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Numerical Solution of Integro-Differential Equation Using Adomian Decomposition and Variational Iteration Methods

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Abstract:*In this paper, a comparative study of Adomian Decomposition Method (ADM) and Variational Iteration Method (VIM) were considered on various types of integrodifferential equation; which are Fredholm, Volterra and Fredholm-Volterra equations. From the examples considered, it was observed that these methods were compared favorably with the exact solution. VIM has an advantage over ADM due to non-requirement of Adomian polynomial and hence converges faster to the exact solution for some nonlinear problems.*
Keywords:*Adomian decomposition method, Adomian polynomial, Integro-Differential equation, Lagragiam multiplier, variational iteration method.*

I. Introduction

Integrodifferential equations are encountered in various fields of sciences. It plays an important role in many branches of linear and non-linear functional analysis and their applications are in the theory of Sciences, Engineering and Social sciences.

Adomian Decomposition Method (ADM) was developed by Adomian in [1, 2] and used heavily in the literature [3-6] and the references therein. Meanwhile, Variational Iteration Method(VIM) as proposed in [7,8] which is a modified general Lagrange multiplier method [9] has been shown to solve effectively, easily and accurately, a large class of nonlinear problems with approximation which converges quickly. Wazwaz [10] compared both methods by solving advection problem, also Ganji et al.[11] solved some highly nonlinear equations using both methods by taking their comparison into account and very recently Siddiqui et al. [12] considered a comparative study of ADM and VIM in solving nonlinear thin film flow problems. In this study, we have applied a comparative study of ADM and VIM to find the approximate solution of various types of integro-differential equations. The methods were also compared with exact solution if it exist.

II. The Methods

2.1 Adomian decomposition method

Consider the differential equation of the form:

$$LU + RU + NU = g(x) \tag{1}$$

Where L is the linear operator which is highest order derivative, R is the remainder of linear operator including derivatives of less order than L , NU represents the non- linear terms and g is the source term. Equation (1) can be rearranged as

$$LU = g(x) - RU - NU \tag{2}$$

Applying the inverse operator L^{-1} to both side of equation (2) and employing given conditions we obtain

$$U = L^{-1}\{g(x)\} - L^{-1}(RU) - L^{-1}(NU) \tag{3}$$

After integrating source term and combining it with the terms arising from given conditions of the problem, a function $f(x)$ is defined in the equation

$$U = f(x) - L^{-1}(RU) - L^{-1}(NU) \tag{4}$$

The non- linear operator $NU = FU$ is represented by an infinite series of specifically generated Adomian polynomials for the specific non linearity. Assuming NU is analytic

$$F(U) = \sum_{k=0}^{\infty} A_k \tag{5}$$

Where $A_{K'S}$ are given as:

$$
\begin{aligned}
A_0 &= F(U_0) \\
A_1 &= U_1 F'(U_0) \\
A_2 &= U_2 F'(U_0) + \frac{1}{2!} U_1^2 F''(U_0) \\
A_3 &= U_3 F'(U_0) + U_1 U_2 F''(U_0) + \frac{1}{3!} U_1^3 F'''(U_0)
\end{aligned}
\tag{6}
$$

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The polynomial $A_{k's}$ are generated for all kinds of non-linearity so that they depend only on U_0 to $U_{K'S}$ by the following algorithm

$$A_k = \frac{1}{k!} \frac{d^k}{d\lambda^k} \left[N\left(\sum_{n=0}^{\infty} \lambda^n y_n \right) \right]_{\lambda=0} \tag{7}$$

Where λ is a parameter introduced for convenience

2.2 Variational iteration method

Inokuti [3] proposed a general Lagrange multiplier method for solving non-linear differential equations where he used non-linear operator of the form as follows

$$LU + NU = g(x) \tag{8}$$

Where L is a linear operator, N is a non- linear operator, $g(x)$ is a known analytic function and U is an unknown that to be determined.

