Objective Properties from Subjective Quantum States: Environment as a Witness

Harold Ollivier,[1,2] David Poulin,[1,3] and Wojciech H. Zurek[1]
[1]*Theoretical Division, LANL, MS B213, Los Alamos, New Mexico 87545, USA*
[2]*INRIA—Projet Codes, BP 105, F-78153 Le Chesnay, France*
[3]*Institute for Quantum Computing, University of Waterloo, Ontario N2L 3G1, Canada*
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We study the emergence of objective properties in open quantum systems. In our analysis, the environment is promoted from a passive role of a reservoir selectively destroying quantum coherence to an active role of amplifier selectively proliferating information about the system. We show that only preferred pointer states of the system can leave a redundant and therefore easily detectable imprint on the environment. Observers who—as is almost always the case—discover the state of the system indirectly (by probing a fraction of its environment) will find out only about the corresponding pointer observable. Many observers can act in this fashion independently and without perturbing the system. They will agree about its state. In this operational sense, preferred pointer states exist objectively.

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The key feature distinguishing the classical realm from the quantum substrate is its objective existence. Classical states can be found out through measurements by an initially ignorant observer without getting disrupted in the process. By contrast, an attempt to discover the state of a quantum system through a direct measurement generally leads to a collapse [1–3]: after a measurement, the state will be what the observer finds out it is, but not, in general, what it was before. Thus, it is difficult to claim that quantum states exist objectively in the same sense as their classical counterparts [4–6].

Decoherence is caused by persistent monitoring of a system by the environment. It can single out a preferred set of states. In the simplest models, such pointer states [7–11] are (often degenerate) eigenstates of the pointer observable which commutes with the system-environment interaction [12]. This concept can be generalized using the predictability sieve: only pointer states evolve predictably in spite of the openness of the system [7,10,11]. They exist in the sense that in absence of any perturbations—save for the monitoring by the environment—they or their dynamically evolved descendants will continue to faithfully describe the system. Thus, when an observer knows what the pointer states are, he can learn which of them represents the system without perturbing it. However, when an observer ignorant of what the pointer states are attempts to find out the state of the system directly, he still faces, even in the presence of decoherence, the danger of collapsing its wave packet.

Here, we build on the idea that a *direct* measurement of the system is not how observers gather data about the Universe: rather, a vast majority (if not all) of our information is obtained *indirectly* by probing a small fraction of the environment [7,9,11]. One may think that this twist in the story can be accounted for by adding a few links to the von Neumann chain [2], but this is not the case: we shall show that the monitoring environment acquires information about the system *selectively*. More importantly, this selective spreading of information through the environment—in essence "quantum Darwinism" [11]—accounts for the objective existence of some preferred quantum states: by probing the system indirectly, hence without perturbing it, many independent observers can obtain reliable information, but only about the pointer states.

This Letter is organized as follows: first, we introduce our operational definition of objectivity. We then state necessary and sufficient conditions for the objective existence of an observable in the context of *einselection* (environment-induced superselection). Next, these requirements are translated into an information theoretic framework, and proven to imply a *unique* observable: the usual pointer observable. This is our key result. Finally, we show that, because of quantum Darwinism, information about pointer states is robust and, hence, objective.

An operational definition of objectivity for a property of a quantum system should not rely on preexistence of an underlying real state as it is presumed in the classical setting. Rather, we demand that an objective property of the system of interest is (i) simultaneously accessible to many observers (ii) who are able to find out what it is without prior knowledge and (iii) who can arrive at a consensus about it without prior agreement. As we already mentioned, the collapse of the wave packet following a direct measurement generally precludes this. However, when the system of interest $\mathcal{S}$ interacts with an environment $\mathcal{E}$ composed of many subsystems, $\mathcal{E} = \bigotimes_{k=1}^{N} \mathcal{E}_k$, the situation changes dramatically. When a property leaves a *complete* and *redundant* imprint on the environment, all three criteria are satisfied: many copies are available, hence simultaneous accessibility (i) follows. Moreover fractions of the environment can be measured without perturbing either $\mathcal{S}$ or the rest of $\mathcal{E}$. Therefore, ignorant observers can select their measurements independently, corroborate their own results, and arrive at a common description of properties of the system. Hence, owing to redundancy, prior knowledge (ii) is not necessary to (iii) reach consensus. The existence of an objective property requires the presence of its complete and redundant imprint in the environment as necessary and sufficient conditions. Our approach will focus on the study of the correlations between parts of the environment and the system of interest.

