Object Categorization using Kernels combining Graphs and Histograms of Gradients

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Abstract. This paper presents a method for object categorization. This problem is difficult and can be solved by combining different information sources such as shape or appearance. In this paper, we aim at performing object recognition by mixing kernels obtained from different cues. Our method is based on two complementary descriptions of an object. First, we describe its shape thanks to labeled graphs. This graph is obtained from morphological skeleton, extracted from the binary mask of the object image. The second description uses histograms of oriented gradients which aim at capturing objects appearance. The histogram descriptor is obtained by computing local histograms over the complete image of the object. These two descriptions are combined using a kernel product. Our approach has been validated on the ETH80 database which is composed of 3280 images gathered in 8 classes. The results we achieved show that this method can be very efficient.

1 Introduction

Object categorization problem is difficult and still presents open issues. Many researches have focused on this topic and yet it has been solved only for particular situations. An object categorization system contains two main parts. First, feature extraction has to be the most exhaustive object representation in order to keep maximum information concerning the object. The second part consists of a classifier which should be able to learn the category of an object from this representation and then to predict the most correctly as possible the category of a new unseen object.

Representing a static object can be done along two ways [14], globally and locally. The first way consists in representing the general shape of an object. The other way can brings information about the object appearance for a stronger discrimination. Then for a categorization purpose these two global representations have to be appropriately combined with an objective of perfomance enhancement. One way for mixing these representations has been presented in Leibe et al. [14] and is based on a decision tree. Another way of combination resides in the properties of the SVM classifier, which can deal with mixture of kernels [5, 1].

Recently, there has been a growing interest around object representation based on graphs. One major interest of graph resides in its property which keeps the object shape topology. This property has been used in different methods for object categorization using graphs : [16, 17, 21, 13], where graph classification is achieved by measuring their similarity [16, 17].

The result of similarity measure obtained from graph matching algorithms [3, 16, 9] is a metric. This metric is then used to find the most similar object, or to cluster objects, according to the distance between them. So that efficiency of these methods depends on the quality of the similarity measure.

A weakness of graph matching algorithms is that they essentially deal with labelled graphs for which labels are a single numerical value. On top of that, information is mainly brought on edges and rarely on vertices [16, 21].

Another way to compare or to classify graphs is to use kernel methods and the so-called kernel trick.

Hence, measuring graph similarity can be addressed by considering kernels function on graphs. In [12], Kashima et al. defined a kernel function for labeled graphs. This function can be interpreted as an inner product on two graphs, obtained by comparing edges and vertices that have been crossed during random walks on the graphs. Then a major particularity of this kernel is the use of kernels between vertices and edges. It means that labels can be complex structures, like vectors, histograms or set of histograms, instead of a single real values, which is the case for most of graph matching algorithms.

One other representation of an object consists in describing the object appearance [11, 14, 8]. Recents works have shown that efficient and robust appearance-based cues can be obtained from histogram of oriented gradient (HOG) in images [15]. One advantage of this method is to bring information on both object appearance and object contours. On top of that, this method is an answer to the variability problem. Representing an image thanks to histogram of gradient is very robust for scale invariance, or different lighting conditions.

Recently, Dalal and Triggs have further developed this idea of histogram of gradient and have achieved excellent recognition rate of human detection in images [7]. This work pointed out the problem of variability, and used an efficient way to solve it.

In this paper we present a method for object categorization using a combination of representations based on kernels and a SVM classifier [20]. We decided to take advantage of graph properties for a global shape representation of an object. However, instead of using graph matching algorithms, we introduce the use of graph kernels for object recognition problem. The object shape representation is combined with an appearance representation based on local histogram of gradients. We build an appropriate kernel that mixes kernels from these two representations by a product. The resulting kernel is then fed into a SVM classifier for categorization. Our paper aims at analysing the categorization performance of the overall approach.

This paper is organized as follows. The first section 2.1 presents our method to design a graph. Starting from an image, the morphological skeleton is obtained thanks to the image binary mask. When the graph structure is complete, we add some labels on both vertices and edges, regard the orignal image. The second part 2.2 presents the HOG descriptor. The third part 2.3 presents briefly the SVM classifier used for multiclass. Next, we describe the graph kernel of Kashima 2.4. Finally, we depict some results in section 3. The test has been accomplished on the ETH-80 database, which has already been used in different approaches to test their efficiency [11, 21, 14].

