# Numerical Solutions of Navier-Stokes Equations for Compressible Turbulent Two/Three Dimensional Flows in the Terminal Shock Region of an Inlet/Diffuser 

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## Summary

The multi-dimensional ensemble-averaged compressible time-dependent Navier-Stokes equations in conjunction with mixing length turbulence model and shock capturing technique have been used to study the terminal shock type of flows in various flight regimes occurring in a diffuser/inlet model. The numerical scheme for solving the governing equations is based on a linearized block implicit approach and the following high Reynolds number calculations have been carried out: (1) $2-D$, steady, subsonic; (2) $2-D$, steady, transonic with normal shock, (3) $2-D$, steady, supersonic with terminal shock, (4) 2-D, transient process of shock development and (5) 3-D, steady, transonic with normal shock. The numerical results obtained for the $2-D$ and $3-D$ transonic shocked flows have been compared with corresponding experimental data; the calculated wall static pressure distributions agree well with the measured data.

## INTRODUCTION

Proper design of the inlet flow region upstream of the compressor face is an important component in the overall design of the aircraft gas turbine for subsonic, transonic and supersonic inlet configurations.

In principle many of the problems of subsonic aircraft inlets are also encountered with transonic and supersonic installations. However, particular complexity is found in practice with transonic and supersonic aircraft, since not only is there the enlarged flight regime to consider, but now the influence of inlet shock structure on flow stability and engine-inlet matching must be taken into account. The importance and complexity of the influence of this inlet shock structure requires detailed investigation, which at present is accomplished by extensive and expensive experimental testing. Recently, however, there have been encouraging developments in the potential use of analyses to reduce the required extensive experimental mapping. In the transonic and supersonic inlet, the flow field can be divided into three main components: a supersonic region in the upstream portion of the inlet which leads into the terminal shock region and finally a subsonic diffusion region downstream of the terminal shock. In regard to the supersonic region Buggeln, McDonald, Levy and Kreskovsky (Ref. 1) have developed a three-dimensional spatial forward marching viscous flow analysis which has been applied successfully to several supersonic inlet configurations (Refs. 1-4). Although this analysis has given very favorable results in the supersonic portion of the inlet, the assumptions required to allow a forward marching calculation are inappropriate in the region of the terminal shock. Downstream of the terminal shock region, the flow is entirely subsonic and in this region the subsonic spatial forward marching analysis of Levy, Briley and McDonald (Ref. 5) is available. However, a portion of the flow field still requiring attention is the terminal shock region where procedures which are based upon a spatial forward marching method are invalid. It is this terminal shock region which is the subject of the present effort.

The terminal shock region is a very difficult problem which impacts upon both the loss characteristics and stability characteristics of the inlet flow. In a practical mixed compression supersonic inlet, the requirement of shock structure stability determines to large extent the normal shock loss in the inlet, itself a major contribution to the overall inlet losses. In essence by allowing some supersonic expansion after the geometric throat i.e. supercritical operation,
with subsequent shock down to subsonic flow via a normal shock, stability margin is obtained at the cost of the normal shock loss. If the normal shock were to occur very near the geometric throat where the local Mach number was unity, the resulting normal shock loss would be minimal but the inlet would be susceptable to unstarting. Having some supersonic reacceleration after the geometric throat places the normal shock downstream of the throat where a degrec of stable upstream shock movement is possible without unstarting the inlet. This upstream shock movement could be unavoidable in practice for instance as the result of changing engine operating conditions or the result of changes in the external flow. Thus, an inlet design in which a terminal shock of some finite strength occurs downstream of the throat is a common occurrence.

The flow in the region of the terminal shock is very complex. First of all it is transonic, secondly shockwave boundary layer interaction with possible accompanying separation occurs and thirdly the flow is very sensitive to area changes, and hence to the three-dimensionality of the geometry. As a result of these properties optimizing the location of the normal shock to maximize stability while minimizing losses is a very demanding, yet very important task for analysis. Further, although the flow downstream of the normal shock may be treated by viscous subsonic forward schemes, nevertheless it has the transonic region as initial conditions, and the forward marching calculation may prove sensitive to the inflow and hence require an accurate definition of the initial conditions. Thus, there exists powerful motivation to develop an analysis of the transonic region of the inlet, which would include threedimensionality and viscous effects. The ability to compute time-dependent flows would also be valuable. With this feature the steady flow (should it exist) would be computed as the time asymptote of the integration from the initial time zero guess of the flow field. Following this, the steady transonic shock structure could be perturbed and the stability of the system determined. The transient perturbation could be introduced by varying the inlet or the exit condition, depending on the physical disturbance being simulated.

Insofar as the governing equations are concerned, the inherent mixed elliptic hyperbolic nature of steady transonic flow does not encourage the use of forward marching in space, except perhaps in some corrector sense once an approximate transonic solution has been obtained. For governing equations one could consider the transonic potential equation, however, in the current problem a knowledge of the shock losses is critical, and this
precludes a potential approach. Turning to the Euler equations, these would permit shock losses to occur; however, the interest in and flow sensitivity to the interaction with the wall boundary layers make a viscous correction mandatory. The prospect of performing a numerical solution of the Euler equations and coupling this in an iterative manner with a threedimensional boundary layer scheme at transonic speeds is not attractive. Even if converged solutions could be obtained the resulting scheme would be unlikely to offer any significant savings in computational expense relative to solving the full Navier-Stokes equations, at least at transonic speeds where the interaction between the core flow and the boundary layer could be very sensitive and difficult to converge. In any event, the resulting procedure would still suffer difficulties with flow separation. The complex fluid mechanics involved in the transonic region of the inlet make the use of the three-dimensional compressible ensemble-averaged time-dependent NavierStokes equations attractive for this problem. Such an approach is described in the present report.

```
LIST OF SYMBOLS
```

Symbols
$A^{+}$

Cp

D

D
d
$d^{+}$
h
$\ell$
$\ell_{\infty}$

```
van Driest damping coefficient
specific heat at constant pressure
determinant of the Jacobian matrix
dissipation function
distance to the nearest wall.
dimensionless distance to the nearest wall
enthalpy, throat height
mixing length
mixing length in the core flow region
static pressure
magnitude of the velocity
turbulent heat flux vector
mean heat flux vector
universal gas constant
Reynolds number
time
temperature
stagnation temperature
velocity vector
```


## LIST OF SYMBOLS (continued)

Symbols
u
$\mathbf{u}_{\tau}$
v
w
${ }^{w}{ }_{e}$
$\mathrm{x}, \mathrm{x}_{1}$
$y, x_{2}$
$y^{1}, y^{2}, y^{3}$
$2, x_{3}$

```
velocity component in x-direction
friction velocity
velocity component in y-direction
velocity component in z-direction
w at the edge of the boundary layer
cartesian coordinate in transverse direction
cartesian coordinate in spanwise direction
computational coordinates
cartesian coordinate in streamwise direction
```

Greek Symbols

| $\delta$ | boundary layer thickness |
| :--- | :--- |
| $\varepsilon$ | turbulence energy dissipation rate |
| $\mu$ | von Karman constant |
| $\nu_{\text {art }}$ | dynamic viscosity |
| $\xi, \Pi, \zeta$ | artificial dissipation |
| $\bar{\pi}$ | computational coordinates |
| $\pi$ | molecular stress tensor |
| $\pi$ | turbulent stress tensor |

```
LIST OF SYMBOLS (continued)
```

Greek Symbols
$\rho$
$\sigma$
$\tau$
$\tau_{\ell}$
$\tau_{x x}, \tau_{x y}$, etc.
$\Phi$

## Subscripts

b
s
t
x
y
z

## Superscripts

T

```
density
artificial dissipation parameter
time
local shear stress
component of stress tensor
meanflow dissipation rate
```

associated with the bottom wall associated with the sidc wall associated with the time or top wall associated with the x -direction associated with the $y$-direction associated with the z-direction
associated with turbulent quantities, transpose of matrix

## Governing Equations

The equations used in the present effort are the ensemble-averaged, time-dependent Navier-Stokes equations which can be written in vector form as Continuity

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}-\nabla \cdot \rho \vec{u}=0 \tag{1}
\end{equation*}
$$

Momentum

$$
\begin{equation*}
\frac{\partial \rho \vec{u}}{\partial t}+\nabla \cdot(\rho \vec{U} \vec{U})=-\nabla p+\nabla \cdot\left(\overline{\bar{\pi}}+\pi^{\top}\right) \tag{2}
\end{equation*}
$$

Energy

$$
\begin{equation*}
\frac{\partial \rho h}{\partial t}+\nabla \cdot(\rho \overrightarrow{\mathrm{u}} \hat{n})=-\nabla \cdot\left(\overline{\bar{व}}+\vec{q}^{T}\right)+\frac{D p}{D t}+\Phi+\rho \epsilon \tag{3}
\end{equation*}
$$

where $\rho$ is density, $\overrightarrow{\mathbf{u}}$ is velocity, $p$ is pressure, $\bar{\pi}$ is the molecular stress tensor, $\pi^{T}$ is the turbulent stress tensor, $h$ is enthalpy, $\overrightarrow{\vec{~}}$ is the mean heat flux vector, $\vec{q}^{T}$ is the turbulent heat flux vector, $\Phi$ is the mean flow dissipation rate and $\varepsilon$ is the turbulence energy dissipation rate. If the flow is assumed at a constant total temperature, the energy equation is replaced by

$$
\begin{equation*}
T^{0}=T+\frac{q^{2}}{2 C_{p}}=\text { constant } \tag{4}
\end{equation*}
$$

where $T^{0}$ is the stagnation temperature, $q$ is the magnitude of the velocity and $C_{p}$ is the specific heat at constant pressure. For the purpose of economy, both in terms of run time and computer storage, calculations presented in this report were run with the constant total temperature assumption. These equations, supplemented by an equation of state,

$$
\begin{equation*}
\mathrm{p}=\rho \mathrm{RT} \tag{5}
\end{equation*}
$$

form the system governing the terminal shock region problem.

The governing equations, Eqs. (1) - (3), are written in general vector form and prior to their application to specific problems it is necessary to decide upon both a set of dependent variables and a proper coordinate transformation. Based upon previous investigations (e.g. Refs. 6 and 7) the specific scalar momentum equations to be solved are the $x, y$ and $z$ Cartesian momentum equations. The dependent variables chosen are the physical Cartesian velocities $u, v, w$ and the density $\rho$.

The equations are then transformed to a general coordinate system in which the general coordinates, $y^{j}$ are related to the Cartesian coordinates, $x_{1}, x_{2}$ and $x_{3}$ by

$$
\begin{align*}
y^{j} & =y^{j}\left(x_{1}, x_{2}, x_{3}, t\right) \quad ; \quad j=1,2,3 \\
\tau & =t \tag{6}
\end{align*}
$$

As implied by Eq. (6), the general coordinate $y^{j}$ may be a function of both the Cartesian coordinates and time. This coordinate time dependence will have an implication in so far as the choice of governing equation form is concerned.

The governing equations can be expressed in terms of the new independent variables $y^{j}$ as

$$
\begin{align*}
& \frac{\partial W}{\partial \tau}+\xi_{t} \frac{\partial W}{\partial \xi}+\xi_{x} \frac{\partial F}{\partial \xi}+\xi_{y} \frac{\partial G}{\partial \xi}+\xi_{z} \frac{\partial H}{\partial \xi} \\
&+\eta_{t} \frac{\partial W}{\partial \eta}+\eta_{x} \frac{\partial F}{\partial \eta}+\eta_{y} \frac{\partial G}{\partial \eta}+\eta_{z} \frac{\partial H}{\partial \eta} \\
&+\zeta_{t} \frac{\partial W}{\partial \zeta}+\zeta_{x} \frac{\partial F}{\partial \zeta}+\zeta_{y} \frac{\partial G}{\partial \zeta}+\zeta_{z} \frac{\partial H}{\partial \zeta}  \tag{7}\\
&=\frac{1}{\operatorname{Re}}\left[\xi_{x} \frac{\partial F_{1}}{\partial \xi}+\eta_{x} \frac{\partial F_{1}}{\partial \eta}+\zeta_{x} \frac{\partial F_{1}}{\partial \zeta}\right. \\
&+\xi_{y} \frac{\partial G_{1}}{\partial \xi}+\eta_{y} \frac{\partial G_{1}}{\partial \eta^{\prime}}+\zeta_{y} \frac{\partial G_{1}}{\partial \zeta} \\
&\left.+\xi_{z} \frac{\partial H_{1}}{\partial \xi}+\eta_{z} \frac{\partial H_{1}}{\partial \eta}+\zeta_{z} \frac{\partial H_{1}}{\partial \zeta}\right]
\end{align*}
$$

through a straight forward application of chain rule differentiation. In Eq. (7)

$$
\begin{aligned}
& \xi=y^{\prime} \\
& \eta=y^{2} \\
& \zeta=y^{3}
\end{aligned}
$$

and

$$
W=\left[\begin{array}{l}
\rho  \tag{8}\\
\rho u \\
\rho v \\
\rho w
\end{array}\right], F=\left[\begin{array}{l}
\rho u \\
\rho u^{2}+p \\
\rho u v \\
\rho u w
\end{array}\right] \quad, \quad G=\left[\begin{array}{l}
\rho v \\
\rho u v \\
\rho v^{2}+p \\
\rho v w
\end{array}\right]
$$

$H=\left[\begin{array}{l}\rho w \\ \rho u w \\ \rho v w \\ \rho w^{2}+p\end{array}\right], F_{1}=\left[\begin{array}{l}0 \\ \tau_{x x} \\ \tau_{x y} \\ \tau_{x z}\end{array}\right], G_{1}=\left[\begin{array}{c}0 \\ \tau_{x y} \\ \tau_{y y} \\ \tau_{y z}\end{array}\right], H_{1}=\left[\begin{array}{l}0 \\ \tau_{x z} \\ \tau_{y z} \\ \tau_{z z}\end{array}\right]$

Since in general the computational coordinates may be a function of time with a time-dependent Jacobian, the equations are recast into the so-called 'strong conservation form' (Ref. 8).

$$
\begin{align*}
& \frac{\partial W / D}{\partial T}+\frac{\partial}{\partial \xi}\left[\frac{W \xi_{t}}{D}+\frac{F \xi_{x}}{D}+\frac{G \xi_{y}}{D}+\frac{H \xi_{z}}{D}\right] \\
& +\frac{\partial}{\partial \eta}\left[\frac{W \eta_{t}}{D}+\frac{\mathrm{F} \eta_{x}}{D}+\frac{\mathrm{G} \boldsymbol{\eta}_{\boldsymbol{y}}}{\mathrm{D}}+\frac{\mathrm{H} \eta_{z}}{\mathrm{D}}\right] \\
& +\frac{\partial}{\partial \zeta}\left[\frac{W \zeta_{t}}{D}+\frac{F \zeta_{x}}{D}+\frac{G \zeta_{y}}{D}+\frac{H \zeta_{z}}{D}\right] \\
& =\frac{1}{\operatorname{Re}}\left[\frac{\partial}{\partial \xi}\left(\frac{F_{1} \xi_{x}}{D}+\frac{G_{1} \xi_{y}}{D}+\frac{H_{1} \xi_{z}}{D}\right)+\frac{\partial}{\partial \eta}\left(\frac{F_{1} \eta_{x}}{D}+\frac{G_{1} \eta_{y}}{D}+\frac{H_{1} \eta_{z}}{D}\right)\right.  \tag{9}\\
& \left.+\frac{\partial}{\partial \zeta}\left(\frac{F_{1} \zeta_{x}}{D}+\frac{G_{1} \zeta_{y}}{D}+\frac{H_{1} \zeta_{z}}{D}\right)\right]
\end{align*}
$$

where

$$
D=\left|\begin{array}{ccc}
\xi_{x} & \xi_{y} & \xi_{z} \\
\eta_{x} & \eta_{y} & \eta_{z} \\
\zeta_{x} & \zeta_{y} & \zeta_{z}
\end{array}\right|
$$

Equation (9) represents the Navier-Stokes equation in strong conservation form and represents the set of equations solved in the present work.

