Hindawi Publishing Corporation
Advances in Difference Equations
Volume 2011, Article ID 190475, 11 pages
doi:10.1155/2011/190475

Research Article

Numerical Solutions of a Fractional Predator-Prey System

Yanqin Liu[1] and Baogui Xin[2, 3]

[1] *Department of Mathematics, Dezhou University, Dezhou 253023, China*
[2] *Nonlinear Dynamics and Chaos Group, School of Management, Tianjin University, Tianjin 30072, China*
[3] *School of Economics and Management, Shandong University of Science and Technology, Qingdao 266510, China*

Correspondence should be addressed to Yanqin Liu, yqlin8801@yahoo.cn

Received 10 December 2010; Accepted 22 February 2011

Academic Editor: Dumitru Baleanu

We implement relatively new analytical technique, the Homotopy perturbation method, for solving nonlinear fractional partial differential equations arising in predator-prey biological population dynamics system. Numerical solutions are given, and some properties exhibit biologically reasonable dependence on the parameter values. And the fractional derivatives are described in the Caputo sense.

1. Introduction

Recently, it has turned out that many phenomena in engineering, physics, chemistry, other sciences [1–3] can be described very successfully by models using mathematical tools form fractional calculus, such as anomalous transport in disordered systems, some percolations in porous media, and the diffusion of biological populations. But most fractional differential equations [4, 5] do not have exact analytic solutions [6, 7]. An effective method for solving such equations is needed. So approximate and numerical techniques must be used. The Homotopy Perturbation Method (HPM) is relatively new approach to provide an analytical approximation to nonlinear problem. This method was first presented by He [8, 9] and applied to various nonlinear problems [10–12]. Recently, the application of the method is extended for fractional differential equations [13–15].

Biological population problems are widely investigated in many papers [16–19]. Dunbar [20] establishes the existence of traveling wave solutions for two reaction diffusion systems based on the Lotka-Volterra model for predator and prey interactions, and discusses some possible biological implications of the existence of these waves. Gourley and Britton [21] investigate stability of coexistence steady-state and bifurcations of a predator-prey system in the form of a coupled reaction-diffusion equations. Petrovskii et al. [22] obtained an exact solution of the spatiotemporal dynamics of a predator-prey community by using an appropriate change of variables, and the properties of the solution exhibit biologically reasonable dependence on the parameter values. Kadem and Baleanu [23] studied the coupled fractional Lotka-Volterra equations using the Homotopy perturbation method.

We consider two-species competitive model with prey population A and predator population B. For prey population $A \rightarrow 2A$, at rate a, $a > 0$ represents the natural birth rate. For predator population $B \rightarrow 0$, at rate $c > 0$, c denotes the natural death rate. The interactive term between predator and prey population is $A + B \rightarrow 2B$, at rate $b > 0$, parameter b denotes the competitive rate. According to a widely accepted knowledge of fractional calculus and biological population, the time-fractional dynamics of a predator-prey system can be described by the equations

$$\begin{aligned} \frac{\partial^\alpha u}{\partial t^\alpha} &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + au - buv, \quad u(x,0) = \varphi(x), \\ \frac{\partial^\beta v}{\partial t^\beta} &= \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + buv - cv, \quad v(x,0) = \phi(x), \end{aligned} \tag{1.1}$$

where $t > 0$, $x, y \in R$, $a, b, c > 0$, and $u(x, y, t)$ denotes the prey population density and $v(x, y, t)$ represents the predator population density, $\varphi(x), \phi(x)$ denote initial conditions of population system; the nonlinear equation of this type has wide applications in the fields of population growth. The derivatives in (1.1) is the Caputo derivative.

In this paper, we consider the fractional nonlinear predator-prey population model. and the paper is organized as follows: in Section 2, a brief review of the theory of fractional calculus will be given to fix notation and provide a convenient reference. In Section 3, we extend the application of the homotopy perturbation method to construct approximate solutions for the nonlinear fractional predator-prey system. In Section 4, we present three examples with different initial conditions to the predator-prey system and show some properties of this fractional nonlinear predator-prey system. Conclusions will be presented in Section 5.

