NUMERICAL SIMULATION OF PARTICLE TRAJECTORIES IN INHOMOGENEOUS TURBULENCE, II: SYSTEMS WITH VARIABLE TURBULENT VELOCITY SCALE

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(Received in final form 12 March, 1981)

Abstract. It is shown that for the purpose of trajectory simulation, the vertical velocity $w_L(t)$ of a fluid element, which is moving in a system (such as a forest canopy, or the unstably stratified atmospheric surface layer) whose turbulent velocity scale σ_w is height-dependent, must be chosen from a frequency-distribution which is asymmetric about $w_L = 0$. If the gradient $\partial\sigma_w/\partial z$ varies only slowly with height, correct trajectories may be obtained by adding a bias $\bar{w}_L = \Lambda_L\, \partial\sigma_w/\partial z$ (where Λ_L is the length scale) to a fluctuating velocity chosen from a symmetric distribution with variance $\sigma_w^2(z)$.

1. Introduction

Atmospheric turbulence close to the ground is always inhomogeneous. Even when there is horizontal homogeneity, the Eulerian scales of the turbulence still exhibit height dependence. This paper is concerned with simulation of particle trajectories in systems in which the Eulerian turbulent velocity scale σ_w (root-mean-square vertical velocity) is height-dependent. Particular examples of such systems are the unstably stratified atmospheric surface layer, and the turbulent motion within an extensive plant or forest canopy.

While the results of this work are applicable to particle motion within the unstable surface layer, the aim was to develop a means of trajectory-simulation within a canopy, with such applications as disease-spore, pollen, and vapor transport in mind. The complex distribution of sources and sinks within a canopy may lead to concentration gradients which change significantly over a distance of the order of the turbulent length scale (Λ_L), invalidating the use of K-theory to relate the turbulent fluxes to average concentration gradients (Tennekes and Lumley, 1972; Corrsin, 1974). Furthermore, it follows from the work of Taylor (1921) that close to a source the effective eddy diffusivity K is a property not only of the turbulence, but also of the distance from the source. Therefore although one may always define an eddy diffusivity by dividing a measured flux by a measured concentration gradient, in a canopy it may be impossible to relate the number thus obtained to properties of the turbulence. A trajectory-simulation approach to dispersion in such systems eliminates the necessity of an a-priori assumption of the relationship between fluxes and gradients.

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Boundary-Layer Meteorology **21** (1981) 423–441. 0006–8314/81/0214–0423$02.85.

In an earlier paper, Wilson *et al.* (1981) described a means of simulation of trajectories in systems in which σ_w is constant. This model will be briefly reviewed and modified to incorporate the effect of height-dependence of σ_w. It is then shown that in order to obtain physically reasonable concentration profiles in a system with variable velocity scale, one must choose the fluctuating Lagrangian vertical velocity from an asymmetric frequency distribution. This may be achieved by adding a bias to a velocity chosen from a symmetric distribution. It is shown that incorporation of a bias velocity

$$\bar{w}_L = \Lambda_L \partial\sigma_w/\partial z$$

leads to concentration distributions in agreement with analytical solutions in systems in which at each z the gradient $\partial\sigma_w/\partial z$ is approximately constant over a distance of several length scales. This restriction on the gradient of σ_w is not satisfied in all systems. In particular, it is not satisfied within a corn canopy, and it is shown that application of the above bias does not lead to correct particle trajectories within a corn canopy.

2. The Trajectory-Simulation Model

Consider a two-dimensional system in which the Eulerian horizontal (x) velocity u is steady and depends only on the height (z), and the Eulerian vertical velocity w is unsteady (turbulent) with a time average value of zero. The Eulerian velocity scale σ_w is constant. The Eulerian time scale is defined by

$$\tau(z) = \int_0^\infty \overline{w(z,t)\, w(z, t + \xi)}\, \mathrm{d}\xi/\sigma_w^2$$

where the overbar denotes a time average. $\tau(z)$ is height-dependent, as is the Eulerian length scale $\Lambda = \sigma_w \tau(z)$.

Wilson *et al.* (1981) showed that each step in a fluid element trajectory in such a system may be calculated using

$$\begin{aligned} \Delta z &= w_L(t_H) \frac{\tau_L(z)}{\tau_L(H)} \Delta t_H \\ &= \frac{\tau_L(z)}{\tau_L(H)} \Delta z_* \\ \Delta z_* &= w_L(t_H) \Delta t_H \\ \Delta x &= u(z) \frac{\tau_L(z)}{\tau_L(H)} \Delta t_H . \end{aligned} \tag{1}$$

Here τ_L is the Lagrangian timescale, a measure of the persistence of the vertical velocity of a marked fluid element, and is assumed to be closely related to the Eulerian timescale, $\tau_L(z) \propto \tau(z)$. t_H is a transformed time, related to real time t by

$$\frac{dt_H}{\tau_L(H)} = \frac{dt}{\tau_L(z)} \tag{2}$$

and z_* is a transformed height axis. To each z_* there corresponds a unique value of z. The fluid element trajectories are calculated in the (x, z_*, t_H) system, in which the scale of the turbulent motion is independent of position. $w_L(t_H)$ is a record of vertical velocity appropriate to a fluid element moving at the reference height $z = H$, having time scale $\tau_L(H)$. This was obtained by applying a single-stage low-pass RC filter to the output from a Hewlett-Packard* random noise generator. The probability density function for $w_L(t_H)$ was approximately Gaussian, with zero mean.

In the case where σ_w is height-dependent, it was expected that satisfactory trajectories would be obtained by writing

$$\sigma_L(z(t)) = \sigma_w(z)$$

and

$$\begin{aligned}\Delta z &= w_L(t_H)\frac{\sigma_w(z)}{\sigma_w(H)}\frac{\tau_L(z)}{\tau_L(H)}\Delta t_H \\ &= \frac{\Lambda_L(z)}{\Lambda_L(H)}\Delta z_* \\ \Delta z_* &= w_L(t_H)\Delta t_H \\ \Delta x &= u(z)\frac{\tau_L(z)}{\tau_L(H)}\Delta t_H.\end{aligned} \tag{3}$$

$w_L(t_H)$ has time scale $\tau_L(H)$ and velocity scale $\sigma_w(H)$, and is again obtained by applying a low-pass filter to the output of a random noise generator (frequency distribution approximately Gaussian with zero mean). This method would satisfy the constraints:

(i) That if a particle travelled a distance δX without moving far from height z, the ratio of travel time to time scale

$$\frac{\delta X/u(z)}{\tau_L(z)},$$

which is a measure of the number of independent velocity 'choices', is preserved in the simulation.

