

Computer Sciences Department
University of Wisconsin
1210 West Dayton Street
Madison, Wisconsin 53706

NUMERICAL STUDIES OF STEADY, VISCOUS,
INCOMPRESSIBLE FLOW IN A CHANNEL WITH
A STEP

by Donald Greenspan

APPENDIX
PROGRAMMING FLOWS IN A CHANNEL WITH
A STEP

by M. McClellan

Computer Sciences Technical Report #17
March 1968

Rerun: July 1972

NUMERICAL STUDIES OF STEADY, VISCOUS, INCOMPRESSIBLE FLOW
IN A CHANNEL WITH A STEP*

by

Donald Greenspan

1. INTRODUCTION.

In this paper we will apply a new digital computer technique to the study of two dimensional, steady, viscous, incompressible flow through a channel with a step. The method is vastly more economical and accurate than time dependent, step-ahead techniques. The power of the method is contained in the structure of the difference equations, which, for all Reynolds numbers \mathcal{R} , yield diagonally dominant systems of linear algebraic equations [1]-[3].

2. THE GENERAL PROBLEM.

The initial problem to be considered will be formulated analytically as follows. Consider the channel with a step which is shown in Figure 2.1. Let S be the polygon ABCDEFGH and let R be the interior of S . On R the equations of motion to be satisfied are the two dimensional, Navier-Stokes equations, that is,

$$(2.1) \quad \Delta \psi = -\omega$$

$$(2.2) \quad \Delta \omega + \mathcal{R} \left(\frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} \right) = 0,$$

*Funds for the computations described in this paper were made available by the Research Committee of the Graduate School of the University of Wisconsin.

where ψ is the stream function, ω is the vorticity, and \mathcal{R} is the Reynolds number. On S the boundary conditions to be satisfied are

$$(2.3) \quad \psi = 1, \quad \frac{\partial \psi}{\partial Y} = 0, \quad \text{on HG};$$

$$(2.4) \quad \psi = 0, \quad \frac{\partial \psi}{\partial Y} = 0, \quad \text{on AB, CD, EF};$$

$$(2.5) \quad \psi = 0, \quad \frac{\partial \psi}{\partial X} = 0, \quad \text{on BC, DE}$$

$$(2.6) \quad \psi = 3Y^2 - 2Y^3, \quad \omega = 12Y - 6, \quad \text{on AH};$$

and

$$(2.7) \quad \frac{\partial \psi}{\partial X} = 0, \quad \frac{\partial \omega}{\partial X} + \mathcal{R} \frac{\partial \psi}{\partial Y} \left(\omega + \frac{\partial^2 \psi}{\partial Y^2} \right) = 0, \quad \text{on FG}.$$

Conditions (2.6) are those of Poiseuille flow [4], while conditions (2.7), formulated by R. E. Meyer and communicated privately, make the flow horizontal and the pressure constant on FG.

3. THE NUMERICAL METHOD.

In this section we will describe in complete generality a numerical method for approximating solutions of boundary value problem (2.1) - (2.7). In the discussion, a reference appearing with a difference equation indicates where a derivation of the equation can be found. Particular examples and actual computations will be described in the next section.

For n a fixed positive integer, determine grid size h from $h = \frac{1}{n}$. Next, on and within polygon $ABCDEFGH$, construct and number in the usual way [1] the set of interior grid points R_h and the set of boundary grid points S_h . With regard to R_h and S_h , it will be assumed, with very little loss of generality, that h can be selected so that $\alpha_1, \alpha_2, \beta, \gamma$ and δ are integral multiples of h .

We will aim at constructing on $R_h + S_h$ a pair of finite sequences of discrete functions

$$(3.1) \quad \psi^{(0)}, \psi^{(1)}, \psi^{(2)}, \dots, \psi^{(k)}, \psi^{(k+1)}$$

$$(3.2) \quad \omega^{(0)}, \omega^{(1)}, \omega^{(2)}, \dots, \omega^{(k)}, \omega^{(k+1)}$$

with the properties that, for some given tolerance ε ,

$$(3.3) \quad |\psi^{(k)} - \psi^{(k+1)}| < \varepsilon$$

$$(3.4) \quad |\omega^{(k)} - \omega^{(k+1)}| < \varepsilon$$

at each point of $R_h + S_h$. All the functions in sequences (3.1) and (3.2) will be called outer iterates and the particular functions $\psi^{(k)}$ and $\omega^{(k)}$ will be taken to be approximations on $R_h + S_h$ to $\psi(x, y)$ and $\omega(x, y)$, respectively.

For the above purpose, we begin by defining $\psi^{(0)}$ and $\omega^{(0)}$ as follows. At each grid point in HG , set $\psi^{(0)} = 1$; at each grid point in $ABCDEF$ set $\psi^{(0)} = 0$; at each grid point in AH determine $\psi^{(0)}$ from (2.6); and on the remaining grid points of $R_h + S_h$ determine $\psi^{(0)}$ by linear interpolation along

the vertical grid lines. At each grid point in AH determine $\omega^{(0)}$ from (2.6) and on the remaining grid points of $R_h + S_h$ set $\omega^{(0)} = 0$.

The second element of sequence (3.1) is now determined as follows. At each grid point in HG set $\psi^{(1)} = 1$; at each grid point in ABCDEF set $\psi^{(1)} = 0$; and at each grid point in AH, determine $\psi^{(1)}$ from (2.6). Next, at each grid point (x, y) in R_h , write down the difference analogue [1]

$$(3.5) \quad -4\psi^{(1)}(x, y) + \psi^{(1)}(x+h, y) + \psi^{(1)}(x, y+h) + \psi^{(1)}(x-h, y) + \psi^{(1)}(x, y-h) \\ = -h^2 \omega^{(0)}(x, y)$$

of differential equation (2.1), while at each point (x, y) of S_h which is interior to FG, write down the difference analogue

$$(3.6) \quad \psi^{(1)}(x, y) = \psi^{(1)}(x-h, y)$$

of the first condition in (2.7). One then solves the linear algebraic system generated by (3.5) and (3.6) by the generalized Newton's method [1] with over-relaxation factor r_ψ and denotes the solution by $\bar{\psi}^{(1)}$, which is of course defined only on R_h and on those points of S_h which are interior to FG. The function $\psi^{(1)}$ is then defined on this point set by the weighted average

$$(3.7) \quad \psi^{(1)} = \rho \psi^{(0)} + (1 - \rho) \bar{\psi}^{(1)}, \quad 0 \leq \rho \leq 1,$$

thus completing the definition of $\psi^{(1)}$ on all of $R_h + S_h$.

The second element of sequence (3.2) is now determined as follows. At each grid point in AH, determine $\omega^{(1)}$ from (2.6). At each grid point

(x, y) which is interior to HG approximate $\omega^{(1)}$ by [3]

$$(3.8) \quad \bar{\omega}^{(1)}(x, y) = \frac{2}{h^2} - \frac{2}{h^2} \psi^{(1)}(x, y-h);$$

at each grid point (x, y) which is interior to AB, CD and EF approximate $\omega^{(1)}$ by

$$(3.9) \quad \bar{\omega}^{(1)}(x, y) = -\frac{2}{h^2} \psi^{(1)}(x, y+h);$$

at each grid point (x, y) interior to BC approximate $\omega^{(1)}$ by

$$(3.10) \quad \bar{\omega}^{(1)} = -\frac{2}{h^2} \psi^{(1)}(x-h, y);$$

at each grid point (x, y) interior to DE approximate $\omega^{(1)}$ by

$$(3.11) \quad \bar{\omega}^{(1)}(x, y) = -\frac{2}{h^2} \psi^{(1)}(x+h, y).$$

At the stagnation points B and E, merely set

$$(3.12) \quad \omega^{(1)} = 0,$$

while at F and G, which will never enter into the computations, we do not define $\omega^{(1)}$ at all.

At the points C and D, assume that (3.5) is valid with $\omega^{(0)}$ replaced by $\omega^{(1)}$, so that at C we approximate $\omega^{(1)}$ by

$$(3.13) \quad \bar{\omega}^{(1)} = -\frac{1}{h^2} [\psi^{(1)}(x, y+h) + \psi^{(1)}(x-h, y)]$$

while at D we approximate $\omega^{(1)}$ by

$$(3.14) \quad \bar{\omega}^{(1)} = -\frac{1}{h^2} [\psi^{(1)}(x+h, y) + \psi^{(1)}(x, y+h)]$$