Information theoretical framework.—A natural way to characterize such correlations is to use the mutual information $I(\sigma:\mathfrak{e})$ between an observable σ of $\mathcal{S}$ and $\mathfrak{e}$ of $\mathcal{E}$. In short, $I(\sigma:\mathfrak{e})$ measures one's ability to predict the outcome of measurement σ on $\mathcal{S}$ after having "looked at the environment" through $\mathfrak{e}$. For a given density matrix $\rho^{\mathcal{S}\mathcal{E}}$ of $\mathcal{S}\otimes\mathcal{E}$, the measurement results are random variables characterized by a joint probability distribution

$$p(\sigma_i, \mathfrak{e}_j) = \mathrm{Tr}\{(\sigma_i \otimes \mathfrak{e}_j)\rho^{\mathcal{S}\mathcal{E}}\}, \tag{1}$$

where σ_i and $\mathfrak{e}_j$ are the spectral projectors of σ and $\mathfrak{e}$. By definition, the mutual information is the difference between the initially missing information about σ and the remaining uncertainty about σ when $\mathfrak{e}$ is known [13]. Using Shannon entropy as a measure of missing information, $H(\sigma) = -\sum_i p(\sigma_i)\log p(\sigma_i)$ and $H(\sigma, \mathfrak{e}) = -\sum_{i,j} p(\sigma_i, \mathfrak{e}_j)\log p(\sigma_i, \mathfrak{e}_j)$, the mutual information is

$$I(\sigma:\mathfrak{e}) = H(\sigma) + H(\mathfrak{e}) - H(\sigma, \mathfrak{e}). \tag{2}$$

The information about observable σ of $\mathcal{S}$ that can be optimally extracted from m environmental subsystems is

$$\hat{I}_m(\sigma) = \max_{\{\mathfrak{e}\in\mathfrak{M}_m\}} I(\sigma:\mathfrak{e}), \tag{3}$$

where $\mathfrak{M}_m$ is the set of all measurements on the Hilbert space of those m subsystems. In general, $\hat{I}_m(\sigma)$ will depend on which particular m subsystems are considered. For simplicity, we will assume that any *typical m* environmental subsystems yield roughly the same information. This may appear to be a strong assumption, but, as we discuss below, relaxing it does not affect our main conclusions. By setting m to the total number N of subsystems of $\mathcal{E}$, we get the information content of the entire environment. Then,

$$\hat{I}_N(\sigma) \approx H(\sigma) \tag{4}$$

expresses the *completeness* prerequisite for objectivity: all (or nearly all) missing information about σ must be in principle obtainable from all of $\mathcal{E}$.

However, as a consequence of basis ambiguity [12,14], information about many observables σ can be deduced by an appropriate measurement on the entire environment. Therefore, completeness, Eq. (4), while a prerequisite for objectivity, is not a very selective criterion [see Fig. 1(a) for illustration]. To claim objectivity, it is not sufficient to have a complete imprint of the candidate property of $\mathcal{S}$ in the environment. There must be many copies of this imprint that can be accessed independently by many observers: *information must be redundant.*

Redundancy and its consequences.—To obtain a measure of redundancy, one must count the number of copies