Insofar as the coordinate system is concerned, the cases considered in the present effort used a simplified coordinate transformation in which;

$$
\begin{align*}
& \xi=f_{1}(x, z) \\
& \eta=f_{2}(y)
\end{align*} \quad \zeta=f_{3}(z)
$$

i.e., a stretched and contour-fitted mon-orthogonal grid was used. The specific grid transformation used in the streamwise direction is that of Oh (Ref. 9), which allows high resolution in user specified regions. In the crosssectional plane hyperbolic tangent transformations were adopted. The reginns of high resolution were taken to be those near solid walls (in the $x$ and $y$ directions) and those near the throat as well as region of sharp contraction of the contour in the $z$ direction.

## Turbulence Model

Since the flows of interest are in the turbulent regime, it is necessary to specify a turbulence model. The present results were obtained from the McDonald's model (Ref. 10) with Van Driest damping (Ref. 11),

$$
\begin{equation*}
l=l_{\infty} \tanh \left[\frac{\kappa d}{l_{\infty}}\right]\left[1-\exp \left(-\frac{d^{+}}{A^{+}}\right)\right] \tag{11}
\end{equation*}
$$

where $k$ is the von Karman constant, $A^{+}$is the van Driest damping coefficient and d is the distance to the nearest solid wall.

$$
l_{\infty}=0.09 \delta \quad, \quad A^{+}=26.0
$$

and $\kappa=0.40$ for two-dimensional calculations, while $\kappa=0.41$ for threedimensional calculations. The nondimensional distance $\mathrm{d}^{+}$is defined as

$$
\begin{equation*}
\mathrm{d}^{+}=\mathrm{d}\left(\frac{\rho \mathrm{u}_{\tau}}{\mu}\right) \tag{12}
\end{equation*}
$$

and the friction velocity $u_{\text {f }}$ in the present analysis is taken as

$$
\begin{equation*}
u_{\tau}=\left(\frac{\tau_{L}}{f}\right)^{1 / 2} \tag{13}
\end{equation*}
$$

where the local shear stress $\tau_{\ell}$ is obtained from

$$
\begin{equation*}
\tau_{l}=(2 \mathbb{D}: \mathbb{D})^{1 / 2} \tag{14}
\end{equation*}
$$

where $\mathbb{D}$ is the dissipation function

$$
\begin{equation*}
\mathbb{D} \equiv \frac{1}{2}\left[(\nabla \vec{U})+(\nabla \vec{U})^{\top}\right] \tag{15}
\end{equation*}
$$

Note that for small d the tanh function in Eq. (11) reduces to kd while for large $d$ it approaches $\ell_{\infty}$.

In boundary layer analysis $\ell_{\infty}$ is usually taken as $0.09 \delta$ where $\delta$ is the boundary layer thickness taken at the location where $w / w_{e}=0.99$. However, this definition of $\delta$ assumes the existence of an outer flow where the velocity $w_{e}$ is independent of distance from the wall at a given streamwise station, i.e., it assumes $w_{e}$ is only a function of the streamwise coordinate. Most Navier-Stokes calculations show no such definitive region to exist and, therefore, an alternate definition is required. In the present effort the boundary layer thickness in the twodimensional region was set by first determining $w_{\text {max }}$, the maximum streamwise velocity, at a given station and then setting $\delta$ via;

$$
\begin{equation*}
\left.\delta=2.0 \mathrm{~d}_{\left(w / w_{\max }\right.}=k\right) \tag{16}
\end{equation*}
$$

i.e., $\delta$ was taken as twice the distance (measured away from the nearest wall) for which $w / w_{\max }=k$. The value of $k$ used in the present effort was 0.90 . The mixing length in the core region was set by linear interpolation between the top and bottom wall boundary layer edge values. The model described above was used in two-dimensional calculations as well as in the nominally twodimensional region of three-dimensional calculation. This nominally twodimensional region was defined as $y \xrightarrow{\geq} y_{s} \approx \delta_{s}$, where $\delta_{s}$ was the side wall boundary layer thickness evaluated, according to Eq. (16), at the midpoint between the top and bottom corners, and was taken as a measure of the overall boundary layer thickness along the side wall. Once $\delta_{s}$ had been determined, $y_{s}$ was then set as the nearest $y$-location of the grid points with $y_{s}$ being slightly larger than $\delta_{s}$. Henceforth, the mixing lengths at each point along $y=y_{s}$,
together with the locations of the top and bottom wall boundary layer edges, were obtained as described before. A schematic of the cross-sectional regions involved in the three-dimensional calculation is depicted in Fig. A, in which $x_{b}$ and $x_{t}$ are the edges of the bottom and top wall boundary layers at $y=y_{s}$. In the bottom wall corner region (i.e., $0 \leq y^{*}<y_{s}$ and $0 \leq x^{*}<x_{b}$ ), the mixing lengths were calculated according to Eq. (11) with a constant $\ell_{\infty}$ specified as the length scale at the point ( $\mathrm{X}_{\mathrm{b}}, \mathrm{y}_{\mathrm{s}}$ ). Similarly, the length scales of points in the top wall corner region (i.e., $0 \leqq y^{*}<y_{s}$ and $x_{t}<x^{*} \leqq x_{\text {max }}$ ) were evaluated according to Eq. (11) with another constant $\ell_{\infty}$ specified as the mixing length at the point $\left(x_{t}, y_{s}\right)$. Finally, the length scales of points in the side wall boundary layer region (i.e. $0 \leqq y^{*}<y_{s}$, $x_{b} \leqq x^{*} \leqq x_{t}$ ) were obtained by using Eq. (11) with $\ell_{\infty}$ specified as the respective mixing lengths at the points ( $x^{*}, y_{s}$ ).


Figure A

The authors' experience in solving Navier-Stokes equations has indicated the important role of boundary conditions in determining accurate solutions and rapid numerical convergence. The boundary conditions used in the present calculations with subsonic inflow and outflow follow the suggestion of Briley and McDonald [12] which specifies upstream total pressure and downstream static pressure conditions. Following this approach the stagnation pressure, transverse velocity and pressure derivative were set on the upstream boundary. In addition, a boundary layer thickness was specified and a dimensionless boundary layer profile set in that region. No-slip conditions in conjunction with zero pressure gradient were set at solid walls. The static pressure and velocity second derivatives were set at the downstream boundary. As mentioned above, this approach is valid for subsonic inflow. If the flow at the upstream boundary is supersonic, then, instead of the total core flow conditions, values of velocity components and density must be specified [27]. A more detailed description of the boundary conditions used for the present calculations will be given later in the section "Test Cases".

## Numerical Procedure

The numerical procedure used to solve the governing equations is a consistently split linearized block implicit (LBI) scheme originally developed by Briley and McDonald [13]. A conceptually similar scheme has been developed for two-dimensional MHD problems by Lindemuth and Killeen [14]. More recently Beam and Warming (Ref. 25) have derived this and other related schemes by the method of approximate factorization. The procedure is discussed in detail in Refs. 13 and 15. The method can be briefly outlined as follows: the governing equations are replaced by an implicit time difference approximation, optionally a backward difference or Crank-Nicolson scheme. Terms involving nonlinearities at the implicit time level are linearized by Taylor expansion in time about the solution at the known time level, and spatial difference approximations are introduced. The result is a system of multidimensional coupled (but linear) difference equations for the dependent variables at the unknown or implicit time level. To solve these difference equations, the Douglas-Gunn [16] procedure for generating alternating-direction implicit
(ADI) schemes as perturbations of fundamental implicit difference schemes is introduced in its natural extension to systems of partial differential equations. This technique leads to systems of coupled linear difference equations having narrow block-banded matrix structures which can be solved efficiently by standard block-elimination methods.

The method centers around the use of a formal linearization technique adapted for the integration of initial-value problems. The linearization technique, which requires an implicit solution procedure, permits the solution of coupled nonlinear equations in one space dimension (to the requisite degree of accuracy) by a one-step noniterative scheme. Since no iteration is required to compute the solution for a single time step, and since only moderate effort is required for solution of the implicit difference equations, the method is computationally efficient; this efficiency is retained for multidimensional problems by using what might be termed block ADI techniques. The method is also economical in terms of computer storage, in its present form requiring only two time-levels of storage for each dependent variable. Furthermore, the block ADI technique reduces multidimensional problems to sequences of calculations which are one dimensional in the sense that easily-solved narrow block-banded matrices associated with one-dimensional rows of grid points are produced. A more detailed discussion of the solution procedure as discussed by Briley, Buggeln and McDonald [17] is given in the Appendix.

## Artificial Dissipation

One major problem to be overcome in calculating high Reynolds number flows using the Navier-Stokes equations is the appearance of spatial oscillations associated with the so-called central difference problem. When spatial derivatives are represented by central differences, high Reynolds number flows can exhibit a saw tooth type oscillation unless some mechanism is added to the equations to suppress their appearance. This dissipation mechanism can be added implicitly to the equations via the spatial difference molecule (e.g. one-sided differencing) or explicitly through addition of a specific term. The present authors favor this latter approach for two reasons. First, if a specific artificial dissipation term is added to the equations, it is clear precisely what approximation is being made. Secondly, if a specific term is added to suppress oscillations, the amount of artificial dissipation added to the equations can be easily controlled in magnitude and location so as to add the minimum amount necessary to suppress spatial
oscillations. Studies can also be easily performed to evaluate the effect of the explicitly added dissipation on the solution.

Various methods of adding artificial dissipation were investigated in Ref. 18, and these were evaluated in the context of a one-dimensional model problem. The model problem used was one-dimensional flow with heat transfer. Flow was subsonic at the upstream boundary, accelerated via heat sources until a Mach number of unity was reached and then accelerated by heat sinks. The exit back pressure was raised to cause a shock to appear in the supersonic region. This basic one-dimensional problem contained many relevant features including strong accelerations and appearance of a normal shock wave and, therefore, it served as a good test case for various forms of artificial dissipation which could be used in the presence of shock waves.

The results of the Ref. 18 investigation led to the conclusion that for the model problem a second order artificial dissipation approach was the best of those considered. This approach adds a term of the form $v_{\text {art }} \frac{\partial^{2} \phi}{\partial Z^{2}}$ or $\frac{\partial}{\partial Z}\left\{\nu_{\text {art }} \frac{\partial \phi}{\partial Z}\right\}$ to each governing equation where $\phi=\rho, u, v, w$ for the continuity, $x$-momentum, $y$-momentum and $z$-momentum equations respectively and $v_{\text {art }}$ is determined by $\frac{\left|U_{Z}\right| \Delta Z}{v+\left(v_{\text {art }}\right)_{Z}} \leq \frac{I}{\sigma_{Z}}$

In the above equation $\Delta Z$ is the distance between grid points in a given coordinate direction, $U_{Z}$ is the velocity in this direction, $\sigma_{Z}$ is the artificial dissipation parameter for this direction and $v$ is the effective kinematic viscosity. The equation determines $v$ art with $v_{\text {art }}$ taken as the smallest non-negative value which will satisfy the expression. It should be noted that in two space dimensions each equation contains two artificial dissipation terms, one in each coordinate direction. For example, the streamwise momentum equation expressed in two-dimensional Cartesian coordinates would contaln the artificial dissipation terms

$$
\left(\nu_{a r t}\right)_{x} \frac{\partial^{2} w}{\partial x^{2}}+\left(\nu_{a r t}\right)_{z} \frac{\partial^{2} w}{\partial z^{2}}
$$

Obviously the desirable condition occurs when sufficient artificial dissipation is added to the equations to suppress spurious oscillations but the amount added does not perceptively change the physical solution. The results of Refs. 18 and 19 indicated that such conditions could be met when the dissipation parameter, $\sigma$, was varied between values of . 10 and .025 and these results were confirmed for the terminal shock problem in the present effort.

Although the original artificial dissipation study was carried out with terms of the form ( $\left.\nu_{\text {art }}\right)_{Z} \partial^{2} \phi / \partial Z^{2}$, the form used in the present case was $\partial\left(\nu_{\text {art }} \partial \phi / \partial Z\right) / \partial Z$. However, recent studies for airfoil and cascade calculations indicate that for low values of $\sigma$ little significant difference occurs as a result of using one form or the other.

## Test Cases

Several test cases were run with the MINT computer code to evaluate the previously described computational procedures for inlet terminal shock flow problems. In general, works aimed at clarifying the fluid mechanical processes involved in the terminal shock region of channel flows are scarce and, in particular, the available data in many cases are not sufficiently complete to form the basis for detailed numerical comparisons. One experimental investigation which gives detailed measurements is that of Bogar, Sajben, Kroutil and Salmon (Refs. 20 and 21) which focuses upon flows in the terminal shock region of inlets/diffusers. More specifically, they investigated transonic flows in nominally two-dimensional, supercritically operated diffusers. These flows exhibit many significant features found in supersonic inlets of aircraft. A detailed description of the diffuser model and results describing both the time-mean and the oscillating flow properties were reported in Ref. 20, while laser Doppler velocimeter measurements were given in Ref. 21. Since these detailed data are considered as reliable and the trends observed are believed to be present in three-dimensional inlet flows as well, this particular data base was selected for designing the test cases for the present effort. The following five cases of different flows have been calculated: (1) two-dimensional subsonic diffuser flow, (2) two-dimensional transonic diffuser flow with a normal shock, (3) two-dimensional supersonic inlet flow with a terminal shock, (4) transient development of normal shock in a two-dimensional convergentdivergent channel and (5) three-dimensional transonic diffuser flow with a normal shock. In all of these calculations the flows are turbulent and, except for case (3), only the asymptotic steady-state solutions are of interest. Furthermore, the selected diffuser/inlet models are either geometrically similar or identical to each other.

A schematic of the inlet/diffuser geometry and the associated coordinate system is shown in Fig. 1. The diffuser/inlet model is a convergent-divergent channel with a flat bottom and a contoured top wall. In addition, the crosssection is rectangular everywhere. A detailed description of this model can be found in Ref. 20 and will not be repeated here. However, it should be noted that the computational domain extends from 3.75 h upstream of the geometric throat to 8.65 h downstream of the throat, where $h$ is the throat height. For the three-dimensional calculation, the throat cross-sectional aspect ratio is 3.0 with the computational domain extending from one side wall to the center plane, and no-slip conditions are applied on all solid walls. This is somewhat different from the experimental conditions in which the throat crosssectional aspect ratio is 4.0 and suction slots are used at several locations to establish the nominal two-dimensionality of the flow.

