

Nutritional Menu Planning: A Hybrid Approach and Preliminary Tests

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Abstract. Menu planning is a process appearing to be straightforward but many complexities arise when it is tried to be solved by computer means. Actually, although there is evidence of previous work since 50 years ago, at present there is no wide know tool which can solve this task in an automated manner. Also, not all proposals deal with full recipes along with considering the user food preferences. In this paper we propose a system architecture based on hybrid optimization: a first module based on mathematical programming, a well known robust approach to this problem; and a second module based on belief merging, a lesser known framework aimed to combine the nutrition scientist advices and policies along with the user food desires. The association of numerical and symbolic approaches will allow us to generate of a more agreeable menu. In order to illustrate our proposal, we present a motivating example detailing the main aspects of the system.

Keywords: Diet planning, menu planning, knowledge representation, belief merging, laws of nutrition.

1 Introduction

A good nutrition is the one that provides to the body the correct amount of energy and the necessary nutrients to maintain the vital functions of an individual and to perform his/her daily activities. Human beings ingest energy and nutrients in the form of a diet consisting of different foods. Dietitians and nutritionist are the scientists who plan food and meal preparations, promote healthy eating habits and help patients with specific nutrition needs to get the essential nutrients for them.

As there is no instruction guide with the steps to generate menus, nor a generic accepted criteria to classify a produced menu as good or bad, we propose to build a model based on the called “Laws of Nutrition” or “Laws of Feeding”:

Law of Quantity, Law of Quality, Law of Harmony, Law of Adequacy [7]. This set of rules were proposed by Pedro Escudero, which is considered the “father of nutrition” in Latin America [14,23,17].

We enforce two of the four laws in our proposal:

Law of Quantity. Individual’s food intake must supply the amount of calories his/her body needs. This value is commonly obtained via anthropometric measurements of the user, mainly gender, age, weight and height; in addition to physical activity. For flexibility, a diet can supply $\pm 10\%$ of the user’s caloric requirement.

Law of Adequacy. The diet must be adequate to each user according not only to his nutritional requirements, but also to his both social and physiological needs. This means that the diet must consider intolerance and preference of aliments. This Law is focused on the final user. For example, it can set restrictions such as cultural manners, climate topics, or geographical issues.

Due to the above restrictions, planning an adequate nutritious menu for individuals and groups is a complex task. There have been multiple attempts to solve it using mathematical algorithms via computational ways since the decade of 60s [1,8]. Since then, there have intensive research in the field, from solutions ranging from mathematical programming ([15,26]) to expert systems ([22,9]) to evolutionary computing and metaheuristics [10,13].

Most of these solutions work mainly with numerical values of the food: amount of calories, carbohydrates, protein and lipids; some proposals consider the food cost. However, the user choices, preferences, acquired tastes or dietary habits usually are not taken into account, since the computational solutions are implemented from the point of view of the nutrition scientist, for whom the goal is mainly to prescribe healthy foods.

Therefore, we propose a hybrid approach to take into account the user desires and preferences in order to generate a menu more according to the user’s point of view, and consequently more acceptable by the user. This inclusion will be done via Belief Merging, a framework used for the integration of knowledge from different sources modeled as propositional formulae.

Finally, some of the approaches work only with basic food items, letting to the nutrition scientist the responsibility of choose the items forming the recipes. In our proposal, we work with basic food items as well with full recipes in order to give a personalized diet considering the user preferences.

2 Belief Merging

Belief merging aims at combining several pieces of (possibly inconsistent) information coming from different sources [12]. The goal is to produce a single consistent set of information, trying to keep the most of the information of the sources.

Belief merging is an important issue in artificial intelligence and databases, and its applications are many and diverse [4]. For example, in multiagent systems

a merging operator defines the beliefs of a group of agents according to the beliefs of each member of the group. When agents have conflicting beliefs about the “true” state of the world, belief merging can be used to determine the “true” state of the world for the group [24].

A belief merging operator is the responsible for making the belief merging. Several merging operators have been defined and characterized in a logical way. Among them, *PS-Merge* is a versatile operator which can be used to real-world problems, although only demonstrated solving “toy examples” due to its exponential complexity [6].

