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Objective Functions and Models of Corporate Optimization

Martin Shubik

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1. Economic Theory and the Goals of the Firm

1.1. Introduction

The development of micro-economic theory has been entering a stage similar to that experienced by twentieth century physics in its break away from the simple comfortable closed systems of its precursor. Homo Oeconomicus operating as an entrepreneur or consumer in smoothly adjusting flexible markets still presents us with a logically and aesthetically satisfying body of theory. However unfortunately there are many facts and facets of an economy which need to be explained, but which cannot be explained using this model. The behavior of individual economic units which still remains not sufficiently explained is of considerable importance to society in general and to the firms in particular. There is a need for an adequate economic theory to serve as a basis for analysis, prediction and as an aid in policy evaluation.

In recent years, linked with the growth of operations research and management science there has been an increasing self-consciousness on the part of many corporations which has lead to a proliferation of investigations and self-analyses concerning goals. The development of certain parts of mathematical economics and industrial engineering have had the paradoxical effect of giving operational meaning to very complex aspects of individual maximization and at the same time illustrating in application, the limited use of models used in explaining behavior in many important areas of decision-making in the firm. The observation by Freud that only small decisions are made "rationally"

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appears to hold true in all adaptive organisms, be they individuals or economic organizations.

The problems of formulation of objectives have given rise to new terminology. For example, the phrase "objective function" has made its way into the literature of economics, operations research and management science by the route of linear programming. In a linear program the objective function is the expression to be maximized or minimized. The typical programming problem is one in which the prices of the final products are given, the capacity limitations on production are specified, technology is assumed to be known and the goal is to maximize a revenue or equivalently, minimize a cost subject to constraints.

Unfortunately in problems involving more than a few aspects of the running of a complex organization such as the modern firm, it becomes most difficult to describe the objective function. As the complexity of the organization grows the so-called objective function becomes more and more subjective and less quantitative as multiple goals, social and political goals and uncertainty and ill-perception of the environment are taken into account.

Does the firm maximize? What does the firm maximize? Should the overall policy of the firm be described as maximization, optimization or something else? The first two questions are possibly of some interest to philosophers, however the third is of direct interest to economists and others involved in shaping policy and predicting behavior of an economy. Preliminary to comment upon the third question, some of the models used in economic theory to describe the behavior of the firm are examined.

1.2. Intent, Behavior and Structure

Throughout this paper an attempt will be made to maintain three distinctions which are clear in the short run but become more difficult to differentiate in the long run. They are the distinctions among intent of the decision-maker, behavior and structure (or environment in which the individual acts). The manifestation of the individual's intent is expressed in his behavior within his environment. We observe his behavior and we observe the environment and from these observations we may be in a position to make statements such as "the individual behaves as though his intent were to maximize monetary profits." The intent per se is not observed. Furthermore many different intents may give rise to the same behavior within certain structures as is amply illustrated in the literature of attempts to control oligopolistic competition.

Another way of phrasing the above distinction is that intent is the operator upon structure which produces behavior.

In the long run, in human affairs, the dichotomy between behavior and environment becomes less distinct than in the short run. If enough individuals do not like the "rules of the game" they may be changed. However as a first approximation, the distinction appears to be worthwhile.

1.3. Pure Competition

The simplest model of the firm is that which is referred to in economic theory as pure competition. The firm is considered to sell a single product to the market. The price for the product is known and fixed "by the play of the market forces" and is not influenced by the size of output, advertising or other acts of the firm. The aim of the firm is assumed to be the maximization

of its short term net revenue. In order to do this it selects a maximal rate of production. The policy can be expressed mathematically as follows: Let p be the price of the product in the market. Let $C(q)$ be the total cost of producing q units, and let $R(q)$ be the net revenue obtained from producing and selling q units.

$$\underset{q}{\text{Maximize}} \quad R(q) = pq - C(q)$$

The q beneath the word maximize indicates that the strategic variable in control of the firm is production.

At best this model can be used to teach us something about the effect on price and output of changes in such parameters as the cost of inputs. It may be used to investigate problems of production, taxation and so forth. It serves as a precursor for the more elegant and complex techniques of programming under conditions of complete information.

It may be argued that this model is adequate to describe the behavior of the small individually owned enterprise, with low capitalization, no (or few and unimportant) market imperfections, no influence in the market and no problems caused by lack of perception, incomplete information or uncertainty.

If the structure of the economy had all of the properties noted above, then indeed the theory of behavior would have much to commend it as it requires almost no consideration of the intent of the entrepreneurs. Provided that we make the weak assumption that the individual prefers greater profits to lesser profits, ceteris paribus, then all additional assumptions concerning intent

(such as the entrepreneurs wish to do each other economic harm or that they are the best of friends and wish to display this in their behavior) are not germane to the prediction of economic behavior. The structure is such that individuals are so weakly interlinked strategically that the interconnectness between any two is below the level of perception. Regardless of the intents of the individuals, the "invisible hand" of the market structure guides their behavior. Unfortunately there are few if any parts of an economy which have enough of the features to make the model of pure competition more than of limited use.

1.4. Monopoly

If a firm is in a monopolistic position its actions can influence the demand for its product (or products). For simplicity we consider only one product. Again economic theory considers that the firm seeks to maximize (short term) net revenue $\frac{1}{}$. Several complications such as pricing, production, advertising and product variation, may all be considered as strategic variables under the control of the firm and relevant to its maximizing process.

The mathematical expression for the behavior of the firm shows a distinct similarity to that of the firm in pure competition. Let $s = (s_1, s_2, s_3, \dots, s_n)$ stand for the strategic variables under the control of the firm, where s_1 is the production rate (the same as q in 1.3 above), s_2 is the advertising budget, s_3 is the price, and so on. The demand for the final product may be a function of all these variables except the production rate and can be written as $D(s_2, s_3, \dots, s_n)$. The cost of operating the firm now includes not

only the costs of producing, but also advertising, product research and so forth. This can be expressed as $C(s_1, s_2, s_4, \dots, s_n)$. The goal or objective of the firm is to:

$$\text{Maximize}_s R(s) = s_3 D(s_2, s_3, \dots, s_n) - C(s_1, s_2, s_4, \dots, s_n).$$

The revenue function is different from that under pure competition, and it is this difference which reflects the difference in the market structure. However the "operator" on this function, or the intent of the individual is the same. The operator is maximization and, as before, this leads to an unambiguous behavior when applied to the market structure. Both the theories of pure competition and monopoly require that we know no more about the intents of the individuals than they have a desire to maximize. Both the pure competitor and the monopolist act in isolation in which the only forces present are their own and the "impersonal market." In the first instance the individual is weak, in the second stronger. In both cases the behavior is that to be expected of a "rational maximizing utilitarian man" acting under conditions of complete information.