Next, at each point (x, y) of R_h , proceed as follows [2]. Determine the values

$$(3.15) \quad \mathfrak{H} = \psi^{(1)}(x+h, y) - \psi^{(1)}(x-h, y)$$

$$(3.16) \quad \mathfrak{K} = \psi^{(1)}(x, y+h) - \psi^{(1)}(x, y-h)$$

and write down, as is appropriate, the following difference analogues of differential equation (2.2):

$$(3.17a) \quad \left(-4 - \frac{\mathfrak{H}\mathfrak{R}}{2} - \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y) + \omega^{(1)}(x+h, y) + \left(1 + \frac{\mathfrak{H}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y+h) \\ + \left(1 + \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x-h, y) + \omega^{(1)}(x, y-h) = 0, \quad (\mathfrak{H} \geq 0, \mathfrak{K} \geq 0),$$

$$(3.17b) \quad \left(-4 - \frac{\mathfrak{H}\mathfrak{R}}{2} + \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y) + \left(1 - \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x+h, y) + \left(1 + \frac{\mathfrak{H}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y+h) \\ + \omega^{(1)}(x-h, y) + \omega^{(1)}(x, y-h) = 0, \quad (\mathfrak{H} \geq 0, \mathfrak{K} < 0),$$

$$(3.17c) \quad \left(-4 + \frac{\mathfrak{H}\mathfrak{R}}{2} - \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y) + \omega^{(1)}(x+h, y) + \omega^{(1)}(x, y+h) \\ + \left(1 + \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x-h, y) + \left(1 - \frac{\mathfrak{H}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y-h) = 0, \quad (\mathfrak{H} < 0, \mathfrak{K} \geq 0),$$

$$(3.17d) \quad \left(-4 + \frac{\mathfrak{H}\mathfrak{R}}{2} + \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y) + \left(1 - \frac{\mathfrak{K}\mathfrak{R}}{2}\right) \omega^{(1)}(x+h, y) + \omega^{(1)}(x, y+h) \\ + \omega^{(1)}(x-h, y) + \left(1 - \frac{\mathfrak{H}\mathfrak{R}}{2}\right) \omega^{(1)}(x, y-h) = 0, \quad (\mathfrak{H} < 0, \mathfrak{K} < 0).$$

In applying (3.17a)-(3.17d), the values of ω at boundary grid points not in GF are to be determined from (3.8)-(3.14). Finally, at each grid

point (x, y) interior to FG , we write down the difference analogue

$$(3.18) \quad \frac{\omega^{(1)}(x, y) - \omega^{(1)}(x-h, y)}{h} + \mathcal{R} \left[\frac{\psi^{(1)}(x, y+h) - \psi^{(1)}(x, y-h)}{2h} \right] [\omega^{(1)}(x, y) \\ + \frac{\psi^{(1)}(x, y+h) - 2\psi^{(1)}(x, y) + \psi^{(1)}(x, y-h)}{h^2}] = 0$$

of the second condition in (2.7).

One then solves the linear algebraic system generated by (3.17a)-(3.17d) and (3.18) by the generalized Newton's method with over-relaxation factor r_ω . This solution is denoted by $\bar{\omega}^{(1)}$.

To determine $\omega^{(1)}$ at those points of $R_h + S_h$ at which only $\bar{\omega}^{(1)}$ has been defined, we use the averaging formula

$$(3.19) \quad \omega^{(1)} = \mu \omega^{(0)} + (1 - \mu) \bar{\omega}^{(1)}, \quad 0 \leq \mu \leq 1,$$

thus completing the definition of $\omega^{(1)}$ on all of $R_h + S_h$.

The numerical method then proceeds by generating $\psi^{(2)}$ from $\omega^{(1)}$ just as $\psi^{(1)}$ was generated from $\omega^{(0)}$ and by generating $\omega^{(2)}$ from $\psi^{(2)}$ just as $\omega^{(1)}$ was generated from $\psi^{(1)}$. The indicated iteration is continued until, for some k , (3.3) and (3.4) are valid. Substitution of $\psi^{(k)}$ and $\omega^{(k)}$ into the difference approximations of (2.1) and (2.2) to assure that these are the desired solutions terminates the method.

4. EXAMPLES.

We shall now try to organize in a comprehensive way the large number of examples run on the CDC 3600 at the University of Wisconsin. Convergent results were obtained readily for $\varepsilon = 10^{-4}$, $h = \frac{1}{10}$, $\beta = 1$, $\delta = 1$, $\gamma = \frac{1}{2}$, $r_\psi = 1.8$, $r_\omega = 1.0$ as indicated in Table 4.1. All stream curves are plotted in Figures 4.1 - 4.6, while typical equivorticity curves are shown in Figures 4.7 - 4.9. A variety of checks were run to determine the validity of the results shown in Figures 4.1 - 4.9. For example, for $\mathcal{R} = 100$ the case $\alpha_1 = 4$, $\alpha_2 = 20$, $\rho = 0.04$, $\mu = 0.7$ was run to verify that the channel length to the right had no effect on the size of the vortex; for $\mathcal{R} = 200$ a more accurate solution was obtained with $\varepsilon = 10^{-6}$ to verify the existence of the vortex on the left; and for $\mathcal{R} = 200$ Poiseuille conditions were assumed on FG in place of (2.7), the result being essentially the same as that obtained with (2.7). Other selected results have been organized in Table 4.2.

From Table 4.1 it can be seen that as the right vortex was increasing in size, the amount of computing time required for $\mathcal{R} > 1000$ would have been exorbitant. Thus, for economy purposes and in order to study better the growth of the left vortex, a modified problem was studied as follows.

5. FLOW UP A STEP.

The channel shown in Figure 2.1 was modified by eliminating the downstream step, as shown in Figure 5.0. The analytical problem formulated in Section 2 was modified by asking that (2.7) now be satisfied on ED in

Figure 5.0 . The numerical approach to this new problem was essentially the same as that described in Section 3 . Convergent results were obtained readily for $\varepsilon = 10^{-4}$, $\alpha = 5$, $r_{\psi} = 1.8$ and $r_{\omega} = 1.0$. In the case $\mathcal{R} = 2000$ with $h = \frac{1}{10}$, convergence was achieved with $\rho = 0.03$, $\mu = 0.3$ in 100 outer iterations in only 2 minutes 31 seconds of running time. In the case $\mathcal{R} = 5000$ with $h = \frac{1}{10}$, convergence was achieved with $\rho = 0.03$, $\mu = 0.3$ in 100 iterations in only 2 minutes 26 seconds of running time. In the case $\mathcal{R} = 10000$ with $h = \frac{1}{10}$, convergence was achieved with $\rho = 0.05$, $\mu = 0.7$ in 230 iterations in 5 minutes of running time. The streamlines and equivorticity curves for these cases are shown in Figures 5.1 - 5.6 .

\mathcal{R}	α_1	α_2	ρ	μ	Approximate number of outer iterates	Approximate running time
10	4	4	0.04	0.7	30	6 minutes
50	4	4	0.04	0.7	60	8 minutes
100	4	4	0.04	0.7	100	10 minutes
200	4	4	0.04	0.7	150	12 minutes
500	4	10	0.03	0.85	470	50 minutes
1000	4	10	0.03	0.9	650	91 minutes

Table 4.1

R	α_1	α_2	h	ρ	μ	r_ψ	τ_ω	Convergent or Divergent
10	4	4	$\frac{1}{4}$	0	0	1.8	1.0	Divergent
10	4	4	$\frac{1}{4}$	0.1	0.5	1.8	1.0	Convergent
10^5	4	4	$\frac{1}{4}$	0.1	0.5	1.8	1.0	Divergent
10	4	4	$\frac{1}{8}$	0.1	0.1	1.8	1.0	Divergent
10	4	4	$\frac{1}{8}$	0.7	0.1	1.8	1.0	Divergent
10	4	4	$\frac{1}{8}$	0.7	0.7	1.8	1.0	Divergent
10^5	4	4	$\frac{1}{8}$	0.1	0.7	1.8	1.0	Divergent
10^5	4	4	$\frac{1}{8}$	0.1	0.3	1.8	1.0	Divergent
10^5	4	4	$\frac{1}{8}$	0.1	0.3	1.8	0.7	Divergent
10^5	4	4	$\frac{1}{8}$	0.1	0.7	1.8	0.7	Divergent
10	4	4	$\frac{1}{16}$	0.1	0.7	1.8	1.6	Divergent
10	4	4	$\frac{1}{10}$	0.1	0.4	1.9	1.0	Divergent
10	4	4	$\frac{1}{10}$	0.2	0.7	1.8	1.0	Divergent
100	4	4	$\frac{1}{8}$	0.1	0.1	1.8	1.3	Divergent
100	4	4	$\frac{1}{10}$	0.1	0.7	1.8	1.0	Convergent
500	4	4	$\frac{1}{10}$	0.07	0.7	1.8	1.0	Divergent
500	4	4	$\frac{1}{10}$	0.03	0.45	1.8	1.0	Divergent
500	4	4	$\frac{1}{10}$	0.04	0.6	1.8	1.0	Divergent
10^3	4	4	$\frac{1}{10}$	0.02	0.5	1.8	1.0	Divergent
10^3	4	4	$\frac{1}{10}$	0.1	0.5	1.8	1.0	Divergent
10^3	4	4	$\frac{1}{10}$	0.5	0.5	1.8	1.0	Divergent
10^3	4	4	$\frac{1}{10}$	0.5	0.9	1.8	1.0	Divergent
10^3	4	4	$\frac{1}{10}$	0.1	0.9	1.8	1.0	Convergent
10^4	4	4	$\frac{1}{10}$	0.05	0.8	1.8	1.0	Divergent
10^4	4	8	$\frac{1}{10}$	0.05	0.6	1.8	1.0	Divergent