Preliminary concepts. In this theory we consider a language $\mathcal{L}(P)$ of propositional logic using a finite ordered set of symbols $P = \{p_1, p_2, \dots, p_n\}$ where the formulae are in Disjunctive Normal Form (DNF). A formula v is in DNF iff v is a disjunction of terms $v = D_1 \vee \dots \vee D_m$, where each term D_i is a conjunction of literals $D_i = l_1 \wedge \dots \wedge l_k$, with $l_i = p_j$ or $l_i = \neg p_j$.

A *belief base* or *knowledge base* (KB) K is a finite set of propositional formulae of \mathcal{L} representing the beliefs from a source. We identify K with the conjunction of its elements.

An *interpretation*, or *state*, or *world* is a function w from P to $\{1, 0\}$. These values are identified with the classical truth values *true* and *false* respectively. The set of all possible interpretations will be denoted as \mathcal{W} , its elements are denoted by vectors of the form $(w(p_1), \dots, w(p_n))$. A model of a propositional formula v is an interpretation such that $w(Q) = 1$ once w is extended in the usual way over the connectives, and the set of models of a formula v will be denoted by $mod(v)$. K is consistent if there exists a model of K .

If v is a propositional formula or a set of propositional formulae then $\mathcal{P}(v)$ denotes the set of atoms appearing in v . $|P|$ denotes the cardinality of set P . A *literal* is an atom or its negation.

A *belief profile* $E = \{K_1, \dots, K_m\}$ is a multiset (bag with possible repeated elements) of m belief bases.

PS-Merge Operator. This belief merging operator is an alternative way to distance-based operators [11,16,18,25] or to syntax-based operators [2,19], as it is based on the notion of Partial Satisfiability [5]. *PS-Merge* produces similar results to other merging approaches, but while other approaches require many merging operators in order to achieve satisfactory results for different scenarios, this approach obtains similar results for all these different scenarios with a unique operator [24].

Let $E = \{K_1, \dots, K_m\}$ be a belief profile and *PS-Merge* be a function which maps a belief profile to a belief base, $PS-Merge : \mathcal{L}(P)^n \rightarrow \mathcal{L}(P)$, then the Partial Satisfiability Merge of E is $PS-Merge(E)$ such that the set of models of the resulting base is:

$$\left\{ w \in \mathcal{W} \mid \sum_{i=1}^m w_{ps}(K_i) \geq \sum_{i=1}^m w'_{ps}(K_i) \text{ for all } w' \in \mathcal{W} \right\} \quad (1)$$

Let K be a belief base, w any interpretation of \mathcal{W} and $|P| = n$, we define the Partial Satisfiability of K for w , denoted as $w_{ps}(Q_K)$, as follows:

- if $Q_K := C_1 \wedge \dots \wedge C_s$ where C_i are literals then:

$$w_{ps}(Q_K) = \max \left\{ \sum_{i=1}^s \frac{w(C_i)}{s}, \frac{n - |\mathcal{P}(\bigwedge_{i=1}^s C_i)|}{2n} \right\} \quad (2)$$

- if $Q_K := D_1 \vee \dots \vee D_r$ where each D_i is a literal or a conjunction of literals then:

$$w_{ps}(Q_K) = \max \{w_{ps}(D_1), \dots, w_{ps}(D_r)\} \quad (3)$$

The intuitive explanation of Partial Satisfiability is as follows [5]: it is natural to think that if we have the conjunction of two literals and just one is satisfied then we are satisfying 50 % of the conjunction. If we generalize this idea we can measure the satisfaction of a conjunction of one or more literals as the sum of their evaluation under the interpretation divided by the number of conjuncts. However, the agent may consider only some atoms of the language, then the agent is not affected by the decision taken over the atoms that are not considered. So, in this case we give a partial satisfaction of 50 % for each atom not appearing in the agent's beliefs. On the other hand, the agent is interested in satisfying the literals that appear in its beliefs, and we interpret this fact by assigning a satisfaction of 100 % to each literal verified by the interpretation and 0 % to those that are falsified.

3 Proposal

We present a hybrid approach to solve the menu planning problem, by means of the system architecture shown in Figure 1. Although significant data preprocessing is done, we divide the overall system functionality in two algorithms, each one performing one main task of the process. The system uses different inputs according to each step, whereas outputs are menus at different level of *adequation*: some menu could have a high level of adequacy for a nutritionist (it meets the user's caloric requirement), but a poor level of adequacy for the final user (it has no favourite foods included).