The Inokuti method is modified by [13] which can be written as

$$U_{n+1}(x_0) = U_n(x_0) + \int_0^{x_0} \lambda \left(L\tilde{U}_n + N\tilde{U}_n - g \right) ds \tag{9}$$

Where U_0 is an initial approximation and $\tilde{U}_n$ is a restricted variation. For arbitrary x_0

$$U_{n+1}(x) = U_n(x) + \int_0^{x} \lambda \left(LU_n(s) + NU_n(s) - g(s) \right) ds \tag{10}$$

Equation (10) is called Variational Iteration Method (VIM). While the integral in equation (10) is called correctional function and index n denotes the nth approximation. With Lagrange multiplier λ defined by [14] as:

$$\lambda = \frac{(-1)^m (s-x)^{m-1}}{(m-1)!} \tag{11}$$

III. Illustration:

Example 1

Consider the Fredholm integro-differential equation

$$y'(x) = 1 - \frac{x}{3} + \int_0^1 xty(t)\,dt \tag{12}$$

With initial condition $y(0) = 0$ and exact solution $y(x) = x$

Table1.(Comparing ADM, VIM with exact for different values of x in example 1)

X	Exact	ADM	VIM	EADM	EVIM
0.1	0.1000000000	0.0999999205	0.0999999205	7.94E-08	7.94E-08
0.2	0.2000000000	0.1999996821	0.1999996821	3.18E-07	3.18E-07
0.3	0.3000000000	0.2999992847	0.2999992847	7.15E-07	7.15E-07
0.4	0.4000000000	0.3999987284	0.3999987284	1.27E-06	1.27E-06
0.5	0.5000000000	0.4999980132	0.4999980132	1.99E-06	1.99E-06
0.6	0.6000000000	0.5999971390	0.5999971390	2.86E-06	2.86E-06
0.7	0.7000000000	0.6999961058	0.6999961058	3.89E-06	3.89E-06
0.8	0.8000000000	0.7999949137	0.7999949137	5.07E-06	5.07E-06
0.9	0.9000000000	0.8999935627	0.8999935627	6.44E-06	6.44E-06
1.0	1.0000000000	0.9999920527	0.9999920527	7.95E-06	7.95E-06

Example 2

Consider the Fredholm integro- differential equation

$$y'(x) = xe^x + e^x - x + \int_0^1 xy(t)dt, \qquad (13)$$

With initial condition $y(0) = 0$ and exact solution $y(x) = xe^x$

Table 2.(Comparing ADM, VIM with exact for different values of x in example 2)

X	Exact	ADM	VIM	EADM	EVIM
0.1	0.1105170918	0.1105170888	0.1105170888	3.0E-09	3.0E-09
0.2	0.2442805516	0.2442805397	0.2442805397	1.19E-08	1.19E-08
0.3	0.4049576424	0.4049576156	0.4049576156	2.68E-08	2.68E-08
0.4	0.5967298792	0.5967298316	0.5967298316	4.76E-08	4.76E-08
0.5	0.8243606355	0.8243605611	0.8243605611	7.44E-08	7.44E-08
0.6	1.0932712800	1.0932711730	1.0932711730	1.07E-07	1.07E-07
0.7	1.4096268950	1.4096267490	1.4096267490	1.46E-07	1.46E-07
0.8	1.7804327420	1.7804325510	1.7804325510	1.91E-07	1.91E-07
0.9	2.2136428000	2.2136425590	2.2136425590	2.41E-07	2.41E-07
1.0	2.7182818280	2.7182815300	2.7182815300	2.98E-07	2.98E-07

Example 3

Considering Volterra equationintegro-differential equation

$$U''(x) = 1 + xe^x - \int_0^x e^{x-t}u(t)dt \qquad (14)$$

With initial conditions $u(0) = 0, u'(0) = 1$ and exact solution $u(x) = e^x - 1$

Table3. (Comparing of ADM, VIM with Exact for different values of x in example 3)

X	Exact	ADM	VIM	EADM	EVIM
0.1	0.105170918	0.1051709278	0.1051709640	9.80E-09	4.60E-08
0.2	0.221402758	0.2214027799	0.2214028205	2.19E-08	6.25E-08
0.3	0.349858808	0.3498587427	0.3498586872	6.53E-08	1.21E-08
0.4	0.491824698	0.4918247080	0.4918245703	6.53E-08	1.28E-08
0.5	0.648721271	0.6487212399	0.6487213115	1.00E-08	4.05E-08
0.6	0.822118800	0.8221188288	0.8221188591	3.11E-08	5.91E-08
0.7	1.013752707	1.013752732	1.013752769	2.88E-08	6.20E-08
0.8	1.225540928	1.225541037	1.225540983	2.50E-08	5.50E-08
0.9	1.459603111	1.459603270	1.459603148	1.09E-07	3.70E-08
1.0	1.718281828	1.718282295	1.718281926	1.59E-07	9.80E-08