1.5. Competition among the Few: Oligopoly

Even in the simplest models involving few players, each with a considerable influence in the market, but none with control, intent and behavior become hard to define and describe. Naively we might suggest that the firm still wishes to maximize its short term net revenue, but this simple intent is no longer sufficient by itself to be used to predict behavior in the market.

In order to predict the behavior of the firm we must simultaneously consider the intents of all competitors. It is possible to define the short term revenue function for a single firm in such a market, but it is a function whose value depends upon the strategic variables of the firm and its competitors. Any statement concerning maximization of revenue by a firm must contain some statement concerning the expected actions of the competitors. Thus instead of the straightforward maximization policies described in 1.3 and 1.4., economists have discussed joint maximization ^{2/}; "live-and-let-live" policies; quasi-maximization and various policies based upon conjectures as to the behavior of all other competitors.

In a market with few competitors, no matter how simple an economic model is constructed, the problems of defining goals and describing behavior become far more difficult and are far different from the simple maximization policies noted in 1.3 and 1.4. Many models have been suggested but not sufficient empirical work has been done to test the relevance of the models. A favorite model has been that of the market with a non-cooperative equilibrium ^{3/}, this, in one form or another is at the basis of the models of Cournot ^{4/}, Stackelberg ^{5/}, Chamberlin ^{6/}, and many others. Some of the market behavior patterns described allow for entry of new firms into competition and the departure of old firms ^{7/}; however, costs of entry and difficulties of exit are not usually dealt with explicitly (although recently the work of Joe Bain has gone far to rectify this omission ^{8/}). Two examples of intent and their resultant behaviors are given below.

Let $s_i = (s_{i1}, s_{i2}, s_{i3}, \dots, s_{in})$ stand for the set of strategic variables under the control of the i^{th} firm. $s = (s_1, s_2, s_3, \dots, s_n)$ stands for the set of all strategic variables under the control of all m competitors. If each firm attempts to maximize its short term net revenue under the assumption that its competitors' actions will remain fixed, then if $R_i(s)$ represents the net revenue function for the i^{th} firm it acts to

$$\begin{aligned} \text{Maximize}_{s_i} R_i(s) = & s_i D(s_{11}, s_{12}, \dots, s_{1n}; \dots; s_{i2}, s_{i3}, \dots, s_{in}; \dots; s_{m1}, \dots, s_{mn}) \\ & - C_i(s_{i1}, s_{i2}, s_{i4}, \dots, s_{in}), \end{aligned}$$

and similarly for the other firms. In general, as the products being sold may be different, the demand function D should be replaced by a D_i which is different for each player. Of course, empirically it would be fantastically difficult to deal with demand functions as complicated as these other than by very crude approximation.

In order to obtain a prediction of behavior in this model we have had to consider the simultaneous effect of m different intents interacting upon m revenue conditions or "payoff functions." Even in order to carry out purely theoretical calculations it is necessary to specify not only the individual's intentions concerning the impersonal environment, but precisely how he intends to utilize his information concerning the behavior of other individuals. Thus a full specification of the intent which leads to the equations above is: "I intend to maximize my short term net revenue, as

defined by fixing the actions of all my competitors at their previous values. I will ignore all information which appears to run contrary to my hypothesis concerning their behavior." If each entrepreneur followed this intent then under most circumstances behavior leading to a non-cooperative equilibrium would be observed.

Without specifying assumptions concerning learning (i.e., the effect of information upon the actor) and implicitly or explicitly the socio-economic aspects of group interaction, it is logically not possible to completely describe a procedure to provide a prediction of behavior in a market with few competitors. The crux of any theory of oligopoly lies in these assumptions. The difficulties encountered and the inadequacy of theory to date has had two main sources. The first has been the logical inconsistency or incompleteness of the models of man in economic competition. This difficulty is being removed by improvements in methodology. The second is caused by lack of knowledge. Although it is easy to supply the necessary assumptions to produce logically consistent and complete models, it is extremely difficult to validate them. A certain amount of empirical work exists in the form of observations from institutional studies and recently experimental work has commenced.

A second example of intent and the resultant behavior is given if we assume that each firm wishes to jointly maximize the net aggregate receipts of all. This is equivalent to:

$$\text{Max.}_{s_1}, \text{Max.}_{s_2}, \dots, \text{Max.}_{s_m} (R_1 + R_2 + R_3 + \dots + R_m) .$$

Here there are m operators, operating simultaneously on one revenue condition. Although the intents in both instances illustrated here are quite different, as can be seen by the two expressions given above; under certain circumstances the observer will not be able to differentiate between them. A simple analogy serves to illustrate this. If two individuals are trapped in a cellar with water rising and a trap door providing the exit which requires two men to lift it, whether they are individual operators or joint maximizers the observed behavior will appear the same. The same "implicit collusion" can be the resultant behavior from many different intentions.

1.6. Long Run Considerations

The examples above all dealt with short term or instantaneous problems of maximization. Attention has been paid to long term maximization problems of the firm. The very simplest assumption is that the firm attempts to maximize the discounted value of its yearly income flow. In other words, the discounted value of the continuous stream of short term net revenues. If R_t is the short term net revenue obtained in period t , ρ is the discount rate, which is equivalent to $1/(1+r)$ where r is the rate of interest and R is the discounted value of the yearly income flow, we can write:

$$R = \sum_{t=0}^{\infty} \rho^t R_t .$$

An immediate problem in time horizons is brought out by this formulation. As written above the income stream is summed (with the discount factor) from the present to infinity. Is the infinity merely a mathematical trick, or does it reflect the theoretically infinite life of a corporation? If the policy is for a firm owned by an individual with no heirs would this be an adequate goal for him to pursue? There is no indication in this formulation of the possible value of terminating operations, selling out or liquidating. Admittedly, if all markets for assets were perfectly competitive there would be no need for this and the discounted income stream would reflect exactly the worth of the firm. However, few markets are perfect and in general this model is too simple to be of more use than as a teaching device.