3.1 Phase 1

First, system takes the user caloric needs and the database with all the available foods to generate a first approximation to a menu in accordance with the energy needed by the user. It is worth noticing that this approximation to a menu may correspond to a possibly incorrect diet according to the Law of Adequacy, that's why we name this result as *list of foods*. Each list of foods contains no element of the user's food intolerance set.

The proposed menu is divided into meals in a day. User specifies the amount of meals he wants to take in a day, and he can also specify which meals are most important by allocating a percentage of intake per meal. System generates 3 lists of foods per meal according to the user caloric needs: the one closer to the needs -10% , another one closer to the exact value of the user needs, and the closer to the needs $+10\%$.

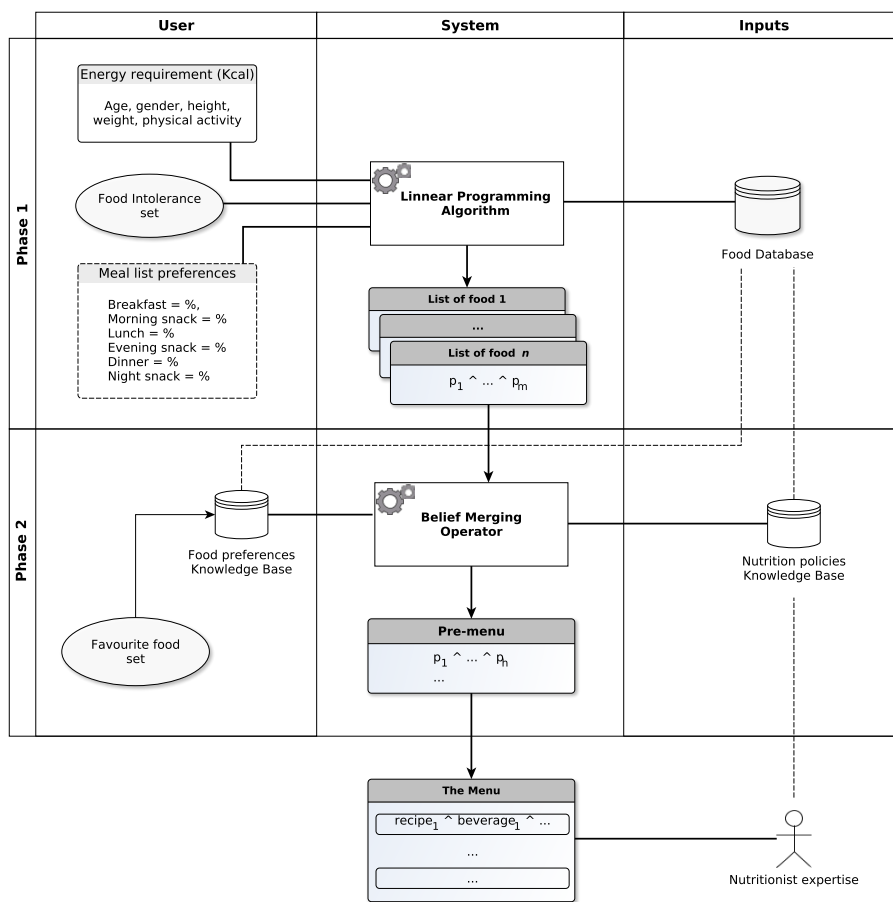


Fig. 1. System architecture. The system (center column) is composed of two algorithms, one for each phase. Input consists of data from the user (first column) and predefined data structures (third column). The final result is an adequate menu.

The main algorithm in this step generates each list of foods via Linear Programming, inspired in the model presented in [21]. Thus we had to translate the problem into a mathematical model. The result of this stage is in knowledge

base format, so we have to translate each food to its corresponding variable.

3.2 Phase 2

At this step, the system tries to meet the user preferences by means of a belief merging operator, since all inputs are symbolic data coming from different sources equally respectable but most likely inconsistent.

We start by translating the user food preferences into a knowledge base format. This KB is merged with the lists of food generated in the previous phase and with a KB of nutritional polices, i.e. “common sense” information (for example the *adequate* combination of foods). The last KB will be filled up by the nutrition scientist knowledge, but it can contain other kind of information. The main algorithm performing this step is the PS-Merge belief merging operator [24]. The output is expected to be a consistent KB per list of food.