(ii) The typical magnitude of Δz is

$$\Delta z \simeq \sigma_w(H)\frac{\sigma_w(z)}{\sigma_w(H)}\Delta t = \sigma_w(z)\Delta t.$$

These were believed to be the most important factors. Equations (3) are compatible with Batchelor's (1957) hypothesis of Lagrangian similarity: $w_L(t_H)$ is a stationary random function of transformed time t_H.

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With this method of calculating fluid element trajectories, it is possible to perform ensemble experiments as described in *I*. The z_* axis is divided into 200 layers of depth Δz_* with midpoints $z_*(I) = (I{-}0.5)\Delta z_*$. Prior to calculation of a set of trajectories, the $z(I)$ corresponding to each $z_*(I)$ are calculated by integrating the relationship

$$\mathrm{d}z = \frac{\Lambda_L(z)}{\Lambda_L(H)}\,\mathrm{d}z_*.$$

In all systems herein, the assumed functional dependence of Λ_L on z led to a simple explicit relationship between z and z_*. For each I, the values of $\Delta x(z(I))$ are tabulated, and each step in a trajectory is given by

$$\Delta z_* = w_L(t_H)\Delta t_H$$

$$\Delta x = \Delta x(I)$$

where I is obtained from the instantaneous value of z_*.

3. Demonstration of the Need for a Bias in the Lagrangian Vertical Velocity

Consider a continuous plane source of strength 1 [marked fluid element per cm of length per cm of crosswind extent per second] at a height zs above a reflecting surface. The source has very long upwind extent, XM. Let σ_w be constant for all $z \geq zs$, and reference height $H = zs$. This will be abbreviated the LFEPS (long-fetch elevated plane source) situation.

The ensemble experiment consists of releasing NP planes to obtain a count of $N(I)$ particles passing the collector (X) in the layer

$$(I - 1)\Delta z_* \leq z_* \leq I\Delta z_*.$$

The quantity $[N(I)/NP]$ is then the average number of particles collected in the Ith layer in 1 second, whence

$$\frac{N(I)}{NP} = F_x(X, z)\Delta z = F_{*x}(X, z_*)\Delta z_*.$$

Here $\Delta z = [\Lambda_L(z)]/[\Lambda_L(H)]\Delta z_*$ and F_x and F_{*x} are the horizontal flux densities in the z and z_* systems,

$$F_x = c(X, z)u(z)$$

$$F_{*x} = c_*(X, z_*)v_*(z_*)$$

where $v_*(z_*) = u(z)\,\tau_L(z)/\tau_L(H)$.

It follows that concentrations in the two systems, c and c_*, are related by

$$c_*(X, z_*) = c(X, z(z_*))\frac{\sigma_w(z)}{\sigma_w(H)}.$$

What does this imply with respect to the outcome of the LFEPS experiment? The turbulent motions in the z_* system have scales independent of height – we therefore have homogeneous turbulence, with wind shear. Figure 1 shows (schematically)

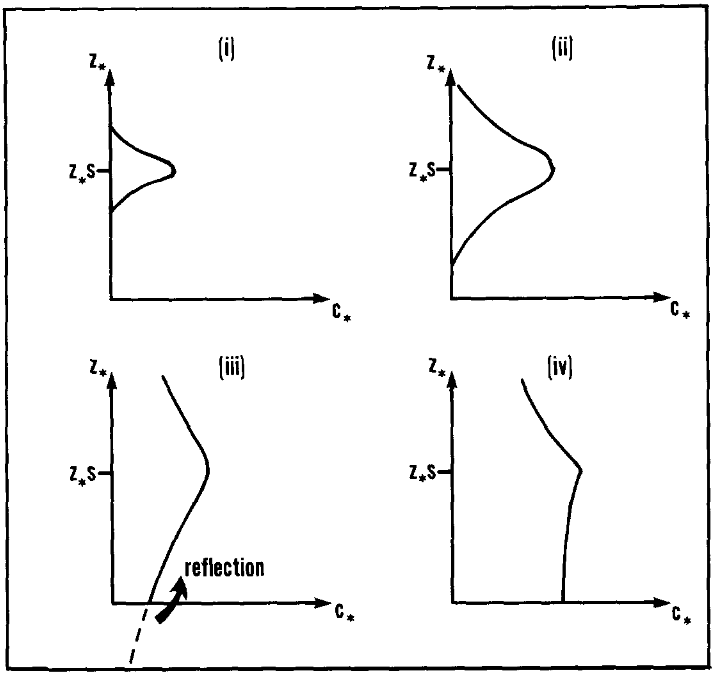

Fig. 1. Schematic diagram of development of the concentration profile with increasing distance from the leading edge of an elevated continuous plane source.
(i) Very near the leading edge. (ii) Farther from the leading edge, presence of surface not yet influential. (iii) Reflection (folding) begins. (iv) Very far from the leading edge. Very small vertical flux and concentration gradient below the source.

the development of the profile of c_* at increasing distances from the leading edge of the source. As the distance from the leading edge becomes very large, the vertical flux below the source becomes small and the concentration gradient below the source becomes small and positive.

$$\frac{\partial c_*}{\partial z_*} \gtrsim 0 \quad \text{for} \quad z_* < z_*s.$$

It follows that

$$\frac{\partial c}{\partial z} \propto -\frac{\partial \sigma_w}{\partial z} \quad \text{for} \quad z < zs.$$

The concentration gradient below the source in the z system is negative (for $\partial\sigma_w/\partial z > 0$). This argument predicts that if we calculate particle trajectories in a system with σ_w height-dependent using Equations (3), then where the vertical flux is zero, the simulation will give rise to a non-zero concentration gradient such that

$$c(z)\sigma_w(z) = \text{constant}.$$

In fact, this was found to be the case. The curve labelled $\bar{w}_L = 0$ in Figure 7 gives concentration predicted by Equations (3) for diffusion from a plane source 320 m in upwind extent at the top of a horizontally homogeneous canopy of corn, with surface reflection (LFEPS case).