2 Method description

2.1 Graph

In this part, we will describe briefly our method used to transform an image into a graph. The aim is to keep the main information contained in a shape, that is to say geometric properties or topologic properties. This last property is particularly interesting in our case. If a shape is made of a set of sub-parts, the skeleton will preserve the connectivity and the shape arrangement.

Graph designing As we mentioned before, we tackle the problem of object recognition represented with an image. In our case, one image corresponds to a single object. The first step consists in extracting the morphological skeleton from the binary mask, that is to say the general shape in white, placed on a dark background. We used the same method which is described in [16]

A skeleton can be defined as a line representation of an object, that is to say it :

– is one-pixel large
– is around the middle of the object
– preserves the geometry and topology object

Given the definition of Lantuejoul [4], a skeleton subset of a black and white image $S_k(A)$ is defined as : $S_k(A) = E(A, kB) - [E(A, kB)\, o\, B]\; k = 0, 1, ...K$ where B is a structuring element, and K is the largest value of k before the set $S_k(A)$ becomes empty. The skeleton is then the union of the skeleton subsets : $S(A) = \cup_{k=0}^{K} S_k(A)$

Skeleton to graph Once the skeleton is obtained, we can build the graph. A graph G is made up of vertices and edges $G = (V, E)$. A vertex is a junction between different edges. We can differentiate two types of vertices : nodes (V_n), which are a junction of many edges and vertices which are edge ending (V_s).
To build the graph we look at the type of each skeleton pixel. If the pixel as only one neighbor, this is an edge ending. If the neighoring corresponds to a defined mask, for example $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, then this pixel is defined as a vertex.

Once all of the skeleton pixels have been tested, we can build the graph. The first step is to search each unique edge associated with each edge ending, since only one vertex is linked with an edge ending. When all edge ending vertices are linked, we can search all links existing between nodes vertices. This method is different, because in the precedent search, the stopping condition was reached when a node was found, but now, if another vertex is reached, it does not mean that all possible ways were tried. So the stopping condition is different, and there exists at least one way to link a node vertex to a pixel of the skeleton, it is possible to reach another node vertex. The course of the skeleton is facilitated, because the skeleton is one pixel thin, so when the skeleton does not fill this condition, it means that a vertex is reached and the path is ended. An edge between the two nodes is established and other ways are continued until they reach a vertex.

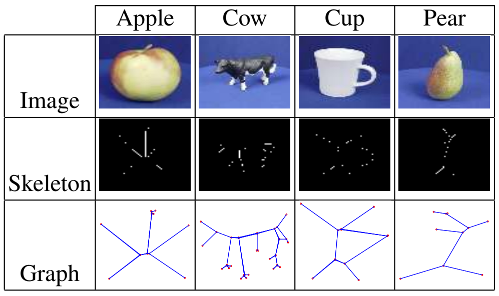


Fig. 1. Example of objects coming from the database ETH-80, their skeleton, and their graph.

Graph labeling One important aspect in our work, is that we deal with attributed graphs. It means that graph components are labeled. A particularity is that we could have a vector of labels for each component, since we have no limitation when we compare two graphs (see 2.4).

- For a vertex, for instance, we can compute the following labels :
  - node coordinates,
  - size of the structured element at node,
  - color mean and variance of the region described by the structured element.
- For edges :
  - length,
  - orientation,
  - area defined by the intersection of all structured elements placed on the edge,
  - luminosity, colour mean, variance and texture characteristics (homogeneity, dissimilarity, contrast, entrophy, energy) of the region defining above.

As we can see, we could obtain various information concerning shape. Some features are able to describe the shape topology, like edge's length, orientation and area. We could complete them with information about the shape texture.

2.2 Histograms of Oriented Gradients

In the context of object recognition, the use of edge orientation histogram has gain popularity [18, 7]. However, the concept of dense and local histogram of oriented gradients (HOG) is a method introduced by Dalal et al.[7]. The aim of such method is to describe an image by a set of local histograms. These histograms count occurences of gradient orientation in a local part of the image. In this work, in order to obtain a complete descriptor of an image, we have computed such local histograms of gradient according to the following steps :

1. compute gradients of the image,
2. build histogram of orientation for each cell,
3. normalize histograms within each block of cells.