An important aspect of almost all numerical calculations is the generation of a suitable computational coordinate system. The present approach uses a contour fitted coordinate system in which both top and bottom as well as side channel walls (for the three-dimensional calculation) fall on coordinate lines. As mentioned earlier, high grid resolution near the walls is obtained by employing a hyperbolic tangent grid packing transformation; the streamwise resolution is obtained by clustering grid points near the location of sharp contraction of the contour as well as near the expected location of the shock. This grid is accomplished by using a versatile grid distribution generator which allows multiple regions of grid packing (Ref. 9). For the present calculations, 31 grid points are used in the transverse direction (x-direction) while 41 grid points are used in the streamwise direction (z-direction). In addition, for three-dimensional case, 16 grid points are used in the spanwise direction (y-direction). Results of all the five test cases were obtained with the same grid distributions.

For all of the test calculations, the Reynolds number based on the inlet core flow condition and the throat height is approximately $4.73 \times 10^{5}$, the inlet core Mach number is approximately 0.46 for cases (1), (2), (4) and (5) while it is approximately 1.90 for case (3). Under the assumption that the flows are at constant total temperature, the equations solved are the continuity equation and momentum equations. The previously described mixing length model
and shock capturing technique are used to provide turbulent viscosity and to locate the shock. As for the boundary conditions, no-slip condition together with zero first derivative of the static pressure (with respect to the transverse computational coordinate) are imposed along the top and bottom walls. For threedimensional case, no-slip condition together with zero first derivative of the static pressure (with respect to the spanwise computational coordinate) are applied along the side wall while the symmetry conditions are used for the center plane. At the exit where the flows are subsonic for all test cases, constant static pressure is specified and the second streamwise (computational coordinate) derivatives of all velocity components are set to be zero. For cases (1), (2), (4) and (5) the flows at the inlet are subsonic, the core flow total conditions together with wall boundary layer thicknesses and profile shapes of the streamwise velocity component are specified. In addition, the second streamwise (computational coordinate) derivatives of static pressure and velocity components in the cross-sectional plane are set to be zero. Experience indicates that it may be beneficial to freeze the cross-sectional velocity components after the initial impulsive transients had passed and this is done for case (5) to obtain the highly damped solution. As for the profile of the streamwise velocity component, the profile suggested by Musker (Ref. 22) supplemented by the Van Driest transformation (Ref. 23) to account for the effects of compressibility is adopted. In case (3), the flow at the inlet section is supersonic except in wall regions of the boundary layer, the velocity components, the density and the static enthalpy (temperature) are specified for the supersonic portion while the second streamwise (computational coordinate) derivatives of the velocity components and the pressure are set to be zero for the subsonic portion of the inflow section. Consequently, the density and temperature in the subsonic portion are calculated in accordance with the specified total enthalpy (temperature) and the equation of state. In this way, the disturbances occurring in the subsonic portions of the internal flow field are allowed to propagate through the upstream inflow section.

Since the governing equations are time-dependent, initial conditions are needcd to start the calculation. In general, a relatively simple approximation to the flow field suffices as an initial condition, however, if a better estimate is easily available it should be used. The construction of the initial
conditions for each test case will be described in the following section. However, a general comment concerning the presence of the discontinuities in the initial conditions should be made here, since it is relevant to the terminal shock type of calculations. One of the most important reasons for the occurrence of surfaces of discontinuity in a gas is the possibility of discontinuities in the initial conditions. These conditions may in general be prescribed arbitrarily. It is known, however, that certain conditions must hold on stable surfaces of discontinuity in a gas; for instance, the discontinuities of pressure, density, etc. in a shock wave are related by the Rankine-Hugoniot relations. It is, therefore, clear that if these conditions are not satisfied in the initial discontinuity, it cannot continue to be a discontinuity at subsequent instants. Instead, it generally splits into several discontinuities (e.g. shock wave, tangential discontinuity and rarefaction wave); in the course of time, these discontinuities of different types move apart. Their propagation, reflection and subsequent interations may cause undesirable transient impulsives with the possible consequences of prolonged computing time or even the instability of the calculation. Therefore, special attention should be paid to the construction of the initial condition for the terminal shock type of problems.

## Computed Results

The previously described test cases cover various flow regimes occurring in a diffuser/inlet model. Depending on the specified upstream and downstream boundary conditions, the resultant internal flow field can be quite different in nature. In most of these cases, asymptotic steady-state solutions are of interest, however, physically meaningful transient solutions for the formation of shock waves have also been obtained. In addition, the effects of artificial dissipation on the numerical solutions have been studied and a three-dimensional calculation has been carried out. A vast amount of information is obtained from the computation of these test cases, and only selected, representative results are to be presented here. The relevant flow parameters describing these cases are given in Table I. These calculations were considered to reach an asymptotic steady-state when there was virtually no change in the wall static pressure
distributions over a (dimensionless) time interval of 2 to 6 , where a dimensionless time of 12 is the time required for a particle moving at the inlet velocity to pass from inlet to exit, and the changes in other flow variables were of very minor significance. In addition, the maximum residual decreased by one to two orders of magnitude, depending upon the initial conditions and the flow problems.

TABLE I - Parameters for Test Cases

| Case | Type | $\mathrm{Re}_{\mathrm{h}}$ | Inlet Core Mach No. | ```Inlet Top Wa11 Boundary Layer Thickness \delta/h``` | ```Inlet Bottom Wall Boundary Layer Thickness \delta/h``` | ```Inlet Side Wall Boundary Layer Thickness \delta/h``` |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2-D, Steady Subsonic | $4.73 \times 10^{5}$ | 0.46 | 0.12149 | 0.060745 | / |
| 2 | ```2-D, Steady Transonic with Shock``` | $4.73 \times 10^{5}$ | 0.46 | 0.12149 | 0.060745 | 1 |
| 3 | 2-D, Steady <br> Supersonic with Terminal Shock | $4.73 \times 10^{5}$ | 1.90 | 0.12149 | 0.060745 | 1 |
| 4 | 2-D, Transient <br> Formation of a <br> Normal Shock | $4.73 \times 10^{5}$ | 0.46 | 0.12149 | 0.060745 | 1 |
| 5 | ```3-D, Steady Transonic with Normal Shock``` | $4.73 \times 10^{5}$ | 0.46 | 0.12149 | 0.060745 | 0.060745 |

Case (1): Steady 2-D Subsonic Diffuser Flow
The calculation was initiated with an initial condition which consisted of a one-dimensional inviscid flow corresponding to the specified diffuser contour with a simple boundary layer correction applied in the vicinity of no-slip surfaces. With this initial condition, it took about 60 time steps to reach an asymptotic steady state solution for $\sigma_{x}=\sigma_{z}=\sigma=0.5$ (which corresponds to a cell Reynolds number of 2). At this stage, the artificial dissipation parameters were then lowered to $\sigma=0.05$ (which corresponds to a cell Reynolds number 20), and it took about another 50 time steps to reach an asymptotic steady state wherein no observable changes occurred over a wide variation in time steps. The calculated top wall pressure distribution is depicted in Fig. 2, while the calculated bottom wall pressure distribution is shown in Fig. 3. For the purpose of reference, some relevant measured data for shocked flow, which is established by a lower exit pressure ( $\mathrm{P}_{\mathrm{e}}=0.933$ as compared to the present 0.96 ), are also given. It is obvious that the choice of the artificial dissipation parameters significantly affect the computed results. Previous experience at SRA with second order artificial dissipation calculations for transonic shock waves has indicated that accurate results and sharp shock representation can be obtained when $\sigma$ is limited to 0.05 . Although experimental data for this case is not available, the results shown in Figs. 2 and 3 are physically realistic. The wall pressures follow the data for a lower back pressure until slightly upstream of the throat as is expected. Since the calculation and the data are for cases with different back pressures, the results diverge as the throat is approached.

## Case (2): Steady 2-D Transonic Diffuser Flow with a Normal Shock

The solutions obtained with exit pressure $P_{e}=0.96$ and the artificial dissipation $\sigma=0.5$ was used as the initial condition for this calculation. At first, the back pressure was dropped to 0.933 over a short period of time and then fixed for the subsequent computations. After approximately 40 time steps an asymptotic steady state solution for $P_{e}=0.933$ and $\sigma=0.5$ was obtained. Then the artificial dissipation parameter $\sigma$ was dropped to 0.05 and after another 40 time steps, the final steady-state solution for $P_{e}=0.933$ and $\sigma=0.05$ was reached. The calculated top wall pressure distribution is given in Fig. 4 and the calculated bottom wall pressure distribution is shown in Fig. 5. The calculated results for $\sigma=0.05$ agree very well with the corresponding experimentally measured data (case denoted by $\mathrm{M}_{\sigma \mathrm{u}}=1.235$ in

Ref. 20). Again, the artificial dissipation parameter plays an important role, the shock is captured with $\sigma=0.05$ while the results are severely smeared with $\sigma=0.5$. In fact, the $\sigma=0.5$ calculation does not even contain a supersonic region (Fig. 6). Further investigation of the sensitivity of the calculated flow fields with respect to the choice of the artificial damping parameter $\sigma$ has been carried out for $\sigma=0.1$. The calculated wall static pressure distributions for this value of $\sigma$ are essentially the same as those for $\sigma=0.05$ and are not presented here. Therefore, it may be concluded that the present mumerical results are insensitive to the choice of the parameter $\sigma$ when $\sigma$ is in the range from 0.1 to 0.05 . It is noted that, based upon previous experience for shocked flow, if no artificial dissipation were used the calculation would be unstable. However, as indicated by the present calculations, if too much artificial dissipation is used the solution would be unrealistically contaminated by its presence.

Case (3): Steady 2-D Supersonic Inlet Flow with Terminal Shock
The construction of the initial condition for this case was essentially the same as that for Case (1) except that, by applying the Rankine-Hugoniot relations, an initial discontinuity of the pseudo two-dimensional flow field was generated in the vicinity and downstream of the geometrical throat. The pressure boundary condition at the exit was specified with its ratio to the pressure at the inflow section being 5.70 ; this value is consistent with the initially assumed shock wave. Such a back pressure was held fixed for subsequent calculations. The calculation required 70 time steps to reach an asymptotic steady-state solution for $\sigma=0.5$, and then, after lowering the artificial dissipation parameter to 0.1 , another additional 50 time steps was needed to reach the final asymptotic steady state solutions where no further observable changes in the solution occurred. In Fig. 7 a schematic flow field is depicted; the difference in the streamwise and normal scales used in this figure should be noted. An oblique shock is formed in the region of the compression corner of the top wall (ramp) while near the bottom wall (cowl) a Mach reflection occurs and the terminal shock stands at approximately one throat height downstream of the geometric throat. The existence of the Mach reflection is consistent with the prediction due to the inviscid theory by noting that the core flow Mach Number near the inflow section is about 1.90 and the deflection angle of the top wall is about $18^{\circ}$, under such conditions a regular reflection of the incident shock wave is not possible. Instead, a Mach reflection
must occur. At the compression corner there exists a separation region induced by the adverse pressure gradients near the wall. A relatively large shockinduced separation zone exists in the Mach reflection region. Further, there are terminal shock-induced separation regions along the top and bottom walls, although the one along the bottom wall is very thin. In Fig. 8 the corresponding dimensionless static pressure distribution along the top wall is presented and in Fig 9 the corresponding dimensionless static pressure distribution along the bottom wall is shown. No experimental data are available for comparison, nevertheless, these results are qualitatively in agreement with the known features of the supersonic inlet flow. The above calculations demonstrate the capability of the MINT code to compute turbulent flows in various flight regimes, as shown by the Mach Number contours depicted in Figs. 10a, $b$ and $c$. In these figures, the main flow direction is from left to right. For Fig. 10a and Fig. 10b, the minimum contour value is 0.432 with constant increment of 0.032 , while for Fig. $10 c$, the minimum contour value is 0.46 with constant increment of 0.06 , the displayed domain extends from the inflow section to 3.4 h downstream of the throat where $h$ is the throat height. Figure 10a shows the Mach Number contours of a subsonic diffuser flow, Fig. 10b illustrates that of a supercritically operated transonic diffuser flow with a normal shock region and Fig. 10c gives the Mach Number contours of a supersonic inlet in which the existence of the oblique shock waves, Mach leg and a terminal shock region is evident. The corresponding static pressure contours are given in Figs. 1la, $b$ and $c$.

Case (4): Unsteady Shock Development in a 2-D Transonic Diffuser
An investigation of the formation of the normal shock by lowering the back pressure ( $P_{e}$ ) from that of a subsonic diffuser flow to that of a supercritically operated transonic diffuser flow also has been performed. The calculation started with the steady-state solution of the subsonic flow ( $\mathrm{P}_{\mathrm{e}}=0.96$ and $\sigma=0.05$ ) and over a very short period of time the back pressure was dropped to 0.933 which was then held as constant. Small artificial dissipation parameter $(\sigma=0.05)$ and (constant) small dimensionless time step ( $\Delta t=0.05$ ) were used. Figure 12 shows the transient development of the static pressure along the top wall and Fig 13 shows the transient development of the static pressure along the bottom wall. As it can be seen, the final asymptotic steady-state solutions agree very well with the corresponding experimentally measured data (case denoted by $M_{\sigma u}=1.235$ in Ref. 20). Although a dimensionless
time interval of 18 units was required for the flow to change from one steadystate (subsonic mode without shock) to another steady-state (transonic mode with a normal shock), the corresponding physical time interval is only about $0.53 \times 10^{-2}$ second. Since its response to the changes of the back pressure are very rapid for this transient flow, very little information about the fluid mechanical process involved in the formation of shock has been provided by most of the relevant experiments for this problem. Some basic features of such a process are revealed by the present numerical investigation and will be presented here. The transient as well as spatial developments of the flow field are illustrated by Fig. 14, which is a history of Mach Number contours in a region which extends from two throat height upstream of the throat to 6 throat heights downstream of the throat. The minimum contour level is 0.432 with a constant increment of 0.032. The corresponding time history of static pressure contours are given in Fig. 15. In the early stages of the development, the disturbances originating at the outflow section propagate in the direction of the upstream; in particular, the propagating speed of the disturbances within the (contoured) top wall boundary layer is relatively large. Once these faster moving disturbances reach the throat region, disturbances transverse to the mean flow are generated, which then continue to propagate up-and downstream as they approach the (flat) bottom wall. In the later stages of shock development, disturbances propagating in upstream, downstream and transverse directions are undoubtedly present and they can interact with each other, but the most important disturbances responsible for the formation of the shock are the transverse waves originating at the boundary layer/core-flow interface, which are strongly influenced by the viscous-inviscid interactions.