Ward (1977) found $\sigma_w \tau = 0.1\ z$ in and above a corn canopy, calculating τ from a measured autocorrelation function. For the simulation we chose the Lagrangian length scale to be $\Lambda_L = 0.5\ z$ (but the conclusions reached from the following work do not depend on the accuracy of this choice). The profiles of σ_w and windspeed used were

$$\begin{aligned} \sigma_w &= 72\,(0.21 + 0.79\exp(-(z - 210)^2/7200)) && z < 210 \\ &= 72 && z \geq 210 \end{aligned}$$

$$\begin{aligned} u &= 220\,(0.1 + 0.81z/230)\exp(3.1(z/230 - 1)) && z < 230 \\ &= 199 + 286\ln(z/230) && 230 \leq z \leq 300 \\ &= 67/.4\ln(z - 145)/30 && z > 300 \end{aligned}$$

These profiles were obtained from measurements of $\sigma_w(z)$, $u(z)$ by Wilson (1980). The source height was chosen as $zs = 230$ cm, and the source strength was 1 cm^{-2} s^{-1}. The trajectories were calculated without interaction with the vegetation, and are thus applicable to a region with the given turbulence and wind profile, but no vegetation.

As expected from the argument above, below the source the trajectory simulation predicts a physically unreasonable concentration gradient ($\partial c/\partial z < 0$) and the concentration obeys $c(z)\sigma_w(z) = $ constant. The same prediction for this LFEPS case is obtained if the vertical steps are calculated using

$$\Delta z = \frac{\tau_L(z)}{\tau_L(H)}\Delta z_*$$

$$\Delta z_* = \frac{\sigma_w(z)}{\sigma_w(H)} w_L(t_H)\Delta t_H,$$

i.e., if the ratio $\sigma_w(z)/\sigma_w(H)$ is not included in the relationship between z and z_*.

Now consider the air within the canopy. Continuity requires zero net flux of air out of the canopy; according to Equations (3) we therefore must have

$$\rho\sigma_w(z) = \text{constant}$$

(where ρ is the density of the air) because the (unmarked) fluid elements are all participating in turbulent motion. The density, ρ, at the bottom of the canopy is predicted to be about five times greater than at the top of the canopy. This violates the ideal gas law, which constrains the density of the air to be height-independent (ignoring hydrostatic lapse of pressure, and assuming isothermal conditions). It is therefore concluded that it is not correct to incorporate the effect of height-dependence of σ_w simply by changing the magnitudes of randomly chosen unbiased velocities in accordance with position. What are the implications of a gradient in σ_w?

(i) In the neutral surface layer σ_w is independent of height except very close to the surface, and fluid element trajectories can be successfully calculated by assuming a discontinuity in σ_w at $z = 0$ at which total reflection occurs, a bias in the velocity choice of each individual fluid element at the discontinuity. If a discontinuity in σ_w at z causes a bias in each individual particle velocity choice at z, perhaps a gradient $(\partial\sigma_w/\partial z)$ between z_2 and z_1 implies a bias in the velocity record of a fluid element between z_2 and z_1.

(ii) The possibility that the Eulerian velocity distribution is skewed should be considered. In a region where $\partial\sigma_w/\partial z > 0$, would we expect that fluid elements crossing z in the downward direction, having some memory of their height of origin, would travel faster than those travelling upward across z and having a memory of being at lower heights? If this were the case, then we could say that a large area A at z consists of two parts,

$$A\uparrow + A\downarrow = A$$

and that continuity is satisfied by saying

$$A\uparrow w\uparrow + A\downarrow w\downarrow = 0$$

with $|w\downarrow| > w\uparrow$. This corresponds to a large proportion of the area at z moving up slowly, and a small proportion moving down quickly. If we now placed one marked fluid element per unit area on A, a small number would move down quickly, a larger number up slowly. This implies a preference for upward movement even for a single marked fluid element.

(iii) Finally it is worthwhile to question whether the constraint of constant pressure has some special implication for turbulent motion in a system with variable σ_w. The absence or presence of a pressure constraint is of importance in the problem of molecular diffusion in the presence of a temperature gradient. If two chambers (A, B) held at temperatures T_A, T_B are separated by a porous plug whose pores are of a characteristic size much smaller than the mean free path in the gas, it is observed (Reynolds, 1879) that for gas at sufficiently low density the equilibrium density distribution is

$$\rho_A\sqrt{T_A} = \rho_B\sqrt{T_B},$$

i.e.,

$$\rho_A\sigma_A = \rho_B\sigma_B$$

where σ_A, σ_B, the root-mean-square molecular velocities, depend on T_A, T_B ($T_A \propto \sigma_A^2$). This density distribution arises because the small pore size in relation to mean free path causes a very high resistance to hydrodynamic flow, and exchange between the two chambers occurs by 'effusion'; for low density in B, the escape of molecules from A through a small pore is equivalent to escape into a vacuum, and the mass per unit area per unit time escaping is therefore proportional to $\rho\sigma$ (Jeans, 1959). However as the density of the gas is increased, the occurrence of collisions reduces the rate of efflux. Alternatively, if the density remains very low but the pore size is increased, hydrodynamic flow of the gas occurs, causing the equilibrium distribution to become

$$p_A = p_B$$

i.e.,

$$\rho_A \sigma_A^2 = \rho_B \sigma_B^2.$$

When the constant-pressure constraint becomes effective, it alters the equilibrium concentration distribution.

It is hoped that this discussion has demonstrated the difficulties which a gradient in σ_w presents. Our understanding of the implication of the changing velocity scale for fluid element trajectories is incomplete, but it seems certain that trajectories must be biased towards the direction of larger values of σ_w.