Gradient computation The gradient of an image has been simply obtained by filtering it with two one-dimensional filters :

– horizontal : $\begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$
– vertical : $\begin{pmatrix} -1 & 0 & 1 \end{pmatrix}^T$

Gradient could be signed or unsigned. This last case is justified by the fact that the direction of the contrast has no importance. In other words, we would have the same results with a white object placed on a black background, compared with a black object placed on a white background. In our case, we have considered an unsigned gradient which value goes from 0 to π.

The next step is orientation binning, that is to say to compute the histogram of orientation. One histogram is computed for each cell according to the number of bins.

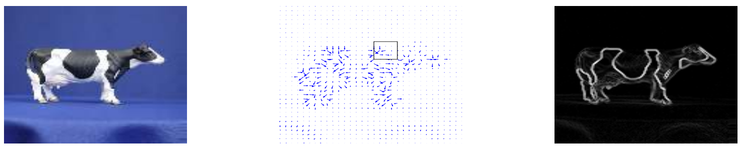


Fig. 2. This figure shows the gradient computation of an image. (left) is the original image, (middle) shows the direction of the gradient, (right) depicts the original image according to the gradient norm.

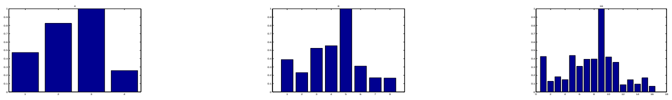


Fig. 3. This figure shows the histograms of gradient orientation for (left) 4 bins, (middle) 8 bins (right) 16 bins.

Cell and block descriptors The particularity of this method is to split the image into different cells. A cell can be defined as a spatial region like a square with a predefined size in pixels. For each cell, we then compute the histogram of gradient by accumulating votes into bins for each orientation. Votes can be weighted by the magnitude of a gradient, so that histogram takes into account the importance of gradient at a given point. This can be justified by the fact that a gradient orientation around an edge should be more significant than the one of a point in a nearly uniform region. Some examples of histogram obtained for the square region given in the middle image of figure 2 are shown in figure 3. As expected, the larger the number of bins is, the more detailed the histogram is.

Block Normalization When all histograms have been computed for each cell, we can build the descriptor vector of an image concatenating all histograms in a single vector. However, due to the illumination variations and other variability in the images, it is necessary to normalize cells histograms. Cells histograms are locally normalized, according to the values of the neighboured cells histograms. The normalization is done among a group of cells, which is called a block.

A normalization factor is then computed over the block and all histograms within this block are normalized according to this normalization factor. Once this normalization step has been performed, all the histograms can be concatenated in a single feature vector.

Different normalization schemes are possible for a vector V containing all histograms of a given block. The normalization factor nf could be obtained along these schemes :

– none : no normalization is applied on the cells, $nf = 1$.
– L1-norm : $nf = \frac{V}{\|V\|_1 + \varepsilon}$
– L2-norm : $nf = \frac{V}{\sqrt{\|V\|_2^2 + \varepsilon^2}}$

ε is a small regularization constant. It is needed as we sometime evaluate empty gradients. The value of ε has no influence on the results.

Note that according to how each block has been built, a histogram from a given cell can be involved in several block normalization. In this case, the final feature vector contains redundant information which has been normalized in a different way. This is especially the case if blocks of cells have overlapping.

2.3 SVM Classifier

Support Vector Machine The Support Vector Machines classifier is a binary classifier algorithm that looks for an optimal hyperplane as a decision function in a high-dimensional space [2, 20, 5]. Thus, consider one has a training data set $\{\mathbf{x}_k, y_k\} \in \mathcal{X} \times \{-1, 1\}$ where $\mathbf{x}_k$ are the training examples and y_k the class label. At first, the method consists in mapping $\mathbf{x}_k$ in a high dimensional space owing to a function Φ. Then, it looks for a decision function of the form : $f(\mathbf{x}) = \mathbf{w} \cdot \Phi(\mathbf{x}) + b$ and $f(\mathbf{x})$ is optimal in the sense that it maximizes the distance between the nearest point $\Phi(\mathbf{x}_i)$ and the hyperplane. The class label of $\mathbf{x}$ is then obtained by considering the sign of $f(\mathbf{x})$. This optimization problem can be turned, in the case of L_2 soft-margin SVM classifier (misclassified examples are quadratically penalized), in this following one :

$$\min_{\mathbf{w},\xi} \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{k=1}^{m} \xi_k \qquad (1)$$

under the constraint $\forall k, \quad y_k f(\mathbf{x}_k) \geq 1 - \xi_k$. The solution of this problem is obtained using the Lagrangian theory and it is possible to show that the vector **w** is of the form :