Table 4.2

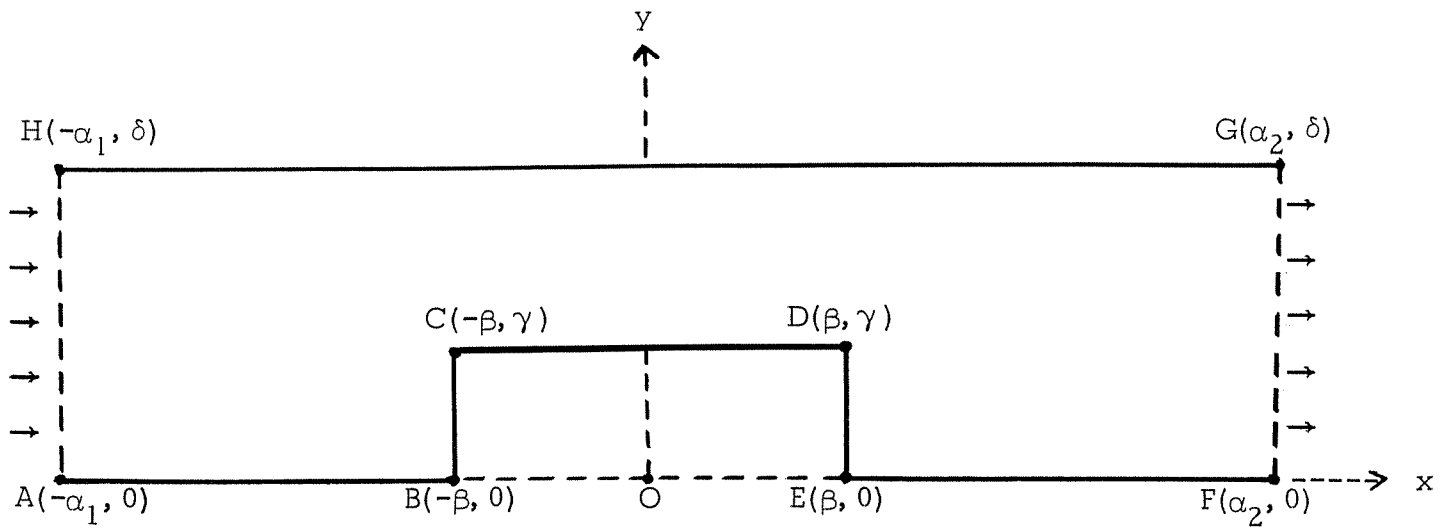


Figure 2.1

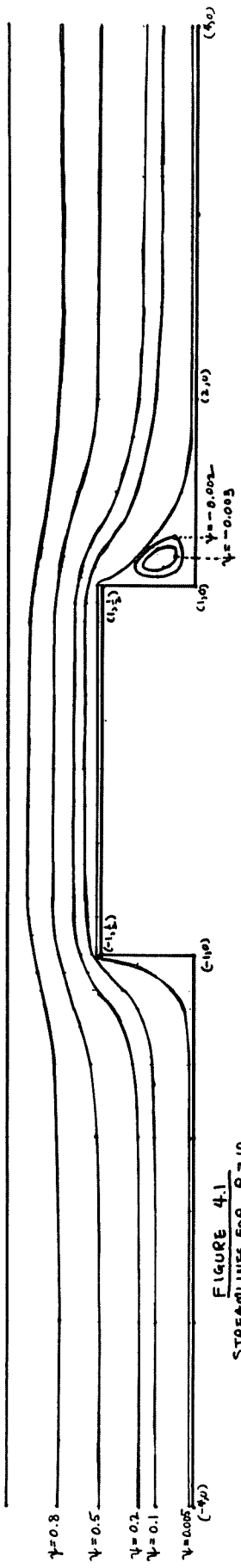


FIGURE 4.1
STREAMLINES FOR $R=10$

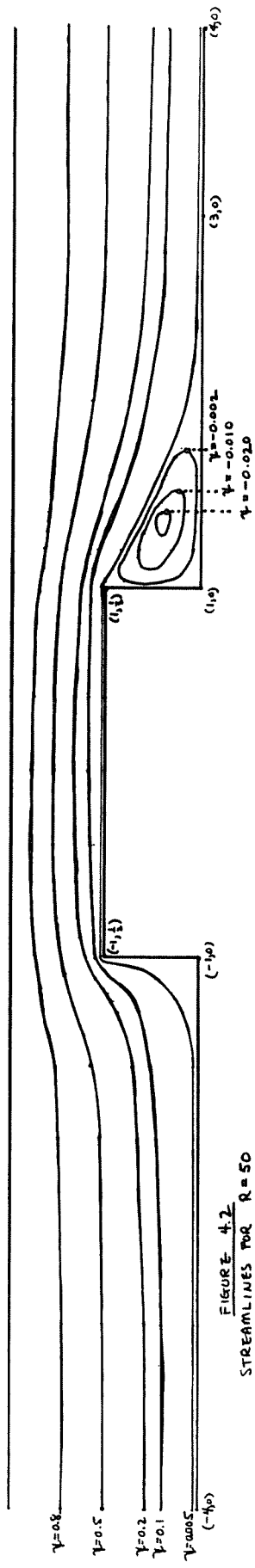


FIGURE 4.2
STREAMLINES FOR $R=50$

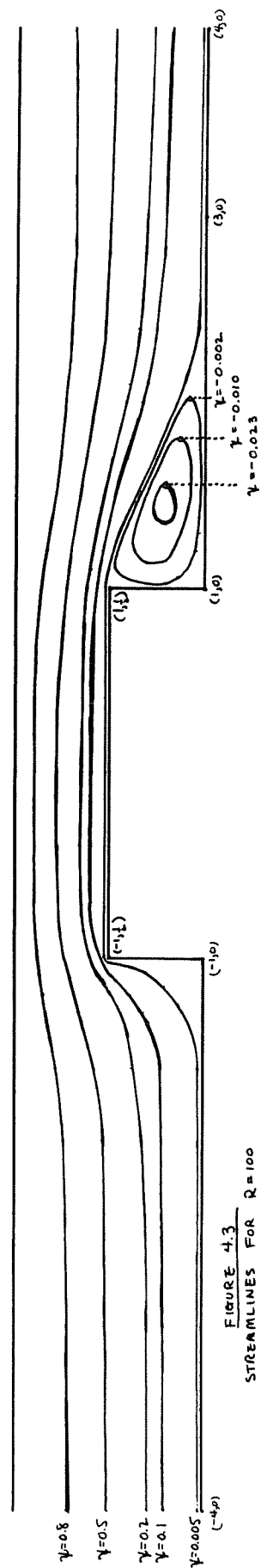


FIGURE 4.3
STREAMLINES FOR $R=100$

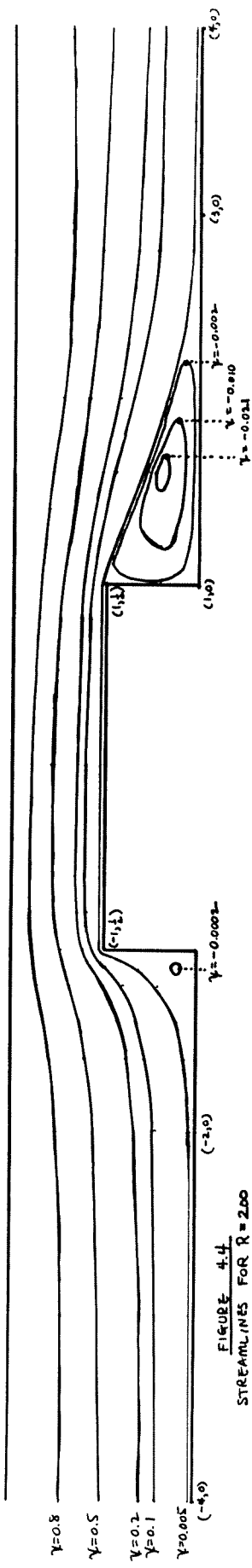


FIGURE 4.4
STREAMLINES FOR R=200

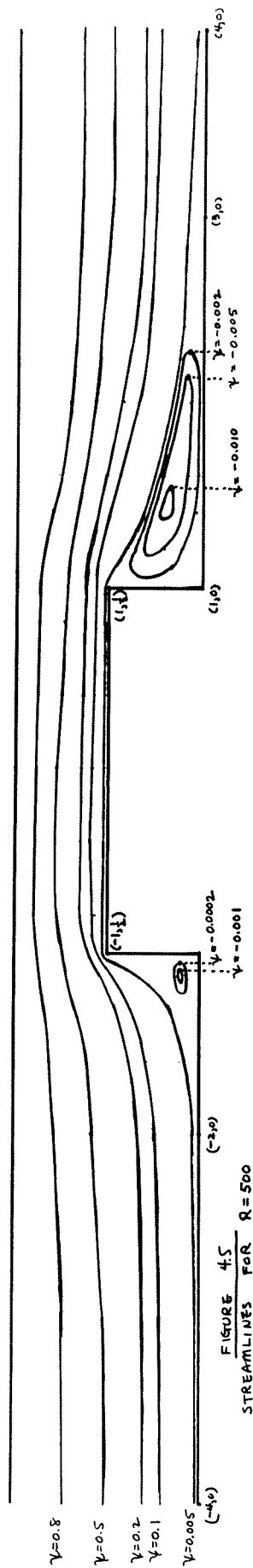


FIGURE 4.5
STREAMLINES FOR R=500

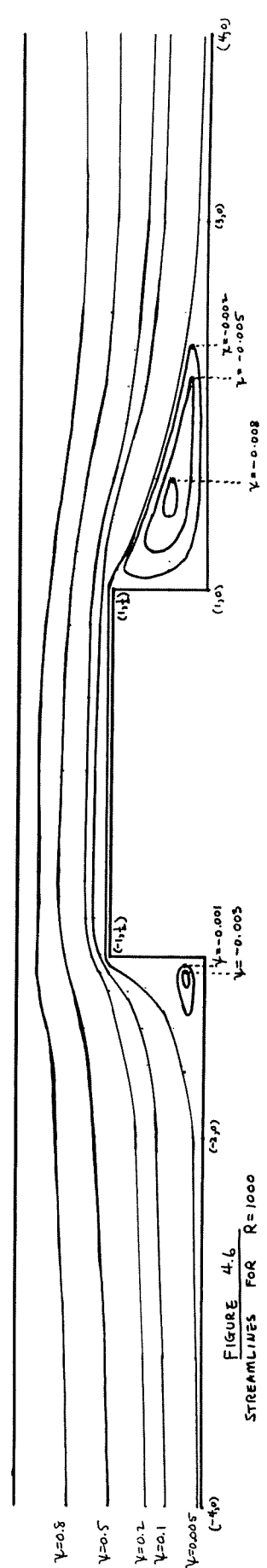


FIGURE 4.6
STREAMLINES FOR R=1000

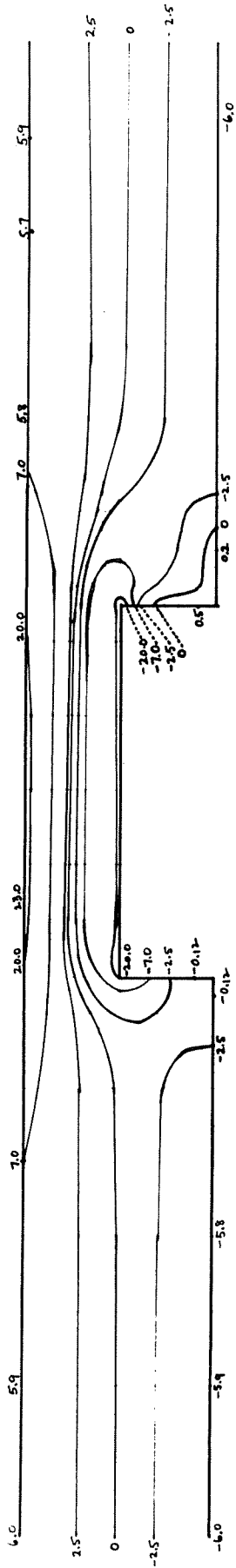


FIGURE 4.7 EQUIPARTICITY CURVES FOR $R=10$

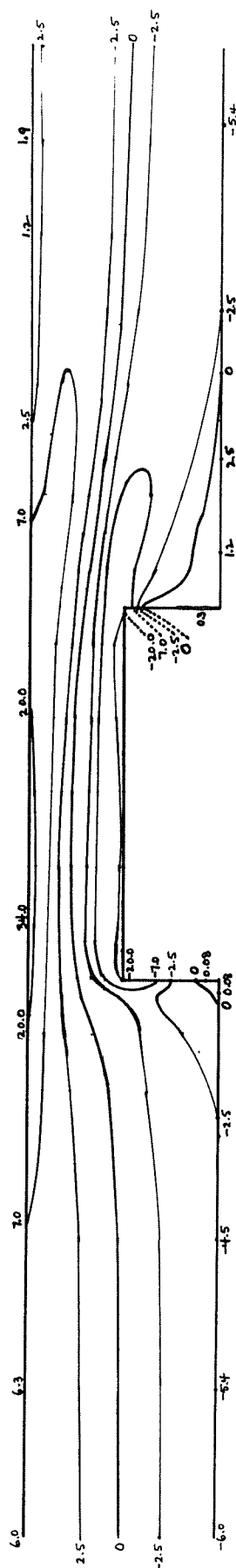


FIGURE 4.8 EQUIPARTICITY CURVES FOR $R=2.00$

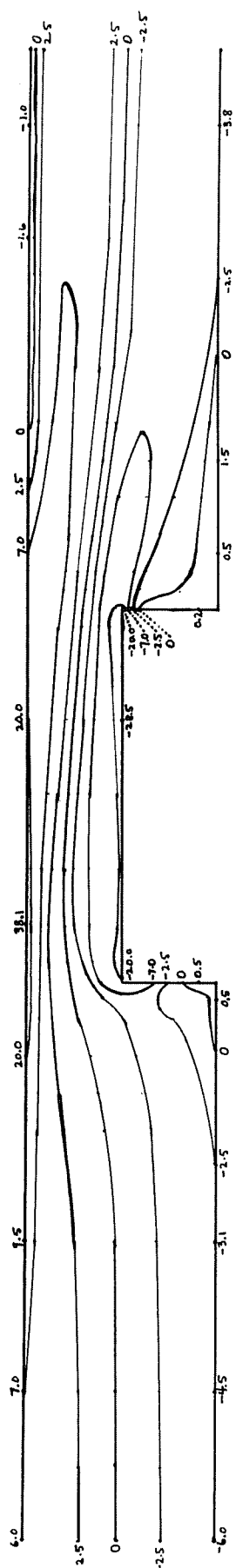


FIGURE 4.9 EQUIPARTICITY CURVES FOR $R=1000$

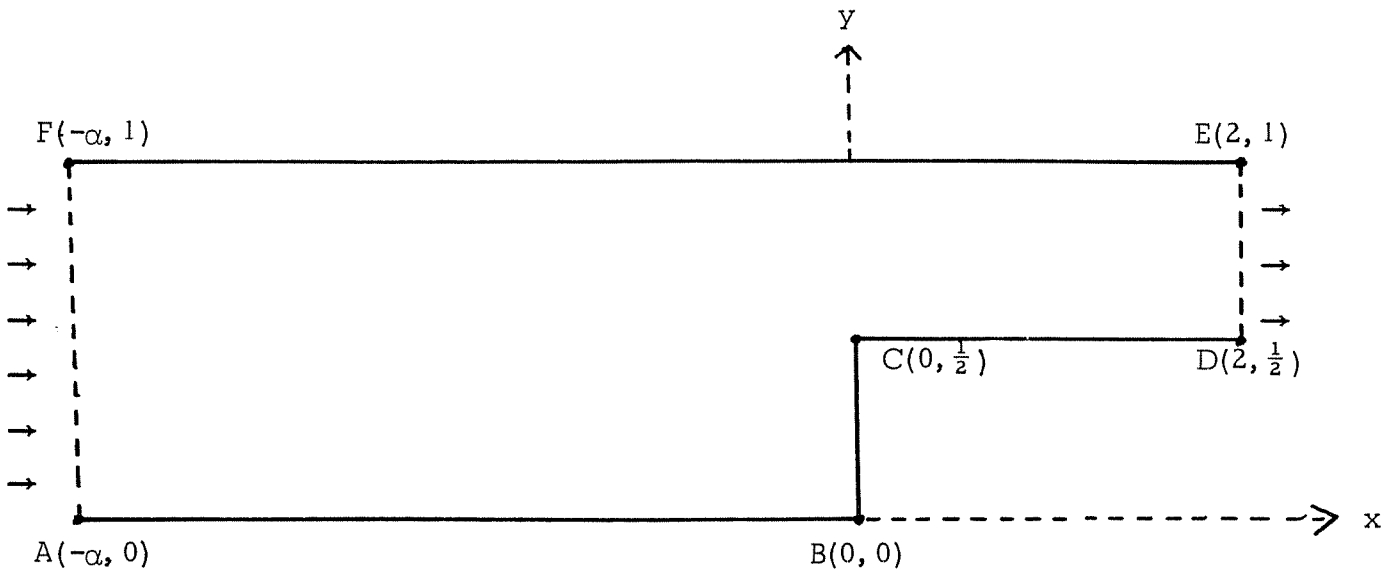


Figure 5.0

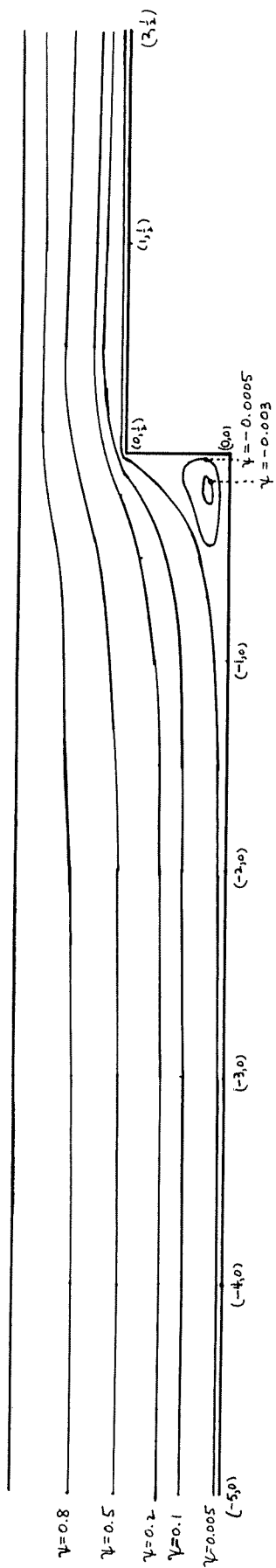


FIGURE S.1. STREAMLINES FOR $R=2000$