4. Biasing Trajectories by Adding a Mean Velocity

The addition of a bias velocity suggests itself on consideration of the relationship between the vertical fluxes in the (z_*, x, t_H) and (z, x, t) systems. In the z_* system the turbulence is homogeneous and we may form a position-independent eddy diffusivity

$$K_* = \sigma_w^2(H)\tau_L(H).$$

Whence

$$\begin{aligned} F_{*z} &= -K_* \frac{\partial c_*}{\partial z_*} \\ &= -\sigma_w^2(H)\tau_L(H)\left[\frac{\partial}{\partial z}\left(c\frac{\sigma_w(z)}{\sigma_w(H)}\right)\right]\frac{\partial z}{\partial z_*} \\ &= -K\frac{\partial c}{\partial z} - c\bar{w}_L \end{aligned}$$

where

$$\bar{w}_L = \Lambda_L \frac{\partial \sigma_w}{\partial z} \quad \text{and} \quad K = \sigma_w^2(z)\tau_L(z).$$

This implies that when $F_{*z} = 0$, we will have non-zero concentration gradient in the real world. If we add to both sides the term

$$c_* \sigma_w(H)\tau_L(z)\frac{\partial \sigma_w}{\partial z} \equiv c\sigma_w(z)\tau_L(z)\frac{\partial \sigma_w}{\partial z},$$

then

$$F'_{*z} = -K_* \frac{\partial c_*}{\partial z_*} + \left[\sigma_w(H)\tau_L(z)\frac{\partial \sigma_w}{\partial z} \right] c_*$$
$$= -K\frac{\partial c}{\partial z}.$$

Therefore if we add a velocity

$$\bar{w}_{*L} = \sigma_w(H)\tau_L(z)\frac{\partial \sigma_w}{\partial z}$$

to the turbulent component $w_L(t_H)$ in the z_* system, then when the resulting flux F'_{*z} is zero, we have

$$\frac{\partial c_*}{\partial z_*} > 0 \qquad \left(\text{for } \frac{\partial \sigma_w}{\partial z} > 0\right)$$

and

$$\frac{\partial c}{\partial z} = 0.$$

How can we rationalize the idea of adding a bias $w_L = \Lambda_L\, \partial\sigma_w/\partial z$ to fluid element trajectories? Taking the example of a forest canopy, perhaps the random motion is acting to build up the density at the bottom of the canopy but the pressure constraint opposes the effect of the random motion and ensures that ρ = constant.

It is hypothesized that for a system in purely random turbulent motion, without a pressure constraint, we may relate the flux to the mean gradient by

$$F_z = -\Lambda_L \frac{\partial(c\sigma_w)}{\partial z}$$
$$= -\sigma_w\Lambda_L\frac{\partial c}{\partial z} - c\left(\Lambda_L\frac{\partial \sigma_w}{\partial z}\right)$$

as long as $(\partial c\sigma_w)/\partial z$ changes only slowly over a length scale.

(a) if σ_w = constant, we recover the flux-mean gradient expression.

(b) if $\sigma_w = \sigma_w(z)$, we obtain $(\partial c/\partial z) \neq 0$ for $F_z = 0$, the solution appropriate in the absence of a pressure constraint.

(c) in a system with $\sigma_w = \sigma_w(z)$ and in which there is a constant-pressure constraint, there exists a bias $\bar{w}_L = \Lambda_L(\partial\sigma_w/\partial z)$ in the fluid element trajectories, and

$$F_z = -\Lambda_L\frac{\partial(c\sigma_w)}{\partial z} + c\left(\Lambda_L\frac{\partial \sigma_w}{\partial z}\right)$$
$$= -K\frac{\partial c}{\partial z}.$$

This is the solution appropriate in a forest or corn canopy.

Following this hypothesis, if we want to simulate particle trajectories where $(\partial\sigma_w/\partial z) \neq 0$ and there is a pressure constraint, we must bias the trajectories with a temporally constant velocity

$$\bar{w}_L = \Lambda_L \frac{\partial\sigma_w}{\partial z}$$

(whose equivalent in the z_* system is

$$\bar{w}_{*L} = \sigma_w(H)\tau_L(z)\frac{\partial\sigma_w}{\partial z}).$$

The appearance of only the first derivative of σ_w suggests that this approach may only work for systems in which $(\partial\sigma_w/\partial z)$ changes very little over a distance of several length scales centred at the height (z_1) at which we are estimating $\bar{w}_L$. This restraint may be written

$$\Lambda_L(z)\left|\left(\frac{\partial^2\sigma_w}{\partial z^2}\right)_z\right| \ll \left|\left(\frac{\partial\sigma_w}{\partial z}\right)_{z_1}\right|$$

for $z_1 - 3\Lambda_L(z_1) \leq z \leq z_1 + 3\Lambda_L(z_1)$.

This is certainly not satisfied if, for any z,

$$\Lambda_L \not< \left|\frac{\partial\sigma_w/\partial z}{\partial^2\sigma_w/\partial z^2}\right| = S(z).$$

The equations governing the trajectories now become

$$\begin{aligned}
\Delta z &= [w_L(t_H) + \bar{w}_{*L}]\frac{\sigma_w(z)}{\sigma_w(H)}\frac{\tau_L(z)}{\tau_L(H)}\Delta t_H \\
&= \frac{\Lambda_L(z)}{\Lambda_L(H)}\Delta z_* \\
\Delta z_* &= [w_L(t_H) + \bar{w}_{*L}]\,\Delta t_H \\
\Delta x &= u(z)\frac{\tau_L(z)}{\tau_L(H)}\Delta t_H.
\end{aligned} \tag{4}$$

The results of ensemble experiments performed using Equations (4) will now be presented. Firstly, a set of artificial systems in which analytical solutions are available will be considered. Secondly, predictions of diffusion from a source within a corn canopy will be presented and discussed.

4.1. Comparison with Analytical Solutions

Philip (1959) gave an analytical solution to the diffusion equation

$$u\frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left(K\frac{\partial c}{\partial z}\right)$$

when

$$u = u(H)\left(\frac{z}{H}\right)^m \qquad K = K(H)\left(\frac{z}{H}\right)^n$$

with the flux boundary condition

$$\left(-K\frac{\partial c}{\partial z}\right) = 1 \quad \text{for} \quad z = 0, x > 0$$
$$= 0 \quad \text{for} \quad z = 0, x \leq 0.$$

This solution has been compared with the concentration profiles predicted by Equations (4) for a continuous plane source at $z = 0$, 320 m in upwind extent. Profiles of $\tau_L(z)$, $\sigma_w(z)$

$$\tau_L(z) = \tau_L(H)\left(\frac{z}{H}\right)^{n_1}$$
$$\sigma_w(z) = \sigma_w(H)\left(\frac{z}{H}\right)^{n_2}$$

are taken to imply that

$$K(z) = K(H)\left(\frac{z}{H}\right)^{2n_2+n_1}$$

Comparison with a K-theory method is felt to be justified in this case because:

(i) The source is at $z = 0$, where the length and time scales become vanishingly small. Therefore the vast majority of material seen at the observation point has had a long diffusion time with respect to the time scale at the source.