1.7. Short Term Uncertain Demand

The short term models in 1.3 and 1.4 were based upon certain information concerning market conditions. The models of 1.5 recognized the problem of interdependence, but did not treat it by assuming a probability distribution for the behaviors of competitors (this can be done, however the reasons for doing so do not appear to be too convincing^{9/}). Fluctuations in demand, especially when there are many customers, can be treated by probability considerations. In recent years consideration of demand fluctuations has led to a considerable literature on inventory theory^{10/}. The simplest assumption is that when faced with uncertainty in the market the firm acts in a manner which maximizes its short run

expected revenue. We assume that p is the market price for the product (the same market form as in 1.3). The value of a unit of inventory is k .

$\phi(t)$ is the probability density that t units will be demanded.

$\Phi(q) = \int_{t=0}^q \phi(t)dt$ is the cumulative probability that q units or fewer

will be demanded. $E[R(q)]$ is the expected revenue. We can express the policy of maximization of expected revenue as:

$$\text{Max}_q E\{R(q)\} = p \left\{ \int_{t=0}^q t d\phi(t) + q[1 - \Phi(q)] \right\} - C(q) + k \int_{t=0}^q (q - t) d\phi(t)$$

This consists of three parts. The first, which has two terms, is the expected revenue obtained from the market if the market demand is less than q (the production rate) and the expected revenue obtained if the demand is q or more. The second is the cost of production of q units and the third is the expected value of the inventory.

Should a businessman maximize his expected revenue? There have been many discussions as to whether or not expected income should be maximized as opposed to other quantities which take into account the variance and other risk considerations^{11/}. The problem amounts to what we are trying to portray? If the firm can be ruined by one bad season this model of maximization of expected income is probably quite inadequate. An explicit consideration of the ruin conditions will be necessary. The model above is certainly a useful first approximation which enables us to carry out studies of inventory policies and goals which were previously beyond analysis.

It enables us to answer problems of the type: "If a businessman wishes to maximize his expected revenue, what should he do?" A normative interpretation may be given to this objective function. We may state that individual owners, partnerships or corporation should maximize their expected short term net revenue when faced with uncertainty of this sort. The third position which can be adopted in interpreting the objective function is that, in fact, businessmen of some class or category do act as though they were trying to maximize this objective function. It appears to be unlikely that this is the case, although with the appropriate specifications testable hypotheses can be formulated.

1.8. Interdependence, Uncertainty, Insensitivity and Misperception

"The firm should attempt to maximize its present value. The firm should attempt to maximize its present value. The firm should attempt to maximize its present value." The Bellman in *The Hunting of the Snark* remarked "What I have said three times is true". This exhortation, however admirable and true, unfortunately provides limited aid either as a predictor to the policy maker or as a maxim for the entrepreneur.

The firm does not consist of homo oeconomicus in vitro, but in general consists of many men embedded in a market in a society. As such, an adequate theory of the behavior of the firm must at least implicitly reflect considerations of (1) the goals of the individual and the individual as a decision-maker; (2) the firm as an organization and economic unit, and (3) the interactions of firms in the socio-economic and politico-economic

environment. This calls for a reconsideration of the properties of a model of economic man and the selection of the relevant variables and parameters in describing his environment.

Economic man is a creature who must act under severe limitations of incomplete information. Even if complete or perfect information were available to him, by choice he must select only a sample of it. His perceptions are poor and his channel capacity for processing information is low. His major asset is time. Total knowledge, even if available must be abstracted and aggregated owing to costs and limitations of processing. The executive who tells the researcher to boil down his report (and thereby distort valuable information according to the researcher) is not always wrong. Economic man, of necessity operates under conditions of low information. Leaving out the problems of interaction with others (which brings in consideration of organization and society) the properties of the individual as an adaptive organism with limited perceptions, limitations on data processing ability and goals dependent upon his learning appear to be of importance to the development of economic theory. The model of "satisficing man" of Herbert Simon^{12/} is a construct which incorporates these features. The new work in problem solving methods, heuristic programming and artificial intelligence^{13/} also involve these features.

For many years students of institutions have noted the role of the firm as a bureaucracy. The distinctions among manager, administrator, entrepreneur or leader are not always clear. Yet from the viewpoint of

socio-economic policy which or what combinations of these roles are required by the heads of large enterprises may have an important economic effect. A broad economic theory of the firm must at least be cognizant of internal organization and the possibility that for many economic questions it cannot be assumed away as a minor imperfection. The works of Argyris^{14/} , Barnard^{15/} , Selznick^{16/} , Simon and March ^{17/} and others have been addressed to the broad problems of organization of the firm. Marschak and Radner, following more closely the developments of mathematical economics and statistics have developed a theory of "teams" which deals with problems of imperfect communication between individuals whose goals are identical, i.e., individuals in the same organization with a perfect harmony of interests^{18/} . In a later Discussion Paper I will present a model of an organization in which the individuals do not have identical goals and in which a basic strategic feature of the operation of the organization is the control of individuals at all levels over the processing, aggregating and disaggregating of information.

Having opened Pandora's box, let us quickly close it, consider that we are in a position to talk about the firm as a unit and consider the problems of the firm in a market. In my estimation, possibly the most important variable left out of economic analysis has been information and its cost. Once even rudimentary considerations are given to it, the wealth of institutional forms found in special markets, distribution systems and communication systems begin to conform to an overall structure of an economic theory rather than remaining as a source for strange examples in a marketing

textbook. As will be discussed further in Section 4 there are good reasons to suspect that parts of economic theories of competition can be made to appear more plausible and can be at least partially validated experimentally if they are interpreted as actions under conditions of very low information.

The firm as a unit acts under a severe handicap of ignorance. Very often it does not know its own costs. It may easily make the assumption that its costs are linear up to a point because it is most likely cheaper to operate under that hypothesis than to pay the large sum required to find out otherwise. It has a birth and very often has a death. These are not continuous processes and hence cost of entry and of exit may be very important. The latter cost has not been given sufficient attention in economics. In the theory of war it has, thus Caesar destroyed the bridges when he crossed the Rubicon. An enemy when cornered may have no other choice but to fight. A dying firm with high investment in specialized equipment may be in the same position.

A long list of variables applying to different trades and markets is not hard to compile. Financial structure is usually important, distribution channels, corporate form and so forth all may play a vital role. These, however, are all factors which are needed to describe what a firm can do, not what it wishes to do, or what it does. Nevertheless structure may cause many intents to lead to the same behavior. As such it is an important part of economics to be able to observe the details of a market and to interpret them in terms of general properties of an organism such as viability, perception, flexibility, vulnerability and other terms used to describe the suitability

of any organism or animal for surviving and possibly flourishing in its environment.