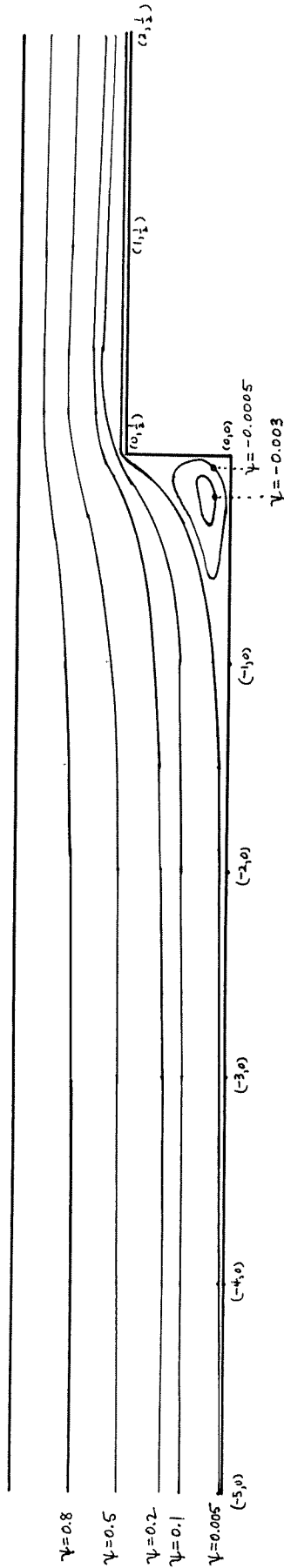


FIGURE S.2. STREAMLINES FOR $R=5000$

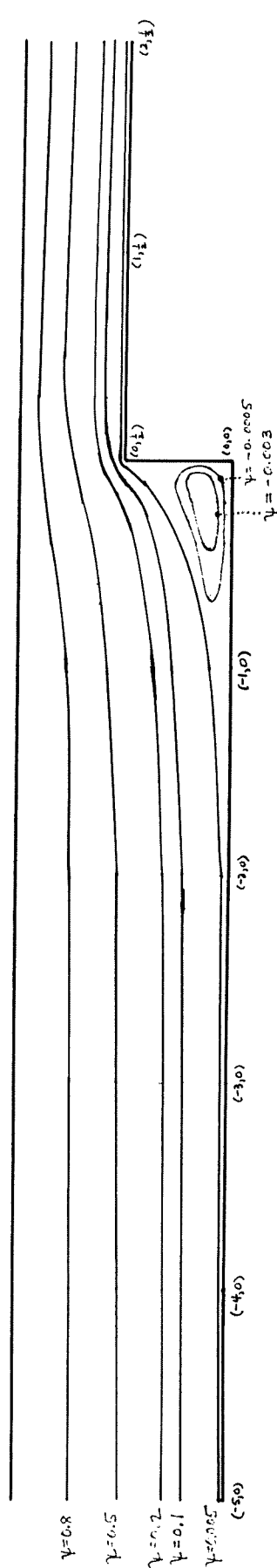


FIGURE S.3. STREAMLINES FOR $R=10000$

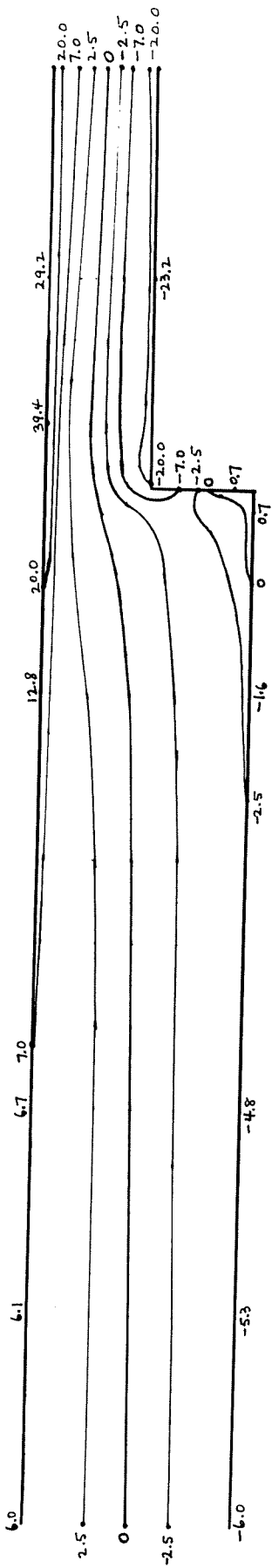


FIGURE 5.4 . EQUIPORTICITY CURVES FOR $R = 2000$

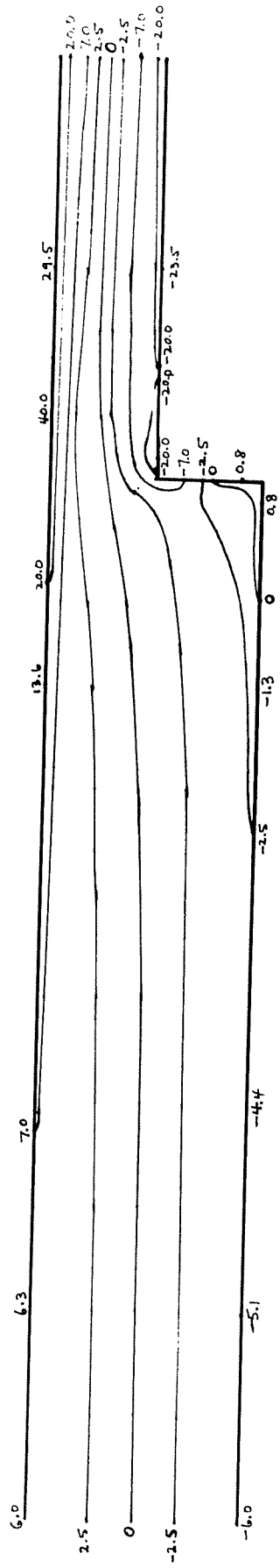


FIGURE 5.5 . EQUIPORTICITY CURVES FOR $R = 5000$

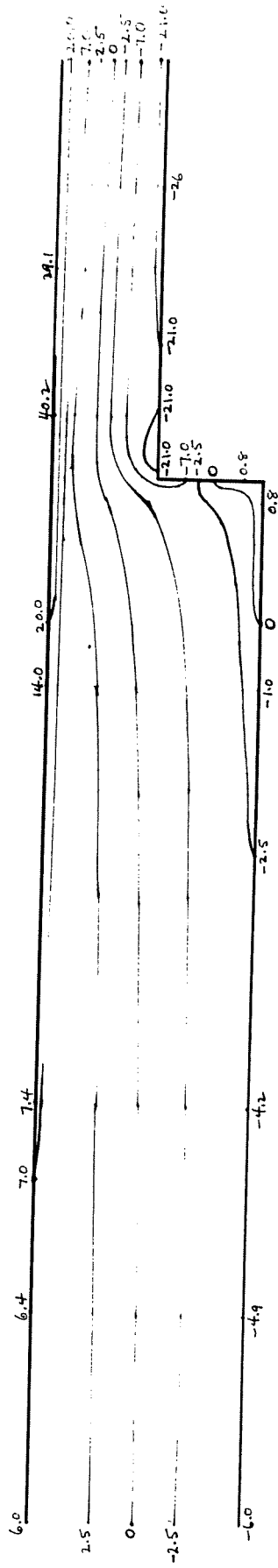


FIGURE 5.6 . EQUIPORTICITY CURVES FOR $R = 10000$

REFERENCES

1. D. Greenspan, Introductory Numerical Analysis of Elliptic Boundary Value Problems, Harper and Row, New York, 1965.
2. D. Greenspan, "Numerical Studies of Two Dimensional Steady State Navier-Stokes Equations for Arbitrary Reynolds Number," Technical Report #9, Department of Computer Sciences, University of Wisconsin, 1967.
3. D. Greenspan, "Numerical Studies of Viscous, Incompressible Flow for Arbitrary Reynolds Number," Technical Report #11, Department of Computer Sciences, University of Wisconsin, 1968.
4. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York, 1960.

APPENDIX

PROGRAMMING FLOWS IN A CHANNEL WITH A STEP
M. McClellanDefinitions of Main Program Variables and Parameters

PSI, W = stream function and vorticity vectors, resp.

XL \leq XSL \leq XSR \leq XR = left end of channel, left end of step, right end of step, right end of channel, resp.

D = width of channel at step ($D < 1.0$, max width).

H = grid size.

M, N = number of vertical and horizontal lines in grid, resp.

OMEGAP = relaxation factor for PSI inner-iterations.

OMEGAW = relaxation factor for W inner-iterations.

RHO, MU = weights for PSI and W, resp.

TOLP = tolerance for PSI inner- and outer-iterations.

TOLW = tolerance for W inner- and outer-iterations.

TOLTEST = number of outer-iterations between tests for problem convergence.

TOLTESTP = number of PSI inner-iterations between tests for convergence.