(ii) Particularly for a long distance from the leading edge, one would expect the concentration gradient to change only slowly along the vertical axis.

In all the following comparisons

$$H = 200 \text{ cm}$$
$$u(H) = \sigma_w(H) = 100 \text{ cm s}^{-1}$$
$$\tau_L(H) = 1 \text{ s}.$$

TABLE I

Parameters for each comparison of the trajectory-simulation method with Philip's analytical solution. (m, n_2, n_3) are the exponents in the power-law (wind, velocity scale, time scale) profiles

Figure	m	n_2	n_1	$\Lambda_L(z)$	S	$\bar{w}_L$ (cm s^{-1} for z in cm)	$\Lambda_L < S$
2	1	1/2	1/2	$0.5\, z$	$2\, z$	$1.77\, z^{1/2}$	for all z
3	1	1	0	$0.5\, z$	∞	$0.25\, z$	for all z
4	1	1/4	0	$27\, z^{1/4}$	$4/3\, z$	$177\, z^{-1/2}$	$z > 55$ cm
5	1	$-1/10$	1/2	$12\, z^{4/10}$	$0.91\, z$	$-204\, z^{-7/10}$	$z > 74$ cm
6	1	1/2	$-1/10$	$12\, z^{4/10}$	$2\, z$	$42\, z^{-1/10}$	$z > 20$ cm

Table I presents the chosen profiles for each comparison, and the corresponding

$$\bar{w}_L = \Lambda_L \frac{\partial \sigma_w}{\partial z}$$
$$S = \left|(\partial \sigma_w/\partial z)/(\partial^2 \sigma_w/\partial z^2)\right|.$$

Figures 2–6, show that in those systems in which

$$\Lambda_L(z) < S(z) \quad \text{for all} \quad z$$

the concentration profiles calculated by the trajectory method using Equations (4) agree precisely with solutions to the diffusion equation. In cases where the inequality above is not satisfied, there is a small difference between the results. In Figure 3 the

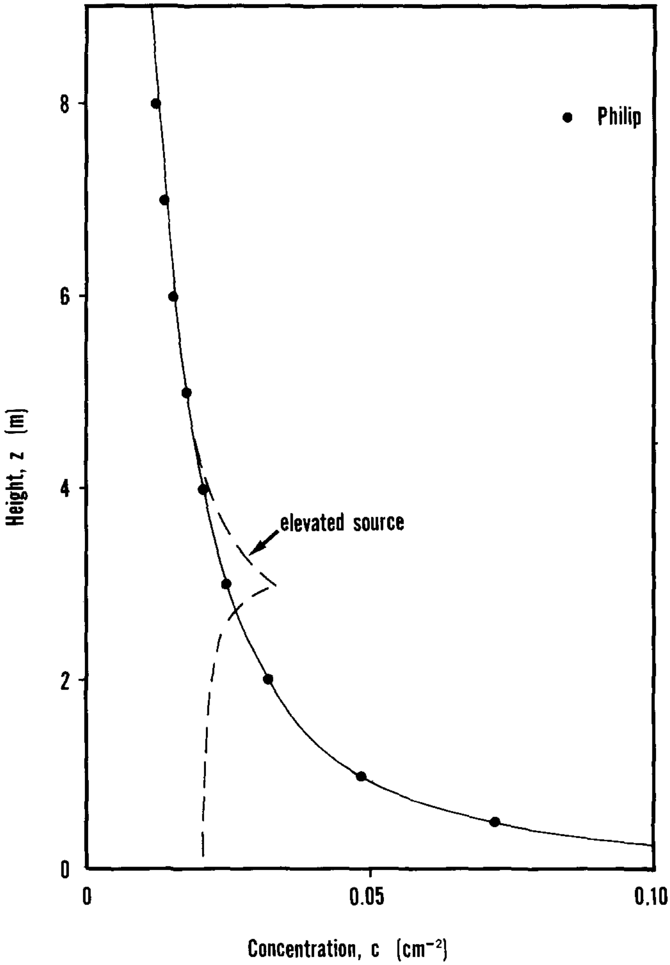

Fig. 2. Prediction of the trajectory-simulation method, and Philip's solution for the concentration profile at the downwind edge of a 320 m long continuous plane source at ground in turbulence with $\sigma_w \propto z^{1/2}$, $\tau \propto z^{1/2}$, $u \propto z$.