2. The Corporate Economy

2.1 Economic Theory and Society

Economic theory in general and Twentieth Century British-U.S. economic theory in particular have tended to bypass the sociological and political aspects of the economy which play a considerable role in determining the goals, nature and behavior of the firm and consumer demand. The micro-economic theory of price has probably suffered the most from this neglect.

By means of great over-simplifications an elegant theory of pure competition has been constructed which serves the purpose of demonstrating many important propositions concerning the allocation of resources and the determination of prices under special conditions. The models at the basis of this theory fit at most very few parts of any economy that exists today. Oligopoly theory leaves even more to be desired.

Even the most casual observations of the economy indicate that the goals, structure and behavior of the small individually own unincorporated firm are sufficiently different from those of the large corporation that a theory which purports to be able to portray or predict behavior and price formation must explicitly take into account some of the differentiating institutional features. When firms employ tens or hundreds of thousands of individuals and have many thousands of stockholders the socio-political aspects of their existence can no longer be implicitly assumed away in an economic study.

In order to develop a theory of the firm in a modern economy, many conditions and roles in relation with the economic aspects of the firm must be specified. The aims and the effect of stockholders, managers and other employees may be predominant in determining the behavior of the present day firm. Financial structure, technological conditions and special institutional limitations largely influence market forms and hence the price structure. Recently there has been a recognition that the empirical content and the number of factors to be taken into account in the construction of a theory of oligopoly must rise.

The classification and differentiation of market forms is beginning to make its way into economic theory as can be seen by the works of Fellner^{19/}, Machlup^{20/}, Bain^{21/}, and others^{22/}. Work such as that of Berle and Means^{23/} and recently Eells^{24/} are of aid in filling the gap in the theory of the firm due to a lack of analysis on the goals and organization of the firm.

2.2. The Firm in the United States

As is well known, the corporate form is the most important type of business organization in the United States. The smaller corporations often are family owned affairs with a close connection between ownership and management. Almost all of the large corporations are run by managers for thousands of stockholders and employ many thousands of workers. The corporate form dominates manufacturing; partnerships are still to be found mainly in the professions and the individually owned unincorporated enterprise is predominant among shopkeepers and farmers.

In 1955 there were 4,180,000 businesses in the United States. Their form of business organization was as follows: 25/

Corporations	560,000
Partnerships	}
Other Unincorporated	
Enterprises	
	3,620,000

If numbers are taken as the criterion, the small individual businesses still dominate the scene, when other criteria such as sales, assets or employees are examined this is not so. Thus, in 1948 less than 1% of the firms classified as industrial enterprises controlled two thirds of all the assets of firms classified as industrial enterprises 26/.

The relative importance of the large corporation to the economy can be seen from the compilation on the 500 largest manufacturing corporations in the United States, whose reports are available. In 1956 the Gross National Product was \$415 billion. During this year the following figures helped to characterize the 500 firms: 27/

Sales	\$174,300,000,000
Assets	139,000,000,000
Net Profits	11,500,000,000
Stockholders	11,100,000
Employees	8,800,000

With commitments of human and financial resources of such a size the large corporation becomes a social as well as an economic entity. Neither the economists studying the theory of the firm nor the managers running a large

corporation can afford to ignore the multi-dimensional aspects of the corporation's goals and actions.

2.3. Statements of Corporate Goals

A study of the statements of corporate aims by twenty-five corporations, seventeen of which are among the five hundred largest manufacturing firms, reveals the blend of economic, political, social and ethical considerations which at least indicates the self-images of the corporations. The sources and detail of the material vary considerably as is indicated below. Titles or captions of the statements of objectives are:

"Our Aim; Basic Objectives; Objectives; Creed;
The Policy; Company Objectives; Company Policy;
Principles; Our Credo; The Company's Philosophy;
General Corporate Policies; Philosophy; Statement
of Policy; and General Objectives of Management."

The sources include statements from chairmen, both in the financial press and elsewhere; statements from management guides and manuals; approved releases by the board of executives; notes by company presidents; and statements from company reports.

The contents have been classified under the headings given below. These are accompanied by the numbers indicating frequency of reference in the 25 statements.

Personnel	21
Duties and Responsibility to Society in General	19
The Consumer	19
Stockholders	16
Profit	13
Quality of Product	11
Technological Progress	9
Supplier Relations	9
Corporate Growth	8
Managerial Efficiency	7
Duties to Government	4
Distributor Relations	4
Prestige	2
Religion as an explicit Guide in Business	1

There are considerable difficulties involved in selecting categories to describe the statements of corporate objectives. The terms used here and the frequency figures are merely indicative of the scope of the corporate statements. Some examples serve to indicate the basis of selection.

"Finally, we shall strive to serve our shareowners, employees, and the customers justly and honestly ... in keeping with our American Tradition of democratic economic freedom."

"We recognize that industry justifies its existence to the degree that it is of service to the community."

"Our fourth responsibility is to the communities in which we live. We must be a good citizen - support good works and charity, and bear our fair share of taxes. We must maintain in good order the property we are privileged to use. We must participate in promotion of civic improvement, health, education and good government, and acquaint the community with our activities."

The three quotations above are classified under Duties and Responsibility to Society in General.

"Therefore, our over-all objective is to conduct our business so that all groups in the community will regard us as the best company in our industry and want us to prosper and grow."

This statement is classified both under Corporate Growth and Prestige.

Throughout all the statements the words: "fair share, fair return, fair prices, equitable wages, fair and equal treatment and proper return to investment" appear with great frequency. Fair and proper may have meanings in law and ethics but unless specified in a welfare measure they do not have meaning in economic theory. The problems of a large corporation involve considerations both of economics and the social environment. In order to be able to assign operational economic meaning to corporate utterances, words such as "fair" must be interpreted in terms of the environment.

3. Problems in the Specification of Goals

3.1. The Purposes of a Statement of Policy

The intermixture of social man and economic man in a large enterprise can scarcely be unscrambled. Bearing this in mind we must examine some of the motivations for the statements of company policy before examining the statements. As public figures, presidents and other members of the upper hierarchy of corporations are often called upon to make public utterances. They talk to many social organizations and are called upon to make statements concerning their attitude towards labor, inflation, competition and so forth. They must prepare press releases and cooperate with the press or risk the consequences of failing to do so. Company policy must be explained to financial writers, to stockholders and creditors. Furthermore, a general policy must be mapped out for long range operational purposes and more detailed policies must be described for the more specific problems of delineating the areas of action for departments or other sub-groupings of a major corporation.