Although the present calculation focuses upon the formation of the shock due to the small changes of the back pressure, the results obtained do strong1y suggest that a one-dimensional inviscid approach is not appropriate for analyzing the response of the terminal shock in a supersonic inlet to the back pressure disturbances (i.e. the hammer shock problem). Such an indication is further supported by the results obtained from a relevant experimental work (Ref. 24) in which the shock motion induced by externally applied disturbances were investigated for a supercritically operated transonic diffuser.

Case (5): Steady 3-D Transonic Diffuser Flow with a Normal Shock
This calculation was initiated with an initial condition which consisted of a 2-D highly damped ( $\sigma=0.5$ ) solution with a simple boundary layer correction applied in the vicinity of the side wall ( $\mathrm{y}=0$ ) . With these initial conditions, it took about 80 time steps to reach an asymptotic steady-state solution for $\sigma_{x}=\sigma_{y}=\sigma_{z}=0.5$. Then the calculation proceded with reduced $\sigma_{z}(=0.05)$ for another 60 time steps. Finally, an additional 50 time steps were advanced with $\sigma_{x}=\sigma_{y}=\sigma_{z}=0.05$ to reach an asymptotic steady state. The calculated top wall pressure distribution is given in Fig. 16 while the computed bottom wall pressure distribution is shown in Fig. 17. As would be expected for this flow, the 3-D results agree quite well with the 2-D numerical results of Case (2) and the nominally $2-\mathrm{D}$ experimental data (Ref. 20), except that the 3-D shock is slightly weaker than, and its position is slightly upstream of the $2-\mathrm{D}$ shock. The variation of the wall static pressure in the spanwise direction is small, as is shown in Fig. 18, which depicts the pressure contours at various spanwise locations. The displayed region extends from two throat heights upstream of the throat to 6 throat heights downstream of the throat. The minimum contour level is 0.502 with a constant increment of 0.02 . Figure 19 presents the streamwise Mach Number distribution. Points A are inside the side wall boundary layer and Points B are on the center (symmetry) plane. Both Points A and B are located slightly below the midplane of each cross-section. The spanwise variation of the Mach Number contours is illustrated in Fig 20, in which the displayed region is the same as that in Fig. 18, but the minimum contour level is 0.432 with a constant increment of 0.032 . It is noted that, contrary to the static pressure distribution, the Mach Number distribution exhibits strong spanwise dependence.

As mentioned above, the strength of the three-dimensional shock is slightly weaker than its two-dimensional counterpart; such a three-dimensional effect on the shock strength is also reported in a recent work on the inviscid transonic flow in an axial compressor rotor (Ref. 28). Further, the position of the weaker $3-D$ shock is slightly upstream of the position of its corresponding stronger $2-\mathrm{D}$ shock; this is consistent with the fact that the flows are in a supercritically operated inlet/diffuser.

Due to the complexity of the fluid mechanics invloved in the terminal shock region of the inlet, the three-dimensional ensemble-averaged compressible time-dependent Navier-Stokes equations in conjunction with suitable turbulence modeling and shock capturing technique have been used to study the terminal shock type of flow problems. The numerical scheme for solving the governing equations is based on a linearized block implicit approach which is embodied in a general computer code termed "MINT". The MINT code has been applied to calculate turbulent flows in various flight regimes occurring in a diffuser/ inlet model. These high Reynolds number calculations are: (1) 2-D, steady, subsonic; (2) $2-D$, steady, transonic with normal shock, (3) $2-D$, steady, supersonic with terminal shock, (4) $2-D$, transient process of shock development and (5) 3-D steady, transonic with normal shock. As an indication of the validity of these computations, the numerical results obtained for the 2-D/3-D transonic diffuser flows have been compared with corresponding experimental data, the calculated wall static pressure distributions agree quite well with the experimentally measured data. Also studied is the role of the artificial dissipation in the shock capturing technique, inappropriate choice of the artificial dissipation will severely smear the shock. These extensive and carefully designed calculations demonstrate the capabilities of the MINT code for predicting the complex flows commonly occurring in the engine inlets. Further investigations should concentrate on the problems concerning the response of the teminal shock to the externally applied disturbances and the effects of the turbulence modeling on the small scale flow properties. In this respect, the turbulence models for three-dimensional terminal shock flows are of particular concern.

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APPENDIX - SOLUTION PROCEDURE [17]
Background
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The solution procedure employs a consistently-split linearized block implicit (LBI) algorithm which has been discussed in detail in [13, 15]. There are two important elements of this method:
(1) the use of a noniterative formal time linearization to produce a fully-coupled linear multidimensional scheme which is written in "block implicit" form; and
(2) solution of this linearized coupled scheme using a consistent "splitting" (ADI scheme) patterned after the Douglas-Gunn [16] treatment of scalar ADI schemes.

The method is thus referred to as a split linearized block implicit (LBI) scheme. The method has several attributes:
(1) the noniterative linearization is efficient;
(2) the fully-coupled linearized algorithm eliminates instabilities and/or extremely slow convergence rates often attributed to methods which employ ad hoc decoupling and linearization assumptions to identify nonlinear coefficients which are then treated by lag and update techniques;
(3) the splitting or ADI technique produces an efficient algorithm which is stable for large time steps and also provides a means for convergence acceleration for further efficiency in computing steady solutions;
(4) intermediate steps of the splitting are consistent with the governing equations, and this means that the "physical" boundary conditions can be used for the intermediate solutions. Other splittings which are inconsistent can have severe difficulties in satisfying physical boundary conditions [15].
(5) the convergence rate and overall efficiency of the algorithm are much less sensitive to mesh refinement and redistribution than algorithms based on explicit schemes or which employ ad hoc decoupling and linearization assumptions. This is important for accuracy and for computing turbulent flows with viscous sublayer resolution; and
(6) the method is general and is specifically designed for the complex systems of equations which govern multiscale viscous flow in complicated geometries.

This same algorithm was later considered by Beam and Warming [25], but the ADI splitting was derived by approximate factorization instead of the Douglas-Gunn procedure. They refer to the algorithm as a "delta form" approximate factorization scheme. This scheme replaced an earlier non-delta form scheme [26], which has inconsistent intermediate steps.

## Spatial Differencing and Artificial Dissipation

The spatial differencing procedures used are a straightforward adaption of those used in [13] and elsewhere. Three-point central difference formulas are used for spatial derivatives, including the first-derivative convection and pressure gradient terms. This has an advantage over one-sided formulas in flow calculations subject to "two-point" boundary conditions (virtually all viscous or subsonic flows), in that all boundary conditions enter the algorithm implicitly. In practical flow calculations, artificial dissipation is usually needed and is added to control high-frequency numerical oscillations which otherwise occur with the central-difference formula.

In the present investigation, artificial (anisotropic) dissipation terms of the form

$$
\begin{equation*}
\sum_{j} \frac{d_{j}}{h_{j}^{2}} \frac{\partial^{2} u_{k}}{\partial x_{j}^{2}} \tag{1}
\end{equation*}
$$

are added to the right-hand side of each ( $k-t h$ ) component of the momentum equation, where $h_{j}$ is the metric coefficient and for each coordinate direction $\mathrm{X}_{\mathrm{j}}$, the dimensionless artificial diffusivity $\mathrm{d}_{\mathrm{j}}$ is positive and is chosen as the larger of zero and the local quantity $\mu_{e}\left(\sigma \operatorname{Re}_{\Delta x_{j}}-1\right) / R e$. Here, $\mu_{e}$ is the effective dynamic viscosity and the local cell Reynolds number $R e_{\Delta x_{j}}$ for the j-th direction is defined by

$$
\begin{equation*}
\operatorname{Re}_{\Delta x_{j}}=\operatorname{Re}\left|\rho u_{j}\right| \Delta x_{j} / \mu_{e} \tag{2}
\end{equation*}
$$

This treatment lowers the formal accuracy to $0(\Delta x)$, but the functional form is such that accuracy in representing physical shear stresses in thin shear layers with small normal velocity is not seriously degraded. This latter property follows from the anisotropic form of the dissipation and the combination of both small normal velocity and small grid spacing in thin shear layers.

## Split LBI Algorithm

## Linearization and Time Differencing

The system of governing equations to be solved consists of three/four equations: continuity and two/three components of momentum equation in three/four dependent variables: $\rho, u, v, w$. Using notation similar to that in [13], at a single grid point this system of equations can be written in the following form:

$$
\begin{equation*}
\partial H(\phi) / \partial t=D(\phi)+S(\phi) \tag{3}
\end{equation*}
$$

where $\phi$ is the column-vector of dependent variables, $H$ and $S$ are column-vector algebraic functions of $\phi$, and $D$ is a column vector whose elements are the spatial differential operators which generate all spatial derivatives appearing in the governing equation associated with that element.

The solution procedure is based on the following two-level implicit timedifference approximations of (3):

$$
\begin{equation*}
\left(H^{n+1}-H^{n}\right) / \Delta t=\beta\left(D^{n+1}+S^{n+1}\right)+(1-\beta)\left(D^{n}+S^{n}\right) \tag{4}
\end{equation*}
$$

where, for example, $H^{n+1}$ denotes $H\left(\phi^{n+1}\right)$ and $\Delta t=t^{n+1}-t^{n}$. The parameter $\beta$ ( $0.5 \leq \beta \leq 1$ ) permits a variable time-centering of the scheme, with a truncation error of order $\left[\Delta t^{2},(\beta-1 / 2) \Delta t\right]$.

A local time linearization (Taylor expansion about $\phi^{n}$ ) of requisite formal accuracy is introduced, and this serves to define a linear differential operator L (cf. [13]) such that

$$
\begin{equation*}
D^{n+1}=D^{n}+L^{n}\left(\phi^{n+1}-\phi^{n}\right)+0\left(\Delta t^{2}\right) \tag{5}
\end{equation*}
$$

Similarly,

$$
\begin{align*}
& H^{\mathrm{n}+1}=H^{\mathrm{n}}+(\partial H / \partial \phi)^{\mathrm{n}}\left(\phi^{\mathrm{n}+1}-\phi^{\mathrm{n}}\right)+o\left(\Delta \mathrm{t}^{2}\right)  \tag{6}\\
& S^{\mathrm{n}+1}=\mathrm{S}^{\mathrm{n}}+(\partial S / \partial \phi)^{\mathrm{n}}\left(\phi^{\mathrm{n}+1}-\phi^{\mathrm{n}}\right)+o\left(\Delta t^{2}\right) \tag{7}
\end{align*}
$$

Eqs. (5-7) are inserted into Eq. (4) to obtain the following system which is linear in $\phi^{n+1}$

$$
\begin{equation*}
\left(A-B \Delta t L^{n}\right)\left(\phi^{n+1}-\phi^{n}\right)=\Delta t\left(D^{n}+S^{n}\right) \tag{8}
\end{equation*}
$$

and which is termed a linearized block implicit (LBI) scheme. Here, A denotes a square matrix defined by

$$
\begin{equation*}
A \equiv(\partial H / \partial \phi)^{n}-\beta \Delta t(\partial S / \partial \phi)^{n} \tag{9}
\end{equation*}
$$

Eq. (8) has $0(\Delta t)$ accuracy unless $H \equiv \phi$, in which case the accuracy is the same as Eq. (4).

## Special Treatment of Diffusive Terms

The time differencing of diffusive terms is modified to accomodate crossderivative terms and also turbulent viscosity and artificial dissipation coefficients which depend on the solution variables. Although formal linearization of the convection and pressure gradient terms and the resulting implicit coupling of variables is critical to the stability and rapid convergence of the algorithm, this does not appear to be important for the turbulent viscosity and artificial dissipation coefficients. Since the relationship between $\mu_{e}$ and $d_{j}$ and the mean flow variables is not conveniently linearized, these diffusive coefficients are evaluated explicitly at $t^{n}$ during each time step. Notationally, this is equivalent to neglecting terms proportional to $\partial \mu_{e} / \partial \phi$ or $\partial d_{j} / \partial \phi$ in $L^{n}$, which are formally present in the Taylor expansion (5), but retaining all terms proportional to $\mu_{e}$ or $d_{j}$ in both $L^{n}$ and $D^{n}$.

It has been found through extensive experience that this has little if any effect on the performance of the algorithm. This treatment also has the added benefit that the turbulence model equations can be decoupled from the system of mean flow equations by an appropriate matrix partitioning (cf. [15]) and solved separately in each step of the ADI solution procedure. This reduces the block size of the block tridiagonal systems which must be solved in each step and thus reduces the computational labor.

In addition, the viscous terms in the present formulation include a number of spatial cross-derivative terms. Although it is possible to treat cross-derivative terms implicitly within the ADI treatment which follows, it is not at all convenient to do so, and consequently, all cross-derivative terms are evaluated explicitly at $t^{n}$. For a scalar model equation representing combined convection and diffusion, it has been shown by Beam and Warming that the explicit treatment of cross-derivative terms does not degrade the unconditional stability of the present algorithm. To preserve notational simplicity, it is understood that all cross-derivative terms appearing in $\mathrm{L}^{\mathrm{n}}$ are neglected but are retained in $D^{n}$. It is important to note that neglecting terms in $L^{\mathrm{n}}$ has no effect on steady solutions of Eq . (8), since $\phi^{\mathrm{nt1}}-\phi^{\mathrm{n}} \equiv 0$ and thus Eq. (8) reduces to the steady form of the equations: $D^{n}+S^{n}=0$. Aside from stability considerations, the only effect of neglecting terms in $L^{n}$ is to introduce an $0(\Delta t)$ truncation error.

## Consistent Splitting of the LBI Scheme

To obtain an efficient algorithm, the linearized system (8) is split using ADI techniques. To obtain the split scheme, the multidimensional operator $L$ is rewritten as the sum of three "one-dimensional" sub-operators $L_{i}(i=1,2,3)$ each of which contains all terms having derivatives with respect to the i-th coordinate. The split form of Eq. (8) can be derived either as in [13, 15] by following the procedure described by Douglas and Gunn [16] in their generalization and unification of scalar ADI schemes, or using approximate factorization. For the present system of equations, the split algorithm is given by

$$
\begin{align*}
& \left(A-\beta \Delta t L_{1}^{n}\right)\left(\phi^{*}-\phi^{n}\right)=\Delta t\left(D^{n}+S^{n}\right)  \tag{10a}\\
& \left(A-\beta \Delta t L_{2}^{n}\right)\left(\phi^{* *}-\phi^{n}\right)=A\left(\phi^{*}-\phi^{n}\right)  \tag{10b}\\
& \left(A-B \Delta t L_{3}^{n}\right)\left(\phi^{n+1}-\phi^{n}\right)=A\left(\phi^{* *}-\phi^{n}\right) \tag{10c}
\end{align*}
$$

where $\phi^{*}$ and $\phi^{* *}$ are consistent intermediate solutions. If spatial derivatives appearing in $L_{i}$ and $D$ are replaced by three-point difference formulas, as indicated previously, then each step in Eqs. ( $10 \mathrm{a}-\mathrm{c}$ ) can be solved by a blocktridiagonal elimination.