2. Fractional Calculus

There are several approaches to define the fractional calculus, for example, Riemann-Liouville, Gruünwald-Letnikow, Caputo, and Generalized Functions approach. Riemann-Liouville fractional derivative is mostly used by mathematicians but this approach is not suitable for real world physical problems since it requires the definition of fractional order initial conditions, which have no physically meaningful explanation yet. Caputo introduced an alternative definition, which has the advantage of defining integer order initial conditions for fractional order differential equations.

Definition 2.1. The Riemann-Liouville fractional integral operator J^α $(\alpha \geq 0)$ of a function $f(t)$ is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} f(\tau) d\tau, \quad (\alpha \geq 0), \tag{2.1}$$

where $\Gamma(\cdot)$ is the well-known gamma function, and some properties of the operator J^{α} are as follows:

$$
\begin{aligned}
J^{\alpha} J^{\beta} f(t) &= J^{\alpha+\beta} f(t), \quad (\alpha \geq 0, \beta \geq 0), \\
J^{\alpha} t^{\gamma} &= \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma+\alpha)} t^{\alpha+\gamma}, \quad (\gamma \geq -1).
\end{aligned}
\tag{2.2}
$$

Definition 2.2. The Caputo fractional derivative D^{α} of a function $f(t)$ is defined as

$$
{}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(t)d\tau}{(t-\tau)^{\alpha+1-n}}, \quad (n-1 < \mathrm{Re}(\alpha) \leq n, \, n \in N). \tag{2.3}
$$

the following are two basic properties of the Caputo fractional derivative.

$$
\begin{aligned}
{}_0D_t^{\alpha} t^{\beta} &= \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} t^{\beta-\alpha}, \\
(J^{\alpha} D^{\alpha}) f(t) &= f(t) - \sum_{k=0}^{n-1} f^{(k)}(0^{+}) \frac{t^k}{k!}.
\end{aligned}
\tag{2.4}
$$

We have chosen the Caputo fractional derivative because it allows traditional initial and boundary conditions to be included in the formulation of the problem. And some other properties of fractional derivative can be found in [1, 3].

3. Homotopy Perturbation Method

The Homotopy analysis method which provides an analytical approximate solution is applied to various nonlinear problems [8, 10, 12–14]. In this section, we extend HPM to (1.1), according to this method, we construct the following simple homotopy:

$$
\begin{aligned}
\frac{\partial^{\alpha} u}{\partial t^{\alpha}} &= p\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + au - buv\right), \\
\frac{\partial^{\beta} v}{\partial t^{\beta}} &= p\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + buv - cv\right),
\end{aligned}
\tag{3.1}
$$

where $p \in [0,1]$ is an embedding parameter. In case $p = 0$, (3.1) is a fractional differential equation, which is easy to solve; when $p = 1$, (3.1) turns out to be the original one (1.1). The basic assumption is that the solutions can be written as a power series in p

$$
\begin{aligned}
u(x,y,t) &= u_0 + pu_1 + p^2 u_2 + p^3 u_3 + \cdots, \\
v(x,y,t) &= v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots.
\end{aligned}
\tag{3.2}
$$

The approximate solutions of the original equations can be obtained by setting $p = 1$, that is,

$$\begin{aligned} u &= \lim_{p\to 1}\sum_{n=0}^{\infty} p^n u_n = u_0 + u_1 + u_2 + u_3 + \cdots, \\ v &= \lim_{p\to 1}\sum_{n=0}^{\infty} p^n v_n = v_0 + v_1 + v_2 + v_3 + \cdots, \end{aligned} \tag{3.3}$$

institute (3.2) into (3.1) and compare coefficients of terms with identical powers of p, then you can get the numerical solutions of the equation. Because of the knowledge of various perturbation methods that low-order approximate solution leads to high accuracy, there requires no infinite series. Then after a series of recurrent calculation by using Mathematica software, we will get approximate solutions of fractional biological population model. In Section 4, we show some examples that the Homotopy perturbation method gives a very good approximation of the exact solution.