As is the case with an army the statement of objectives may serve both as a morale builder and a general "heuristic" or overall rule for all members of the firm or corporation. There must be some sense of identification on the part of the members with the whole. Dewing has noted "that the association of human beings, in order to achieve a purpose is the fundamental and teleological basis for the coming into being and continuing existence of a corporation"^{28/}.

How specific or how general must the statement of purpose be? Selznick notes that:

"to make a profit" is widely accepted as a statement of business purpose, but this is too general to permit responsible decision-making. Here again the more marginal the business, that is, the greater its reliance upon quick returns, easy liquidation, and highly flexible tactics, the less need there is for an institutionally responsible and more specific formulation of purpose. Indeed, the very generality of the purpose is congenial to the opportunism of these groups. But when institutional continuity and identity are at stake, a definition of mission is required that will take account of the organization's distinctive character, including present and prospective capabilities, as well as the requirements of playing a desired role in a particular industrial or commercial context²⁹.

In light of this we can expect a hierarchy of statements in the specification of goals. For example consider a statement of goals by a large diversified manufacturing organization. (1) Any organization will usually have survival as a goal. (2) Any manufacturing organization may have as its intent the designing, manufacturing and selling of products at a profit. If, in a general sense, the concept of an "ethical organization" is introduced then (3) Any ethical economic organization will consider as part of its intent, the maintenance of its reputation in the minds of employees, customers, investors, stockholders, vendors and the public at large. (4) The goals of a large diversified manufacturing organization, if it is to have a raison d'être must include the intent to contribute economic and social values best generated by this form of organization.

Some of the larger U. S. corporations have tended to assume the properties of institutions over and above their properties as economic organizations. It

has been suggested that: "once an organization becomes a 'going' concern with many forces working to keep it alive, the people who run it can readily escape the task of defining its purposes"^{30/}. This conforms with the results of a study indicating that especially for large companies goals tend to reflect the company characteristics rather than the image of the management^{31/}.

In cultures other than that of the United States, the institutional aspects of industry may be even more dominant over the simpler economic purpose of the economic organization than it is here. Abegglen, in writing about the Japanese Factory states:

"In a purely economic definition of employment terms, where the financial success of the factory is the overriding goal of management's policy, the national well-being and workers' welfare would be secondary considerations in much policy formation. Underlying the specific points made in support of the Japanese policy, and quite aside from the validity of these points, is the tacit recognition by management that the relationship between the company and the worker is not simply a function of the economic convenience of the two parties. The worker, whether laborer or manager, may not at his convenience leave the company for another position. He is bound, despite potential economic advantage, to remain in the company's employ. The company, for its part, must not dismiss the worker to serve its own financial ends. Loyalty to the group and an interchange of responsibilities -- a system of share obligation -- take the place of the economic basis of employment of worker by the firm."^{32/}

Given that economic organizations are complex and have the many facets we have discussed, what should the economist do about them? Should he base his predictions or recommendations upon a relatively austere economic model with traditional economic variables, should he modify the model or leave the field to the sociologist and anthropologist? In some instances the selection of the

location for a monumental new plant or laboratory may have more to do with the mausoleum building propensities of a childless vice-president about to retire, than with the economics of transportation. In other instances actions may best be explained by looking at the firm as a club dedicated primarily to providing a satisfactory existence for its executives.

The actions of the firm have been of traditional interest to economists, and as an economist it is natural to suggest that the field should not be relinquished in toto. However there is little doubt that economic model-building must take into account the many instances where other societal variables dominate the economic effects.

The fourteen categories noted in 2.3. include many vague and ill-defined classifications which serve as a basis to set the general degree of cooperation internal to the organization, descriptive of the institution and the general milieu within which it operates. Within them however, heavily disguised by slogans and catch words such as "trade ethics", the "American way of enterprise", "fair-dealing" and so forth, probably lie a few parameters which characterize the inherent degree of cooperation or limitations on the weapons of economic conflict which call for the American, or Japanese, or German large industrialist to conform to somewhat different rules.

There are great difficulties attendant upon attempting to formalize and obtain these measures, however the task does not seem to be impossible. This writer has attempted to devise a criterion for the degree of stability or cooperation within a market, although even from a theoretical standpoint

it is not altogether satisfactory^{33/}. There are markets and customs which should give some insight into the formalization of parameters for cooperation which have received little if any treatment by economic theory. They are phenomena such as tipping, customs of bargaining and specialized features of trading be they in used-car wholesaling, commodity markets, money-lending in under-developed areas or credit-rating in over-developed areas which reflect the relationship of economic man to the market and his society.

There are good reasons to suspect that in almost all oligopolistic markets, any reasonable economic theory will give rise to the possibility of multiple equilibria^{34/}. In a specific instance involving only two or three individuals in a relatively institution-free situation, psychological variables may be successfully introduced to remove the indeterminacy; however, in large markets shared primarily by large institutions a socio-economic measure of degree of cooperation is needed. The size and make up of the "police-force" will tend to vary with the degree of communality or cooperation perceived of by the individuals.

3.2. Problems of Measurement and Aggregation

Returning to narrower methodological problems, the economist faced with reading and deciphering a general statement of corporate goals; or even advising a corporation in light of them; or helping to write them; may find it useful to apply five criteria to the examination of a statement of goals. They are: (1) Logical consistency, (2) Completeness, (3) Generality, (4) Measurability, and (5) Aggregation.

Statements which call for the simultaneous maximization of two or more objectives which are functionally interrelated are usually logically inconsistent. Thus it may be impossible to maximize profits together with obtaining the largest share of the market. Often, however, these two goals may be fulfilled in concert, in which case the directive to maximize market share may be more specific for members of an organization to follow than the directive to maximize profits. An old favorite among statements of simultaneous optimization is the intent "to obtain the greatest output at least cost". This is nonsense inasmuch as it can be given no operational meaning. It is meaningful to ask for the greatest output for a fixed cost; or the least cost for a fixed output; but the greatest output at least cost is operationally undefined.

Corporate goals which call for: "fair returns to stockholders and employees, technological superiority over all competitors and a high rate of return to sales", are incomplete and general to the point of vagueness for the purposes of the economist. They must be regarded as general directives which serve to delegate responsibility. The individuals who have to act in accordance with the statements of general intent have the additional burden of interpreting these statements. Thus from the viewpoint of operations they are incomplete. Additional restrictions must be read in before action can be taken. For example, a general statement calling for "ethical conduct consistent with the competitive practises of U. S. enterprise" is sufficiently undefined that there are many paths to the criminal courts to be found by an individual in an oligopolistic market trying to give it operational meaning.

