Numerical Simulation of Heat Transfer Enhancement in Laminar Flow of Viscoelastic Fluids through a Rectangular Channel

N. Peres[a] , A. Afonso[b], M.A. Alves[b] and F.T. Pinho[a,c]

[a]*CEFT, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal*
[b]*Faculdade de Engenharia da Universidade do Porto, Departamento de Engenharia Química, CEFT, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal*
[c]*Universidade do Minho, Largo do Paço, 4704-533 Braga, Portugal*

Abstract. A finite-volume method is applied to the three-dimensional simulation of heat transfer of a viscoelastic fluid in laminar flow through a rectangular channel with an aspect ratio of 2 and constant heat flux at the channel walls. The objectives of the present work are twofold: (a) to compare the numerical results with the heat transfer experimental data of Hartnett and Kostic [1]; (b) to analyze the influence of the secondary flow on the heat transfer enhancement on account of the non-zero second normal-stress difference. The rheology of the viscoelastic fluid is represented by the Phan-Thien-Tanner constitutive equation with non-zero second normal-stress difference. The simulations show the strong effect of secondary flows on the correct prediction of experimental results. The predictions confirm the enhancement of local and mean Nusselt numbers, as found experimentally by Hartnett and Kostic [1], and show that free convection has a major influence in the experimental heat transfer results for Newtonian fluid, but less so as fluid elasticity is increased.

Keywords: Enhanced heat transfer; Viscoelastic fluid flow; Channel flow; Secondary flows.
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INTRODUCTION

Laminar flows of viscoelastic fluids in ducts of non-circular cross section are characterized by enhanced levels of heat transfer relative to the corresponding Newtonian flows, due to the existence of secondary flows, even though the pressure drop may not be significantly different, as demonstrated experimentally by Hartnett and Kostic [1]. The objective of this work is to quantify and understand the extent of this effect by a systematic numerical investigation. A finite-volume method is used to simulate the laminar flow and heat transfer of viscoelastic fluids in a duct of rectangular cross-section under hydrodynamic fully-developed and thermally developing conditions (large Prandtl numbers). The rectangular duct has an aspect ratio of 2, constant heat fluxes are imposed at the walls and the viscoelastic fluids are represented by the complete linear form of the Phan-Thien-Tanner (PTT) model. Heat transfer in the rectangular duct flow is investigated as a function of fluid rheology to show the enhanced heat transfer levels as a function of secondary flow intensity, as quantified in terms of local Nusselt numbers. Discrepancies between the predictions and the experimental data of [1] are discussed and related to the selection of rheological constitutive equation (only steady shear viscosity data are available for fitting the rheological model), but especially to free convection effects that are present in the experiments when the imposed heat flux is at the bottom wall of the duct. Free convection effects are not included in the present calculations, but will be accounted for in future works.

GOVERNING EQUATIONS AND NUMERICAL METHOD

The experiments in the rectangular channel have a thermal development length of 533.3 D_h (hydraulic diameter) [1], the duct has an aspect ratio of 2 and constant heat fluxes are applied at the top and bottom walls, whereas the

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side walls are adiabatic. For incompressible fluids the equations to be solved are the conservation of mass, $\partial u_i / \partial x_i = 0$, the momentum equation,

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_j u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \eta_s \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \tag{1}$$

and the thermal energy equation,

$$\frac{\partial (\rho C_P T)}{\partial t} + \frac{\partial (\rho C_P u_j T)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j} \right) + \tau_{ij} \frac{\partial u_i}{\partial x_j} \tag{2}$$

where u_i represents the velocity vector, p is the pressure, k the thermal conductivity, C_P the specific heat, ρ the density, T the temperature, t the time and τ_{ij} polymer extra-stress tensor. The polymer contribution is modeled by the full form of the Phan-Thien Tanner constitutive equation (PTT), with linear kernel for the stress function,

