned} a &= \gamma_3 + (\gamma_2 + \gamma_4)\cos[hk] + (\gamma_1 + \gamma_5)\cos[2hk], \\ b &= (\gamma_4 - \gamma_2)\sin[hk] + (\gamma_5 - \gamma_1)\sin[2hk]. \end{aligned} \tag{20}$$

The modulus of $|g|$ is 1, for this reason the linearized scheme is unconditionally stable.

4. Numerical experiments and discussion

In this section, to show the accuracy of the numerical simulation of dispersive shallow water waves with Rosenau-KdV equation and to compare obtained results with both exact values and other results given in the literature, the L_2 and L_∞ error norms are calculated by using the analytical solution in Eq. (21). We take into consideration the motion of single solitary wave solution for three test problems. Accuracy and efficiency of the method is reckoned by the error norm L_2

$$L_2 = \left\| U^{exact} - U_N \right\|_2 \simeq \sqrt{h \sum_{J=1}^{N} \left| U_j^{exact} - (U_N)_j \right|^2}, \tag{21}$$

and the error norm L_∞

$$L_\infty = \left\| U^{exact} - U_N \right\|_\infty \simeq \max_j \left| U_j^{exact} - (U_N)_j \right| \quad , \quad j = 1, 2, ..., N-1. \tag{22}$$

Rosenau-KdV equation satisfies only two conservation laws given by Ref.[16]

$$\begin{aligned} I_1 &= \int_a^b U dx \simeq h \textstyle\sum_{J=1}^N U_j^n, \\ I_2 &= \int_a^b [U^2 + (U_{xx})^2] dx \simeq h \textstyle\sum_{J=1}^N [(U_j^n)^2 + (U_{xx})_j^n]. \end{aligned} \tag{23}$$

In the simulation of solitary wave motion, the invariants I_1 and I_2 are observed to check the conversation of the numerical algorithm.

4.1. The motion of single solitary wave

For this problem, the Rosenau-KdV Eq. (3) is considered for the boundary conditions $U \to 0$ as $x \to \pm\infty$ and the initial condition

$$U(x,0) = (-\frac{35}{24} + \frac{35}{312}\sqrt{313}) \sec h^4 (\frac{1}{24}\sqrt{-26 + 2\sqrt{313}}x). \tag{24}$$

Note that the analytical solution of the equation can be written as

$$U(x,t) = (-\frac{35}{24} + \frac{35}{312}\sqrt{313}) \sec h^4 [\frac{1}{24}\sqrt{-26 + 2\sqrt{313}}\left(x - \left(\frac{1}{2} + \frac{1}{26}\sqrt{313}\right)t\right). \tag{25}$$

In this section, to apply numerical method we have considered three sets of parameters. First of all, we have used the parameters $h = 0.1$ and $\Delta t = 0.1$ over the interval $[-70, 100]$ to coincide with those of Ref.[26]. So, the solitary wave has an amplitude 0.52632 and the computations are done up to time $t = 40$ to obtain the invariants and error norms L_2 and L_∞ at various times. Error norms L_2, L_∞ and two invariants of the Rosenau-KdV equation are listed in Table 1. It is seen from the table that the error norms are found to be small enough. The percentage of the relative error of the conserved quantities I_1 and I_2 are calculated with respect to the conserved quantities at $t = 0$. Percentage of relative changes of I_1 and I_2 are found to be 1.177×10^{-7} %, 9.86×10^{-8} %, respectively. Fig. 1 shows the motion of solitary wave with $h = 0.1$ and $\Delta t = 0.1$ at various time levels. The distributions of the errors at time $t = 40$ are illustrated for solitary waves amplitudes 0.52632 in Fig. 2.