Combining Eqs. ( $10 \mathrm{a}-\mathrm{c}$ ) gives

$$
\begin{align*}
& \left(A-B \Delta t L_{1}^{n}\right) A^{-1}\left(A-B \Delta t L_{2}^{n}\right) A^{-1}\left(A-B \Delta t L_{3}^{n}\right)\left(\phi^{n+1}-\phi^{n}\right)  \tag{II}\\
& =\Delta t\left(D^{n}+S^{n}\right)
\end{align*}
$$

which approximates the unsplit shceme (8) to $0\left(\Delta t^{2}\right)$. Since the intermediate steps are also consistent approximations for Eq. (8), physical boundary conditions can be used for $\phi^{*}$ and $\phi^{* *}[13,15]$. Finally, since the $L_{i}$ are homogeneous operators, it follows from Eqs. (10a-c) that steady solutions have the property that $\phi^{\mathrm{nt1}}=\phi^{*}=\phi^{* *}=\phi^{\mathrm{n}}$ and satisfy

$$
\begin{equation*}
D^{n}+s^{n}=0 \tag{12}
\end{equation*}
$$

The steady solution thus depends only on the spatial difference approximations used for (12), and does not depend on the solution algorithm itself.

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Fig. 1 - Schematic of the inlet/diffuser model.

Subsonic Diffuser


Fig. 2 - Top wall static pressure distribution


Fig. 3 - Bottom wall static pressure distribution

Transonic Diffuser


Fig. 4 - Top wall static pressure distribution.

Transonic Diffuser


Fig. 5 - Bottom wall static pressure distribution.


Fig. 6 - Core Mach number distribution and effects of artificial dissipation.


Note: The scale in transverse direction is 5 times larger than the scale in streamwise direction.

Fig. 7 - Schematic flow field.


Fig. 8 - Top wall static pressure distribution.


Fig. 9 - Bottom wall static pressure distribution.

$z / h=-3.75$
$z / h=0$
$\mathrm{z} / \mathrm{h}=3.4$
(a) Subsonic diffuser

(b) Transonic diffuser

(c) Supersonic inlet

Fig. 10 - Mach number contours.

(a) Subsonic diffuser

(b) Transonic diffuser

(c) Supersonic inlet

Fig. 11 - Dimensionless static pressure contours.


Fig. 12 - Time history of the top wall static pressure distribution. ( 1 time unit $\approx 0.3 \times 10^{-3} \mathrm{sec}$ )

Formation of the shock (transonic diffuser)


Fig. 13 - Time history of the bottom wall static pressure distribution. ( 1 time unit $\approx 0.3 \times 10^{-3} \mathrm{sec}$ )

(b) $t=10.0$ units


Fig. 14 - Time history of Mach number contours (transonic diffuser).

(a) $\mathrm{t}=5.0$ units


Fig. 15 - Time history of dimensionless static pressure contours (transonic diffuser)


Fig. 16 - Top wall static pressure distribution

Transonic Diffuser (3-D)


Fig. 17 - Bottom wall static pressure distribution

(a) $y / h=0.216 \times 10^{-1}$

(b) $y / \mathrm{h}=0.732 \times 10^{-1}$


Fig. 18 - Dimensionless static pressure contours (3-D, transonic diffuser)


Fig. 19 - Streamwise Mach No. distribution

(a) $y / h=0.216 \times 10^{-1}$

(b) $y / h=0.732 \times 10^{-1}$


Fig. 20 - Mach Number contours (3-D, transonic diffuser)

## User's Manual

The present manual is prepared for the INLTS.GO1 CODE, which is the first version of the MINT INLET CODE. This particular version of the code is being stored on the Lewis IBM 370-3033 with TSS operating system and is written to solve the multi-dimensional ensemble-averaged time-dependent Navier-Stokes equations for turbulent, shocked flows in contoured, straight ducts with rectangular cross-sections. The coordinate system is nonorthogonal, contourfitted and the equations are cast into the so-called strong conservation form. For the present time, the solution of the energy equation is replaced by the assumption that the total temperature is constant throughout the flowfield, although an energy equation can be activated. The effects of turbulence are represented by a mixing length model and the shock is captured by a second order artificial dissipation technique. The numerical procedure solves the time-dependent equations beginning with a specified initial condition and appropriate boundary conditions. Detailed descriptions of these various items can be found in the previous sections and will not be repeated here.

The INLTS.GO1 CODE combines a BLOCK DATA program (BLKDAT) containing pertinent data statements, a main program (DAL) and a series of subroutines to perform the required calculations. Chart 1 shows the overall program flow, Chart 2 illustrates the input and initialization procedures, Chart 3 is a global description of the execution control. These program flow charts only provide a broad picture of the code. The interested user should consult the program listing about the details. Since the contour of the inlet varies from case to case according to user's interest, the user must set up the particular contour by slightly modifying the following subroutines: TIMGEO, INVICD and SPREAD.

In SUBROUTINE TIMGEO, the variables RBMAX and RBMIN must be specified by the user. RBMAX is the $x$-coordinate of the top wall at a given streamwise location and RBMIN is the $x$-coordinate of the bottom wall at the same streamwise location. In SUBROUTINE INVICD, the variable MZSHK must be given by the user. This variable indicates the initial location of the normal shock in the starting flow field, e.g., MZSHK $=10$ means that at the 10 th streamwise grid point a normal shock will be generated according to the RankineHugoniot relation. Obviously, if the inflow is not supersonic and the
one-dimensional inviscid theory does not indicate an internal supersonic region, then MZSHK must be set to be an integer greater than the total number of streamwise grid points. In SUBROUTINE SPREAD, the variables PINVCD and HEIT must be specified by user. PINVCD is the dimensionless static pressure at the outflow section obtained by the one-dimensional inviscid theory under the condition that the dimensionless static pressure at the inflow section is 1 . It should be noted that PINVCD may not be the actual static pressure used as boundary condition at the outflow section. HETT is the height of the channel at the streamwise location denoted by KZ . Further, the variables MZSHK, PINVCD and HEIT are used only for setting up the starting flow field. The SUBROUTINE WRPLOT deals with the construction of plot files, since this also depends on the specific interest of the user, the coding of this subroutine must also be modified by the user to accommodate the user's interest. Nevertheless, since the structure of this subroutine is consistent with the NASA-Lewis inhouse plotting routines, the modifications should be quite straightforward.

The card input data is all in NAMELIST format: READ1, READ9, DATA1 and INFLW. The NAMELIST READ1 defines the restart option and input/output units. The NAMELIST READ9 specifies the grid parameters, reference quantities, time-step parameters, boundary conditions and print parameters. The NAMELIST DATAI sets up the streamwise grid distribution, and the NAMELIST INFLW specifies the turbulent compressible boundary layer profiles at the inlet section. In the case of a restart run, the NAMELIST INFLW should not be supplied.


Chart 1. - Overa11 Program Flow, PROGRAM DAL


Chart 2. - Program flow chart for SUBROUTINE READA


Chart 3. - Program flow chart for SUBROUTINE EXEC

Namelist or
Variable Name

## Description

\&READ1

| IREST | $=0$ |
| :--- | :--- |
|  | $=1$ |
| IOTAPE | $=10$ |
|  | $=9$ |
| INTAPE | $=20$ |
| IOTAP1 |  |
| INTAP1 | $=19$ |

A new calculation is being started.
Case is being run from restart files.

Output unit number for dependent variable array restart data.

Input unit number for dependent variable array restart data.

Output unit number for namelist restart data.

Input unit number for namelist restart data.
\&READ9

NUMDX

NUMDY

Number of interior grid points in the transverse direction ( $x$ or $y^{1}$ direction). Total number of points in this direction $=$ NUMDX +2 . NUMDX $\leq 29$.

Number of interior grid points in the spanwise direction ( $y$ or $\mathrm{y}^{2}$ direction). Total number of points in this direction $=$ NUMDY +2 . NUMDY $\leq 29$.

| Namelist or |  |
| :---: | :---: |
| Variable Name | Description |
| NJMDZ | Number of interior grid points in the streamwise direction ( $z$ or $y^{3}$ direction). <br> Total number of points in this direction $=$ NUMDZ + 2. NUMDZ $\leq 59$. |
| XGMIN (1) | Dimensionless value of $x$-coordinate of the bottom wall at the inflow boundary. |
| XGMIN(2) | Dimensionless value of $y$-coordinate of the side wall. |
| XGTMIN (3) | Dimensionless value of $z$-coordinate at the inflow boundary. |
| XGMAX (1) | Dimensionless value of $x$-coordinate of the top wall at the inflow boundary. |
| XGMAX (2) | Dimensionless value of $y$-coordinate of the (spanwise) symmetry plane. |
| XGMAX (3) | Dimensionless value of $z$-coordinate of the nutflow boundary. |
| $\operatorname{GRID}(\mathrm{I}), \mathrm{I}=1-6$ | $\text { Define } \begin{aligned} \tau_{1} & \equiv \operatorname{GRID}(2 \mathrm{~K}-1) \\ \tau_{2} & \equiv \operatorname{GRID}(2 \mathrm{~K}) \end{aligned}$ <br> For each coordinate direction $y^{k}$ ( $\mathrm{K}=1,2,3$ ). $\quad \tau_{1}$ and $\tau_{2}$ are the grid stretching parameters for controlling grid spacing near the computational domain boundaries. In the present code this grid distribution technique is used for the $x$ and $y$ directions. The streamwise, z-direction, is constructed via variables in \&DATAl. The limits on $\tau_{1}$ and $\tau_{2}$ are: $-1<\tau_{1} \leq 0,0 \leq \tau_{2}<1$. If $\tau_{1}=\tau_{2}=0$, the grid spacing is uniform. If $\tau_{1}=0$, $\mathrm{t}_{2}>0$, the grid points will be more dense near the XGMAX (K) boundary. <br> If $\tau_{1}<0, \tau_{2}=0$, the grid points will be more dense near the XGMIN(K) boundary. The transformation is singular if $\tau_{1}=$ -1 or ${ }^{\tau} 2=1$. |


| Namelist or |  |
| :---: | :---: |
| Variable Name | Description |
| $\operatorname{XCENTR}(\mathrm{I}), \mathrm{I}=1-3 .=0$ | Not used. |
| $\operatorname{DAMPG}(\mathrm{I}), \mathrm{I}=1-3 .=0$ | Not used. |
| IGEOM $\quad=1$ | The height of the channel is not a function of streamwise coordinate. <br> Contoured channel. |
|  | Logical variable for two-dimensional calculation. <br> Logical variable for three-dimensional calculation. |
| $\begin{array}{ll}\text { IGTYPE } & =1 \\ & =2\end{array}$ | ```Two-dimensional calculation. Used with TWOD = T Three-Dimensional calculation. Used with TWOD = F``` |
| LXSPLT | x grid point location for summary print. |
| LYSPLT | y grid point location for summary print. Default is 1. |
| REY | Reynolds number (calculated). |
| CLENG | Reference length, meters (throat height). |
| WREF | Reference velocity, m/sec (core-flow velocity at the inflow boundary). |

Namelist or
Variable Name
$\operatorname{XCENTR}(\mathrm{I}), \mathrm{I}=1-3 .=0$
$\operatorname{DAMPG}(I), I=1-3 .=0$

LXSPRLT

LYSPLT

REY

CLENG

WREF

Reference velocity, m/sec (core-flow velocity at the inflow boundary).

Namelist or
Variable Name
DENSR

TREF

PREF

VISCR

CMACH
LAMFLO $=0$
$\neq 0$

AVISC(IDIR,IEQ)

## Description

Reference density, $\mathrm{kg} / \mathrm{m}^{3}$ (core-flow density at the inflow boundary).

Reference temperature, ${ }^{\circ} \mathrm{K}$ (core-flow temperature at the inflow boundary).

Reference pressure, Pa (calculated).

Reference dynamic viscosity, $\mathrm{kg} / \mathrm{m}-\mathrm{sec}$ (core-flow dynamic viscosity at the inflow boundary).

Reference Mach Number (calculated)

Turbulent flow
Obsolete.

Artificial dissipation parameter $\sigma$ (See Eq. (1) of the Appendix).
IEQ $=1-5$, and $\operatorname{IDIR}=1-3$.
IEO $=1$ indicates the $x$-momentum equation.
IEQ $=2$ indicates the $y$-momentum equation.
IEQ $=3$ indicates the $z$-momentum equation.
IEQ $=4$ indicates the continuity equation.
IEQ $=5$ indicates the energy equation.
IDIR=1 indicates the x second derivative term
IDIR=2 indicates the $y$ second derivative term
IDIR=3 indicates the $z$ second derivative term
For example, $\operatorname{AVISC}(3,1)$ is the value of $\sigma$ used for the artificial dissipation term
$\frac{\partial^{2} U}{\partial z^{2}}$ in the $x$-momentum equation.
Note that even if the $y$-momentum equation is not solved, the corresponding AVISC must be supplied. Default values are 0.0. Recommended values are 0.50 initially followed by runs at 0.10 .

| Namelist or |  |
| :---: | :---: |
| Variable Name | Description |
| NT | Number of time steps to be run |
| DT | Initial nondimensional time step. If DT is omitted on a restart DT will be set to value at termination of last run. |
| DTMIN | Minimum nondimensional time step for this run |
| DTMAX | Maximum nondimensional time step for this run |
| IDTADJ $\quad=0$ | Constant DT is used for this run |
| $=1$ | Time step adjusted. If maximum relative change in any flow variable is less than 0.04 , DT is multiplied by 1.25. If maximum relative change in any flow variable is greater than 0.06, DT is divided by 1.25 . |
| $=2$ | Time step is cycled between DTMIN and DTMAX using an acceleration parameter concept. A sequence of NTSTEP time steps is used under this option. |
| NTSTEP | Number of time steps used in cycling. Default value is 3 . |
| ITEST | Steady state test is performed every ITEST time steps. Default value is 1. |
| SSEPS | Steady-state convergence criteria. Default is 0.001. |
| IPRINT | Complete flow field printouts are provided every IPRINT time steps. |

Namelist or
Variable Name

## LZPRNT(LZ)

$=0$
$=1$

IVARPR (IV)
$=0$
$=1$

IDUMP1
$=1$
$=2$

IPLOT

$$
=0
$$

$>0$

NFPLOT
$=0$

Optional print control flag for threedimensional calculations only. No printout at streamwise station number LZ

Normal printout at Station LZ. Default is 1.

Optional print control flag for variable IV
Suppress printout of variable IV
Normal printout of variable IV
IV IVARPR(IV)
1 transverse velocity, u
2 spanwise velocity, v
3 streamwise velocity, w
4 density, $\rho$
5 enthalpy, h
26 pressure, $p$
27 temperature, T
28 effective viscosity, $\mu_{e f f}$
33 mixing length, \&
35 Mach number, M
36 total pressure, Po

No plot file (TAPE1) written.

Plot file written at time step increment IPLOT.

NFPLOT must be set to zero.

Namelist or
Variable Name
$\operatorname{IGPRT}(1)=0$
$=1$
$\operatorname{IGPRT}(2)=0$

$$
=1
$$

$\operatorname{IGPRT}(3)=0$
$=1$

ICASE
$=1$
$=2$

PRESS6
\&DATAI

YFERST $=$ XGMIN (3)

YLAST $=$ XGMAX (3)

NCLUST

## Description

No $x$-coordinate printout.
Print $x$-coordinate distribution.