$$f(\tau_{kk})\tau_{ij} + \lambda \left[\frac{\partial \tau_{ij}}{\partial t} + u_k \frac{\partial \tau_{ij}}{\partial x_k} - \tau_{jk} \frac{\partial u_i}{\partial x_k} - \tau_{ik} \frac{\partial u_j}{\partial x_k} + \xi \left(\tau_{ik} S_{kj} + S_{ik} \tau_{kj} \right) \right] = \eta_p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{3}$$

where λ is the relaxation time, η_p is the polymer viscosity coefficient, S_{ij} the rate of deformation tensor and ξ the slip parameter. This slip parameter is responsible for a non-zero second normal-stress difference in shear, leading to secondary flows in ducts of non-circular cross-sections.

A fully-implicit finite-volume method (FVM) was used to solve the set of equations and this is described in detail elsewhere [2-5]. Here, the energy equation was incorporated into the FVM code and its validation was done against analytical solutions. Several progressively refined uniform meshes were used to ensure the accuracy of the calculations. The minimum cell spacing was varied from $\Delta x_{\min}/D_h = 0.667$ and $\Delta y_{\min}/D_h = 0.0375$ for the coarse mesh, to $\Delta x_{\min}/D_h = 0.333$ and $\Delta y_{\min}/D_h = 0.025$ for the finest mesh.

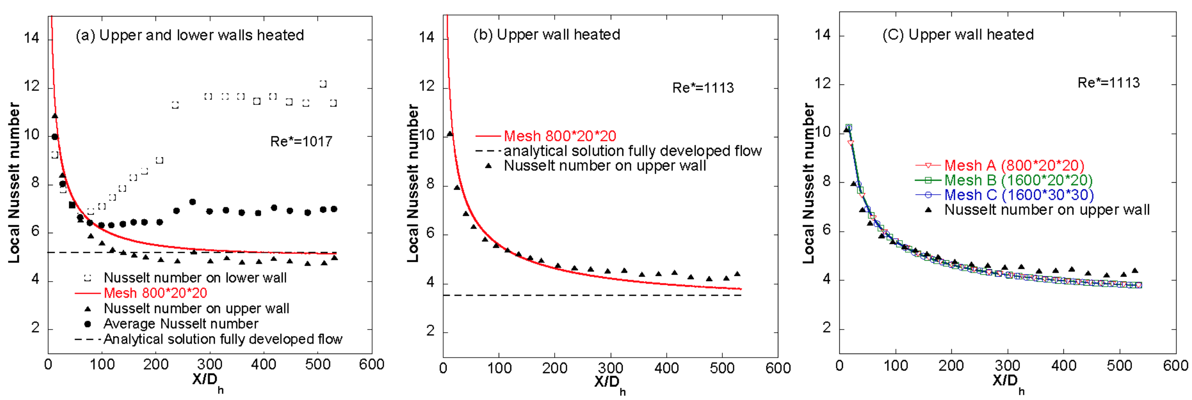
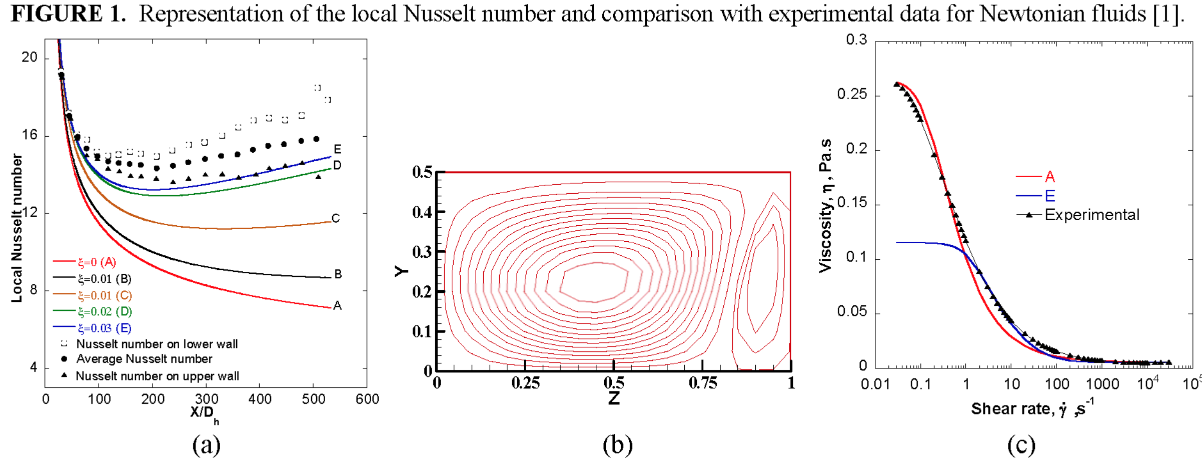