Completeness, generality and aggregation are somewhat different aspects of the hierarchy of goals within an adaptive organism such as a corporation. Armies and corporations tend to approach the problems of specification in the same way. There is an overall statement of "mission" or corporate goals; then there may be more specialized statements of sub-missions or directives for special segments of the corporation, and so on down the chain to individual job descriptions.

The question of completeness is best illustrated in terms of a computer program example. The program or the statement of policy is complete if for every conceivable circumstance it will provide a course of action. In other words, in some sufficient sense it will instruct the machine how to handle any situation. A directive may be very general and yet complete if it contains a blanket statement concerning any situation which falls into a residual category such as "when in doubt as to what to do, ask your manager". The goals in this instance will provide a rule for behavior under all circumstances if they are consistent and if the president or chief officer to whom all unresolved problems will come has a method for dealing with them that is implicitly covered in the statement of the goals.

The test for completeness is fundamental to all models in the behavioral sciences. Many of the difficulties and much of the confusion in writings on the theory of oligopoly have been caused by inconsistent or incomplete models. In other words by models which purport to cover a class of events, but which either produce logically inconsistent outcomes for some of them, or which are unable to provide a specification of the outcome for certain events.

Generality and aggregation are different aspects of the same phenomenon. A corporation, its goals and its environment are all multivariate systems. Statements concerning goals are, of necessity, general or aggregative statements concerning multivariate systems. It has been fashionable to talk about "the aggregation problem" in certain branches of economics. There is no "the aggregation problem". It is the general problem of model building and abstraction which underlies all scientific endeavor.

There is a possible distinction between a general statement and an aggregative statement. The latter usually refers to a simplification of a complex, but nevertheless known or completely knowable situation. The former may include instructions as to how to handle the unknown; it has learning and adaptation built into it. The vague and general statements of companies contain elements of aggregation to avoid a detailed specification of known objectives, but much of the generality is an implicit recognition that they are fundamentally searching organisms who neither fully know their environment nor totally know what they are after.

3.3. Some Techniques for the Economic Formulation of Goals

In this section a few of the formal problems involved in an attempt to specify goals are sketched. These are primarily aimed at the "second level" of organization, in other words at the managers, engineers or economists faced with making the broad utterances operative in a narrow economic sense.

3.3.1. Multiple Objectives

Let there be n items which enter into the goals of the firm. Leaving out the difficulties of measurement we assume, for the moment, that the intensity of fulfillment of the i^{th} goal is x_i where $i = 1, 2, 3, \dots, n$.

Let there also be m means by which the goals are achieved. The intensity of application of the j^{th} mean is denoted by y_j , $j = 1, 2, 3, \dots, m$.

We may write:

$$x_i = \Phi_i (y_1, y_2, \dots, y_m) \quad i = 1, 2, \dots, n.$$

where not all the y_j necessarily appear in every Φ_i . We also must specify the boundary conditions which apply to the y_j . At its simplest the form may be:

$$A_j \leq y_j \leq B_j.$$

The bounds may be limits on production rates, labor supply and so forth.

If we wish to account for all n items directly in the goals of the firm we can write, generally as the aim of the firm:

$$\max_{y_1} \max_{y_2} \dots \max_{y_m} U(\Phi_1(y_1, y_2, \dots, y_m), \Phi_2, \Phi_3, \dots, \Phi_n)$$

subject to:

$$A_j \leq y_j \leq B_j .$$

where U may be some complicated function of the x_i . It is the general utility function for the firm.

3.3.2. Functionally Related Incompatible Goals

Often a statement may be made that "we wish to maximize profits and market share." These items may be negatively correlated through the effect of the independent variables. In this case the production rate and the cost of the sales effort necessary to increase market share may decrease profits. This is so basic and evident that a formal demonstration will be omitted.

3.3.3. Measurable Goals

The multiple goals of a firm may only be specified by an ordering relationship, equivalent to the indifference relations encountered in the theory of consumer behavior. Thus if profits and market share are the multiple goals, we can say that one state is preferred to another but cannot specify by how much. This gives us a well defined method for determining policy providing both the "transformation function", which delineates the relationship between market share and profits for a given level of resources of the firm, and the utility or value function are known. Figure 1 illustrates this.

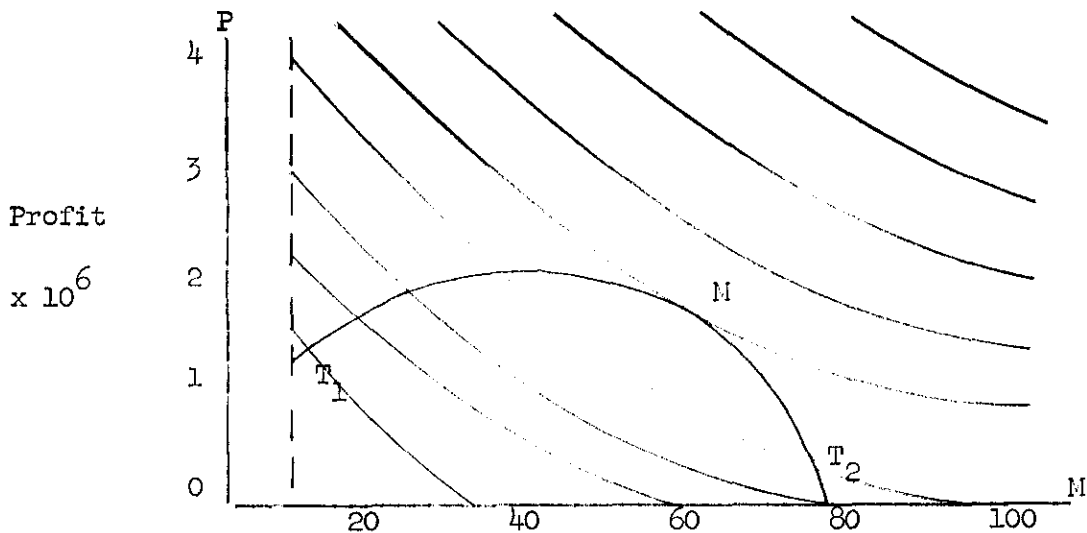


Figure 1. Market Share

$T_1 T_2$ is the transformation function which indicates the maximum profit that can be made for a specific market share. The point M is the solution point which specifies the most preferable profit, market share combination which is attainable. The dotted line on which T_1 lies is drawn to indicate that it is not meaningful to define the preference relations -- when market share is 0, and it is not particularly useful or relevant to define the relationship for very small market shares.