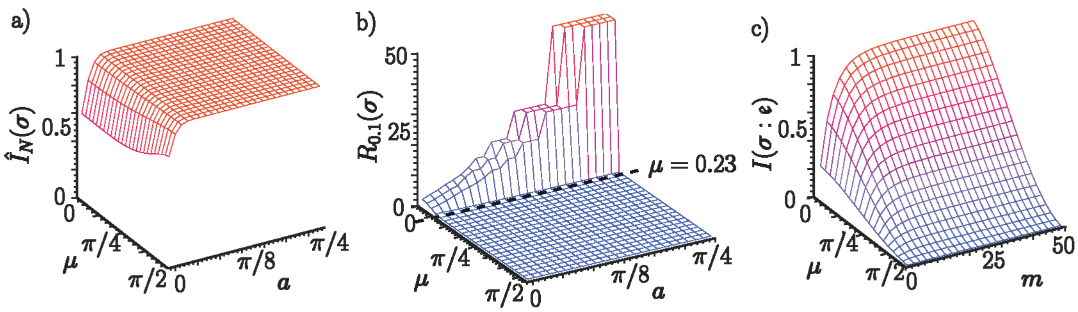

FIG. 1 (color). Quantum Darwinism can be illustrated using a model introduced in [12]. The system $\mathcal{S}$, a spin-$\frac{1}{2}$ particle, interacts with $N = 50$ two-dimensional subsystems of the environment through $\hat{H}^{\mathcal{S}\mathcal{E}} = \sum_{k=1}^{N} g_k \sigma_z^{\mathcal{S}} \otimes \sigma_{yk}^{\mathcal{E}}$ for a time t. The initial state of $\mathcal{S}\otimes\mathcal{E}$ is $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle_1^{\mathcal{E}} \otimes \cdots \otimes |0\rangle_N^{\mathcal{E}}$. All the plotted quantities are functions of the system's observable $\sigma(\mu) = \cos(\mu)\sigma_z + \sin(\mu)\sigma_x$, where μ is the angle between eigenstates of $\sigma(\mu)$ and the pointer states of $\mathcal{S}$—here the eigenstates of $\sigma_z^{\mathcal{S}}$. (a) Information acquired by the optimal measurement $\mathfrak{e}$ on the whole environment, $\hat{I}_N(\sigma)$, as a function of the inferred observable $\sigma(\mu)$ and the action $a_k = g_k t = a$ for all k. A large amount of information is accessible in the *whole* environment for any observables $\sigma(\mu)$ except when the interaction action a_k is very small. Thus, complete imprinting of an observable of $\mathcal{S}$ in $\mathcal{E}$ is not sufficient to claim objectivity. (b) Redundancy of the information about the system as a function of the inferred observable $\sigma(\mu)$ and the action $a_k = g_k t = a$. It is measured by $R_{\delta=0.1}(\sigma)$, which counts the number of times 90% of the total information can be "read off" independently by measuring distinct fragments of the environment. For all values of the action $a_k = g_k t = a$, redundant imprinting is sharply peaked around the pointer observable. Redundancy is a very selective criterion. The number of copies of relevant information is high only for the observables $\sigma(\mu)$ falling inside the theoretical bound (see text) indicated by the dashed line. (c) Information about $\sigma(\mu)$ extracted by an observer restricted to local random measurements on m environmental subsystems (e.g., $\mathfrak{e} = \mathfrak{e}_1^{\mathcal{E}} \otimes \cdots \otimes \mathfrak{e}_m^{\mathcal{E}}$, where each $\mathfrak{e}_k^{\mathcal{E}}$ is chosen at random). The interaction action $a_k = g_k t$ is randomly chosen in $[0, \pi/4]$ for each k. Because of redundancy, pointer states—and only pointer states—can be found out through this far-from-optimal measurement strategy. Information about any other observable $\sigma(\mu)$ is restricted by our theorem to be equal to the information about it contained in the pointer observable $\sigma_z^{\mathcal{S}}$, Eq. (6).

of the information about σ present in $\mathcal{E}$. Redundancy is thus quantified by the number of disjoint subsets of $\mathcal{E}$ containing almost all—all but a fraction δ—of the information about σ available from the entire $\mathcal{E}$:

$$R_\delta(\sigma) = N/m_\delta(\sigma). \quad (5)$$

Above $m_\delta(\sigma)$ is the smallest number m of typical environmental subsystems that contain almost all the information about σ, i.e., $\hat{I}_m(\sigma) \geq (1-\delta)\hat{I}_N(\sigma)$.

The key question now is: What is the structure of the set $\mathcal{O}$ of observables that are completely, $I_N(\sigma) \approx H(\sigma)$, and redundantly, $R_\delta(\sigma) \gg 1$ with $\delta \ll 1$, imprinted on the environment? The answer is provided by the following theorem.