FIGURE 1. Representation of the local Nusselt number and comparison with experimental data for Newtonian fluids [1].



(a) (b) (c)

FIGURE 2. (a) Comparison of predicted local Nusselt number against experimental data, (b) illustration of secondary flow and (c) Different fittings of shear viscosity.

RESULTS AND DISCUSSION

Predictions for local and mean Nusselt numbers for Newtonian fluid flow are compared with experimental data [1] in Figure 1. The comparison is good, except when the bottom wall is heated (Figure 1a), because the numerical simulations do not account for free convection effects, which are strong in this case. When only the upper wall is heated the free convection effects are minimized and the comparison is quite good as shown in figure 1(b). For fully-developed conditions the predictions agree with the horizontal lines in figures 1(a)-(b), which pertain to the corresponding theoretical solution by Shah and London [6]. For heating through all walls this value is 4.12, while the prediction with the coarse mesh was 4.005. For 2D channel flow the value of the Nusselt number obtained with our code was 8.2397 against 8.2353 for the analytical result [6] corresponding to an error of 0.054%. Figure 1(c) shows that results for the three finest meshes are in close agreement, thus indicating the good quality of the predictions.

The shear viscosity of the aqueous solutions of PAA used in [1] is represented in Figure 2(a) together with the viscosity of the various PTT models that were fitted to the experimental data (only a single mode was used). Table 1 lists the numerical values of the parameters used in these fittings. Note that [1] did not present data on N_1 and N_2. The best viscosity fitting is for fluids A and B, without and with second normal stress differences, respectively. The other models in Table 1 were used to explore the effect of various rheological parameters on heat transfer characteristics. Models with $N_2 \neq 0$ lead to secondary flow as shown in the projected streamlines of the secondary flow of Figure 2(b). Predictions of the variation in local Nu for the A and E fluids in Table 1 are plotted in Figure 2(c) and show the significant impact of the second normal-stress coefficient in the heat transfer enhancement, which is proportional to ξ. In order for the predictions to approach the experimental data it was necessary to reduce the polymer viscosity coefficient for reasons of iterative convergence, at the expense of less good predictions of the shear viscosity. This will have to be addressed in the near future together with the inclusion of free convection effects.

TABLE 1. Parameters used in the viscoelastic simulations.

Simulation	η_p [Pa s]	η_s [Pa s]	ξ	ε	λ [s]	U [m/s]	De
A	0.26	0.0055	0	0.25	5.0	0.80	333.33
B	0.26	0.0055	0.01	0.25	5.0	0.80	333.33
C	0.11	0.0055	0.01	0.25	0.5	0.80	33.33
D	0.11	0.0055	0.02	0.25	0.5	0.80	33.33
E	0.11	0.0055	0.03	0.25	0.5	0.80	33.33

CONCLUSIONS

The heat transfer in a rectangular channel flow was studied by means of a fully-implicit finite-volume method. The present work showed that the local Nusselt number obtained in the several simulations with a Newtonian fluid are in good agreement with the experimental results of [1], except for conditions that enhance the role of free convection effects, which are not accounted for in the numerical simulations. The PTT predictions showed the enhanced heat transfer to be proportional to the intensity of the secondary flow and it was possible to approach the experimental data by increasing ξ to 0.03.

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