Numerical simulation of linear wave propagation, wave-induced circulation, sediment transport and beach evolution

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Abstract

In the present work two numerical models for linear wave propagation, wave induced circulation and sediment transport are presented. The wave model WAVE-L is based on the hyperbolic type mild slope equation and is valid for a compound wave field near coastal structures where the waves are subjected to the combined effects of shoaling, refraction, diffraction, reflection (total and partial) and breaking. Radiation stress components estimated from the hyperbolic wave model drive the depth averaged circulation model COAST for the description of the nearshore currents and sediment transport in the surf and swash zone. The model COAST is coupled with a 3D bed evolution model or with an one-line model to provide bathymetry or shoreline changes.

1. Introduction

Many models exist for the evaluation of wave deformation in the coastal region but the most of them are based on the progressive wave assumption (period averaged refraction wave models) and employ elliptic or parabolic type differential equations which are in general difficult to numerically solve. Besides they are not valid for a compound wave field near coastal structures where the waves subject to the combined effects of shoaling, refraction, diffraction, reflection and breaking.

The evaluation of the wave field only is not sufficient for the design of coastal structures. The current pattern, the sediment transport and the bottom topography changes also play an important role in the design. Basic to the description of these currents is the incorporation of the wave breaking (into the wave model) and the formulation of the driving forces (radiation stress) from the wave model results.

In the present work the wave model WAVE-L, based on the hyperbolic type mild slope equation, valid for a compound wave, is presented. The model, after the incorporation of breaking and the evaluation of the radiation stress, drives the depth-averaged circulation and sediment transport model COAST for the description of the nearshore currents and beach deformation. A new one-line model, with additional terms, is proposed in order to calculate shoreline position taking into account the cross-shore related seasonal shoreline variation.

2. Wave model WAVE-L

The breaking and non breaking wave model is based on the hyperbolic type mild slope equation without using the progressive wave assumption. The model consists of the following pair of equations[5]:

$$\frac{\partial \zeta_w}{\partial t} + \frac{c}{c_g} \nabla \frac{c_g}{c} \mathbf{Q}_w = 0$$

$$\frac{\partial \mathbf{U}_w}{\partial t} + \frac{c^2}{d} \nabla \zeta_w = 0$$

$$(1)$$

where ζ_w is the surface elevation, $\mathbf{U}_w$ the mean velocity vector $\mathbf{U}_w = (U, V)$, d the depth, $\mathbf{Q}_w = \mathbf{U}_w h_w = (Q_w, P_w)$, h_w the total depth ($h_w = d + \zeta_w$), c the celerity and c_g the group velocity.

The above equations, derived by Copeland[5], are able to compute the combination of wave refraction, diffraction and reflection (total or partial).

The numerical model is adapted for engineering applications:

1. The input wave is introduced in a line inside the computational domain according to Larsen and Dancy[14].

2. A sponge layer boundary condition is used to absorb the outgoing waves in the four sides of the domain[14].

3. Total reflection boundary condition (U_w or $V_w = 0$) is incorporated automatically in the model. The existence of a vertical structure with 100% reflection coefficient is introduced from the depth file (depth d=-1).

4. Submerged structures are incorporated as in Karambas and Kriezi[11].

5. Partial reflection is introduced from an artificial eddy viscosity file. The values of the eddy viscosity coefficient are estimated from the method

developed by Karambas and Bowers[9], using the values of the reflection coefficients proposed by Bruun[4].

The model is extended in the surf zone in order to include breaking effects providing the equations with a suitable dissipation mechanism by the introduction of a dispersion term in the right-hand side of momentum eqn (1):

$$v_h = \nabla^2 \mathbf{U}_w$$

(2)

where v_h is an horizontal eddy viscosity coefficient estimated from[1]:

$$v_h = 2d \left(\frac{D}{\rho} \right)^{1/3}$$

(3)

in which ρ is the water density and D is the energy dissipation given by[2]:

$$D = \frac{1}{4} Q_b f \rho g H_m^2$$

(4)

with f the mean frequency, H_m the maximum possible wave height and Q_b the probability that at a given point the wave height is associated with the a breaking or broken wave. For a Rayleigh type probability distribution[2]:

$$\frac{1 - Q_b}{\ln Q_b} = \left(\frac{H_{rms}}{H_m} \right)^2$$

in which H_{rms} is the mean square wave height: $H_{rms} = 2 \ (<2\zeta_w^2>)^{1/2}$ and the brackets $<>$ denote a time mean quantity.