4. Fractional Predator-Prey Equation

In order to assess the advantages and the accuracy of the Homotopy perturbation method presented in this paper for nonlinear fractional Fisher's equation, we have applied it to the following several problems.

Case 1. In this case, we consider the fractional predator-prey equation and subject to the constant initial condition

$$u(x, y, 0) = u_0, \qquad v(x, y, 0) = v_0. \tag{4.1}$$

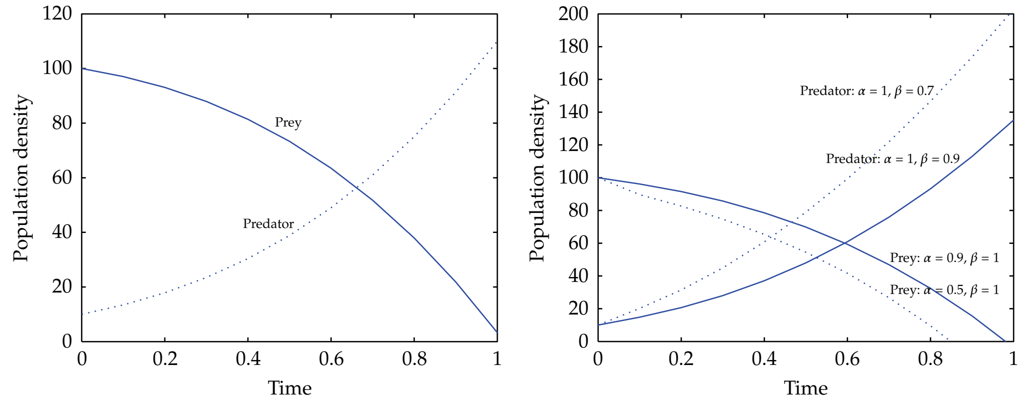
Substituting (3.2) into (3.1) and equating the terms with the same powers of p lead to the following two sets of linear equation:

$$\begin{aligned}
p^0 &: \frac{\partial^\alpha u_0}{\partial t^\alpha} = 0, \\
p^1 &: \frac{\partial^\alpha u_1}{\partial t^\alpha} = \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} + a u_0 - b u_0 v_0, \\
p^2 &: \frac{\partial^\alpha u_2}{\partial t^\alpha} = \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + a u_1 - b(u_1 v_0 + u_0 v_1), \\
p^3 &: \frac{\partial^\alpha u_3}{\partial t^\alpha} = \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + a u_2 - b(u_2 v_0 + u_1 v_1 + u_0 v_2), \\
p^4 &: \frac{\partial^\alpha u_4}{\partial t^\alpha} = \frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} + a u_3 - b(u_3 v_0 + u_2 v_1 + u_1 v_2 + u_0 v_3), \\
&\vdots
\end{aligned}$$

$$
\begin{aligned}
p^0 &: \frac{\partial^\beta v_0}{\partial t^\beta} = 0,\\
p^1 &: \frac{\partial^\beta v_1}{\partial t^\beta} = \frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial y^2} + bu_0v_0 - cu_0,\\
p^2 &: \frac{\partial^\beta v_2}{\partial t^\beta} = \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} + b(u_1v_0 + u_0v_1) - cv_1,\\
p^3 &: \frac{\partial^\beta v_3}{\partial t^\beta} = \frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} + b(u_2v_0 + u_1v_1 + u_0v_2) - cv_2,\\
p^4 &: \frac{\partial^\beta v_4}{\partial t^\beta} = \frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial y^2} + b(u_3v_0 + u_2v_1 + u_1v_2 + u_0v_3) - cv_3,\\
&\vdots
\end{aligned}
\tag{4.2}
$$