Example 4

Consider Volterra integro differential equation

$$U'(x) = -1 + \int_0^x U^2(t)dt \qquad (15)$$

With initial condition $U(0) = 0$ and exact solution $u(x) = -x$

Table4. (Comparing ADM, VIM for different value of x in example 4)

X	Exact	ADM	VIM	EADM	EVIM
0.1	-0.1000000000	-0.09999166707	-0.09999166707	8.33E-06	8.33E-06
0.2	-0.2000000000	-0.19986671750	-0.19986671750	1.33E-04	1.33E-04
0.3	-0.3000000000	-0.29932586690	-0.29932586690	6.74E-04	6.74E-04
0.4	-0.4000000000	-0.39787315100	-0.39787315100	2.13E-03	2.13E-03
0.5	-0.5000000000	-0.49482250800	-0.49482250780	5.18E-03	5.18E-03
0.6	-0.6000000000	-0.5893100941	-0.5893100927	1.07E-02	1.07E-02
0.7	-0.7000000000	-0.6803138597	-0.6803138492	1.97E-02	1.97E-02
0.8	-0.8000000000	-0.7666814593	-0.7666814003	3.33E-02	3.33E-02
0.9	-0.9000000000	-0.8471669260	-0.8471666589	5.28E-02	5.28E-02
1.0	-1.0000000000	-0.9204757108	-0.9204746882	7.95E-02	7.95E-02

Example 5

Consider the Fredholm-Volterra integrodifferential equation

$$Y''(t)+Y'(t)-Y(t)=e^{t-1}-e^{t}-1+\int_0^1 e^{s+t}(Y(s))^2\,ds+\int_0^t Y(s)\,ds \qquad (16)$$

With initial condition $y(0)=1, y'(0)=-1$ and exact solution $y(t)=e^{-t}$

Table5.(Comparing ADM, VIM with Exact for different values of t in example 5)

T	Exact	ADM	VIM	EADM	EVIM
0.1	0.904837418	0.896160501	0.904350694	0.008676917	0.000486724
0.2	0.818730753	0.783594511	0.817029618	0.035136242	0.001701135
0.3	0.740818221	0.660685557	0.737405770	0.080132664	0.003412451
0.4	0.670320046	0.525762821	0.664765171	0.144557225	0.005554875
0.5	0.606530659	0.377106516	0.598327465	0.229424144	0.008203194
0.6	0.548811636	0.212950800	0.537264396	0.335860836	0.011547240
0.7	0.496585303	0.031483915	0.480719900	0.465101388	0.015865403
0.8	0.449328964	-0.169154690	0.427831430	0.618483654	0.021497534
0.9	0.406569659	-0.390880529	0.377751090	0.797450188	0.028818569
1.0	0.367879441	-0.635673673	0.329664630	1.003553114	0.038214811

Example 6

Considering nonlinear Volterra-Fredholm integro-differential equation

$$t^2 y''(t)+2y'(t)=2-\frac{5t}{6}+\frac{te^{-t^2}}{2}+\int_0^t tse^{-y^2(s)}ds+\int_0^1 ty^2(s)ds \qquad (17)$$

With initial condition $y(0)=0, y'(0)=1$ and exact $y(t)=t$

IV. Discussion Of Result And Conclusion:

From table 1, table 2, table 4, and table 5, it clearly shows how effective these methods are compared to the analytical solution and example 6 gives exact solution when applying both methods.From table 3 and table 5, both methods gives favorable solution and as the value of x increases, VIM converges faster than ADM and both methods gives exact solution to example 6.

Our results shows both methods are powerful and efficient method that give approximations of higher accuracy and closed form solution. It was also discovered that ADM provide the component of the exact solution whereas VIM gives several successive iterations through using the iteration of correction functional.

Moreover, VIM requires the evaluation of Lagrangian multiplier λ , while ADM requires the evaluation of Adomian polynomial that mostly requires tedious algebraic calculations. For nonlinear equation that arises frequently to express nonlinear phenomenon, VIM facilitates the computational work and gives the solution rapidly if compared with ADM.

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