In order to show a human-readable menu, the resulting KB is translated from propositional variables to foods names, and the final configuration is left to the specialist. This resulting menu meets the user’s caloric requirement, as well as it complies with the two Laws of Nutrition considered.

3.3 Food Database

Figure 2 shows the relational model of the food database proposed. Main tables are **food** and **patient**, whose main relations are about the user’s food intolerance set (**patient_intolerant_food**), user favourite food (**patient_favourite_food**), and the final menu for the day (**menu**).

4 A Motivating Example

Now we will show a simple instance that illustrates the hybrid approach of our proposal. We will describe the entire process by generating a breakfast, one single meal from a complete menu. Each element of the system architecture will be described in terms of the example instance, emphasizing in the belief merging process, the main contribution of this work.

4.1 User Data

Energy Requirement. We need to plan a menu for a 33 year old male, weight 84 kg, 186 cm tall, and an intense physical activity. To obtain his caloric requirement we have to calculate the Basal Metabolic Rate (BMR), which is the minimum amount of energy that the body needs without realizing any activity using the following formula [3]:

$$\text{BMR}_{\text{men}} = (10 \times \text{weight}) + (6.25 \times \text{height}) - (5 \times \text{age}) + 5 = 1680 \text{ kcal}$$

Afterwards, we calculate the Total Energy Expenditure (TEE) representing the amount of energy needed to maintain the vital functions of the body and

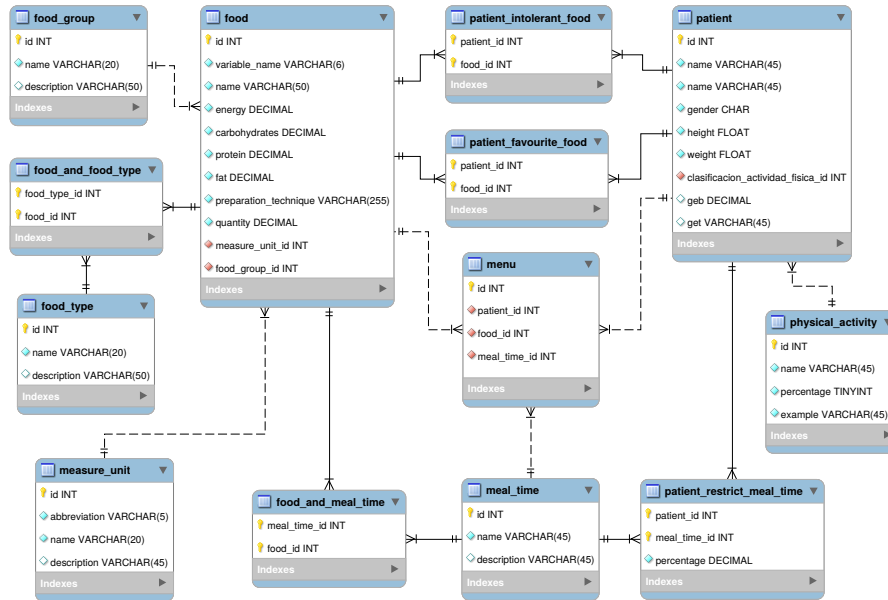


Fig. 2. Foods database relational diagram.

perform the daily activities. This is obtained from the BMR plus two values: adding a 10 % due to the digestive effect (Thermic Effect of the Food, TEF), and adding a percentage depending the user’s physical activity (Physical Activity Factor, PAF) according to $\text{Repose} = 20\%$, $\text{Sedentary} = 37.5\%$, $\text{Moderate} = 55\%$, $\text{Intense} = 72.5\%$ [3]. So the TEE for this patient is as follows:

$$\text{TEE} = \text{BMR} + \text{TEF} + \text{PAF} = 1680 + 168 + 1218 = 3066 \text{ kcal}$$

Food Intolerance. The user claims to be lactose intolerant.

Menu Distribution. The individual can only take three meals per day, so we must distribute all his food intake into three meals, according to the following distribution:

Meal	Percentage	Energy
Breakfast	30 %	920 kcal
Lunch	40 %	1226 kcal
Dinner	30 %	920 kcal

Favourite Foods. Apples and bread in the breakfast, and he always eats pepper sauce in each meal. He also dislikes papaya fruit. According to foods shown in Table 1, these beliefs can be modeled in the Food Preferences KB as:

$$K_1 = (a \wedge b \wedge m \wedge \neg p) \quad (4)$$

4.2 Input Data Structures

Food Database. Table 1 considers the foods from the database related to breakfast. Data were obtained from a local nutrition lab. For the sake of space, we only shown the relevant information related to this example.