Consequently, by applying the Riemann-Liouville fractional operator J^α and J^β to the above sets of linear equations, which is the inverse operator of Caputo derivative D^α and D^β respectively, the first few terms of the Homotopy perturbation method series for the system (1.1) are obtained as follows:

$$
\begin{aligned}
u_0 &= u(x,y,0) = u_0, \qquad v_0 = v(x,y,0) = v_0,\\
u_1 &= \frac{(au_0 - bu_0v_0)t^\alpha}{\Gamma(1+\alpha)}, \qquad v_1 = \frac{(bu_0v_0 - cv_0)t^\beta}{\Gamma(1+\beta)},\\
u_2 &= \frac{u_0(a-bv_0)^2t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{bu_0v_0(c-bu_0)t^{\alpha+\beta}}{\Gamma(1+\alpha+\beta)},\\
v_2 &= \frac{v_0(c-bu_0)^2t^{2\beta}}{\Gamma(1+2\beta)} + \frac{bu_0v_0(a-bv_0)t^{\alpha+\beta}}{\Gamma(1+\alpha+\beta)},\\
u_3 &= \frac{u_0(a-bv_0)^3t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{\Gamma(1+\alpha+\beta)b(c-bu_0)(a-bv_0)u_0v_0t^{2\alpha+\beta}}{\Gamma(1+\alpha)\Gamma(1+\beta)\Gamma(1+2\alpha+\beta)}\\
&\quad - \frac{b(c-bu_0)^2u_0v_0t^{\alpha+2\beta}}{\Gamma(1+\alpha+2\beta)} + \frac{b(c-2bu_0)(a-bv_0)u_0v_0t^{2\alpha+\beta}}{\Gamma(1+2\alpha+\beta)},\\
v_3 &= -\frac{v_0(c-bu_0)^3t^{3\beta}}{\Gamma(1+3\beta)} + \frac{\Gamma(1+\alpha+\beta)b(a-bv_0)(c-bu_0)u_0v_0t^{\alpha+2\beta}}{\Gamma(1+\alpha)\Gamma(1+\beta)\Gamma(1+\alpha+2\beta)}\\
&\quad + \frac{b(a-bv_0)^2u_0v_0t^{2\alpha+\beta}}{\Gamma(1+2\alpha+\beta)} - \frac{b(a-2bv_0)(c-bu_0)u_0v_0t^{\alpha+2\beta}}{\Gamma(1+\alpha+2\beta)}.
\end{aligned}
\tag{4.3}
$$



(a) (b)

Figure 1: Time evolution of population of $u(x, y, t)$ and $v(x, y, t)$ when $\alpha = 1$, $\beta = 1$ in (a) for (4.4).

Table 1: Comparison of the numerical values with Homotopy perturbation method and Variational iteration method when $a = 0.05$, $b = 0.03$, and $c = 0.01$ for (1.1), and (4.1).

t	$\alpha = \beta$	Numerical value (u, v) by HPM	Numerical value (u, v) by VIM
0.02	1	(99.4831,10.6146)	(99.4834,10.6323)
	0.9	(99.1865,10.9633)	(99.3065,10.8375)
0.2	1	(93.0910,17.8514)	(93.3908,17.7382)
	0.9	(90.5735,20.5567)	(92.4584,18.8198)
0.3	1	(87.9348,23.4430)	(88.9466,22.7237)
	0.9	(83.7933,27.7785)	(87.8005,24.0532)

Then the approximate solution in a series form is

$$u(x, y, t) = u_0 + u_1 + u_2 + u_3 + \cdots, \qquad v(x, y, t) = v_0 + v_1 + v_2 + v_3 + \cdots. \tag{4.4}$$

Figure 1 shows the approximate solutions for (4.4) by using the HPM when choosing the constant initial condition $u_0 = 100$, $v_0 = 10$ and $a = 0.05$, $b = 0.03$, **and** $c = 0.01$. From the figures, it is clear to see the time evolution of prey-predator population density and we also know that the numerical solutions of fractional prey-predator population model is continuous with the parameter α and β.