$$\mathbf{w} = \sum_{k=1}^{m} \alpha_k^* y_k \varPhi(\mathbf{x}_k) \tag{2}$$

where α_i^* is the solution of the following quadratic optimization problem :

$$\max_{\alpha} W(\alpha) = \sum_{k=1}^{m} \alpha_k - \frac{1}{2} \sum_{k,\ell}^{m} \alpha_k \alpha_\ell y_k y_\ell K(\mathbf{x}_k, \mathbf{x}_\ell) \tag{3}$$

subject to $\sum_{k=1}^{m} y_k \alpha_k = 0$ and $\forall k, 0 \leq \alpha_k \leq C$, where $K(\mathbf{x}_k, \mathbf{x}_\ell) = \langle \mathbf{x}_k, \mathbf{x}_\ell \rangle$. According to equation (2) and (3), the solution of the SVM problem depends only on the Gram matrix K. Hence, in our case, the classification with SVMs only needs a kernel, which is, in our case, a combined kernel.

Kernels combination In our method, we use a combination of kernels [5, 1]. A kernel can be defined as a combination of positive-definite kernels.

Let K_1 and K_2 be kernels over $\mathcal{X} \times \mathcal{X}$, $0 \leq \lambda \leq 1$, $a \geq 0$, the following functions are kernels :

- $K(x, y) = \lambda K_1(x, y) + (1 - \lambda) K_2(x, y)$
- $K(x, y) = a K_1(x, y)$
- $K(x, y) = K_1(x, y) \times K_2(x, y)$

2.4 Graph Kernel

We use the inner product between graphical representations based on Kashima et al. paper's [12]. The idea is to compare two label sequences generated by two synchronized random walks on the two graphs. The operation is repeated until there is convergence of the result.

Given :

1. h_i^1 and h_j^2 the nodes i and j of graphs G^1 and G^2,
2. $p(h_i | h_{i-1})$ the transition probability from node h_{i-1} to node h_i,
3. $p_q(h_\ell)$ the probability to stop at node ℓ,
4. $K_n(h_k^1, h_k^2)$ the inner product between two nodes of the graphs G^1 and G^2,
5. et $K_a(a^1_{h^1_{k-1} h^1_k}, a^2_{h^2_{k-1} h^2_k})$ the inner product between 2 arcs a^1 and a^2 of G^1 and G^2.

The comparison of all paths, of all lengths, from all nodes in the two graphs, weighted by the path probability leads to :

$$\begin{aligned} K(G^1, G^2) = \sum_{\ell=1}^{\infty} \sum_{h^1} \sum_{h^2} p(h_1^1) \prod_{i=2}^{\ell} p(h_i^1 | h_{i-1}^1) p_q(h_\ell^1) \times p(h_1^2) \\ \textstyle\prod_{j=2}^{\ell} p(h_j^2 | h_{j-1}^2) p_q(h_\ell^2) \times K_n(h_1^1, h_1^2) \prod_{k=2}^{\ell} K_a(a^1_{h^1_{k-1} h^1_k}, a^2_{h^2_{k-1} h^2_k}) K_n(h_k^1, h_k^2) \end{aligned} \tag{4}$$

Computation of this formula gives the comparison of each vertices values and edges values, which are encountered for each ways starting from each vertex of graphs. This formula shows that computation could be exhaustiv and test every possible path combination.

The detailed computation of $K(G^1, G^2)$ is in the paper of Kashima et al. [12] .

The complexity of this computation is $\mathcal{O}\left((|G^1||G^2|)^2\right)$, with $|G^i|$ the number of nodes in graph G^i. For this reason the number of vertices in each graph have to be as small as possible.

The graph kernel suggests that a kernel between vertices and a kernel between edges have to be defined. In our case, we have chosen to use a classical gaussian kernel since nodes and edges are labeled with vectorial values :

$$K_n(x,y) = K_a(x,y) = \exp\left(-\frac{||x-y||^2}{2\sigma^2}\right) \tag{5}$$

where σ is the bandwidth of the gaussian kernel.

Note that the graph kernel depends only on the probability transition between vertices and kernels between vertices and kernel between edges. This means that the label information of edges and vertices can be richer than it is at the present time. In fact, since we only need an inner product values, labels can be a non-vectorial data which admit a kernel.