3. Wave-induced circulation and sediment transport model COAST

3.1. Radiation stress and wave-induced current submodel

Taking the horizontal axes x_1 and x_2 on the still water surface, and the z axis upward from the surface, the definition of the radiation stress S_{ij} component is:

$$S_{ij} = < \int_{-d}^{\zeta_w} \left(p\delta_{ij} + \rho u_i u_j \right) dz > -0.5\rho g \left(d + <\zeta> \right)^2 \delta_{ij}$$

(5)

where δ_{ij} is the Kroneker's delta, $u_i(z)$ is the wave horizontal velocity component in direction x_i, ζ is the mean sea level, p the pressure and $<>$ denotes a time average.

The total pressure p is obtained from the vertical momentum equation:

$$p = \rho g(\zeta - z) - \rho u_3{}^2 + \frac{\partial}{\partial x_1}\int_z^0 \rho u_1 u_3 dz + \frac{\partial}{\partial x_2}\int_z^0 \rho u_2 u_3 dz + \frac{\partial}{\partial t}\int_z^\zeta \rho u_3 dz$$

(6)

where u_3 is the z-velocity component.

Based on the above eqn (6) and after the substitution of u_i and p, from model results (eqn 1) using linear wave theory, Copeland[6] derived the expressions for S_{ij} without the assumption of progressive waves. Those expressions are used in the present model.

The radiation stresses are the driving forces of a 2D horizontal wave-induced current model:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (Uh)}{\partial x} + \frac{\partial (Vh)}{\partial y} = 0$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + g\frac{\partial \zeta}{\partial x} =$$

$$-\frac{1}{\rho h}\left(\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}\right) + \frac{1}{h}\frac{\partial}{\partial x}\left(v_h h\frac{\partial U}{\partial x}\right) + \frac{1}{h}\frac{\partial}{\partial y}\left(v_h h\frac{\partial U}{\partial y}\right) - \frac{\tau_{bx}}{\rho h}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + g\frac{\partial \zeta}{\partial y} =$$

$$-\frac{1}{\rho h}\left(\frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y}\right) + \frac{1}{h}\frac{\partial}{\partial x}\left(v_h h\frac{\partial V}{\partial x}\right) + \frac{1}{h}\frac{\partial}{\partial y}\left(v_h h\frac{\partial V}{\partial y}\right) - \frac{\tau_{by}}{\rho h}$$

(7)

where h is the total depth $h = d + \zeta$, U, V are the current horizontal velocities and τ_{bx} and τ_{by} are the bottom shear stresses.

In the current model the treatment of the bottom stress is critical (all longshore current models employing radiation stress solve for the mean current

velocity through its role in the bottom friction term). The general expression for the time-average bottom shear stress in the current model is written:

$$\tau_{bx} = \rho C_f < \left(U + u_b\right)\sqrt{\left(U + u_b\right)^2 + \left(V + v_b\right)^2} >$$

$$\tau_{bx} = \rho C_f < \left(V + v_b\right)\sqrt{\left(U + u_b\right)^2 + \left(V + v_b\right)^2} >$$

$$(8)$$

where C_f is the friction coefficient which depends on the bottom roughness and on the orbital amplitude at the bed[12], and u_b and v_b are the wave velocities at the bottom.

Inside surf zone the existence of the undertow current that is directed offshore on the bottom cannot predicted by a depth averaged model. However, representing the cross-shore flow is essential for a realistic description of the sediment transport processes. The present model calculates local vertical distribution of the horizontal velocity using the analytical expression for the cross-shore flow below wave trough level proposed by Stive & Wind[18]:

$$v_u = \frac{1}{2}\left[\left(\xi - 1\right)^2 - \frac{1}{3}\right]\frac{h - \zeta_t}{\rho\,v_\tau}\frac{dR}{dy} + \left(\xi - \frac{1}{2}\right)\frac{\left(h - \zeta_t\right)\tau_s}{\rho v_\tau} - \frac{M\cos\Theta}{h - \zeta_t}$$