No $y$-coordinate printout.
Print y-coordinate distribution. Default is 0 .

No z-coordinate printout.
Print z-coordinate distribution.

Inflow is not supersonic.
Inflow is supersonic.

Dimensionless static pressure at the outflow section. Note that the reference condition is the inflow core condition.

See \&READ9.

See \&READ9.

Total number of the interior cluster points. A cluster point is the sequential number of the selected grid point which must coincide with particular predetermined value of the $z$-coordinate. Accordingly, pairs of (1, YFIRST) and (NUMDZ+2, YLAST) are also cluster points, but they are boundary cluster points.

Namelist or
Variable Name
$\operatorname{CLPY}(\mathrm{I}), \mathrm{I}=1-61$

CLPX(I), $I=1-61$
$\operatorname{ETAP}(J), J=1-61$

ALPH(J), J=1-61
$>0$
$<0$

NEND

$$
\begin{aligned}
& =0 \\
& =1 \\
& =2 \\
& =3
\end{aligned}
$$

RHW

$$
\text { always > } 0.0
$$

$=1.0$

The z-coordinate of the cluster point. Note that both of boundary and internal cluster points must be specified.

The sequential number of the grid point corresponding to CLPY(I).

The sequential number of the grid point defined as pivot point. The grid spacing will have the fastest variation at a pivot point. For each of the interior cluster points there shall be a pair of pivot points: one ahead of the cluster point, the other one after the cluster point. However, only one pivot point shall be associated with each of the boundary cluster points.

Width parameter specifying width (in terms of the number of grid points) in which 90 per cent of grid-size variation takes place around the pivot point ETAP (J).
decreasing grid-size.
increasing grid-size.

No stretching at YFIRST and YLAST Stretching at YFIRST only Stretching at YLAST only Stretching at YFIRST and YLAST

Approximate ratio of grid-size at CLPY(2) to the maximum grid-size in the interval CLPY(1) <Z< CLPY(2).

No stretching at YFIRST. Used with NEND $=0$ or 2 .

| Namelist or Variable Name | Description |
| :---: | :---: |
| RATIO ( K ), $\mathrm{K}=1-40$. always $>0.0$ | Approximate ratio of grid-size at CLPY $(K+1)$ to the maximum grid-size in the interval CLPY ( $K$ ) $<Z<\operatorname{CLPY}(K+1)$ |
| $=1.0$ | No grid-variation at CLPY $(\mathrm{K}+1)$. |
| BETAO always > 0.0 | Calculated. It indicates the first derivative of Z -coordinate with respect to the computational coordinate at YFIRST. |
| $\operatorname{BET}(\mathrm{L}), \mathrm{L}=1-61$. alwyas > 0.0 | Calculated. It indicates the ratio between the grid sizes on both sides of the pivot point ETAP (L+1). |
| INFLW |  |
| CFLW | Coefficient of skin friction at the botton wall of inflow boundary. |
| DELTAL | Dimensionless boundary layer thickness at the bottom wall of inflow boundary. |
| PRDL | Prandtl Numer used only for generating the bottom wall velocity pro£ile. |
| CFUW | Coefficient of skin friction at the top wall of inflow boundary. |
| deltau | Dimensionless boundary layer thickness at the top wall of inflow boundary. |
| PRDU | Prandtl Number used only for generating the top wall velocity profile. |
| TINF $=$ TREF | See \&READ9 |

## LIST OF MAJOR FORTRAN VARIABLES

| FORTRAN | COMMON | DESCRIPTION |
| :---: | :---: | :---: |
| SYMBOL | BLOCK |  |
| $\operatorname{AC}(\mathrm{I}, \mathrm{J}, \mathrm{K})$ | BLK1 | DEPENDENT VARIABLE ARRAY |
| $\operatorname{ACG}(J, J)$ | BLK1 | GEOMETRY DATA ARRAY |
| AN ( $\mathrm{I}, \mathrm{J}$ ) | BLKM | ARRAY STORING TIME TERM LINEARIZED COEFFICIENTS |
| $\operatorname{APR}(\mathrm{I}, \mathrm{J})$ | PRNT | PRINT OUTPUT ARRAY |
| AVANDR | TURB | DAMPING CONSTANT |
| AVISC(I, J) | MISC2 | ARTIFICIAL DISSIPATION PARAMETER |
| $C(I, J, K)$ | BLKM | COUPLED MATRIX ARRAY STORAGE |
| CLENG | CREF | REFERENCE LENGTH |
| CMACH | MISC2 | REFERENCE MACH NUMBER |
| D | VARNO | INDEX FOR DIVERGENCE |
| D1 ( $\mathrm{I}, \mathrm{J}, \mathrm{K}$ ) | BLKM | ARRAY STORING FIRST SWEEP LINEARIZED COEFFICIENTS |
| D2 ( $\mathrm{I}, \mathrm{J}, \mathrm{K}$ ) | BLKM | ARRAY STORING SECOND SWEEP LINEARIZED COEFFICIENTS |
| D3 ( $\mathrm{I}, \mathrm{J}, \mathrm{K}$ ) | BLKM | ARRAY STORING THIRD SWEEP LINEARIZED COEFFICIENTS |
| DENSR | CREF | REFERENCE DENSITY |
| DFW ( $\mathrm{I}, \mathrm{J}, \mathrm{K}$ ) | ADI7 | DIFFERENCE WEIGHT ARRAY |
| DIM1 | NOND | INVERSE REYNOLDS NUMBER |
| DIM2 | NOND | REFERENCE PRESSURE/REFERENCE DYNAMIC HEAD |
| DIM3 | NOND | REFERENCE PRESSURE/(REFERENCE DENSITY * REFERENCE ENTHALPY) |
| DIM4 | NOND | $1.0 /\left(\right.$ REY * $\mathrm{P}_{\mathbf{r}}$ ) |
| DIM12 | NOND | $2.0 *$ DIMI |
| DS | VARNO | INDEX FOR DISSIPATION |
| DT | MISC2 | TIME STEP |


| FORTRAN | COMMON | DESCRIPTION |
| :---: | :---: | :---: |
| SYMBOL | BLOCK |  |
| DTCON | MISC2 | INVERSE STEP |
| DTMAX | MISC2 | MAXIMUM ALLOWABLE TIME STEP |
| DTMIN | MISC2 | MINIMUM ALLOWABLE TIME STEP |
| E(I, J, K) | BLKM | COUPLED MATRIX ARRAY STORAGE |
| GRID (I) | GTRAN | GRID DISTRIBUTION PARAMETER (SEE \&READ9) |
| H | VARNO | INDEX FOR ENTHALPY |
| I1 | MGAUS | LOWER LIMIT FOR MATRIX INVERSION |
| IADI | ADI1 | ADI SWEEP NUMBER |
| IBC | ADII | BOUNDARY CONDITION BOUNDARY PARAMETER |
| IDT | MISC2 | TIME STEP INDEX |
| IDTADJ | MISC2 | TIME STEP CONTROL PARAMETER (SEE \&READ9) |
| IDUMP1 | OUTA | PARAMETER CONTROLLING INITIAL STATION PRINT (SEE \&READ9) |
| IEQ | ADI1 | EQUATION NUMBER |
| IGPRT (I) | GEO1 | GEOMETRY PRINT CONTROL (SEE \&READ9) |
| IL | MGAUS | UPPER LTMIT FOR MATRIX INVERSION |
| IPRINT | MISC2 | PRINT INTERVAL PARAMETER (SEE \&READ9) |
| IRERUN | MISC2 | RESTART WRITE CONTROL PARAMETER (SEE\&READ1) |
| IREST | MISC2 | RESTART READ CONTROL PARAMETER (SEE \&READ1) |
| IVARPR(I) | MISC2 | PRINT PARAMETER (SEE \&READ9) |
| JADI | ADII | ADI SWEEP PARAMETER |
| $J E Q B C(I, J, K)$ | ADII | BOUNDARY CONDITION TYPE PARAMETER |
| JX | ADI2 | DIRECTION-1 GRID POINT INDEX |
| KZ | ADI2 | DIRECTION-3 GRID POINT INDEX |


| FORTRAN | COMMON | DESCRIPTION |
| :---: | :---: | :---: |
| SYMBOL | BLOCK |  |
| LX | ADI2 | DIRECTION-1 GRID POINT INDEX |
| LX1 | ADI3 | FTRST DIRECTION-1 INTERIOR POINT |
| LX2 | ADI3 | LAST DIRECTION-1 INTERIOR POINT |
| LY | ADI2 | direction-2 Grid point index |
| LYI | ADI3 | FIRST DIRECTION-2 INTERIOR POINT |
| LY2 | ADI3 | LAST DIRECTION-2 INTERIOR POINT |
| LZ | ADI2 | DIRECTION-3 GRID POINT INDEX |
| L21 | ADI3 | FIRST DIRECTION-3 INTERIOR POINT |
| L22 | ADI3 | LAST DIRECTION-3 INTERIOR POINT |
| LZPRNT | MISC2 | THREE DIMENSIONAL PRINT CONTROL (SEE \&READ9) |
| MEQS | ADII | NUMBER OF EQUATIONS TO BE SOLVED |
| NT | MISC2 | NUMBER OF TIME STEPS TO BE RUN |
| NUMDX | MISC2 | NUMBER OF INTERIOR DIRECTION-1 POINTS |
| NUMDY | MISC2 | NUMBER OF INTERIOR DIRECTION-2 POINTS |
| NUMDZ | MISC2 | NUMBER OF INTERIOR DIRECTION-3 POINTS |
| NXI | ADI4 | FIRST GRID POINT - DIRECIION 1 |
| NX2 | ADI4 | LAST GRID POINT - DIRECTION 1 |
| NY1 | ADI4 | FIRST GRID POINT - DIRECTION 2 |
| NY2 | ADI. 4 | LAST GRID POINT - DIRECTION 2 |
| NZ1 | ADI4 | FIRST GRID POINT - DIRECTION 3 |
| NZ2 | ADI4 | LAST GRID POINT - DIRECTION 3 |


| FORTRAN | COMMON | DESCRIPTION |
| :---: | :---: | :---: |
| SYMBOL | BLOCK |  |
| P | VARNO | INDEX FOR PRESSURE |
| PCNT1 | MISC2 | TIME STEP CONTROL PARAMETER |
| PCNT2 | MISC2 | TIME STEP CONTROL PARAMETER |
| PREF | CREF | REFERENCE PRESSURE |
| PRNDL | CREF | PRANDTL NUMBER |
| PTOT | BCCON | TOTAL PRESSURE |
| R | VARNO | INDEX FOR DENSITY |
| REY | CREF | REYNOLDS NUMBER |
| SN (I) | BLKM | ARRAY STORING SOURCE TERM LINEARIZED COEFFICIENT |
| SSTEST | MISC2 | MAXIMUM CHANGE IN VARIABLE ACROSS TIME STEP |
| T | VARNO | INDEX FOR TEMPERATURE |
| TAUW | TURB | WALL SHEAR |
| TREF | CREF | REFERENCE TEMPERATURE |
| TTIME | MISC2 | CUMULATIVE TIME |
| TTOT | BCCON | TOTAL TEMPERATURE |
| U | VARNO | INDEX FOR DIRECTION-1 VELOCITY |
| USTAR | TURB | DIMENSIONLESS VELOCITY |
| V | VARNO | INDEX FOR DIRECTION-2 VELOCITY |
| VISCL | TURB | LAMINAR REFERENCE VISCOSITY |
| VISCR | CREF | REFERENCE VISCOSITY |
| VS | VARNO | INDEX FOR VISCOSITY |

FORTRAN SYMBOL

| W | VARNO | INDEX FOR DIRECTION-3 VELOCITY |
| :--- | :--- | :--- |
| WREF | GREF | REFERENCE VELOCITY |
| XGMAX(I) | GRID1 | MAXIMUM COORDINATE VALUE (SEE \&READ9) |
| XGMIN (I) | GRID1 | MINIMUM COORDINATE VALUE (SEE \&READ9) |
| YPLUS | TURB | DIMENSIONLESS DISTANCE FROM SURFACE |

## FILE INPUT/OUTPUT

To read restart files, the following commands must be given at the beginning of a run:

| RMDS | A10, R9 |
| :--- | :--- |
| RELEASE | RMDS |
| RMDS | A20, R19 |
| RELEASE | RMDS |

To write restart files, the following command must be given at the end of a run:
CATALOG SC10, U, , A10
MDS A10
CATALOG SC20, U, , A20
MDS A20
Where A10 and A20 are some given file names, SC10 and SC20 are scratch files defined in PROCDEF RUNMT.

Remarks on Storage Requirements and Run Time
When stored in data pool of the Lewis IBM 370, the files INLTS, INLTB and INLTCM (see page 83) occupy 171,476 and 10 pages, respectively. The sizes of the restart files are problem dependent; as an example, for $2-\mathrm{D}$ problems with $31 \times 41$ grid points, the file A10 requires 78 pages and the file A20 requires 2 pages. For 3-D problems with $31 \times 41 \times 16$ grid points, the file Alo requires 640 pages, while the file A20 needs 3 pages. As for the run time, in terms of CPU sec per time-step per grid-point, approximately 0.013 sec is needed in a $2-D$ problem and 0.028 sec is required for a $3-\mathrm{D}$ problem.

As a further indication of the storage requirements, information obtained from executing another version of the MINT Code on a CDC machine is also given here. With 3500 grid points, 225000 decimal words are needed; however, by using overlay and out-of-core-option, the storage requirement has been reduced to 90000 decimal words. Although this out-of-core-option is not included in the present INLTS.GO1 version, the implementation of this option can be carried out in a straightforward manner.

```
PROCDEF MOD
PARAM $A
DDEF X,VI,SOURCE. $AN,RET=T
DEFAULT SYSINX=E
EDIT SOURCE.$AN
-REVISE 100,LAST
-EXCERPT M,$A,100,LAST
PROCDEF MODN
PARAM $A
-END
FIN $AN,LISTDS=N,SLIST=N,CRLIST=N
DEFAULT SYSINX=G
ERASE SOURCE.$AN
```

PROCDEF RUNMT
ERASE M
DDEF FT01F001,VS,SC01,RET=T
DDEF FTO2F001,VS, SCO , $\mathrm{RET}=\mathrm{T}$
DDEF FT03F001,VS,SC03,RET=T
DDEF FT04F001,VS,SC04,RET=T
DDEF FT09F001,VS,R9,RET=T
DDEF FT10F001,VS,SC10, RET=T
DDEF FT11F001,VS,SC11,RET=T
DDEF FT12F001,VS,SC12,RET=T
DDEF FT13F001,VS,SC13,RET=T
DDEF FT14F001,VS,SC14,RET=T
DDEF FT15F001,VS,SC15, RET $=\mathrm{T}$
DDEF FT16F001, VS, SC16, RET=T
DDEF FT17F001, VS, SC17, RET=T
DDEF FT19F001,VS,R19,RET=T
DDEF FT20F001,VS,SC20,RET=T
DDEF FT21F001,VS,SC21, RET $=T$
LOAD BLKDATN
LOAD DALN
DALN

A sample case was run to illustrate the set up of input parameters and typical printouts of the INLTS.GO1 code. The given input card deck is for a new calculation. In the case of restarted calculation, all parameters of \&READl and \&DATAl must be specified, however, in \&READ9, only NT, PRESS6 and ICASE must always be specified through input cards for restarted calculation, unless the user wants to change other parameters for other purposes. The Namelist \&INFLW should be omitted for restarted calculation. Also note that, parameters associated with the grid point distribution should not be changed since the computational coordinate system is time-independent.