Table 1. Subset of breakfast foods in database.

Var.	Name	Energy	Group	Type	Time	Meal	Unit	Qty.
<i>a</i>	Apple	70	F	F	1,2,3	B,S,D	Piece	1
<i>n</i>	Banana	96	F	F	1,2,3	B,S,D	Piece	1
<i>p</i>	Papaya	43	F	F	1,2	B,S,D	gr	200
<i>s</i>	Eggs with ham	310	P	E	4	B	Portion	1
<i>u</i>	Lima beans soup	328	P	C	4	B,L	Portion	1
<i>e</i>	Eggs with beans	446	P	E,L	4	B,L	Portion	1
<i>h</i>	Hot cakes	362.5	C	C	4	B	Portion	1
<i>i</i>	Cappuccino coffee	157	C	C	5	B,D	Cup	1
<i>c</i>	Caffè Americano	21	C	C	5	B,D	Cup	1
<i>o</i>	Orange juice	197.5	F	F	5	B	Glass	1
<i>k</i>	Hot milk	116	P	K	5	B,S	Glass	1
<i>b</i>	Bread roll	301	C	C	2	B,L,D	Piece	1
<i>t</i>	Tortilla	246	C	C	2	B,L,D	Piece	1
<i>g</i>	Green salad	79.5	V	V	4	B,S,D	Portion	1
<i>l</i>	Accompanying salad	26	V	V	2	B,L,D	Portion	1
<i>m</i>	Tomato spicy sauce	45	V	V	2	B,L,D	ml	50

Table 1 uses the following acronyms:

- **Type:** C = Cereal, D = Dairy, E = Egg, F = Fruit, L = Legumes, M = Meat, V = Vegetables.
- **Group:** C = Cereal, F = Fruit, P = Protein, V = Vegetables.
- **Time:** 1 = Entrance, 2 = Accompanying, 3 = Dessert, 4 = Main dish, 5 = Drink.
- **Meal:** B = Breakfast, S = Snack, L = Lunch, D = Dinner.

Nutrition Policies KB. For this case, we consider the following expert advises: (i) not to eat bread and tortilla in the same meal, because both are high carbohydrates foods; and (ii) it is recommended to eat apple, orange or papaya in breakfast due to their digestive properties. With the foods shown in Table 1, these two beliefs can be modeled in the Nutrition Policies KB as:

$$K_2 = (\neg(b \wedge t) \wedge (a \vee o \vee p)) \quad (5)$$

4.3 Phase 1

In this step we employ Linear Programming to generate the lists of foods. First we get rid of the intolerance foods using a subset from Table 1 without food from dairy type (no hot milk, cappuccino coffee nor hot cakes in this case). Within this subset we find three lists of foods whose energy complies the breakfast's energy required by the user (920 kcal):

List of foods 1	List of foods 2	List of foods 3
Apple 70	Papaya 43	Green salad 79.5
Eggs with beans 446	Apple 70	Apple 70
Caffè Americano 21	Eggs with ham 310	Lima beans soup 348
Bread roll 301	Orange juice 197.5	Orange juice 197.5
Total energy: 838	Bread roll 301	Bread roll 301
	Total energy: 921.5	Total energy: 996

Energy of the first list of foods is under the breakfast's exact energy requirement, but over the -10% . Energy of the second list of foods is close to the exact breakfast's energy requirement. Energy of the third list of foods is above the breakfast's energy requirement but lower than the $+10\%$.

4.4 Phase 2

In this step we employ Belief Merging to generate menus. Translating list of foods 1 as KB:

$$K_3 = (a \wedge e \wedge i \wedge b) \quad (6)$$

Gives us the belief profile to be merged:

$$E = (K_1, K_2, K_3) \quad (7)$$

PS-Merge operator requires normalizing each KB to DNF:

$$Q_{K_1} = (a \wedge b \wedge m \wedge \neg p) \quad (8)$$

$$Q_{K_2} = (\neg b \wedge a) \vee (\neg b \wedge o) \vee (\neg b \wedge p) \vee (\neg t \wedge a) \vee (\neg t \wedge o) \vee (\neg t \wedge p) \quad (9)$$