Table 1 shows the approximate solutions of predator-prey system for (1.1) and initial condition (4.1) by using the Homotopy perturbation method and Variational iteration method when parameter $a = 0.05$, $b = 0.03$, $c = 0.01$, $u_0 = 100$, and $v_0 = 10$. It is noted that only the forth-order of the Homotopy perturbation solution were used in evaluating the approximate solutions for Table 1 Unlike the Variational iteration method, in this method, we do not need the Lagrange multiplier, correction functional, stationary conditions, or calculating integrals, which eliminate the complications that exist in the VIM. So, it is evident that HPM used in this paper has high accuracy. And from the comparison of the numerical values with HPM and VIM, we also know that, as the time t and the parameter α, β increase, the error between the two methods is growing.

Case 2. In this case, the initial conditions of systems (1.1) are given by

$$u(x,y,0) = e^{x+y}, \qquad v(x,y,0) = e^{x+y}. \tag{4.5}$$

By using (3.1) and (3.2), we now successively obtain

$$u_0 = e^{x+y}, \qquad v_0 = e^{x+y}, \tag{4.6}$$

$$u_1 = \frac{e^{x+y}(2+a-be^{x+y})t^{\alpha}}{\Gamma(1+\alpha)}, \qquad v_1 = \frac{e^{x+y}(2-c+be^{x+y})t^{\beta}}{\Gamma(1+\beta)}, \tag{4.7}$$

$$u_2 = \frac{e^{x+y}\left[(2+a-be^{x+y})(a-be^{x+y})+2(2+a-4be^{x+y})\right]t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{be^{2x+2y}(2-c+be^{x+y})t^{\alpha+\beta}}{\Gamma(1+\alpha+\beta)}, \tag{4.8}$$

$$v_2 = \frac{e^{x+y}\left[(2-c+be^{x+y})(be^{x+y}-c)+2(2-c+4be^{x+y})\right]t^{2\beta}}{\Gamma(1+2\beta)} + \frac{be^{2x+2y}(2+a-be^{x+y})t^{\alpha+\beta}}{\Gamma(1+\alpha+\beta)}, \tag{4.9}$$

$$\begin{aligned}
u_3 = {} & \frac{\left[be^{2x+2y}\left((8+a)(c-2)-b(18+2a+c)e^{x+y}+2b^2e^{2x+2y}\right)\right]t^{2\alpha+\beta}}{\Gamma(1+2\alpha+\beta)} \\
& + \frac{\left[e^{x+y}\left((2+a)^2(2+a-b)-(10+2a)(8+a-b)be^{x+y}+(18+a-b)b^2e^{2x+2y}\right)\right]t^{3\alpha}}{\Gamma(1+3\alpha)} \\
& + \frac{\left[-be^{2x+2y}\left((2-c)^2+b(10-2c)e^{x+y}+b^2e^{2x+2y}\right)\right]t^{\alpha+2\beta}}{\Gamma(1+\alpha+2\beta)} \\
& + \frac{\Gamma(1+\alpha+\beta)\left[-be^{2x+2y}(2+a-be^{x+y})(2-c+be^{x+y})\right]t^{2\alpha+\beta}}{\Gamma(1+\alpha)\Gamma(1+\beta)\Gamma(1+2\alpha+\beta)},
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
v_3 = {} & \frac{\left[be^{2x+2y}\left((2+a)(8-c)+b(a-18+2c)e^{x+y}-2b^2e^{2x+2y}\right)\right]t^{\alpha+2\beta}}{\Gamma(1+\alpha+2\beta)} \\
& + \frac{\left[e^{x+y}\left((2-c)^2(2+b-c)+(10-2c)(8+b-c)be^{x+y}+(18+b-c)b^2e^{2x+2y}\right)\right]t^{3\beta}}{\Gamma(1+3\beta)} \\
& + \frac{\left[be^{2x+2y}\left((2+a)^2-b(10+2a)e^{x+y}+b^2e^{2x+2y}\right)\right]t^{2\alpha+\beta}}{\Gamma(1+2\alpha+\beta)} \\
& + \frac{\Gamma(1+\alpha+\beta)\left[be^{2x+2y}(2+a-be^{x+y})(2-c+be^{x+y})\right]t^{\alpha+2\beta}}{\Gamma(1+\alpha)\Gamma(1+\beta)\Gamma(1+\alpha+2\beta)}.
\end{aligned} \tag{4.11}$$