TOLTESTW = number of W inner-iterations between tests for convergence.

ITERMAX = maximum number of outer-iterations for both PSI and W.

ITERMAXP = maximum number of inner-iterations for PSI.

ITERMAXW = maximum number of inner-iterations for W.

Program switches

PCONV = 1, if PSI outer-iterations have converged.

TAPEUSE = 1, if PSI and W are to be saved on tape.

INPTAPE = 1, if PSI and W are to be initialized from tape.

RET = return address from W relaxation-coefficients block.

PROGRAM NS2AB

DIMENSION PSI(241,11),W(241,11),PSISAV(241,11),WSAV(241,11)

COMMON/PARAMS/ M,N,M1,MSL,MSR,MR,MSLP1,NB,NST,MP,NP,XSL,XSR,XL,

1 H,MS

TYPE REAL MU,MU1

TYPE INTEGER TOLTEST,TOLTESTP,TOLTESTW,RET

TYPE LOGICAL PCNV,TAPEUSE,INPTAPE

C

C

READ INPUT.

READ 901, NPROBS

901 FORMAT(I5)

DO 70 IPROB=1,NPROBS

READ 904, XL,XSL,XSR,XR,D,H,OMEGAP,OMEGAW,RHO,MU,TOLP,TOLW,R

904 FORMAT(10F5,2E5,E10)

READ 906, MP,NP,ITERMAX,TOLTEST,ITERMAXP,TOLTESTP,

1 ITERMAXW,TOLTESTW,TAPEUSE,INPTAPE

906 FORMAT(8I5,2L5)

C

C

COMPUTE INITIAL PARAMETERS.

XCH=XR-XL

XCHL=XSL-XL

XCHR=XR-XSR

XS=XSR-XSL

M=XCH/H+1.5

MSL=XCHL/H+1.5

MSR=(XSR-XL)/H+1.5

MS=XS/H+1.5

MR=XCHR/H+1.5

MSLP1=MSL+1

MSL1=MSL-1

MSRP1=MSR+1

MSR1=MSR-1

N=1.0/H+1.5

NB=(1.0-D)/H+1.5

NBP1=NB+1

NB1=NB-1

NST=D/H+1.5

M1=M-1

N1=N-1

NPTS=M*N-(MS-2)*NB1

H2=H*H

CP1=1.0-OMEGAP

CP2=0.25*OMEGAP

CW00=1.0-OMEGAW

CW5=2./H2

CW6=1.0/H2

CW7=H2*OMEGAW

R2=0.5*R

CW=1.0

RHO1=1.-RHO

MU1=1.-MU

RMAX=RMAXPSV=1.E+5

ITER=ITOL=0

C

C

PRINT INITIAL PARAMETERS.

PRINT 909, XCH,XCHL,XCHR,XS,CW,D,H,N,M,NPTS,OMEGAP,OMEGAW,

```

1 TOLP,TOLW,TOLTEST,ITERMAX
909  FORMAT(1H1,10X,13HPROBLEM NO. 2,10X,82HNAVIER-STOKES EQUATIONS FOR
1  FLOW IN A CHANNEL WITH STEP (LPSTREAM-DOWNSTREAM CASE) //
2  20X,19HLENGTH OF CHANNEL =,F8.4,31H WITH LENGTH TO LEFT OF STEP
2  =,F8.4 / 49X,29HAND LENGTH TO RIGHT OF STEP =,F8.4 /
2  58X,20HAND LENGTH OF STEP =,F8.4 /
3  20X,22HWIDE PART OF CHANNEL =,F8.4 /
4  20X,24HNARROW PART OF CHANNEL =,F8.4 /
4  20X,15HGRID SIZE (H) =,F10.6 /
5  20X,25HNO. OF HORIZONTAL LINES =,15,28H AND NO. OF VERTICAL LINES
6  =,15 / 20X,19HTOTAL GRID POINTS =,17 /
7  20X,39HRELAXATION FACTOR FOR STREAM FUNCTION =, F6.2 /
7  20X,33HRELAXATION FACTOR FOR VORTICITY =,F6.2 /
8  20X,19HTOLERANCE FOR PSI =,E10.1 / 20X,17HTOLERANCE FOR W =,E10.1
8  / 20X,43HTOLERANCE TEST CYCLE FOR OUTER-ITERATIONS =15 /
9  20X,30HMAXIMUM NUMBER OF ITERATIONS =,16 )
    PRINT 9091, ITERMAXP,TOLTESTP,ITERMAXW,TOLTESTW,R,RHO,MU
9091  FORMAT(20X,43HMAXIMUM ITERATIONS FOR PSI-INNER-ITERATIONS,16 /
1  20X,47HTOLERANCE TEST CYCLE FOR PSI INNER-ITERATIONS =,16 /
2  20X,41HMAXIMUM ITERATIONS FOR W-INNER-ITERATIONS,16 /
3  20X,45HTOLERANCE TEST CYCLE FOR W INNER-ITERATIONS =,16 /
4  20X,17HREYNOLDS NUMBER =,F10.2 /
5  20X,25HWEIGHTING (RHO) FOR PSI =F6.2 /
6  20X,22HWEIGHTING (MU) FOR W =,F6.2 //)

```

C

```

C  INITIALIZE VECTORS W AND PSI.
    IF(INPTAPE) 5,8

```

```

C  INITIALIZE FROM INPUT TAPE.

```

```

5  REWIND 5
    DO 6 J=1,N
6  READ (5) (PSI(I,J),I=1,M)
    DO 7 J=1,N
7  READ (5) (W(I,J),I=1,M)
    GO TO 9

```

```

C  INITIALIZE BY STANDARD PROCEDURE.

```

```

8  CALL INIT2AB(W,PSI)

```

```

C  PRINT INITIAL VECTORS W AND PSI.

```

```

9  PRINT 911, ITER,RMAX
911  FORMAT(///10X,16HAT ITERATION NO.,16,20H MAXIMUM RESIDUAL =,E12.4
1  /20X,15HSTREAM FUNCTION)

```

```

    CALL PRMAT(PSI)

```

```

    PRINT 912

```

```

912  FORMAT(//20X,9HVORTICITY )

```

```

    CALL PRMAT(W)

```

C

C

```

C  BEGIN MAIN LOOP.

```

C

```

C  TEST IF VECTORS TO BE SAVED ON TAPE.

```

```

10  IF(TAPEUSE) 1003,101
1003 P1=ITER/100
    P2=ITER/100,
    IF(P1 .NE. P2) GO TO 101
    REWIND 5

```

```

    DO 1005 J=1,N

```

```

1005 WRITE (5) (PSI(I,J),I=1,M)

```

```

      DO 1006 J=1,N
1006 WRITE (5) (W(I,J),I=1,M)
C
101  ITER=ITER+1
      ITOL=ITOL+1
      DO 102 J=2,N1
      DO 102 I=2,M
102  PSISAV(I,J)=PSI(I,J)
105  RMAXP=1.E94
      ITERP=0
106  ITOLP=0
12   ITERP=ITERP+1
      ITOLP=ITOLP+1
      IF(ITOLP .LT. TOLTESTP) 15,25
C
C   SWEEP STREAM FUNCTION IN REGIONS TO LEFT AND RIGHT BELOW STEP.
15   DO 20 J=2,NB
      DO 17 I=2,MSL1
17   PSI(I,J)=CP1*PSI(I,J) +CP2*(PSI(I+1,J)+PSI(I,J+1)+PSI(I-1,J)
      1 + PSI(I,J-1)+H2*W(I,J))
      DO 20 I=MSRP1,M1
20   PSI(I,J)=CP1*PSI(I,J) +CP2*(PSI(I+1,J)+PSI(I,J+1)+PSI(I-1,J)
      1 + PSI(I,J-1)+H2*W(I,J))
C   SWEEP STREAM FUNCTION IN REGION ABOVE STEP.
      DO 22 J=NBP1,N1
      DO 22 I=2,M1
22   PSI(I,J)=CP1*PSI(I,J) +CP2*(PSI(I+1,J)+PSI(I,J+1)+PSI(I-1,J)
      1 + PSI(I,J-1)+H2*W(I,J))
C   COMPUTE STREAM FUNCTION ON RIGHT BOUNDARY, FG.
      DO 23 J=2,N1
23   PSI(M,J)=PSI(M1,J)
      GO TO 12
C
25   RMAX1P=0.0
C   SWEEP STREAM FUNCTION IN REGIONS TO LEFT AND RIGHT BELOW STEP,
C   AND COMPUTE RESIDUALS.
      DO 30 J=2,NB
      DO 27 I=2,MSL1
      PSIOLD=PSI(I,J)
      PSINEW=CP1*PSI(I,J) +CP2*(PSI(I+1,J)+PSI(I,J+1)+PSI(I-1,J)
      1 + PSI(I,J-1)+H2*W(I,J))
      PSI(I,J)=PSINEW
      RES=ABSF(PSINEW-PSIOLD)
      IF(RES .GT. RMAX1P) 26,27
26   RMAX1P=RES
27   CONTINUE
      DO 30 I=MSRP1,M1
      PSIOLD=PSI(I,J)
      PSINEW=CP1*PSI(I,J) +CP2*(PSI(I+1,J)+PSI(I,J+1)+PSI(I-1,J)
      1 + PSI(I,J-1)+H2*W(I,J))
      PSI(I,J)=PSINEW
      RES=ABSF(PSINEW-PSIOLD)
      IF(RES .GT. RMAX1P) 29,30
29   RMAX1P=RES
30   CONTINUE
C   SWEEP STREAM FUNCTION IN REGION ABOVE STEP, AND COMPUTE RESIDUALS.