Table 1. The invariants and the error norms for single solitary wave with amplitude= 0.52632, $h = 0.1$, $\Delta t = 0.1$, $-70 \le x \le 100$.

t		0	10	20	30	40
I_1	Present	5.4981750556	5.4981750556	5.4981750556	5.4981750555	5.4981750621
	[26]	5.4977225480	5.4977249365	5.4977287449	5.4977319638	5.4977342352
I_2	Present	1.9897841615	1.9897841624	1.9897841629	1.9897841633	1.9897841635
	[26]	1.9845533653	1.9845950759	1.9846459641	1.9846798272	1.9847015013
$L_2 \times 10^3$	Present	0.000000	0.370348	0.665684	0.924741	1.187411
	[26]	0.000000	1.641934	3.045414	4.241827	5.297873
$L_\infty \times 10^3$	Present	0.000000	0.149073	0.253418	0.336342	0.422656
	[26]	0.000000	0.631419	1.131442	1.533771	1.878952

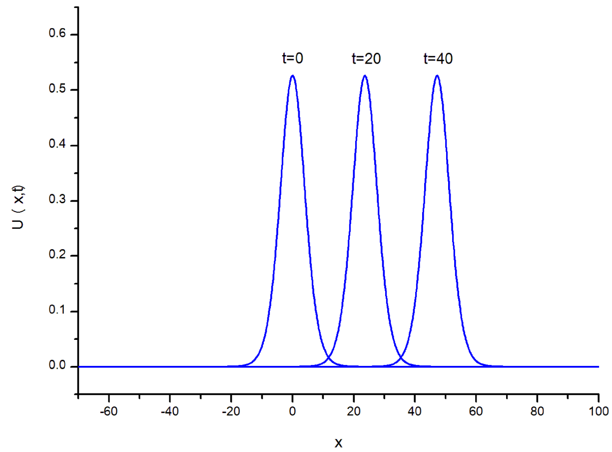

Fig. 1. Single solitary wave with $h = 0.1$, $\Delta t = 0.1$, $-70 \le x \le 100$, $t = 0, 20$ and 40.

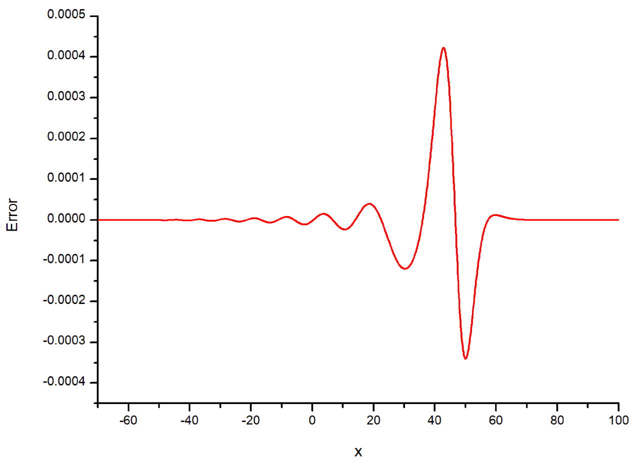

Fig. 2. Error $h = 0.1$, $\Delta t = 0.1$, $-70 \le x \le 100$, $t = 40$.

For the second case, the parameters $h = 0.05$ and $\Delta t = 0.05$ with interval $[-70, 100]$ are taken. Hence, the solitary wave has amplitude 0.52632 and the simulations are run up to time $t = 40$ to obtain the invariants and the error norms at several times. Error norms L_2 and L_∞ and conserved quantities are reported in Table 2 together with the results obtained in Ref.[26]. It can be easily seen from the table that the obtained error norms are smaller than those given in Ref.[26]. The agreement between numerical and analytic solution is excellent. Percentage of relative changes

Table 2. The invariants and the error norms for single solitary wave with amplitude= 0.52632, $h = 0.05$, $\Delta t = 0.05$, $-70 \le x \le 100$.