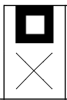
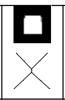
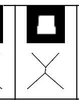
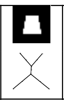
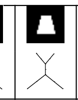
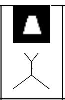
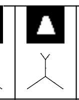
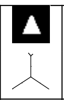
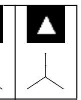
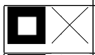
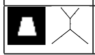
									
	1.000	0.954	0.946	0.932	0.911	0.892	0.856	0.866	0.943
	0.911	0.975	0.989	0.996	1.000	0.997	0.983	0.979	0.925

Fig. 4. Inner product between a square graph and a trapezoid graph coming from the transform of a square to a triangle.

In order to evaluate the pertinence of this method, we have computed values of the inner product between the graph of a square with graphs of objects coming from the progressive transformation of a square (■) to a triangle (△). Results are shown on figure 4. We conclude from that :

- when we compare the inner product $K(G(■), G(■))$ with $K(G(■), G(▲))$ and $K(G(■), G(⏢))$, we can see that the triangle graph seems to be closer of the square graph than the trapezoid graph, but the trapezoid graph seems to be closer of the square graph. This can be explained by the fact that the graphs of square and triangle are adjacent, compared with the graph of trapezoid. In fact, trapezoid graph has a central branch which is not present in the other graphs. So the triangle graph differs from square graph only by one missing branch.

- We also can notice a symmetry concerning the trapezoid result. In our example, the triangle and the square are equally placed from the central trapezoid.
- the result is symmetric : $K(G_1, G_2) = K(G_2, G_1)$.

Compared with graph matching algorithms, [3, 16, 9], the method proposed by Kashima has a great interest. It can be used by a SVM classifier, since some similarity measures have not the required properties, in particular symmetry of the measure. This method can also deal with complex structures as labels, when other methods only treat single numerical value.

3 Results

We now have to evaluate the efficiency of this method.

The test was accomplished on a complete database : ETH-80 (`http://www.mis.informatik.tu-darmstadt.de/Research/Projects/categorization/index_html`). This base contains 80 objects, dispatched over 8 classes : apple, pear, cow, dog, horse, cup, car, tomatoe. Each object has been captured from different points of view to produce 41 views for each object.

Our SVM is used for multi-class, with one-against-one method: we trained $(\frac{n(n-1)}{2})$ binary classifiers for n classes. The weight for missclassified points C (1) was established at 1000. For classification, vectors are tested in all models giving a vote of belonging to a class, finally, it will be labeled as the class that has more votes. The following results are given for a leave-one-object-out crossvalidation method. We remove 41 images corresponding to the same object at a time. The learning set is composed of all remaining objects, and we classify each view of the tested object. To evaluate each efficiency of the different methods, we first studied independantly the graph kernel and the HOG method.

3.1 Graph kernel

The aim of graph is mainly to discriminate object shape. To fill this condition, we chose to use the labels which are pertinent for a shape description. In other words, we conserve labels which give information about the object shape topology.

We retain the following parameters :

- for vertices : size of structured element, coordinates,
- for edges : orientation, length, strength, area,

Each characteric was normalized to have a mean equal to 0, and a standard deviation equal to 1.

Figure 5 shows our results. We obtained a recognition rate of 78%. This low rate can be explained by the fact that classes dog, horse and cow are strongly mixed.

3.2 HOG results

As we saw in section 2.2, the HOG descriptor actually involves many parameters. To tune these parameters correctly, we completed a test to evaluate optimal set of parameters, retrieve this following set :

- image size : 96 × 96 pixels,
- size of cell : 4 × 4 pixels,
- size of block : 2 × 2 cells,
- overlap of blocks : 1 cell,
- normalization factor for block : L2,
- number of bins for histogram : 4.

It should be noticed that the majority of these parameters have a small influence, like block overlap, size of block, number of bins for histogram, and results are less than 5% better compared with non-optimal set of parameters. On the contrary, the normalization factor and size of cell have more influence on the result which are up to 10% better.

The rate obtained with the HOG descriptor is up to 90%.