```



```

DO 31 J=NBP1,N1
DO 31 I=2,M1
PSIOLD=PSI(I,J)
PSINNEW=CP1*PSI(I,J) +CP2*(PSI(I+1,J)+PSI(I,J+1)+PSI(I-1,J)
1 + PSI(I,J-1)+H2*W(I,J))
PSI(I,J)=PSINNEW
RES=ABSF(PSINNEW-PSIOLD)
IF(RES .GT. RMAX1P) 305,31
305 RMAX1P=RES
31 CONTINUE
RMAXP=RMAX1P
C COMPUTE STREAM FUNCTION ON RIGHT BOUNDARY WITHOUT RESIDUALS.
DO 315 J=2,N1
315 PSI(M,J)=PSI(M1,J)
C
C TEST PSI INNER-ITERATIONS FOR DIVERGENCE.
IF(RMAXP.GT. 1.E+5 ) 32,35
32 PRINT 9017, RMAXP,ITERP
9017 FORMAT(/77H ***** DIVERGENCE IN PSI-INNER-ITERATIONS. PROBLEM AB
1ANDONED. MAX RESIDUAL =E15.6,8H AT ITER,16 )
MP=NP=1
PRINT 9009
9009 FORMAT(/ 20X,20HSTREAM FUNCTION, PSI )
CALL PRTMAT(PSI)
PRINT 9050
9050 FORMAT(/ 20X,12HVORTICITY, W )
CALL PRTMAT(W)
GO TO 70
C TEST PSI INNER-ITERATIONS FOR CONVERGENCE.
35 IF(RMAXP.LE. TOLP) 40,45
40 PRINT 915, ITERP,TOLP,RMAXP,ITER
915 FORMAT( 26H ***** AT INNER-ITERATION,16,10H TOLERANCE,E10.1,
1 34H SATISFIED WITH MAXIMUM RESIDUAL =,E15.6,10H FOR PSI(,15,1H))
C WEIGHT STREAM FUNCTION IN INTERIOR AND ON RIGHT BOUNDARY.
DO 402 J=2,NB
DO 401 I=2,MSL1
401 PSI(I,J)=RHO*PSISAV(I,J)+RH01*PSI(I,J)
DO 402 I=MSRP1,M
402 PSI(I,J)=RHO*PSISAV(I,J)+RH01*PSI(I,J)
DO 403 J=NBP1,N1
DO 403 I=2,M
403 PSI(I,J)=RHO*PSISAV(I,J)+RH01*PSI(I,J)
IF(ITOL.LT. TOLTEST) 50,405
405 ITOL=0
IF(ITER .EQ.1) GO TO 50
RMAX1=0.0
C
C COMPUTE OUTER-ITERATION RESIDUALS FOR STREAM FUNCTION.
DO 42J=2,NB
DO 414 I=2,MSL1
RES=ABSF(PSI(I,J)-PSISAV(I,J))
IF(RES .GT. RMAX1) 413,414
413 RMAX1=RES
414 CONTINUE
DO 42 I=MSRP1,M
RES=ABSF(PSI(I,J)-PSISAV(I,J))

```

```

IF(RES .GT. RMAX1) 416,42
416 RMAX1=RES
42 CONTINUE
DO 423 J=NBP1,N1
DO 423 I=2,M
RES=ABSF(PSI(I,J)-PSISAV(I,J))
IF(RES .GT. RMAX1) 422,423
422 RMAX1=RES
423 CONTINUE
RMAX=RMAXPSV=RMAX1
C
C TEST PSI OUTER-ITERATIONS FOR DIVERGENCE.
IF(RMAX .GT.1.E+5 ) 432,435
432 PRINT 9432, ITER,RMAX
9432 FORMAT(/56H ***** DIVERGENCE IN STREAM FUNCTION AT OUTER-ITERATI
10N,I6,20H MAXIMUM RESIDUAL =,E12.4 )
MP=NP=1
PRINT 9009
CALL PRIMAT(PSI)
PRINT 9050
CALL PRIMAT(W)
GO TO 70
C TEST PSI OUTER-ITERATIONS FOR CONVERGENCE.
C IF CONVERGENCE, GO TO COMPUTE AND TEST VORTICITY.
435 PCONV=0
IF(RMAX .LE. TOLP) 440,448
440 PRINT 9440, ITER,TOLP,RMAX
9440 FORMAT( 26H *** AT OUTER-ITERATION,I6,14H PSI-TOLERANCE,E10.1
1 ,34H SATISFIED WITH MAXIMUM RESIDUAL =,E15.6 /)
PCONV=1
GO TO 50
C
C FOR VORTICITY
C COMPUTE OUTER-ITERATION RESIDUALS EVERYWHERE BUT LEFT BOUNDARY, AH.
4402 RMAX1=0.0
DO 442 J=1,NB
DO 441 I=2,MSL
RES=ABSF(W(I,J)-WSAV(I,J))
IF(RES .GT. RMAX1) 4405,441
4405 RMAX1=RES
441 CONTINUE
DO 442 I=MSR,M
RES=ABSF(W(I,J)-WSAV(I,J))
IF(RES .GT. RMAX1) 4415,442
4415 RMAX1=RES
442 CONTINUE
DO 443 J=NBP1,N
DO 443 I=2,M
RES=ABSF(W(I,J)-WSAV(I,J))
IF(RES .GT. RMAX1) 4425,443
4425 RMAX1=RES
443 CONTINUE
RMAX=RMAX1
C
C TEST VORTICITY OUTER-ITERATIONS FOR DIVERGENCE.
IF(RMAX .GT.1.E+5) 4432,4435

```

```

4432 PRINT 9532, ITER,RMAX
9532 FORMAT(/50H ***** DIVERGENCE IN VORTICITY AT OUTER-ITERATION,I6,
1 20H MAXIMUM RESIDUAL =,E12.4 )
      MP=NP=1
      PRINT 9009
      CALL PRTMAT(Psi)
      PRINT 9050
      CALL PRTMAT(W)
      GO TO 70
C   TEST VORTICITY OUTER-ITERATIONS FOR CONVERGENCE.
C   IF CONVERGENCE, AND IF PSI HAS CONVERGED, SOLUTION OBTAINED.
4435 IF(RMAX .LE. TOLW) 4439,445
4439 PRINT 9443, ITER,TOLW,RMAX
9443 FORMAT( 26H *** AT OUTER-ITERATION,I6,12H W-TOLERANCE,E10.1
1 ,34H SATISFIED WITH MAXIMUM RESIDUAL =,E15.6 /)
      IF(PCONV) 444,48
444  MP=NP=1
      PRINT 9009
      CALL PRTMAT(Psi)
      PRINT 9050
      CALL PRTMAT(W)
C   SAVE SOLUTION ON TAPE, IF REQUIRED.
4441 IF(TAPEUSE) 4442,70
4442 REWIND 5
      DO 4444 J=1,N
4444 WRITE (5) (Psi(I,J),I=1,M)
      DO 4445 J=1,N
4445 WRITE (5) (W(I,J),I=1,M)
      GO TO 70
C   TEST IF MAXIMUM OUTER ITERATIONS EXCEEDED.
445  PRINT 9445, ITER,RMAX
9445 FORMAT(26H *** AT OUTER-ITERATION,I6,34H MAXIMUM RESIDUAL FOR
1 VORTICITY =,E15.6 /)
      IF(ITER .GE. ITERMAX) 447,48
447  PRINT 913
913  FORMAT(/65H ***** MAXIMUM NUMBER OF OUTER-ITERATIONS USED. ABAND
1 ON PROBLEM. )
      MP=NP=1
      PRINT 9009
      CALL PRTMAT(Psi)
      PRINT 9050
      CALL PRTMAT(W)
      GO TO 4441
C
448  PRINT 9448, ITER,RMAXPSV
9448 FORMAT(26H *** AT OUTER-ITERATION,I6,40H MAXIMUM RESIDUAL FOR
1 STREAM FUNCTION =,E15.6 /)
      GO TO 50
C   TEST IF MAXIMUM PSI INNER-ITERATIONS EXCEEDED.
45  IF(ITERP.GE. ITERMAXP) 47,106
47  PRINT 9013, RMAXP,ITERP
9013 FORMAT(/ 88H ***** MAXIMUM NUMBER OF INNER-ITERATIONS USED FOR S
1 TREAM FUNCTION. MAXIMUM RESIDUAL =E12.4,8H AT ITER,I6 )
      MP=NP=1
      PRINT 9009
      CALL PRTMAT(Psi)

```

```

PRINT 9050
CALL PRTMAT(W)
GO TO 70

```

```

C
48 PRINT 9009
CALL PRTMAT(PSI)
PRINT 9050
CALL PRTMAT(W)
PRINT 9480
9480 FORMAT(///)
GO TO 10

```

```

C
C BLOCK TO COMPUTE CW-COEFFICIENTS FOR VORTICITY.