$$(9)$$

where v_u is the undertow velocity in the y (shore-normal) direction, $\xi = z/(h - \zeta_t)$, ζ_t is the wave trough level, $dR/dy = 0.14\,\rho\,gdh/dy$, τ_s is the shear stress at the wave trough level[18], M is the wave mass flux above trough level, Θ is the direction of the wave propagation and v_τ the eddy viscosity coefficient given by equation (1). The value of the coefficient in eqn (3) is now taken equal to 0.03 (instead of 2). The direction of the wave propagation Θ is given by:

$$\Theta = \arctan\left[\left(<Q_w^2>/<P_w^2>\right)^{1/2}\right]$$

$$(10)$$

3.2. Sediment transport submodel

3.2.1. Sediment transport in the surf zone

The prediction of the sediment transport is based on the energetics approach, in which the submerged weight transport rates, i_{xt} in the x direction and i_{yt} in the y direction, are given by Karambas[12]:

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$$i_{xt} = < \left[\frac{\varepsilon_b}{\tan\phi} \left(\frac{u_o}{u_{ot}} + \frac{d_x}{\tan\phi} \right) \omega_b + \varepsilon_s \frac{u_{ot}}{w} \left(\frac{u_o}{u_{ot}} + \varepsilon_s d_x \frac{u_{ot}}{w} \right) \omega_t \right] >$$

$$i_{yt} = < \left[\frac{\varepsilon_b}{\tan\varphi} \left(\frac{v_o}{u_{ot}} + \frac{d_y}{\tan\phi} \right) \omega_b + \varepsilon_s \frac{u_{ot}}{w} \left(\frac{v_o}{u_{ot}} + \varepsilon_s d_y \frac{u_{ot}}{w} \right) \omega_t \right] >$$

$$(11)$$

where w is the sediment fall velocity, ϕ is the angle of internal friction, ε_b and ε_s are the bed and suspended load efficiency factors respectively ($\varepsilon_b = 0.13$, $\varepsilon_s = 0.01$), $u_{ot} = \sqrt{u_o^2 + v_o^2}$ (u_o, v_o are the total flow velocities at the bottom), d_x and d_y are the bottom slopes $\omega_b = C_f \rho u_{ot}^3$, and ω_t is the total rate of energy dissipation given by Leont'yev[15]:

$$\omega_t = \omega_b + De^{3/2(1-h/H)}$$

$$(12)$$

in which H is the wave height ($H = H_{rms}$), D is the mean rate of breaking wave energy dissipation per unit area given by eqn (4).

In eqn (12) the first term express the power expenditures due to bed friction while the second due to excess turbulence penetrating into bottom layer from breaking waves.

The above method had been applied using a non linear dispersive wave model based on the Boussinesq equations (Karambas et al.[8], Karambas[12, 13]). A Boussinesq model automatically includes the existence of the mean wave-induced current and consequently there is no need to separate the bottom velocities into a mean and a oscillatory part. However, since the present model is a linear one, the total flow velocity at the bottom is considered as a sum of the steady U, V, v_u and the oscillatory u_b, v_b components which include two harmonics:

$$u_o = U + u_{bm} \cos(\omega t) + u_{b2m} \cos(2\omega t)$$

$$v_o = V + v_u + v_{bm} \cos(\omega t) + v_{b2m} \cos(2\omega t + a)$$

$$(13)$$

in which ω is the wave frequency, a is the phase shift and u_{bm}, u_{b2m}, v_{bm} and v_{b2m} are the velocity amplitudes given by Leont'yev [15,16].

The above sediment transport formula has been derived directly form the Bailard primitive equations without the assumption that the only

dissipation mechanism is the bed friction. This is the most important limitation of the Bailard theory and precludes the use of the original formula within the surf zone, where the dissipation of energy associated with the process of wave breaking is largely dominant.

3.2.2. Sediment transport in the swash zone

Adopting the procedure proposed by Leont'yev[15] the submerged weight transport rates i_{ys} near the shoreline, in the y (shore-normal) direction, is given by:

$$i_{ys} = \frac{\varepsilon_b f_R}{2 \tan^2 \phi} \rho \left| < u_R^3 > \right| \left(\tan \beta_{eq} - \tan \beta \right)$$

(14)

where f_R is the run-up friction coefficient (of order 10^{-1}-10^{-3}), u_R is the flow velocity in the swash zone, $\tan \beta$ is the actual slope gradient[15] and $\tan \beta_{eq}$ is the slope under equilibrium state approximated by (Yamamoto et al.[19]):

$$\tan \beta_{eq} = \left(\frac{0.0864 \, s \, g \, d_{50} \, T^2}{H_b^2} \right)^{2/3}$$

(15)

where s is the specific gravity of sediment in water, d_{50} is the median grain size, H_b is the breaker height and T the wave period.