The code output first prints out a series of dimensionless parameters DIM1 - DIM10, DIM12 and DIM14, and the dimensionless total temperature, total pressure and total enthalpy. This is followed by the finite difference coefficients for first and second derivatives in both directions. In each direction three lines are written. The first and third lines give one-sided difference weights at the lower and upper boundaries; the second line of each set gives central differences used for the interior points. Six numbers are written on each line; the first set of three values represent the first derivative coefficients and the second set of three values represent the second derivative coefficients.

The next output item is the grid distribution data. They are quite self-explanatory. The first part indicates the results of streamwise coordinate transformation (see SUBROUTINES OHGRID, STCLST and FIXBYI). The second part gives the $x$-coordinates at each $z$ location and the third part gives the z-coordinates of each streamwise grid point.

These geometry data are followed by the printout of NAMELTST INFLW and the results of the inflow boundary layer profiles calculation. These are printed out in the SUBROUTINE PROFIL, the user should consult this subroutine if detailed informations are desired. It is only noted here that the first part deals with the bottom wall boundary layer profile calculation and the second part deals with the top wall boundary layer profile calculation.

Following the boundary layer profiles printouts, the results of one-dimensional inviscid calculation are printed in the SUBROUTINE INVICD. The inviscid solutions are given at each streamwise grid point. The SUBROUTINES INVICD and SPREAD should be consulted about these results. The following item is the printouts of the NAMELISTS READI, READ9 and DATAl. Note that values of some parameters may be changed by the internal operations.

The output of the namelist data is followed by estimations of the total mass and total energy within the computational domain, as well as the estimated mass flux at each streamwise section. This is then followed by the printouts of the flowfield variables at the starting time-step of this calculation. These printouts are quite self-explanatory and will not be described in detail here. At each time-step, a summary print is uritten, in which the maximum relative change over a time-step is given by SSTEST along with the location. Also given are the maximum relative changes of each dependent variable. RESMAX is the maximum residual of the equations solved and indicates how well is the steady-state equation being satisfied. Finally, flow variables at (LXSPLT, LYSPLT) are written for each streamwise station. Note that PBOT, PTOP and DP represent static pressure at the bottom wall, top wall, and at the point indicated by LXSPLT. A typical set of output along with the corresponding given input will be presented in the following pages.

Note that the JOB CONTROL LANGUAGE (JCL) streams, as presented in pages 81 and 82 , are for the IBM 370 and the command DDEF defines devices which must be assigned for any other type machine. Also note that all the level 1 errors appearing in pages 83 and 84 are due to the non-optimal arrangements of the common blocks involved in the subroutines and they do not adversely affect execution of the code.

ELASE HIG
EHASE MINT
EHASE PROCM
ERASE PR
ERASE M
HMDS INLTH，MINT
HELEASE RMISS
UDEF $X X X \cdot V H$ ．DSINAME＝MINT，OP $I I U N=J U B L I H, O I S H=U L O$
RNUS INLTSMM
HELEASE RMUS
LHIJS INLTCM，PRUCM
HFLEASE MiHIS
MOU SPREA1）
－पEVISE 54300
PlWVCU $=1.0$
NOUN SPHEAU
HUNAT
8READI
［HEST $=0$ ．
10 PAPF＝10，
INTAPF＝ 0 ．
LOTAPI＝2U．
INTAPI＝19．
GEivU READI
\＆READG
NT＝3，
ICASE＝1．
PRESSK＝0．943．
YUMUX $=1 \mathrm{H}$, NUMU $L=1 \mathrm{H}$ ，
$x(; M I N=0.0 \cdot 0.0 .0 .0$ ．
$X G M A X=1.0 .0 .0, C .0$ ，
（FK（I）＝－0．9945－0．9995＊4＊0．0．
XCENTR（1）$=$ U．O． $0.0,0.0$ ，

［TEOVA $=1$ ．
TWUO＝T．IGTYPE＝1．
LXSNLT＝10．
CLENG $=0.0441$ ．
WREF＝14Y．17075．
リビン5H＝1．2ヵり212H，
THEF $=247.623$ ．
VISCK $=1.76415 H R U-05$ ．
$\mathrm{LA} \| F(0)=0$ ．
AvISC（1．1）＝154 U．5．
リT＝＝0．005，I）TMIN＝0．001，UTMAX $=0.10$ ，
［DIAIJJ $=1$ ．
ITEST $=1$ ．
1PHINT＝10，
IVAHPL $=$ ？ HE $^{6}$
IVARPK（2प）$=$ U．
IVAKFR（33）$=1 . \quad$ IVAKPK（35）$=1$.

IPLUT＝い。
IGOHT＝100．1．
－ENU HEADO

## RDATAI

NCLJうT $=$ の。
NEN 1 ＝ 0 ，
GETAO＝1）． 0 。
ETAP＝？．$n .2$ U．U．
$\Delta L P H=1.0 \cdot 1.0$ ，
YFLHST＝0．0．
YLAS1＝？．0．
CLPi＝1．0．2U．U

Gar！いこ1－11．

HH $\alpha=1$－ 1 ．
AEIVI）IMATAI
\＆IVFL
CFLiv＝0．00106．
JELTAL＝0．12144．
NHUL＝ 1.01 －
CFUN＝0． 001 万6，
DELTAIJ＝11．121440
PRUU＝1．0．
IINF＝？ $1 / 27.523$ ，
オEIN（）INFLW
HELEASF FT
ERASE NA
EMASE N19
EHASE MINT
FHASE PHNCM
EHASE M
EHASE SCOI
EHASE SCli
ERAjE SCPO LUGUFF



OINITIAL INPUY StREAM

ORESTART OUTPUT UNIT TP2O POSITIONED AT RESTART DUMP NO. 0 O PARTITIONS SKIPPED.
ORESTART OUTPUT UNIT TPIO POSITIONED AT RESTART DUMP NO. $0 \quad 0$ PARTITIONS SKIPPED.
DIM1-DIM10,DIM12,DIM14
T, P, H-TOTAL $=1.03850 E$ OO 1.14139 E 001.03850 E 00

## ODIRECTION-1 DIFFERENCE OPERATORS


DIRECTION-3 DIFFERENCE OPERATORS
-1.50000000E 002.00000000 E 00-5.00000000E-
$-5.00000000 \mathrm{E}-012.00000000 \mathrm{E}$ OO-5.0000000E-01

1. 00000000 E 00-2.00000000E DO 1.00000000E O




$$
0.0000000000 \times \text { LAST }=
$$

$$
\begin{array}{r}
20.00000000 \\
2.0000000000
\end{array}
$$


ametry data at istep no. outte = 0.00000000



 $\begin{array}{ll}16 & 0.99210 E \\ 15 & 0.93055 \mathrm{E}\end{array}$

$\qquad$




 gTRY DATA AT TSTEP NO. 0 GTIME $=0.00000000$



 ZINFLW

CFLW= $0.1660 \mathrm{D}-02$
ELTAL $=0.121490$
CFUW= $0.16600-02$
PRDU $=1.0 .014$
TINF $=287.6230$


```
PREF=104784.09706382
CMSCR= 0.1764158800-048
LAMFEO= 0.43880492276718
NT=3
DT= =50D-02
DTMIN= 0.10D-02
MTDADJ=1
NTSEST=1
\
VARPR=28*1, 4*0, 1, 0, 1, 5*0
MPMP1=0
NFPLOT=0,0,1, 17*0
ICASE= 1.0.933
&END
DATA1
MCLUSS=1
BETAO= 0.10526315789474
EIAP=2.0,200.0% 59*0.
LPH=2*1.0, 59*0.0
YFIRST=0.0
M,
CLPY=0,0, 20.0,59*0.0
RATIO= 1.0,
BET= 61*
8END
1 START-- TOTAL MASS = 0.1985675364E O1 TOTAL ENERGY = 0.1561919368E 01 
```




O***** U-VL
*****


| 20 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 18 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 17 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |








$012=$
$2=$
20
0.0000000 .105




#### Abstract

$842 E 000.842105 E 000.947388 E 00$


| 1 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.000 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0 L z=$ | $0.105263 E 0$ | $15789 \mathrm{E}$ | $26316 \mathrm{E}$ | $136842 E$ | $147368 \mathrm{E}$ | $.57895 \mathrm{E}$ | $168421 \mathrm{E}$ | $178947 E$ | $.189474 E$ | $200000 \mathrm{E}$ |
| 20 |  |  |  |  |  |  |  |  |  |  |
| 19 | $0.18239 \mathrm{E}-04$ | 18236E-04 | 18233E | 0.18229E-04 | 0.18226E-04 | 0.18223 | 0.18220E-04 | $0.18217 \mathrm{E}-04$ | 82 | 21 |
| 18 | $0.12028 \mathrm{E}-03$ | .12027E-03 | $0.12025 \mathrm{E}-03$ | $0.12023 \mathrm{E}-03$ | 0.12021 E | 0.12019E | $0.12017 \mathrm{E}-03$ | 0.12015 E | 101 |  |
| 17 | $0.41927 E-03$ | $0.41921 \mathrm{E}-03$ | $0.41915 \mathrm{E}-03$ | $0.41908 \mathrm{E}-03$ | 0.41902E-03 | 0.41896E-03 |  | 883 E | 0. 41877 | 03 |
| 16 | . $14369 \mathrm{E}-02$ | 0.14367E-02 | 0.14365 EE 02 | $0.14363 \mathrm{E}-02$ | 0.14361 EE 02 | $0.14360 \mathrm{E}-02$ | $0.14358 \mathrm{E}-02$ | $0.14356 \mathrm{E}-02$ | 0.14354E-02 | 35 |
| 15 | . $49124 \mathrm{E}-02$ | . $49120 \mathrm{E}-02$ | $0.49115 \mathrm{EE}-02$ | 0.49110E-02 | 0.49105E-02 | 0.49101E-02 | $0.49096 \mathrm{E}-02$ | 0.49091E-02 | 0.49086E-02 | 0.49082 |
| 14 | $0.11682 \mathrm{E}-01$ | 01 |  | 0.11680E-01 | $0.11680 \mathrm{E}-01$ <br> $0.14825 \mathrm{E}-01$ | $\begin{aligned} & 0.11679 \mathrm{E}-01 \\ & 0.14824 \mathrm{E}-01 \end{aligned}$ | $\begin{aligned} & 0.11679 \mathrm{E}-01 \\ & 0.14824 \mathrm{E}-01 \end{aligned}$ | $\begin{aligned} & 0.11678 E-01 \\ & 0.14824 E-01 \end{aligned}$ | $\begin{aligned} & 0.11678 \mathrm{E}-01 \\ & 0.14823 \mathrm{E}-01 \end{aligned}$ | 0.11677E-01 |
| 12 | $0.15467 \mathrm{E}-01$ | $15467 \mathrm{E}-01$ | 15467E-01 | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | 0.15467E-01 | $0.15467 \mathrm{E}-01$ | $0.15467 E-01$ | $0.15467 \mathrm{E}-01$ |  |
| 11 | $0.15467 \mathrm{E}-01$ | 0.15467E-01 | $0.15467 \mathrm{E}-01$ | $0.15467 E-01$ | $0.15467 \mathrm{E}-01$ | 0.15467E-01 | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | - 15456 - | O. 15467 E -01 |
| 10 | $0.15467 \mathrm{E}-01$ | .15467E-01 | 15467E-01 | . 15467 E-01 | 0.15467E-01 | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ |
| 9 | 0.15467 E 01 | .15467E-01 | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | $0.15467 \mathrm{E}-01$ | 0.15467E-01 |
|  | $0.14826 \mathrm{E}-01$ | . $14825 \mathrm{E}-01$ | $0.14825 E-01$ | $0.14825 \mathrm{E}-01$ | $0.14825 \mathrm{E}-01$ | $0.14824 \mathrm{E}-01$ | $0.14824 \mathrm{E}-01$ | $0.14824 E-01$ | $0.14823 \mathrm{E}-01$ | $0.14823 \mathrm{E}-01$ |
|  | $0.11682 E-01$ | $11681 \mathrm{E}-01$ | 1681E-01 | 01 | 0.11680 EE 01 | E-01 | 0.11679E-01 | $0.11678 \mathrm{E}-01$ | $0.11678 \mathrm{E}-01$ | $0.11677 \mathrm{E}-01$ |
| 5 | 0.49124E-02 | 0.49120E-02 | $0.49115 \mathrm{E}-02$ $0.14365 \mathrm{E}-02$ | 0.49110E-02 | $0.49105 \mathrm{E}-02$ $0.14361 \mathrm{E}-02$ | 0.14360E-02 | $0.49096 \mathrm{E}-02$ | $0.49091 \mathrm{E}-02$ | $0.49086 \mathrm{E}-02$ | 0.49082E-02 |
| 4 | $0.41927 \mathrm{E}-03$ | 0.41921E-03 | $0.41915 \mathrm{E}-03$ | $0.41908 \mathrm{E}-03$ | 0.41902E-03 | $0.41896 \mathrm{E}-03$ | 0.41889 E-03 |  | 0.41877E-03 | 0.41870 |
|  | 12028E-03 | $0.12027 E-03$ | 0.12025E-03 | 0.12023E-03 | 0.12021E-03 | 0.12019E-03 | $0.12017 \mathrm{E}-03$ | 0.12015E-03 | 0.12013E-03 | 0.12011 -03 |
|  | 18239E-04 | $0.18236 \mathrm{E}-04$ | $18233 \mathrm{E}-04$ | 18229E-04 | 18226E-04 | 0.18223E-04 | 0.18220E-04 | $0.18217 E-04$ | $0.18214 \mathrm{E}-04$ | 210E-04 |
|  | 0.00000 |  | . 00000 | 0.00000 | 00 | 0000 | 0000 | 0.00000 | 0.00000 | 0.000 |