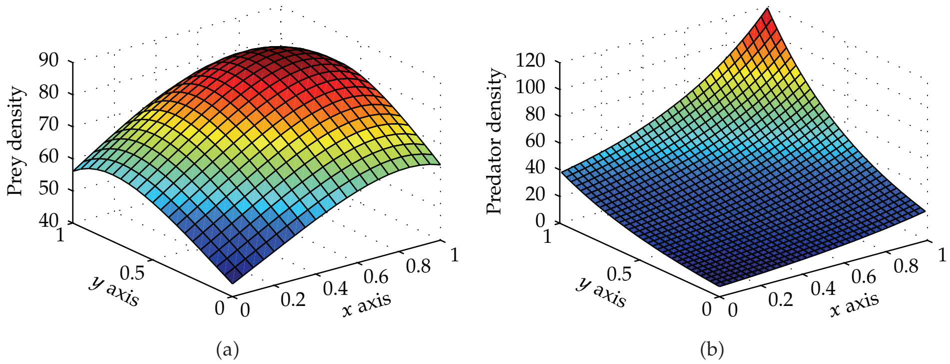


Figure 2: The surface shows the solution of $u(x,y,t)$ and $v(x,y,t)$ when $\alpha = 0.88$, $\beta = 0.54$, $a = 0.7$, $b = 0.03$, $c = 0.3$, $t = 0.53$ in (a) and $c = 0.9$, $t = 0.6$ in (b) for (4.11).

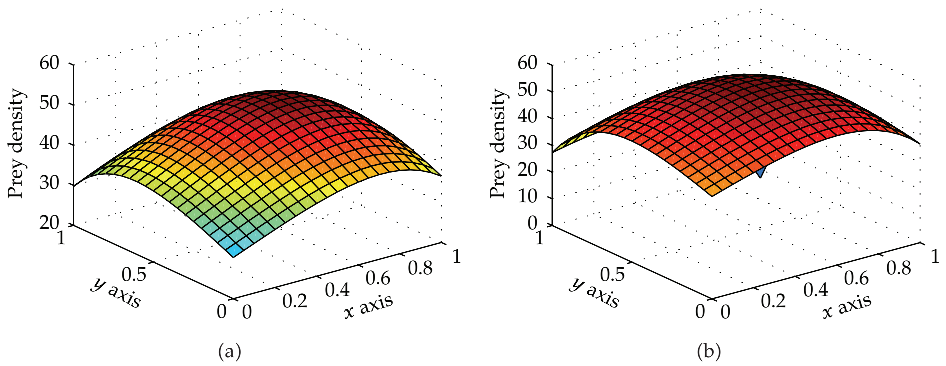


Figure 3: The surface shows the solution of $u(x,y,t)$ when $\alpha = 0.88$, $\beta = 0.54$, $c = 0.3$, $t = 0.53$, $a = 0.5$, $b = 0.03$ in (a) and $a = 0.7$, $b = 0.04$ in (b) for (4.11).

Figure 2 shows the numerical solutions for prey-predator population system with appropriate parameter. From the figures, we know that prey population density first increases with the spatial variables, then decreases. although the predator population density always increase with the spatial variables with the parameter we choose here. Analysis and results of prey-predator population system indicate that the fractional model match the anomalous biological diffusion behavior observed in the field.

Figure 3 shows the numerical solutions for prey population density with different values of parameter a, b, that is, natural birth rate of prey population and competitive rate between predator and prey population. Comparing Figures 2 and 3, we concluded that the parameter a, b infects the increase speed, the Maximum value, and the decrease speed of the prey population. In the same way, the parameter b, c infects predator population growth. This behavior in agreement with realistic results.