$$Q_{K_3} = (a \wedge e \wedge i \wedge b) \quad (10)$$

PS-Merge operator is used to find KB with no inconsistencies and with most of the food preferences and nutrition policies satisfied. For example, taking Q_{K_2} and the interpretation $w' = (1, 1, 1, 1, 1, 1, 0, 0)$ corresponding to the propositional values $(w(a), w(b), w(e), w(i), w(m), w(o), w(p), w(t))$ in that order, we have:

$$w'_{ps}(Q_{K_2}) = \max \{w'_{ps}(\neg b \wedge a), w'_{ps}(\neg b \wedge o), w'_{ps}(\neg b \wedge p), w'_{ps}(\neg t \wedge a), w'_{ps}(\neg t \wedge o), w'_{ps}(\neg t \wedge p)\}$$

Obtaining the partial satisfiability of the first element:

$$w'_{ps}(-b \wedge a) = \max \left\{ \frac{w'(-b) + w(a)}{2}, \frac{8 - |\mathcal{P}(-b, a)|}{2(8)} \right\} = \max \left\{ \frac{0 + 1}{2}, \frac{8 - 2}{16} \right\}$$

$$= \max \left\{ \frac{1}{2}, \frac{3}{8} \right\} = \frac{1}{2}$$

Computing the partial satisfiability of the remaining elements we can obtain the partial satisfiability of the KB with respect to w' :

$$w'_{ps}(Q_{K_2}) = \max \left\{ \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, 1, 1, \frac{1}{2} \right\} = 1$$

For the sake of space and clarity, below we only show the interpretations corresponding to the best partial satisfiability. Both w_1 and w_2 contain the views of the majority, representing the best possible menu according to the input KBs:

Belief merging result via PS-Merge												
	<i>a</i>	<i>b</i>	<i>e</i>	<i>i</i>	<i>m</i>	<i>o</i>	<i>p</i>	<i>t</i>	$w_{ps}(Q_{K_1})$	$w_{ps}(Q_{K_2})$	$w_{ps}(Q_{K_3})$	Sum
w_1	1	1	1	1	1	1	0	0	1	1	1	3.0
w_2	1	1	1	1	1	0	0	0	1	1	1	3.0

The final process of this stage is to translate these KBs to menus, and then verify if the menu is still consistent according to the energy requirement of the user:

Menu 1		Menu 2	
Apple	70	Apple	70
Eggs with beans	446	Eggs with beans	446
Caffè Americano	21	Caffè Americano	21
Bread roll	301	Bread roll	301
Tomato spicy sauce	45	Tomato spicy sauce	45
Orange juice	197.5	Total energy:	883
Total energy:	1080.5		

We can notice that energy in Menu 1 is above the user's needs, so this menu must be discarded. On the other hand, Menu 2 has a right amount of energy, as well as satisfies the user's food preferences and also complies with the nutrition policies.

Lastly, the adequate menu, or maybe a set of menus, are presented to the nutrition scientist, who will choose or modify the proposals to finally prescribe the diet to the user.

5 Conclusions

Diet planning is considered as an art, but we consider the need of a set of strict rules to give a reference of the "correctness" of a menu. In order to produce

adequate menus, we use the “Laws of Nutrition” as a formal reference to validate our results. So, our proposal enforces the Law of Quantity as well as the Law of Adequacy, the latter with emphasizing the user preferences.

We have presented a novel approach for dealing with the menu planning problem. The proposed architecture is based on two formal frameworks used in two phases to solve the different views of the diet planning problem.

In the first step, the Linear Programming method has been proven to solve the problem properly and on time. For this purpose, we focus on build a mathematical model containing the quantitative information of the problem. In this way we obtain menus in accordance with the user’s nutritional needs.

In the second step, we implement a Belief Merging operator, a relatively new framework which few applications to real-world problems are known (i.e. [20]). For this purpose, we focus on build a logical model containing the qualitative information of the problem. In this way we obtain menus in accordance with the user’s food desires. PS-Merge belief merging operator provides good results, as it tries to satisfy the majority opinion and so minimize the level of total dissatisfaction.

We have shown that this hybrid proposal can be a viable approach to solve the problem at a very specific level. Further research is required to provide evidence of the effectiveness and feasibility of this proposal to complete menus with bigger input data.

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