HOG								
Prediction / True								
	402	0	0	0	0	0	0	8
	0	409	0	0	1	0	0	0
	0	5	323	0	38	43	1	0
	0	0	0	410	0	0	0	0
	0	2	30	0	354	24	0	0
	0	1	59	0	24	326	0	0
	0	0	0	0	0	0	410	0
	5	0	0	0	0	0	0	405

Graph								
P / T								
	296	0	0	28	0	0	0	86
	0	386	12	1	3	1	7	0
	0	28	258	0	83	41	0	0
	13	6	1	379	0	0	1	10
	0	8	66	0	231	105	0	0
	0	6	34	0	90	280	0	0
	0	10	0	1	0	0	399	0
	100	0	0	25	0	0	1	284

Fig. 5. Left :Results obtained for the HOG Kernel, with leave-one-object-out crossvalidation test. Good recognition rate : 90,1%. Right :Results obtained for the Graph Kernel, with leave-one-object-out crossvalidation test. Good recognition rate : 78%

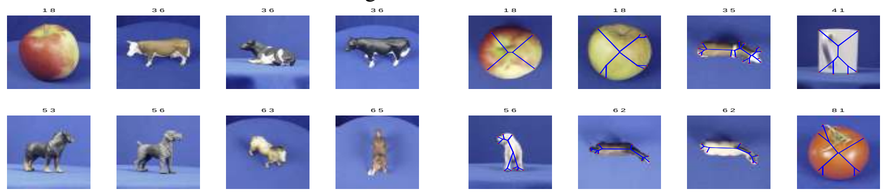


Fig. 6. Left : misclassified object with HOG method. Right : misclassified objects with graph method.

3.3 Kernel combination

We notice a certain complementarity between these methods, since some objects are badly discriminated by a method, but well recognized with the other. Figure 6 shows some examples of misclassified objects with the graph representation and with the HOG descriptor.

We now describe the final test realized over the complete database. We combine kernels with a product : $K(x, y) = K_{HOG}(x, y) \times K_{graph}(x, y)$.

Each kernel K_{HOG} and K_{graph} was normalized previously : $k_n(x, y) = \frac{k(x,y)}{\sqrt{k(x,x) \times (y,y)}}$

Figure 7 shows results for this test, which gives 94,1% of good recognition.

Prediction / True								
	403	0	0	0	0	0	0	7
	0	409	1	0	0	0	0	0
	0	1	345	0	37	27	0	0
	0	0	0	410	0	0	0	0
	0	1	31	0	353	25	0	0
	0	0	35	0	21	354	0	0
	0	0	0	0	0	0	410	0
	7	0	0	0	0	0	0	403

Fig. 7. Results obtained for the combined kernel, with leave-one-object-out crossvalidation test. Recognition rate : 94,1%.

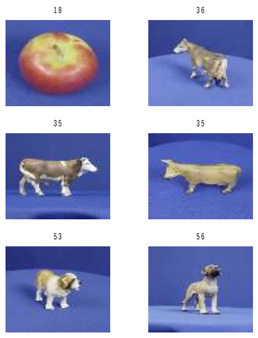


Fig. 8. Examples of miss-classified objects for complete test.

We can compare our results with other methods tested on the same database. In [14], Leibe and Schiele used 7 classifiers in an optimal-decision tree. They obtained 93% of good recognition rate for a leave-one-object-out crossvalidation method. Results are comparable, but their method depends on the classes used for the classification, and may not be as efficient with additional classes. In our case, we could add other categories, without redefining completely our classification method.

In [21], results are given for a leave-one-image-out crossvalidation method for only 32 objects (4 from each class). This query method gave a recognition rate of 95%. In our case, we tested our method over the complete database. Moreover, we remove completely an object, no image of the tested object was present in the learning set.

4 Conclusion

This paper presents a method for object categorization. The aim is to depict images thanks to labeled graphs and histograms of oriented gradients.

The first representation is a labeled graph, which enables us to describe the global shape of an object. The second representation is based on histograms of oriented gradients, which brings more information concerning the appearance of the object. A graph kernel is obtained by random walk on graphs, and we combine this kernel with a linear kernel obtained from HOG descriptors. We combined these kernels to use them with the SVM classifier.

The advantage of this method is to combine two kinds of representations to categorize an object. Using a classifier like SVM is well adapted to this combination which clearly improves recognition performance, compared when only one representation is used.

A complete test of this method on the ETH-80 database has proved that this approach is very promising, with 94% of good recognition rate for a leave-one-object-out crossvalidation test. This result proved that our method is efficient compared with existing methods [14, 21]. Our results could also be improved by combining additional object representations.

We have also some perspectives to improve this method. First, we would like to integrate histograms into the graph, and define a kernel for the vertices which deals with these histogramms. Another point resides in using multiple kernels. This point could help us to improve classification results.

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