3.3.4. Aggregated Goals

In some situations the correlation between the factors which control the values of two of the stated goals of the firm may be sufficiently close to 1 that maximization of the value of one goal maximizes the other to a good enough approximation. An example of this would be provided if, as has already been noted, growth and profitability or (in a market of fixed size, with a fixed price and constant average costs of production and marketing) profitability and market share were both stated as goals.

3.3.5. Goals and Boundary Conditions

A firm may state that it wishes to "maximize profits, maintain growth and treat employees and stockholders fairly". This statement contains no evaluation of the worth of fulfillment of the different aims and does not indicate the interrelationship that may exist between them. If, as is invariably the case, an overall valuation or utility function for the many features of corporate aims does not exist we have to devise methods to give

operational meaning to them. In the example above we can select one feature and assume that the firm wishes to maximize it, subject to boundary conditions which require that the other corporate aims meet certain specifications. We can represent the aim of the firm as:

- (1) Maximize profits subject to maintaining a specific growth program, dividend rate and employment policy which will satisfy stockholders and employees sufficiently that they do not act to change our environment.

Alternatively, if the dominant interest of the firm is to take care of its employees, its goal may be stated as:

- (2) Maximize disbursement to employees subject to maintaining a specified growth pattern and dividend and profit policy which will satisfy stockholders.

3.3.6. Goals of the Adaptive Organism

The recent work in psychology ^{35/}, artificial intelligence and problem solving^{36/} has dealt with models of organisms as feed-back systems with levels of aspiration varying according to their success. A formal expression of such a system is far more easily given by means of a flow diagram or a programming language such as FORTAN^{37/} than it is in either English or mathematics. These methods are more precise than English and more flexible and "richer" in expression than mathematics. It is my belief that they are sufficiently important to be mentioned here, but an adequate development of the above would require a separate paper.

4. Some Recent Approaches to the Study of Economic Intent, Structure and Behavior.

In the recent past developments have taken place in several diverse yet important aspects of the study of the firm. They have been in the methods and techniques of observation and data processing; experimentation and methodology. Two topics which might be termed "experimental economics" and "mathematical institutional economics" utilizing respectively the techniques of gaming and of simulation, in my opinion give promise of becoming extremely important in the investigation of the behavior of the firm, its structure and the structure of its environment. These combined with advances in methodology should serve to provide a basis for further developments in theory.

4.1. Experimental Economics: Gaming

Until recently it has been all but impossible to validate conjectures concerning oligopolistic intent and behavior in vivo. Formal theories concerning oligopolistic behavior have been in existence for well over a hundred years, yet it has only been in the last three or four years that strict formal experimental attempts have been made to see if at least the conjectures concerning intent and behavior hold up in the highly simplified and overly-antiseptic atmosphere of a laboratory.

Two major types of gaming are currently being employed. The first is commonly referred to as business gaming. A complex model of several firms interlinked in a synthetic or simulated oligopolistic market is constructed. Usually three to five or six teams consisting of anywhere from two or three to seven or eight persons per team participate in the game. Each team simulates

the activities of the management of one of the firms. In the course of anywhere between twenty minutes to several hours it is possible to simulate the activity of a fiscal quarter. In the course of a day several years of simulated history can be experienced. The prime purpose of these games to date has been training and teaching^{38/}. Their extensive use in business schools and industry has already indicated that they are very useful as aids to the understanding of organizational and market structures. Furthermore they provide illustrations of oligopolistic problems of intent and behavior.

Although these games are still considerable simplifications of actual industrial situations, they are very complex and "environment rich" as compared with strict laboratory games. Most of them require a high speed digital computer to perform the calculations required after every move. In at least one instance it has been possible to examine the structure of a relatively complex four team business game and to calculate predicted behavior given certain hypotheses concerning the intent of the competitors. An investigation of the structure of the business game constructed by George Feeney^{39/} and the behavior patterns exhibited by many teams playing this game indicates that the average payoff received by the players if they exhibit a non-cooperative behavior (in the sense of Cournot or Chamberlin or more generally in the sense of Nash^{40/}) serves as a good predictor of the upper bound of the average payoffs achieved by teams playing the game.

The second type of gaming is strictly controlled experimental economics. M. Flood and others^{41/} instigated and carried out experimental investigations of general n-person games. Sociologists^{42/} social-psychologists and psychologists

have all used experimental gaming techniques. In 1948 E. H. Chamberlin reported on an oligopoly experiment^{43/}. Since then the work of Jeremy Stone^{44/}, Austin Hoggatt^{45/}, David Stern^{46/}, Seigal and Fouraker^{47/} and the current experiments of Seigal, Fouraker and Shubik have been aimed at formal controlled experiments to test hypotheses concerning economic behavior. Stone and Seigal and Fouraker were primarily concerned with bargaining. The remaining investigations have been aimed at duopolistic or oligopolistic situations.

In the experiments to date, actions and payoffs associated with a short term non-cooperative behavior appear to be fairly good predictors, at least in symmetric games, although Hoggatt's results are for non-symmetric games. This also appears to be the case in the complex business game constructed by Feeney. There is one important difference between the roles of the players in the "environment poor" or strictly experimental games and in the business game. In the experimental games competition was observed under conditions of incomplete information (or in game theoretic terminology, lack of complete knowledge of the rules of the game). Player 1 was not aware of the payoff to Player 2 and vice-versa. In the business game the players knew that all firms had the same payoff structure. In this instance, however, the amount of information processing required to take advantage of this knowledge was such as to swamp the channel capacities of the teams. It appears that this overburden served to make the information conditions in this game equivalent to the others. Supplying one team with a computer to do the data processing for it, changes the characteristics of this game.

In the simple experimental games it appears that complete information by both players changes the behavior and the outcome considerably. The tendency (which should be of surprise to no-one) is to move above the distribution obtainable at the non-cooperative equilibrium state towards (but not necessarily at) a point on the Pareto Optimal Surface.

As is well known, under reasonable conditions it can be shown that as the number of participants in an oligopolistic market becomes large and the relative size of the individual small, the non-cooperative equilibrium point (or points) tend to the equilibrium of pure competition. The experimental work at this time appears to indicate that competitive or non-cooperative theories are probably reasonably good provided that they are interpreted as theories postulating low levels of information of the players. J. M. Clark apparently appreciated this^{48/}, although sight was lost of it in the philosophical discourses which ensued on the topic of "perfect information".