O***** MACH

| $\begin{array}{r} 012= \\ 2= \end{array}$ | $00$ |  | $26 E$ | 789 E |  | $3316 E 0$ | E | E | $21051$ | $7368 \mathrm{E}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 |  |  |  |  |  |  |  |  |  |  |
| 19 | $0.55944 \mathrm{E}-01$ | $0.55944 E-01$ |  |  | 0.55944E-01 | $0.55944 \mathrm{E}-01$ | $0.55944 \mathrm{E}-01$ | $0.55944 \mathrm{E}-01$ | 0.55944E-01 |  |
| 18 | 13348 E 00 | 0.13348 E 00 | 0.13348 E 00 | 0.13348 E 00 | 0.13348 E 00 | 0.13318 E 00 | 0.13348 E 00 | 0.13348 E 00 | 0.13348 E 00 | 0.13348500 |
| 17 | 17698 E 208390 | 0.17698 E 00 |  | $98 \mathrm{E} 0$ | $00$ | $\begin{aligned} & \text { E OO } \\ & \text { F O } \end{aligned}$ | $0.17699 E 00$ | $0.17693 \mathrm{E} 00$ | E | 0.17698 E 00 |
| 15 | 00 | 0.24407E 00 | O.24407E 00 | $\begin{aligned} & 0.20839 E 00 \\ & 0.24407 E O D \end{aligned}$ |  | $000$ | - | $0.20839 E 00$ |  | 0.20839 ED |
| 14 | 30778 OD | 0.30778 E 00 | 0.30778 E 00 | 0.30778 E 00 | 78 E 00 | . 30778 E 00 | 0.30778 E | 0.30778 E | 0778E |  |
| 13 | 42553E 00 | 42553E 00 | 53E 00 | 42553E 00 | $3 E$ | 0.42553 E | 0.42553 E | 0.42553 E | - | 0.42553E 0 |
| 12 | OE 00 | OE DO | 3880E 00 | 43880 E 0 | 80E 00 | 0.43880 E 00 | 0.43880 E 00 | 0.43880 E |  |  |
| 11 | 43880E 00 | 43880 E 00 | 880E 00 | OE 00 | OE OO | OE OO | 80E | 0.438 | E | $0.43386 E$ |
| 10 | 43880 E |  |  |  | . 43838 | 0.43380 E 00 | 80 E | 0.438 | E | . 4 |
|  | 43880 EO | 0.43880 E 00 | OE 00 | 0.43880 E 00 | . 43880 E 00 | 0.43880 E 00 | 0.43880 E | 0.43880E | 0.43380 E | 0.43830 E DO |
| 8 | 42553 E 00 | 0.42553 E 00 | 3 E | . 42553 E 00 | . 4 | . | 0 | 0.42553 E | - 42553 F | $0.42553 E 00$ |
| 7 | 30778 E 00 | 30778E 00 | $0.30778 E 00$ | . 30778 E 00 | . 30778 E 00 | 0.30778 E 00 | 0.30778 E 00 | 0.30778 E 00 | 0.30778 E 00 | 0.30778 E 0 |
| 6 | $24407 E 00$ | 24407 E 00 | $0.24407 E 00$ | 4407E 00 | 7E 00 | $0.24407 E 00$ | 0.24407 E | 0.24407 E | 0.24407 E | $0.24407 E 0$ |
| 5 | 839E 00 | 20839E 00 | 20839E 00 | 20839E 00 | 0 |  |  |  |  |  |
|  | 698E 00 |  | 7698E 00 |  |  | 0.17693 E | 0.17698 E | 0.17698 E | 0.17698 E | 0. |
| 3 | 8 E 00 | 8 E DO | 3348 E 00 | . 13348 E 00 | 348 E OO | 0.13348 E 00 | 0.13348 E 00 | 0.13348 E 00 |  |  |
| 2 | $944 \mathrm{E}-01$ | $4 \mathrm{E}-01$ | 45E-01 | 44E-01 | $944 \mathrm{E}-01$ | 5944E-01 | 0.55944E-01 | 0.55944E-01 | 01 |  |
|  |  |  |  |  |  |  |  |  |  |  |
| $2=$ | $\text { E } 0$ | E |  |  |  | $7895 E$ | $8421 E$ |  | $189474 E$ |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  | 01 | 01 | -01 | $55944 \mathrm{E}-01$ | 0.55944E-01 | $0.55944 \mathrm{E}-01$ | $0.55944 \mathrm{E-01}$ | 0.55944E-01 | $0.55944 \mathrm{E}-01$ |  |
|  | 00 |  | 8 E 00 | 48 E 00 |  | 0.13348 E 00 |  |  |  |  |
|  |  |  |  |  |  |  | 8 O | 6 | 769 |  |
| 15 | 407 O 00 | O.24407E 00 | $\begin{aligned} & 839 E 00 \\ & 407 E O D \end{aligned}$ |  | . 2440839 E 00 | O.20839E 00 |  |  |  |  |
| 14 | $0.30778 E 00$ | 0.30778 E 00 | 778E 00 | 30778E 00 | . 30778 E 00 | 0.30778 E. 00 |  |  |  |  |
| 13 | 42553E 00 | 0.42553 E 00 | . 42553 E 00 | . 42553 E 00 | 42553E 00 | $0.42553 E 00$ | 3 E 00 | 0.42553 E | 553 E |  |
| 12 | 43880 E 00 | OE 00 | OE 00 | 43880 E 00 | . 43880 E 00 | 0.43880 E 00 | 0.43880 E 00 | 0.43880 E 00 | 0.43880 E | 0.43830 E |
| 11 | 43830 E 00 | 43880 E 00 | 3880 E 00 | 43880 E 00 | . 43880 E 00 | 0.43880 E 00 | 0.43880 E 00 | 0.43880 E 00 | - | -. 4 , |
| 10 | OE 00 | 43880E 00 | 3880E OO | 43880 E 00 | . 43880 E 00 | 0.43880 E 00 | 0.43880 E 00 | 0.43880 E OO | 0.43880 E 00 | 0.43 |
|  | 43880 E 00 | 0.43880 E OO | 0.43880 E 00 | . 43880 E 00 | . 43880 E 00 | 0.43880 E 00 | 0.43880 E 00 | 0.43890 E 0 | 0.43830 E 00 |  |
| 8 | 42553E 00 | 0.42553 E 00 | 0.42553 E 00 | 0.42553 E 00 | . 42553 E 00 | 0.42553 E 00 | 0.42553 E 00 | 0.42553 E 00 | 0.42553E 00 | . 42553 E |
|  | 30778E 00 | 0.30778 E 00 | $0.30778 E^{0} 0$ | 0.30778 E 00 | 0.30778 E 00 | 0.30778 E 00 | 0.30778 E 00 | $0.30778 E 00$ | 0.30778 E |  |
| 6 | 24407E 00 | 0.24407 E 00 | $0.24407 E 00$ | 0.24407 E 00 |  | $0.24407 E 00$ | $0.24407 E 00$ | $0.24407 E 00$ | 0.24407E 00 | . |
|  | 0.20839 EOO | 0.20839 E 00 | 0.20839 E 00 | . 20839 E 00 | . 20839 E 00 | 0.20839 E 00 | 0.20839 E 00 | 0.20839 E 0 | 0.20839 E 00 | 0.20839 E 0 |
| 4 | 0.17698 E 00 | 17698E 00 | 0.17698 E 00 | .17698E 00 | .17698E 00 | 0.17698 E 00 | 0.17698 E 00 | 0.17698 E 00 | 0.17698 E | 0. |
|  | $\begin{gathered} 8 E 00 \\ 4 E-01 \end{gathered}$ | BE DO | E 00 | $8 \mathrm{E} 00$ | $\begin{aligned} & E O O \\ & E-01 \end{aligned}$ | $\begin{aligned} & 00 \\ & 01 \\ & 01 \end{aligned}$ | $00$ | $00$ | 00 | 0.13348 E 0 |
|  |  |  |  |  |  |  |  |  |  |  |
| cur | total ma total | $\begin{aligned} & \text { ASS }=0.19856 \\ & \text { MASS }=0.198 \end{aligned}$ | $675364 E 01$ $5706868 \mathrm{E} 01$ | total ENERGY <br> total energ | $Y=0.156199$ | $368 \mathrm{E} \text { Oi }$ |  |  |  |  |

O\#* TIME STEP NO $\mathcal{I}$ CPTIME $=0.00$ MIN. TPHYS $=0.50000 E-02$ DT $=0.50000 E-02$ VISC STAB $=0.122 E-06$ IDTADJ $=1$









$\begin{array}{llllllllll}\text { EQ } & 4 & \text { VAR } & 1 & 0.00000 & 0.00000 & 0.16050 E-04 & 0.00000-0.47840 E-02 & 0.000000 & 0.00000 \\ \text { EQ } & 4 & \text { VAR } & 3 & 0.00000 & 0.30944 E 00-0.12851 E-02 & 0.00000 & 0.00000 & 0.00000 & 0.10980 E-03\end{array}$



O** IIME SIEP NO. 2 CPTIME $=0.00$ MIN. TPHYS $=0.11250 \mathrm{E}-01$ DT $=0.62500 \mathrm{E}-02$ VISC STAB $=0.152 E-06$ IDTADJ=















 0***** PRES



 ORESULTS AT TSTEP NO. $\quad 3 \quad$ TIME $=0.190625 E-01$


10.10385 E 010.10385 E O1 0.10385 E 01 0.10385E 01 0.10385E 01 0.10385E 01 0.10385E 01 0.10385E 01 0.10385E 01 0.10385E 01

 ORESULTS AT TSTEP NO. $\quad 3$ TME 01 O.
*****






0***** MACH


|  | 0 | - 0.00000 | 0 |  |  |  |  |  | 000 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 01 | $0.45294 \mathrm{E}-01$ |  | $0.45140 \mathrm{E}-01$ |  |  |  | $0.45125 \mathrm{E}-01$ | E-01 | 0.45118E-0 |
| 18 | 0.13347 E 00 | 0.13130 E 00 | 0.13125 E 00 | 0.13124 E 00 | 0.13124E 00 | $0.13124 E 00$ | $0.13124 \mathrm{E}^{00}$ | $0.13124 E 00$ | 0.13123800 |  |
| 17 |  | $1 E 00$ |  | 2E OO | 2 E 00 | 0.17692 F 20 | 0.17692 E 00 | 0.17652 E 00 | 0.17692 E 0 | 0.17692 E |
| 16 | 0.20838 E 000 | 0.20876 E 00 | 0.20879 E 00 | 0.20879 E 00 | 0.20879 E 00 | 0.20879 E 00 | 0.20879 E 00 | $0.20879 \mathrm{E} 00$ | $0.20879 \mathrm{E}$ |  |
| 15 | O. 24406 E 00 | 0.24473 E 00 | O.24478E 0.3080 0.308 | $\begin{aligned} & 24479 \mathrm{E} \\ & 00 \\ & 0.20910 \mathrm{O} \end{aligned}$ | $0.24479 E 00$ 0.30810 E | $0.24479 E 00$ 0.30310 E | $0.24479 E \quad 00$ 0.30810 E | $\begin{aligned} & 0.24479 \mathrm{E} 00 \\ & 0.30810 \mathrm{E} 00 \end{aligned}$ | $\begin{aligned} & 0.24479 \mathrm{E} 00 \\ & 0.30310 E 00 \end{aligned}$ | $\begin{aligned} & 0.24479 E \\ & 0.30510 E \end{aligned}$ |
| 14 | $\begin{aligned} & 0.30776 E 00 \\ & 0.42551 E 00 \end{aligned}$ | $0.30806 E$ 0.425390 | ${ }_{0}^{0.358095}$ | ${ }_{0} .42537 E 00$ | 0.42537 E OO | 0.42537 E 00 | 0.42537 E 00 | $0.42537 E 00$ | $0.42537 E$ oo | $0.42537 E$ |
| 12 | 43878 E 00 | 0.43590 E 00 | $0.43391 E 00$ | 0.43891 E 00 | 0.43891 E 00 | 3991E 00 | 391 E 0 | 0.43891 E | 91E | 891 E |
| 11 | 3878E 00 | 0.43890 E 00 | 3891E 00 | 0.43892 E 00 | 0.43892 E 00 | 0.43892 EO | 892 E | 0.43592 E 00 |  | 0.43372 E |
| 10 | 8818E 00 |  | 91E 00 | 3892E 00 | 3892 E 0 | JE92E 00 | $0.43592 E 00$ | 3852 E 00 |  |  |
| 9 | 43878E 00 | 0.43890 E 00 | . 43891 E 00 | 0.43371 E 00 | $0.43391 E 00$ | $0.43891 E 00$ | $0.43891 E 00$ | 0.43391 E | 0.43291200 | 0.43891 E |
| 8 | 42551 E 00 | $0.42539 E 00$ | . 42537 E 00 | $0.42537 E 00$ | $0.42537 E 00$ | $0.42537 E 00$ | $0.42537 E 00$ | $0.42537 E 00$ | 0.42537 E 00 | 0.42537 E |
|  | 30776E 00 | 0.30806 E DO | 0.30839 E 08 | . 30810 [ 00 | 0810E 00 | 0.30810 E 00 | $0.30810 E^{00}$ | 0.30810 E 00 | 0.30810E 00 | 0.30810 E |
|  | 24406E 00 | 0.24473 E 00 | 0.24478 E 00 | 0.24479 E 0 | 0.24479 E 0 | 0.24479 E 00 | $0.24479 E 00$ | 0.24479 E 0 | 0.24479 E 00 | 0.24479 E |
|  | 20838E 00 | $0.20376 E 00$ | 20879E 00 | 0879E 00 | 20879E 00 | 0.20879 E 00 | 0.20879 E 00 | 0.20879 E 00 | 0.20879 E 00 | 0.20879 E |
| 4 | 17697E 00 | $0.17691 \mathrm{O}^{0} 0$ | 17692E 00 | 0.17692 E 00 | 17692 E 00 | 17692 E 0 | 17692 E | $0.17692 E 00$ | $0.17692 \mathrm{E} 00$ |  |
|  | 13347 E 0 | 0.13130 E 00 | $0.13125 E 00$ | 0.13124 E 00 | $0.13124 E^{00}$ | 0.13124 E 00 | $0.13124 E 00$ $0.45129 E-01$ | $\begin{aligned} & 0.13124 E 00 \\ & 0.45125 E-01 \end{aligned}$ | $\begin{aligned} & 0.3123 E 00 \\ & 0.45121 E-01 \end{aligned}$ | $\begin{aligned} & 0.15123 \mathrm{E} \\ & 0.45110 \mathrm{E} \end{aligned}$ |
| 2 | $\begin{array}{r} 55942 \mathrm{E}-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 0.45294 \mathrm{E}-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 5177 \mathrm{E}-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 45140 \mathrm{E}-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 45136 E-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 45133 \mathrm{E}-01 \\ 0.000000 \end{array}$ | $\begin{array}{r} 0.45129 \mathrm{E}-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 0.45125 \mathrm{E}-01 \\ 0.000000 \end{array}$ | $\begin{array}{r} 45121 E-01 \\ 0.00000 \end{array}$ | $\begin{array}{r} 45110 \mathrm{E}-01 \\ 0.00000 \end{array}$ |
|  | $5263 \mathrm{E}$ |  | $\begin{gathered} 13 \\ 26316 E \end{gathered}$ | $\begin{gathered} 14 \\ 6842 E \end{gathered}$ | $\begin{gathered} 15 \\ 17368 \mathrm{E} \end{gathered}$ | $\begin{gathered} 16 \\ 57895 \mathrm{E} \end{gathered}$ | $\begin{gathered} 17 \\ 68421 E \end{gathered}$ | $78947 \mathrm{E}$ | $189474 E$ | 200000 E |
|  |  |  |  |  |  |  |  |  |  |  |




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[^0]:    *For sale by the National Technical Information Service, Springfield, Virginia 22161