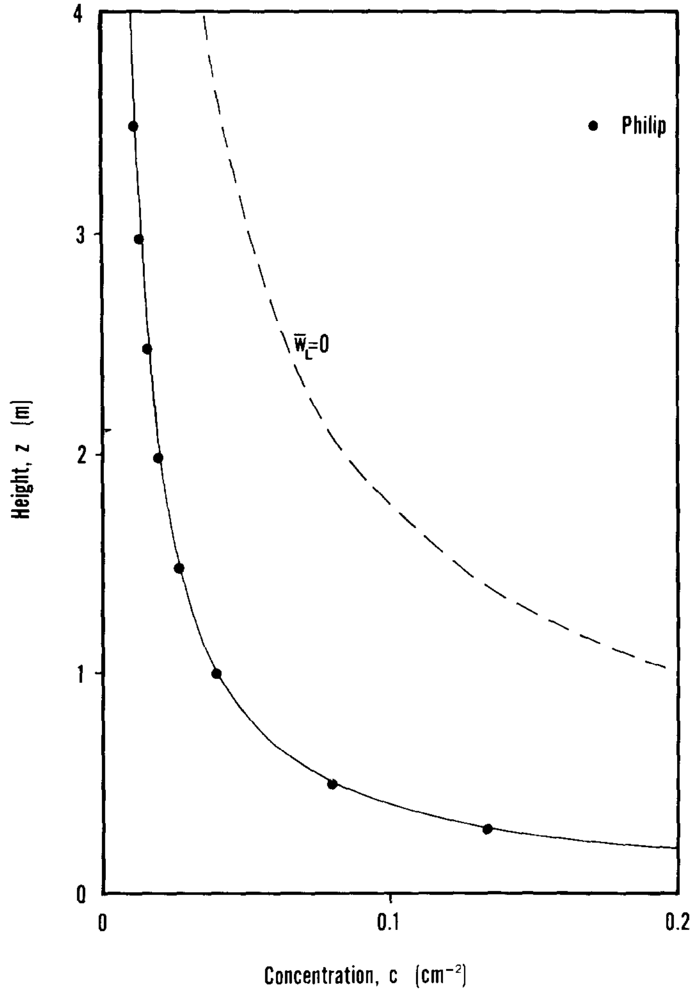

Fig. 3. Prediction of the trajectory-simulation method and Philip's solution for the concentration profile at the downwind edge of a 320 m long plane source at ground in turbulence with $\sigma_w \propto z$, $\tau =$ constant, $u \propto z$. The dashed curve is the prediction without incorporation of the bias velocity to compensate for the gradient in σ_w.

prediction with $\bar{w}_L = 0$ is plotted (dashed curve) in order to show the dramatic alteration the bias introduces. The dashed curve in Figure 2 shows the concentration predicted by the trajectory-simulation method for an elevated plane source, with all parameters (except source height) unchanged. No analytical solution is available. However the prediction agrees with the intuitive expectation of the effect of increasing the source height. It should be remembered in comparing this solution with the result for the ground-level source that the quantity

$$\int_0^\infty c(z)u(z)\,\mathrm{d}z$$

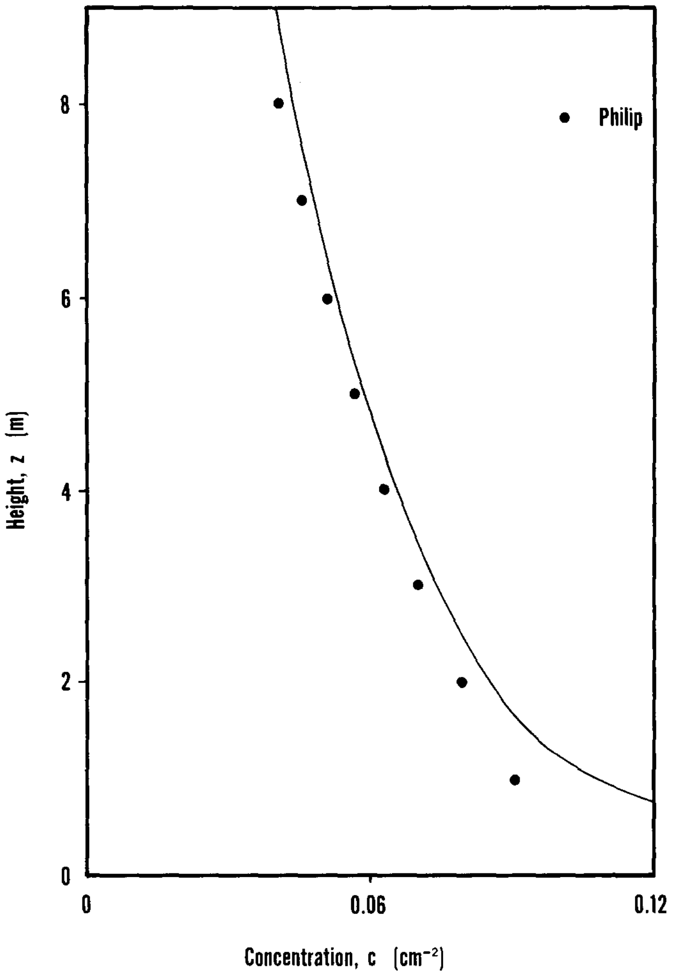

Fig. 4. Prediction of the trajectory-simulation method and Philip's solution for the concentration profile at the downwind edge of a 320 m long plane source at ground in turbulence with $\sigma_w \propto z^{1/4}$, τ = constant, $u \propto z$.

is a constant depending only on the length of the source, but there is no such restriction on

$$\int_0^\infty c(z)\,\mathrm{d}z.$$

4.2. Predictions of diffusion within a corn canopy

Equations (4) have also been used to calculate fluid element trajectories within a corn canopy. The windspeed, length scale, and velocity scale profiles used were given in Section 3. The offset velocity $\bar{w}_L$ was derived from the chosen length and

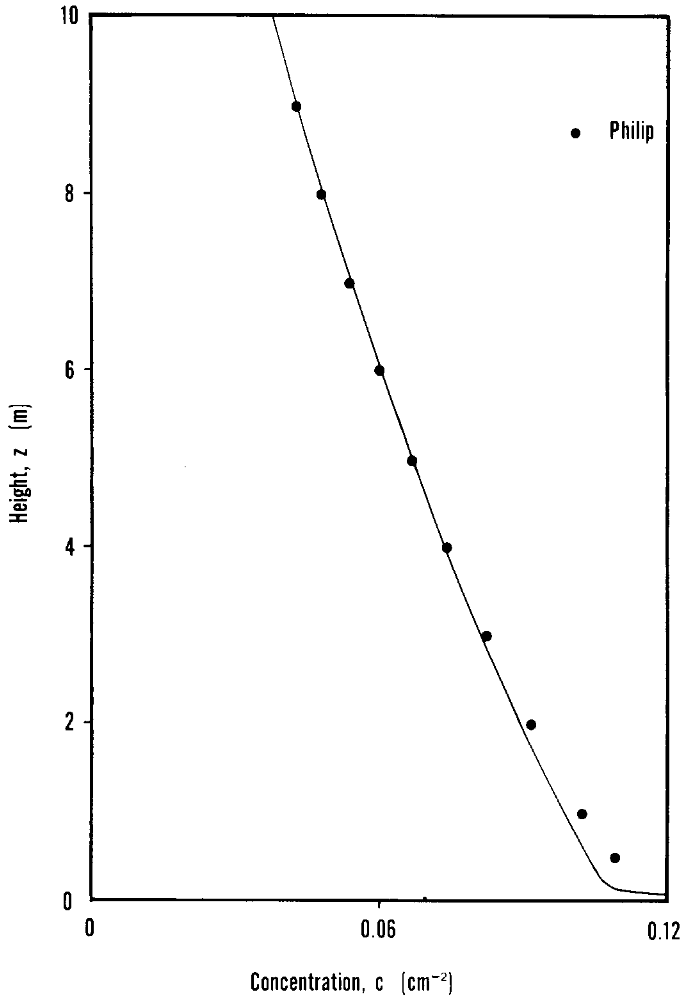

Fig. 5. Prediction of the trajectory-simulation method and Philip's solution for the concentration profile at the downwind edge of a 320 m long plane source at ground in turbulence with $\sigma_w \propto z^{-0.1}$, $\tau \propto z^{1/2}$, $u \propto z$.