The flow velocity in the swash zone u_R is parameterized in terms of the run-up height R according to Leont'yev[15]: $u_R = (2g \, (R - z_c) \,)$, where z_c is the height of water mass above the water level which increases proportionally to the distance from the upper run-up boundary.

If the bottom gradient exceeds the equilibrium value then $i_{ys} < 0$ (erosion). In opposite case $i_{ys} > 0$ (accretion).

The longshore (x direction) total swash sediment transport i_{xs} is calculated by the global expression proposed by Briad & Kamphuis[3].

4. 3D bed evolution and one-line models

The model COAST is coupled with a 3D bed evolution model or with an one-line model to provide bathymetry or shoreline changes.

The nearshore morphological changes are calculated by solving the conservation of sediment transport equation:

$$\frac{\partial d}{\partial t} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}$$

(16)

where d is the still water depth and q_x, q_y are the volumetric longshore and cross-shore sediment transport rates, related to the immersed weight sediment transport through:

$$q_{x,y} = \frac{i_{x,y}}{(\rho_s - \rho)gN}$$

(17)

in which N is the volume concentration of solids of the sediment (N= 0.6) and ρ_s and ρ are the sediment and fluid densities.

Under certain assumptions eqn (16) can be transformed into an 1D equation (one-line model). The one-line models find wider engineering use as they are much less costly to run.

Let us define the total longshore sediment transport Q and the mean (cross-shore) water depth $\bar{d}$ by the equations:

$$Q = \int_0^{y_s} q_x dy \qquad \bar{d} = \frac{1}{y_s} \int_0^{y_s} ddy$$

(18)

where y_s is the width of the nearshore zone.

The integration of eqn (17) over the width of the nearshore zone from its outer boundary (y=0) to the shoreline (y=y_s), using the Leibnitz relation, leads to the following equation:

$$\frac{\partial\left(y_s\bar{d}\right)}{\partial t} = \frac{\partial Q}{\partial x} - q_x(y_s)\frac{\partial y_s}{\partial x} + q_y(y_s)$$

(19)

where we have supposed that the following conditions are valid: d=0 at shoreline (y=y_s) and the transport rates $q_x(0)=0$, $q_y(0)=0$ at the outer boundary (y=0) are zero.

Eqn (19) differs from a standard one-line model in the last two terms. The second term of the right hand side of the equation is related to the longshore transport rate near the shoreline while the last term incorporates the cross-shore related seasonal shoreline variation.

The cross-shore transport rate near the shoreline $q_y(y_s)$ is given Sunamura formula (Yamamoto et al.[19]):

$$q_y(y_s)=K \ U_r^{0.2} \ \Phi \ (\ \Phi -0.13 \ Ur) \ w \ d_{50}$$

(20)

where U_r is the Ursell parameter $U_r=gHT^2/h^2$ (H is the wave height and h is the wave set-up at shoreline), $\Phi =H^2/shd_{50}$ (s is the specific gravity of sediment) and K is a coefficient of sediment transport rate:

$$K=Ae^{-Bt/T}$$

(21)

where the coefficient A and B are given by[19]:

$$A=1.61 \ 10^{-10} \ (d_{50}/H_o)^{-1.31}$$

$$B=4.2 \ 10^{-3} \ (\tan \beta)^{1.57}$$

(22)

where H_o is the deep water wave height.

The coefficient K of eqn (21) is a function of time since the rate of cross-shore sediment transport decreases with the lapse of time and the beach profile approaches the equilibrium state.

Also it can be expected that the mean depth $\overline{d}$ is relatively conservative characteristic in comparison with the local shoreline position y_s, and consequently, it can be considered as a constant in eqn (19).