```

```

4800 A=PSI(I+1,J)-PSI(I-1,J)
      B=PSI(I,J+1)-PSI(I,J-1)
      R2A=R2*A
      R2B=R2*B
      IF(R2A .GE. 0.) 4851,4855
4851 IF(R2B .GE. 0.) 4852,4853
4852 CW0=4.0+R2A+R2B
      CW1=CW4=1.0
      CW2=1.+R2A
      CW3=1.0+R2B
      GO TO 4860
4853 CW0=4.0+R2A-R2B
      CW3=CW4=1.0
      CW2=1.0+R2A
      CW1=1.0-R2B
      GO TO 4860
4855 IF(R2B .GE. 0.) 4856,4857
4856 CW0=4.0-R2A+R2B
      CW1=CW2=1.0
      CW3=1.0+R2B
      CW4=1.0-R2A
      GO TO 4860
4857 CW0=4.0-R2A-R2B
      CW2=CW3=1.0
      CW1=1.0-R2B
      CW4=1.0-R2A

```

```

C
4860 CW0=OMEGAW/CW0
      GO TO (525,53,54,639,6403,648), RET

```

```

C
50 RMAXW=1.E91
C SAVE VORTICITY OF PREVIOUS OUTER-ITERATION.
DO 502 J=1,N
DO 502 I=1,M

```

```

502 WSAV(I,J)=W(I,J)

```

```

C COMPUTE VORTICITY ON TOP AND BOTTOM BOUNDARIES FOR THIS OUTER-ITER.

```

```

C THE LINE AB.
DO 5021 I=2,MSL1

```

```

5021 W(I,1)=-CW5*PSI(I,2)

```

```

C THE LINE CD.
DO 5022 I=MSLP1,MSR1

```

```

5022 W(I,NB)=CW5*PSI(I,NBP1)

```

```

C THE LINE EF.

```

```

DO 5023 I=MSRP1,M1
5023 W(I,1)=-CW5*PSI(I,2)
C THE LINES CB AND ED.
DO 5024 J=2,NB1
W(MSR,J)=-CW5*PSI(MSRP1,J)
5024 W(MSL,J)=-CW5*PSI(MSL1,J)
C THE LINE HG.
DO 5025 I=2,M1
5025 W(I,N)=CW5*(1.0-PSI(I,N1))
C THE CORNER POINTS C AND D.
W(MSR,NB)=-CW6*(PSI(MSRP1,NB)+PSI(MSR,NBP1))
W(MSL,NB)=-CW6*(PSI(MSL,NBP1)+PSI(MSR1,NB))
C
C BEGIN VORTICITY INNER-ITERATIONS.
ITERW=0
505 ITOLW=0
506 ITERW=ITERW+1
ITOLW=ITOLW+1
IF(ITOLW.LT.TOLTESTW) 507,63
C THE RECTANGULAR REGION BELOW THE STEP ON THE LEFT.
507 RET=1
DO 525 J=2,NB
DO 525 I=2,MSL1
GO TO 4800
525 W(I,J)=CW00*W(I,J)+CW0*(CW1*W(I+1,J)+CW2*W(I,J+1)+CW3*W(I-1,J)
1 + CW4*W(I,J-1))
C THE RECTANGULAR REGION BELOW THE STEP ON THE RIGHT.
RET=2
DO 53 J=2,NB
DO 53 I=MSRP1,M1
GO TO 4800
53 W(I,J)=CW00*W(I,J)+CW0*(CW1*W(I+1,J)+CW2*W(I,J+1)+CW3*W(I-1,J)
1 + CW4*W(I,J-1))
C THE RECTANGULAR REGION ABOVE THE STEP.
RET=3
DO 54 J=NBP1,N1
DO 54 I=2,M1
GO TO 4800
54 W(I,J)=CW00*W(I,J)+CW0*(CW1*W(I+1,J)+CW2*W(I,J+1)+CW3*W(I-1,J)
1 + CW4*W(I,J-1))
C COMPUTE VORTICITY ON RIGHT BOUNDARY.
DO 56 J=2,N1
B=PSI(M,J+1)-PSI(M,J-1)
CW8=R2*B
56 W(M,J)=CW00*W(M,J)+(OMEGAW/(1.+CW8))*(W(M1,J)-(CW8/H2)*(PSI(M,J+1)
1-2.*PSI(M,J)+PSI(M,J-1)))
GO TO 506
C
63 RMAX1W=0.0
C COMPUTE ONE SWEEP OF VORTICITY IN INTERIOR, AND RESIDUALS.
C THE RECTANGULAR REGION BELOW THE STEP ON THE LEFT.
RET=4
DO 640 J=2,NB
DO 640 I=2,MSL1
GO TO 4800
639 WOLD=W(I,J)

```

```

WNEW =CW00*W(I,J)+CW0*(CW1*W(I+1,J)+CW2*W(I,J+1)+CW3*W(I-1,J)
1 + CW4*W(I,J-1))
W(I,J)=WNEW
RES=ABSF(WNEW-WOLD)
IF(RES .GT.RMAX1W) 6395,640
6395 RMAX1W=RES
640 CONTINUE
C THE RECTANGULAR REGION BELOW THE STEP ON THE RIGHT.
RET=5
DO 641 J=2,NB
DO 641 I=MSRP1,M1
GO TO 4800
6403 WOLD=W(I,J)
WNEW =CW00*W(I,J)+CW0*(CW1*W(I+1,J)+CW2*W(I,J+1)+CW3*W(I-1,J)
1 + CW4*W(I,J-1))
W(I,J)=WNEW
RES=ABSF(WNEW-WOLD)
IF(RES .GT.RMAX1W) 6405,641
6405 RMAX1W=RES
641 CONTINUE
C THE RECTANGULAR REGION ABOVE THE STEP.
RET=6
DO 650 J=NBP1,N1
DO 650 I=2,M1
GO TO 4800
648 WOLD=W(I,J)
WNEW =CW00*W(I,J)+CW0*(CW1*W(I+1,J)+CW2*W(I,J+1)+CW3*W(I-1,J)
1 + CW4*W(I,J-1))
W(I,J)=WNEW
RES=ABSF(WNEW-WOLD)
IF(RES .GT.RMAX1W) 649,650
649 RMAX1W=RES
650 CONTINUE
C COMPUTE VORTICITY ON RIGHT BOUNDARY WITH RESIDUALS.
DO 652 J=2,N1
B=PSI(M,J+1)-PSI(M,J-1)
CW8=R2*B
WOLD=W(I,J)
WNEW =CW00*W(M,J)+(OMEGAW/(1.+CW8))*(W(M1,J)-(CW8/H2)*(PSI(M,J+1)
1-2.*PSI(M,J)+PSI(M,J-1)))
W(I,J)=WNEW
RES=ABSF(WNEW-WOLD)
IF(RES .GT.RMAX1W) 6515,652
6515 RMAX1W=RES
652 CONTINUE
RMAXW=RMAX1W
C
C TEST VORTICITY INNER-ITERATIONS FOR DIVERGENCE.
IF(RMAXW.GT. 1.E+5 ) 665,666
665 PRINT 9665, RMAXW,ITERW
9665 FORMAT(// 55H ***** DIVERGENCE IN W-ONLY ITERATIONS. MAX RESIDUAL
1 =,E12.4,8H AT ITER,I6 )
MP=NP=1
PRINT 9009
CALL PRMAT(PSI)
PRINT 9050

```

```
      CALL PRMAT(W)
      GO TO 70
C     TEST VORTICITY INNER-ITERATIONS FOR CONVERGENCE.
C     IF CONVERGENCE, THEN WEIGHT AND GO TO TEST OUTER-ITERATIONS.
666  IF(RMAXW .LE. TOLW) 67,675
67   PRINT 9067, ITERW,TOLW,RMAXW,ITER
9067 FORMAT( 26H ***** AT INNER-ITERATION,I6,10H TOLERANCE,E10.1,
1 34H SATISFIED WITH MAXIMUM RESIDUAL =,E15.6,8H FOR W(,15,1H) /)
C     WEIGHT VORTICITY EVERYWHERE EXCEPT LEFT BOUNDARY.
      DO 672 J=1,NB
      DO 6715 I=2,MSL
6715 W(I,J)=MU*WSAV(I,J)+MU1*W(I,J)
      DO 672 I=MSR,M
672  W(I,J)=MU*WSAV(I,J)+MU1*W(I,J)
      DO 6725 J=NBP1,N
      DO 6725 I=2,M
6725 W(I,J)=MU*WSAV(I,J)+MU1*W(I,J)
      IF(ITOL .EQ. 0) 4402,10
C     TEST IF MAXIMUM VORTICITY INNER-ITERATIONS EXCEEDED.
675  IF(ITERW .GE.ITERMAXW) 677,505
677  PRINT 9677 , RMAXW,ITERW
9677 FORMAT(/ 80H ***** MAXIMUM NLMBER OF ITERATIONS USED FOR W-INNER
1-ITERATIONS. MAX RESIDUAL =,E12.4,8H AT ITER,I6 )
      MP=NP=1
      PRINT 9009
      CALL PRMAT(PSI)
      PRINT 9050
      CALL PRMAT(W)
C
C     END OF MAIN LOOP
70   CONTINUE
      STOP
      END
```