velocity scales. The gradient in σ_w changes very rapidly near the top of the canopy, and it may be shown that the restriction $\Lambda_L < S$ is not everywhere satisfied.

Figure 7 shows the predicted concentration profile for a plane source of strength 1 cm^{-2} s^{-1} extending 320 m upstream of the observation point, with source height $zs = 230$ cm, and with surface reflection (LFEPS case). The concentration gradient below the source is incorrect, and this was found not to be a consequence of insufficient fetch. Also included in Figure 7 is the profile predicted using Equations (4) with $\bar{w}_L = 0$, (i.e., in effect using Equations (3)).

If τ'_L is defined by

$$\sigma_w^2(zs)\tau'_L = \sigma_w^2(z)\tau_L(z),$$

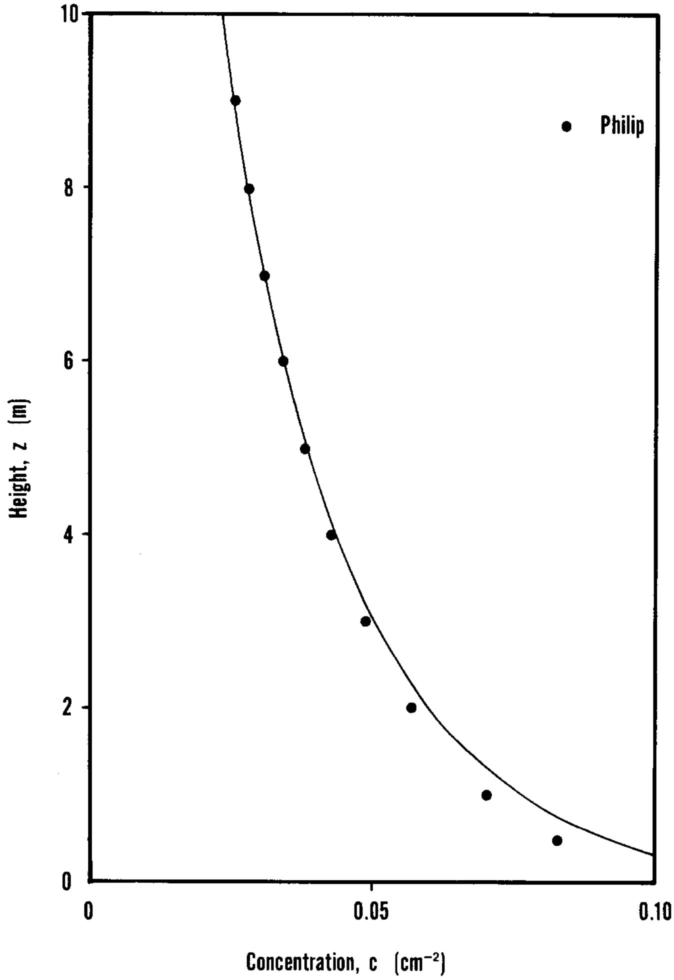

Fig. 6. Prediction of the trajectory-simulation method and Philip's solution for the concentration profile at the downwind edge of a 320 m long plane source at ground in turbulence with $\sigma_w \propto z^{1/2}$, $\tau \propto z^{-0.1}$, $u \propto z$.

one may calculate trajectories in the system

$$\sigma_w = \text{constant} = \sigma_w(zs)$$

$$\tau'_L(z) = \tau_L(z)\left(\frac{\sigma_w(z)}{\sigma_w(zs)}\right)^2$$

$$u(z) = \text{observed profile}$$

using the equations appropriate to constant-σ_w systems, Equations (2). In this latter system, the 'artificial canopy', the eddy diffusivity is everywhere equal to that in the real canopy, but the length, time, and velocity scales are individually correct only at the source height: we therefore would expect quite different trajectories in