5. Applications

The wave and the wave-induced circulation model had been tested against experimental data for diffraction, refraction, reflection, shoaling, breaking, dissipation after breaking of regular waves, as well as for the breaking wave induced current in previous works (Karambas & Koutitas[7], Karambas & Kitou[10]).

In order to verify the one-line model (coupled with the models WAVE-L and COAST), we try to simulate the beach evolution at Angelochori coast (Macedonia, Greece) due construction of the breakwater. Since wave data were not available, a hindcast procedure is used. The incident direction is the NW, the duration is 29 days each year, the incident wave height is 0.85 m (H_o=0.85 m), the wave period is 3.6 sec (T=3.6 sec) and the grain size is about 0.2mm (d_{50}=0.2 mm). The mean (cross-shore) water depth $\overline{d}$ is assumed to be equal to 1m ($\overline{d}$=1 m).

Fig. 1 shows the comparison between model results and field measurements 5 years after the construction of the breakwater. The shoreline contour is very well reproduced by the model.
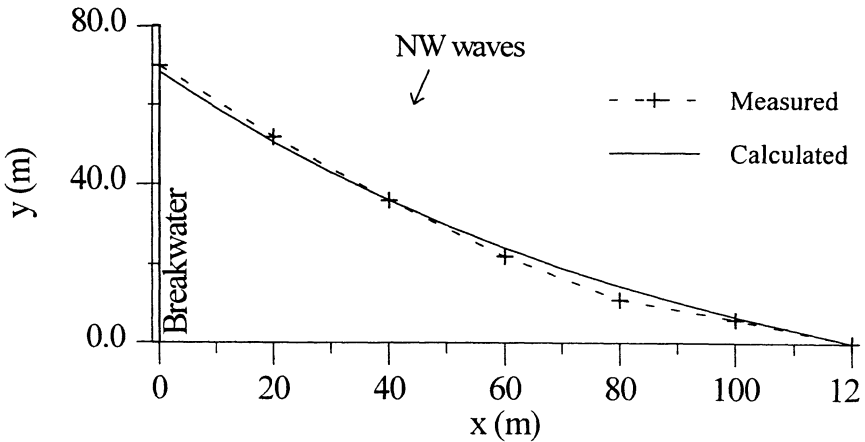
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Figure 1. Result of shoreline change simulation in comparison with field
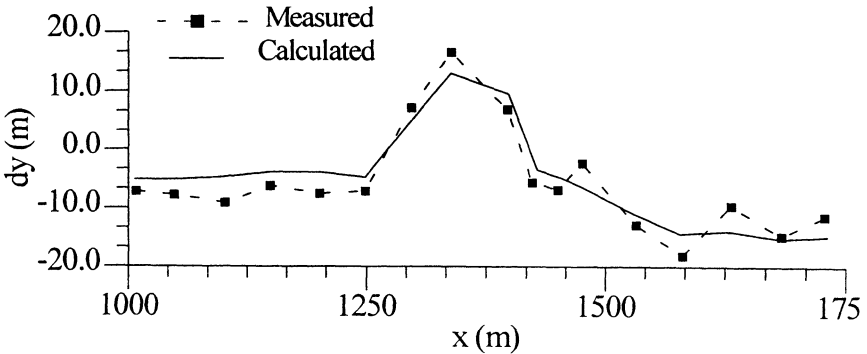measurements (initial shoreline position at y=0).

Figure 2. Total erosion and accretion (dy) over the period 1994-1997 (de la
Pena[17]): Model results in comparison with field measurements behind the
detached breakwater.

The models were also applied to estimate the shoreline change at the
Malagueta beach (Spain). The area behind the detached breakwater was chosen
for comparison. The breakwater is placed approximately at the depth of 5 m
(d=5 m) from x=1300m to x=1500m. The main incident directions are: E, ES,
SE, S and SW. From the available wave data different wave heights and periods
were derived. The grain size is about 0.4mm (d_{50}=0.4 mm). Model results over
the period 1994-1997, in comparison with field measurements are shown in Fig.
2. This application verifies the inclusion of the cross-shore effects through the

last term of the eqn (19) since the total lost of sediment to deeper water is estimated to result erosion of order 10 m.

6. Conclusion

Based on the results of the wave and the wave-induced circulation models shoreline changes can be simulated well using a simple one-line model and incorporating the cross-shore related seasonal shoreline variation.

Acknowledgements

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