t		0	10	20	30	40
I_1	Present	5.4981692134	5.4981692136	5.4981692136	5.4981692134	5.4981692116
	[26]	5.4980606845	5.4980608372	5.4980610805	5.4980612870	5.4980613985
I_2	Present	1.9897831853	1.9897831855	1.9897831855	1.9897831854	1.9897831852
	[26]	1.9843901753	1.9844010295	1.9844143675	1.9844232703	1.9844289740
$L_2 \times 10^4$	Present	0.000000	0.888297	1.823510	2.862236	3.842086
	[26]	0.000000	4.113510	7.631169	10.62971	13.27645
$L_\infty \times 10^4$	Present	0.000000	0.362314	0.649564	1.000742	1.320897
	[26]	0.000000	1.582641	2.835874	3.843906	4.709118

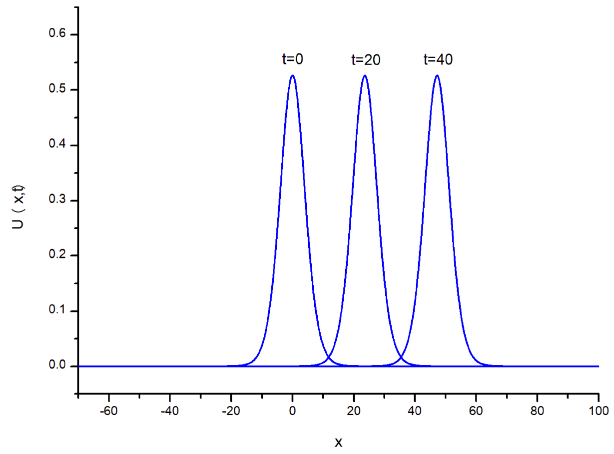

Fig. 3. Single solitary wave with $h = 0.05$, $\Delta t = 0.05$, $-70 \le x \le 100$, $t = 0, 20$ and 40.

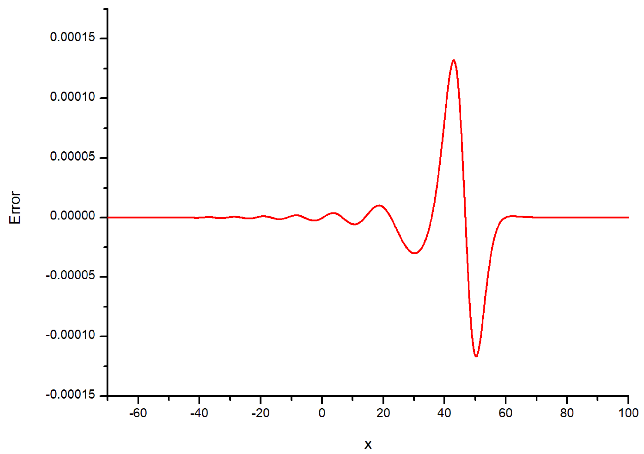

Fig. 4. Error $h = 0.05$, $\Delta t = 0.05$, $-70 \le x \le 100$, $t = 40$.

of I_1 and I_2 are found to be 3.30×10^{-8} %, 6.7×10^{-9} %, respectively. Perspective views of the traveling solitons are graphed at diverse time levels in Fig. 3. The distributions of the errors at time $t = 40$ are drawn in Fig. 4.

Finally, for the third case, the parameters $h = 0.025$ and $\Delta t = 0.025$ with interval $[-70, 100]$ are chosen. Therefore, the solitary wave has amplitude 0.52632 and the experiments are run from the time $t = 0$ to the time $t = 40$ to obtain the invariants and the error norms L_2 and L_∞ at different times. Error norms L_2 and L_∞ and conserved quantities are listed

Table 3. The invariants and the error norms for single solitary wave with amplitude= 0.52632, $h = 0.025$, $\Delta t = 0.025$, $-70 \le x \le 100$.