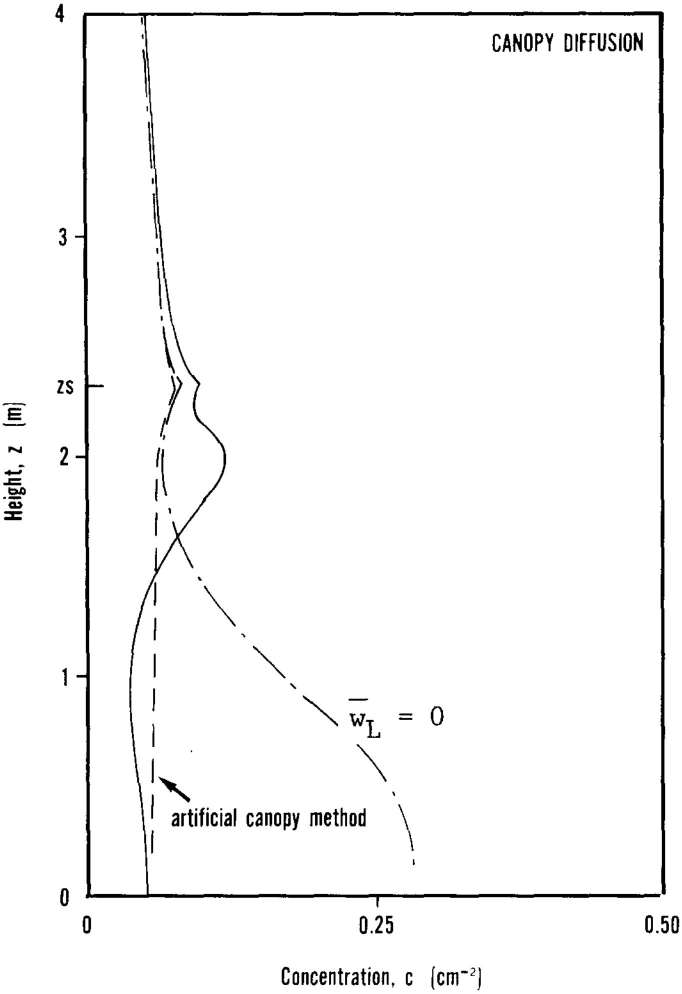

Fig. 7. Predictions of the trajectory-simulation method for the concentration profile at the downwind edge of a 320 m long continuous plane source above a corn canopy. Solid line obtained by biasing the vertical velocity with a mean velocity $\bar{w}_L = \Lambda_L \, \partial\sigma_w/\partial z$; dot-dashed line with $\bar{w}_L = 0$; dashed line obtained using correct $\sigma_w^2\tau_L$ at all heights, but with $\sigma_w = \sigma_w(zs)$ for all z and $\tau'_L = \tau_L\sigma_w^2/\sigma_w^2(zs)$.

the artificial and real canopies. If the only important parameter for the diffusion was $K = \sigma_w^2(z)\tau_L(z)$, then this method should give the correct solution. In view of the elevated source height ($\tau_L(zs) \neq 0$), there is no sound reason to believe it is the correct solution in this case, but it is included in Figure 7 for comparison.

Figure 8 shows predicted concentration in the canopy at the downwind edge of a ground level plane source, 320 m in length, and of source strength 1 cm^{-2} s^{-1}, using Equations (4). The dashed curve was generated by integrating the relationship

$$\frac{\Delta c}{\Delta z} = -\frac{F_z(z)}{\sigma_w^2(z)\tau_L(z)}$$

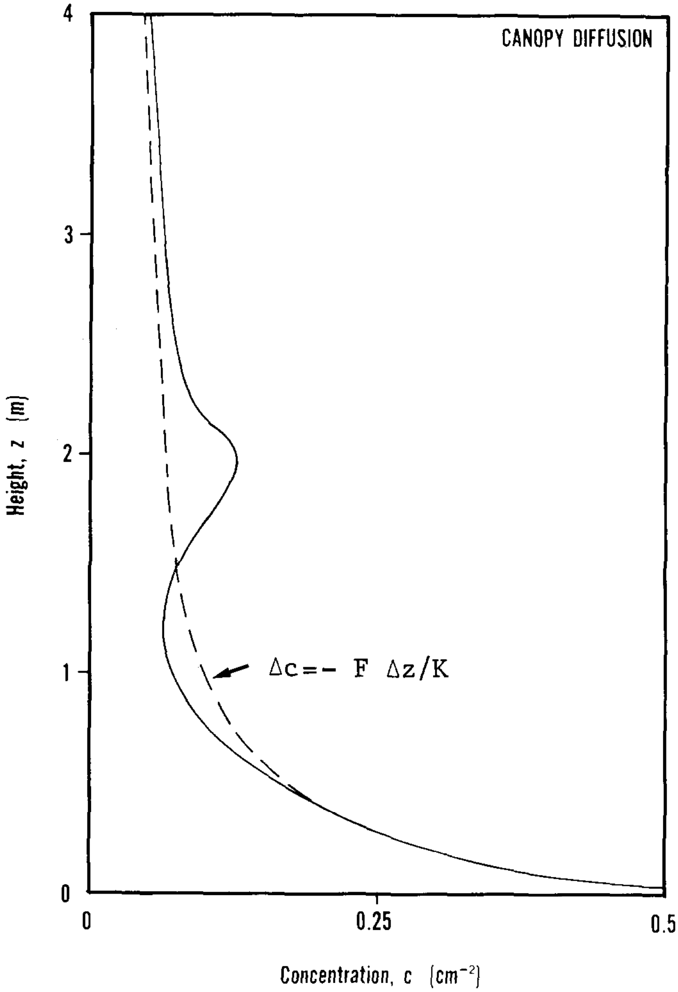

Fig. 8. Prediction of the trajectory-simulation method for the concentration profile at the downwind edge of a 320 m long plane source at ground in a corn canopy. The dashed line is a K-theory solution.

with a starting point on the profile predicted by the trajectory method, and using $F_z(z) = 1$ (according to the trajectory method, $F_z > 0.99$ for $z < 200$ cm). There is some basis for expecting this relationship to be correct because the source is at ground ($\tau_L(zs) \simeq 0$) and the fetch is very long.

Biasing trajectories by adding an offset

$$\bar{w}_L = \Lambda_L \partial\sigma_w/\partial z$$

does not lead to correct predictions of diffusion within the corn canopy. It seems reasonable to attribute this to the fact that the restriction $\Lambda_L < S$ is not obeyed by the turbulence scales within the canopy.

An alternative means of biasing particle trajectories is to reflect some proportion of downward-moving particles at each of a number of elevated planes. This approach is currently under investigation.

5. Conclusion

In a horizontally homogeneous system with σ_w = constant, the vertical velocity $w_L(t)$ of a marked fluid element may be chosen at random from a frequency distribution which has zero mean. When σ_w is height-dependent, the same approach leads to concentration distributions which are physically unreasonable, and it is suggested that in this case $w_L(t)$ is biased, and cannot be chosen from a distribution with zero mean.

In those systems investigated in which at each z the gradient $\partial\sigma_w/\partial z$ is approximately constant over several length scales (slowly varying gradient in σ_w), the addition of a bias velocity

$$\bar{w}_L = \Lambda_L \frac{\partial \sigma_w}{\partial z}$$

to $w_L(t)$ gave trajectories which lead to concentration distributions in agreement with analytical solutions. However, within a corn canopy the gradient in σ_w changes rapidly over a distance of one length scale, and application of the bias velocity did not lead to correct trajectories.

In a later paper predictions of the trajectory-simulation method are compared with experimental data for the atmospheric surface layer. The bias velocity given above has been incorporated in the case of highly unstable stratification.

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