Case 3. We will consider the initial conditions of fractional predator-prey equation (1.1)

$$u(x,y,0) = \sqrt{xy}, \qquad v(x,y,0) = e^{x+y}. \tag{4.12}$$

We now successively obtain by using (3.1) and(3.2)

$$u_0 = \sqrt{xy}, \qquad v_0 = e^{x+y},$$

$$u_1 = \frac{\left(-x^2 - y^2 + 4ax^2y^2 - 4be^{x+y}x^2y^2\right)t^{\alpha}}{4xy\sqrt{xy}\Gamma(1+\alpha)}, \qquad v_1 = \frac{e^{x+y}\left(2 - c + b\sqrt{xy}\right)t^{\beta}}{\Gamma(1+\beta)},$$

$$u_2 = \frac{\left(a - be^{x+y}\right)\left(-x^2 - y^2 + 4ax^2y^2 - 4be^{x+y}x^2y^2\right)t^{2\alpha}}{4xy\sqrt{xy}\Gamma(1+2\alpha)} - \frac{be^{x+y}\sqrt{xy}\left(2 - c + b\sqrt{xy}\right)t^{\alpha+\beta}}{\Gamma(1+\alpha+\beta)},$$

$$+ \frac{\sqrt{xy}\left(15y^4 + 4\left(a - be^{x+y}\right)x^2y^4 + 16be^{x+y}x^3y^4 - x^4\left(15 + 4ay^2 + 4be^{x+y}y^2(4y-1)\right)\right)t^{2\alpha}}{16x^4y^4\Gamma(1+2\alpha)},$$

$$v_2 = \frac{e^{x+y}\left(c^2 + b\left(bxy + 2\sqrt{xy}\right) - 2c\left(1 + b\sqrt{xy}\right)\right)t^{2\beta}}{4xy\sqrt{xy}\Gamma(1+2\beta)}$$

$$+ \frac{be^{x+y}\left(-x^2 - y^2 + 4ax^2y^2 - 4be^{x+y}x^2y^2\right)t^{\alpha+\beta}}{4xy\sqrt{xy}\Gamma(1+\alpha+\beta)}$$

$$- \frac{e^{x+y}\left[(-16+8c)xy\sqrt{xy} + b\left(y^2 - 4xy^2 + x^2\left(1 - 4y - 8y^2\right)\right)\right]t^{2\beta}}{16x^4y^4\Gamma(1+2\beta)}. \tag{4.13}$$

Because of the knowledge of various perturbation methods that low-order approximate solution leads to high accuracy, there requires no infinite series (mostly 2–4 terms are enough). The corresponding solutions are obtained according to the recurrence relation using Mathematica.

5. Conclusion

In this letter, we implement relatively new analytical techniques, the Homotopy perturbation method, for solving nonlinear fractional partial differential equations arising in prey-predator biological population dynamics system. Comparing the methodology HPM to ADM, VIM and HAM have the advantages. Unlike the ADM, the HPM is free from the need to use Adomian polynomials. In this method we do not need the Lagrange multiplier, correction functional, stationary conditions, or calculating integrals, which eliminate the complications that exist in the VIM. In contrast to the HAM, this method is not required to solve the functional equations in each iteration the efficiency of HAM is very much depended on choosing auxiliary parameter. We can easily conclude that the Homotopy perturbation method is an efficient tool to solve approximate solution of nonlinear fractional partial differential equations.

Acknowledgments

The authors thank to the referees for their fruitful advices and comments. This work was supported partly by the National Science Foundation of Shandong Province (Grant nos. Y2007A06 & ZR2010Al019) and the China Postdoctoral Science Foundation (Grant no. 20100470783).

References

[1] I. Podlubny, *Fractional Differential Equations*, vol. 198 of *Mathematics in Science and Engineering*, Academic Press, New York, NY, USA, 1999.

[2] R. Metzler and J. Klafter, "The random walks guide to anomalous diffusion: a fractional dynamics approach," *Physics Reports A*, vol. 339, pp. 1–77, 2000.

[3] R. Hilfer, *Applications of Fractional Calculus in Physics*, World Scientific, Singapore, 2000.

[4] A. K. Golmankhaneh, A. K. Golmankhaneh, and D. Baleanu, "On nonlinear fractional Klein-Gordon equation," *Signal Processing*, vol. 91, pp. 446–451, 2011.

