Numerically reliable identification of complex systems

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Frequency domain identification

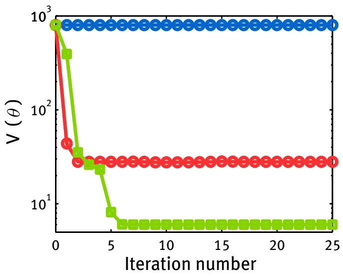
Ingredients for parametric identification:

- Data
- Model structure
- Cost function

} $\xrightarrow{\text{minimise cost}}$ Model

$$\mathcal{V}(\theta) = \left| W(\xi) \left(P_0(\xi) - \hat{P}(\xi, \theta) \right) \right|_2^2, \qquad \hat{P}(\xi, \theta) = \frac{n(\xi, \theta)}{d(\xi, \theta)},$$

Algorithm	Computation
Levy	$A\theta = b$
SK	iterate: $A\theta = b$
IV [1]	iterate: $C^T A\theta = C^T b$



Numerical aspects

Conditioning:

- SK: $\kappa(A)$
- IV: $\kappa(C^T A) \approx \kappa(A)^2$
- $\kappa(A)$ is very high (beyond 10^{16}) $\Rightarrow$ inaccurate solution
- partial solutions
  - frequency scaling and scaling of columns of A
  - use orthonormal/rational bases [2]: OBF, FLBF, ...
- full solution: (bi-)orthonormal basis

Benchmarking and comparison of multiple methods

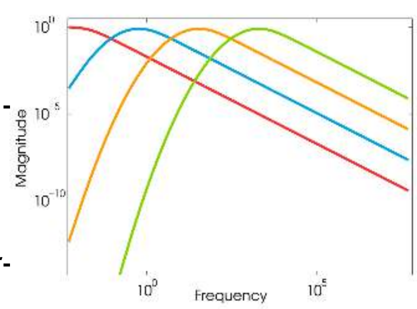
Methods

Rational bases:

- Frequency localising basis [3]:

$$\phi_{\mathrm{FL},p} = \frac{|a_p|}{s + a_p} \prod_{l=1}^{p-1} \left(\frac{s}{s + a_l} \right)$$

- band-pass filters: approximate orthogonality
- Vector fitting [4]



Open issues:

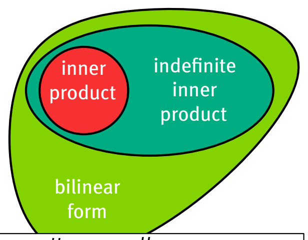
- poles ϕ_{FL} cancelled by iterative SK/IV reweighting
- convergence properties VF

Proposed solutions:

- SK-FLBF using pole relocation [4]
- IV-Vector fitting

Data dependent bases:

- scalar Forsythe polynomials:[5] data dependent inner product
- orthonormal block-polynomials [6] $\rightarrow$ optimal conditioning SK
- bi-orthonormal block-polynomials $\rightarrow$ optimal contioning IV bi-bilinear form. [7] $\langle \phi_i, \psi_j \rangle = \sum_{k=1}^{m} \psi_j(\xi_k)^H W_{2,(k,k)}{}^H W_{1,(k,k)} \phi_i(\xi_k)$



Experimental results

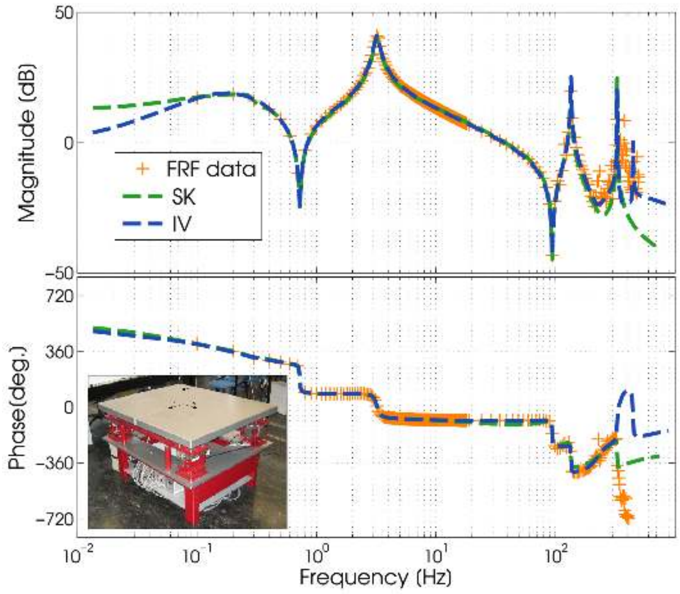


Figure 1: Frequency response measurements and identified models

Table 1: Average conditioning

	κ_{SK}	κ_{IV}
Orth	1.000 ...	1.000 ...
FLBF	$4.8 \cdot 10^4$	$1.1 \cdot 10^{10}$
VF	$9.2 \cdot 10^3$	$3.6 \cdot 10^8$
Mon	$2.8 \cdot 10^{22}$	$1.0 \cdot 10^{72}$
$\mathrm{Mon}_{\mathrm{sc}}$	510	$7.0 \cdot 10^6$

Table 2: Convergence

	$V(\theta^\star)$
SK	$5.6 \cdot 10^1$
IV	$2.8 \cdot 10^1$

Bi-orthonormal basis is promising $\rightarrow$ optimal conditioning.

Ongoing research

- theoretical properties of bi-orthonormal basis
- efficient computation of bi-orthonormal basis
- implementation in a MIMO toolbox

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