4.2. Mathematical Institutional Economics: Simulation

Even if intent were known, without a sufficient understanding of environment it would not be possible to predict behavior. Microeconomics has been severely limited by difficulties in the description of environment. In general, mathematical economic models have been too "simple" and restricted in their descriptions (the aggregations have been too drastic owing to problems of analysis or to the inherent lack of richness of mathematics as a language). On the other hand descriptive works and institutional studies have tended to be too discursive for analytical manipulation. The new techniques of high

speed digital computer simulation provide a means for large scale data-organization and detailed model building which preserve at least some of an institutional "richness" and at the same time permit manipulation and experimentation. It is too early at this time to do very much more than to call attention to the work done so far and to express the belief that this approach will be of increasing importance. The symposium on simulation which has recently appeared in the American Economic Review^{49/} provides a sketch of the applications of simulation relevant to economics.

4.3. Models of the Firm: A Suggestion

A few of the techniques for handling multiple goals dependent upon interrelated means have been noted. At the level of the economy, Tinbergen's masterful essay on the Theory of Economic Policy^{50/} covers most of them more generally. However more specifically at the level of the firm, the desire for survival (and the decision to come into existence) are heavily economic decisions and merit explicit introduction into models of the firm.

In an economy with frictions, indivisibilities, uncertainty, costs of entry, losses upon exit and various institutional forms for business firms the decision to come into being or to continue to exist is often a vitally important decision. The objective function of the group in control of the firm must be defined and costs of entry and exit calculated if specific consideration is to be made of birth and death processes. One simple model of the firm with these additional features is suggested below.

The fortunes of the firm are divided into two distinct accounts. The first is called the Corporate Account, and the second the Withdrawal or Stockholder Account. The distinction between the two accounts is clearest in a corporation where the stockholders have limited liability. A forced liquidation may kill the firm and wipe out the stockholder's equity in the corporate account, however the previous income stream into the withdrawal account still remains. The firm may die eventually, but during its lifetime it may yield enough to merit owning.

Let the corporate account at the start of period t be C_t . The net revenue of the firm during period t is denoted by R_t . The amount paid into the withdrawal account is w_t . This gives the condition:

$$C_{t+1} = C_t + R_t - w_t$$

Bankruptcy and liquidation conditions can be specified immediately. If $C_t \leq A$ (where $C_{t-n} > A$ for $n = 1, 2, 3, \dots$) then the firm is bankrupt and is forced to liquidate. The liquidation value may be some number $B = B(C_t) \leq A$. The difference between C_t and B is a measure of the liquidation loss caused by market imperfections.

Money paid into the withdrawal account represents immediate income to ownership. In order to value the income stream a discount factor is introduced.

$$p = \frac{1}{1 + q} \quad \text{where } q \text{ is the rate of interest.}$$

Three different corporate goals are now defined in which the attitudes of risk towards the possible death of the firm and the possibility of paying out income are illustrated.

(1) The Individual Entrepreneur

The objective function of an individual entrepreneur whose goal is to maximize the value of the discounted income stream from his incorporated firm will be:

$$\text{Maximize } \left\{ \sum_{t=0}^r p^t w_t + p^r B \right\}$$

where r is the period at which the firm is finally bankrupted.

The examples in 3.3.5 can now be formalized.

(2) The Unincorporated Enterprise and the Cautious Corporation

In some cases a small businessman may place a high value on his "independence". He will be willing to run his own enterprise as long as he is able to pay himself a sum sufficient to compensate for the wages he could earn in an alternative employment. As there will be a large disutility to ruin, (probably greater than the mere monetary loss) the goal of the businessman can be described as:

$$\text{Maximize } \prod_{t=1}^r \{1 - P_t(c_t \leq A)\}$$

$$\text{subject to } w_t \geq a_t \quad \text{for } t = 1, 2, \dots, r.$$

$P_t(c_t \leq A)$ is the probability that the firm will be bankrupted in the t^{th} period. a_t is the minimum withdrawal required by the businessman in the t^{th} period. The above expression indicates that the probability of survival for r periods is maximized subject to certain withdrawals being made each period.

This policy could be also adopted by a corporation whose managers wish to keep their survival chances as high as possible subject to having to declare dividends sufficient to satisfy the stockholders.

(3) Dividends and Risk

A management-run corporation may express its policy by attempting to maximize the value of the discounted income stream paid to stockholders over some time period subject to the boundary condition that the risk of bankruptcy is kept below a given level.

$$\begin{aligned} \text{Maximize} \quad & \sum_{t=0}^r p^t w_t \\ \text{subject to:} \quad & \prod_{t=1}^r \{1 - P_t(c_t \leq A)\} \geq k. \end{aligned}$$

The boundary condition indicates that the probability of survival for r periods will be at least k .

A simple modification of the above model suffices to distinguish between firms in existence and "firms-in-being"^{49/}. It is evident that the models of maximization by the firm suggested here are related in some ways to the

Gambler's Ruin problems of statistics. This relation is explored elsewhere in an investigation of "games of economic survival"^{50/}. A necessary but only preliminary attempt is made to consider oligopolistic markets in terms of many person games of economic survival^{51/}.

5. Power, Intent and Policy

The need for an understanding of intent of the firm beyond weak conditions of maximization varies with the power of the firm in its market places and society. The central problem of political science and of administration becomes the central problem of the economics of the oligopolistic firm and the industries within which it operates. That is the meaning and measurement of power. The less the firm is able to influence its environment, the less needs to be known about the motivation of the management of the firm for most purposes of policy.

Current industrial organization is such, and future industrial organization will probably be such that under any reasonable criterion of power, powerful firms will exist. The multiple goals of the powerful firm are of importance both to it and to society. Its very possession of power prevents it from ignoring them. Although few economists or political scientists would deny that General Motors is a powerful firm and Russia is a powerful country, the measurement of power (or even its definition) in general is hardly adequate. This is easily seen in the difficulties encountered in attempts to measure the influence of multi-market firms or financial entities.

It is possible that the needs of society are best served by inherently powerful firms provided that their behavior falls within certain bounds. If this is so then the control of behavior may precede the control of power. Concomitant with this is the burden to be borne by the firms that impeccable intentions acted upon by the powerful may result in undesirable behavior.

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