[5] S. Z. Rida, H. M. El-Sherbiny, and A. A. M. Arafa, "On the solution of the fractional nonlinear Schrödinger equation," *Physics Letters A*, vol. 372, no. 5, pp. 553–558, 2008.

[6] X. Y. Jiang and M. Y. Xu, "Analysis of fractional anomalous diffusion caused by an instantaneous point source in disordered fractal media," *International Journal of Non-Linear Mechanics*, vol. 41, pp. 156–165, 2006.

[7] S. Wang and M. Xu, "Axial Couette flow of two kinds of fractional viscoelastic fluids in an annulus," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 2, pp. 1087–1096, 2009.

[8] J.-H. He, "Homotopy perturbation technique," *Computer Methods in Applied Mechanics and Engineering*, vol. 178, no. 3-4, pp. 257–262, 1999.

[9] J.-H. He, "A coupling method of a homotopy technique and a perturbation technique for non-linear problems," *International Journal of Non-Linear Mechanics*, vol. 35, no. 1, pp. 37–43, 2000.

[10] J.-H. He, "The homotopy perturbation method nonlinear oscillators with discontinuities," *Applied Mathematics and Computation*, vol. 151, no. 1, pp. 287–292, 2004.

[11] J. H. He, "Application of homotopy perturbation method to nonlinear wave equations," *Chaos, Solitons & Fractals*, vol. 26, pp. 695–700, 2005.

[12] X. Li, M. Xu, and X. Jiang, "Homotopy perturbation method to time-fractional diffusion equation with a moving boundary condition," *Applied Mathematics and Computation*, vol. 208, no. 2, pp. 434–439, 2009.

[13] Q. Wang, "Homotopy perturbation method for fractional KdV-Burgers equation," *Chaos, Solitons & Fractals*, vol. 35, no. 5, pp. 843–850, 2008.

[14] S. Momani and Z. Odibat, "Homotopy perturbation method for nonlinear partial differential equations of fractional order," *Physics Letters A*, vol. 365, no. 5-6, pp. 345–350, 2007.

[15] Z. Odibat and S. Momani, "Modified homotopy perturbation method: application to quadratic Riccati differential equation of fractional order," *Chaos, Solitons & Fractals*, vol. 36, no. 1, pp. 167–174, 2008.

[16] F. Shakeri and M. Dehghan, "Numerical solution of a biological population model using He's variational iteration method," *Computers & Mathematics with Applications*, vol. 54, no. 7-8, pp. 1197–1209, 2007.

[17] S. Z. Rida and A. A. M. Arafa, "Exact solutions of fractional-order biological population model," *Communications in Theoretical Physics*, vol. 52, no. 6, pp. 992–996, 2009.

[18] Y. Tan, H. Xu, and S.-J. Liao, "Explicit series solution of travelling waves with a front of Fisher equation," *Chaos, Solitons & Fractals*, vol. 31, no. 2, pp. 462–472, 2007.

[19] S. Petrovskii and N. Shigesada, "Some exact solutions of a generalized Fisher equation related to the problem of biological invasion," *Mathematical Biosciences*, vol. 172, no. 2, pp. 73–94, 2001.

[20] S. R. Dunbar, "Travelling wave solutions of diffusive Lotka-Volterra equations," *Journal of Mathematical Biology*, vol. 17, no. 1, pp. 11–32, 1983.

[21] S. A. Gourley and N. F. Britton, “A predator-prey reaction-diffusion system with nonlocal effects,” *Journal of Mathematical Biology*, vol. 34, no. 3, pp. 297–333, 1996.

[22] S. Petrovskii, H. Malchow, and B.-L. Li, “An exact solution of a diffusive predator-prey system,” *Proceedings of The Royal Society of London A*, vol. 461, no. 2056, pp. 1029–1053, 2005.

[23] A. Kadem and D. Baleanu, “Homotopy perturbation method for the coupled fractional Lotka-Volterra equations,” *Romanian Journal of Physics*, vol. 56, 2011.