t		0	10	20	30	40
I_1	Present	5.4981698357	5.4981698365	5.4981698322	5.1981698290	5.4981698203
	[26]	5.4981454184	5.4981454791	5.4981455454	5.4981456095	5.4981456591
I_2	Present	1.9897809062	1.9897809077	1.9897809038	1.9897809019	1.9897808975
	[26]	1.9843493353	1.9843521098	1.9843555206	1.9843578113	1.9843592922
$L_2 \times 10^4$	Present	0.000000	0.357060	0.925408	1.057023	1.183710
	[26]	0.000000	1.028173	1.905450	2.650990	3.306738
$L_\infty \times 10^5$	Present	0.000000	1.421479	3.264848	4.742297	4.846861
	[26]	0.000000	3.965867	7.097948	9.610332	11.76011

in Table 3 together with the results obtained in Ref. [26]. The agreement between numerical and analytic solutions is perfect. Percentage of relative changes of I_1 and I_2 are found to be 2.812×10^{-7} %, 4.369×10^{-7} %, respectively. The profiles of the solitary wave at different time levels are shown in Fig. 5. The distributions of the errors at time $t = 40$ are depicted graphically for solitary waves amplitudes 0.52632 in Fig. 6.

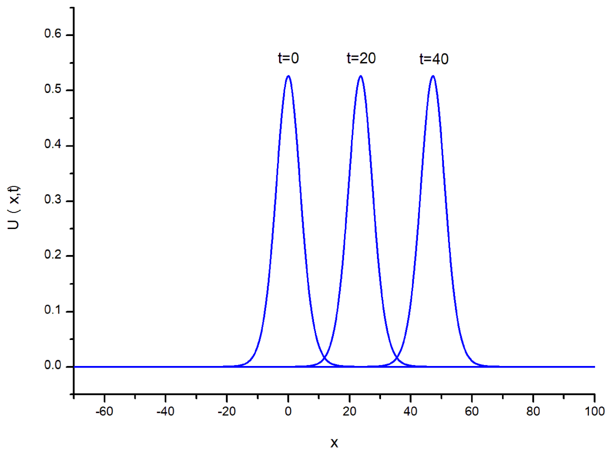

Fig. 5. Single solitary wave with $h = 0.025$, $\Delta t = 0.025$, $-70 \le x \le 100$, $t = 0, 20$ and 40.

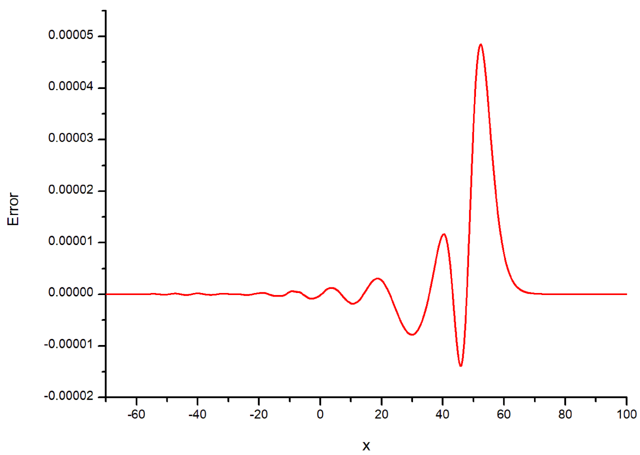

Fig. 6. Error $h = 0.025$, $\Delta t = 0.025$, $-70 \le x \le 100$, $t = 40$.

Consequently, as seen from the three cases, the changes of the invariants are reasonable small and the quantity of obtained error norms are better than the ones in earlier numerical methods.

5. Conclusion

In this study, finite element method has been successfully applied to show numerical simulaton of dispersive shallow water waves with Rosenau-KdV equation. Here, collocation method with quintic B-spline basis functions has been used. To show the accuracy of the method, we have calculated the error norms L_2, L_∞ and the invariants I_1, I_2. Since the error norms are satisfactorily small during the simulations, single solitary wave motion is well presented and conservation laws have been held satisfactorily constant in the course of the computer run. The obtained results show that the presented method is more precise than results in previous numerical method. Thus, numerical results demonstrate that presented method is a promising and powerful tool for solving the Rosenau-KdV equation. Main advantages of the present technique are: it is simple, effective and moreover easy to understand.

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