Theorem: *The set $\mathcal{O}$ is characterized by a unique observable π, called by definition the* maximally refined observable: *the information $\hat{I}_m(\sigma)$ about any observable σ in $\mathcal{O}$ obtainable from a fraction of $\mathcal{E}$ is equivalent to the information about σ that can be obtained through its correlations with the maximally refined observable π:*

$$\hat{I}_m(\sigma) = I(\sigma{:}\pi) \quad (6)$$

for $m_\delta(\pi) \leq m \ll N$.

Outline of the proof for perfect records, $\delta = 0$.—Let $\sigma^{(1)}$ and $\sigma^{(2)}$ be two observables in $\mathcal{O}$ for $\delta = 0$. Since $\sigma^{(1)}$ and $\sigma^{(2)}$ can be inferred from two disjoint fragments of $\mathcal{E}$, they must commute. Similarly, let $e^{(1)}$ [respectively $e^{(2)}$] be a measurement acting on a fragment of $\mathcal{E}$ that reveals all the information about $\sigma^{(1)}$ [respectively $\sigma^{(2)}$] while causing minimum disturbance to $\rho^{S\mathcal{E}}$. Then, $e^{(1)}$ and $e^{(2)}$ commute, and can thus be measured *simultaneously.* This combined measurement gives complete information about $\sigma^{(1)}$ *and* $\sigma^{(2)}$. Hence, for any pair of observables in $\mathcal{O}$, it is possible to find a more refined observable which is also in $\mathcal{O}$. The maximally refined observable π is then obtained by pairing successively all the observables in $\mathcal{O}$. By construction π satisfies Eq. (6) for any σ in $\mathcal{O}$. ■

Note that the Theorem does not guarantee the existence of a *nontrivial* observable π: when the system does not properly correlate with $\mathcal{E}$, the set $\mathcal{O}$ will only contain the identity operator.

This Theorem can be extended to nearly perfect records for assumptions satisfied by usual models of decoherence [15]. The proof is based on the recognition that only the already familiar pointer observable can have a redundant and robust imprint on $\mathcal{E}$. The Theorem can be understood as a consequence of the ability of the pointer states to persist while immersed in the environment. This resilience allows the information about the pointer observables to proliferate, very much in the spirit of the "survival of the fittest."

Two important consequences of this theorem follow. (i) An observer who probes only a fraction of the environment is able to find out the state of the system as if he measured π on S. (ii) Information about any other observable σ of S will be inevitably limited by the available correlations existing between σ and π. In essence, our theorem proves the uniqueness of redundant information, and therefore the selectivity of its proliferation.

Quantum Darwinism—the idea that the perceived *classical reality* is a consequence of the selective proliferation of information about the system [11]—is consistent with previous approaches to einselection, such as the predictability sieve, but goes beyond them. The existence of redundant information about the system, induced by specific interactions with the environment, completely defines how and what kind of information can be retrieved from $\mathcal{E}$: Equation (6) shows that the most efficient strategy for inferring σ consists in estimating π first, and deducing from it information about σ. It also explains the emergence of a consensus about the properties of a system. Observers that gain information about π—the only kind of information available in fragments of $\mathcal{E}$—will agree about their conclusions: their measurement results are directly correlated with π, and are therefore correlated with each other. Hence, observers probing fractions of the environment can act *as if* the system had a state of its own—an *objective* state (one of the eigenstates of π). By contrast, such consensus cannot arise for superpositions of pointer states, e.g., Schrödinger cats, since information about nonlocal superpositions can only be extracted by probing the whole environment, and thus cannot be obtained independently by several observers. Objectivity comes at the price of singling out a preferred observable of S whose eigenstates are redundantly recorded in $\mathcal{E}$. Cloning of quanta is not possible [16], but amplification of a preferred observable happens almost as inevitably as decoherence, and leads to objective classical reality. The impossibility of cloning and the capacity for amplification imply selection—Darwinian survival of the fittest.

Emergence of objectivity exemplified.—Figure 1(b) illustrates the redundancy $R_{\delta=0.1}$ for a specific model as a function of the inferred observable $\sigma(\mu)$ (whose eigenstates are tilted by an angle μ from the pointer ones) and of the interaction action, $a_k = g_k t$ [where $\sin(a_k)$ characterizes the strength of the correlations between S and $\mathcal{E}$]. By carefully tracking all orders of δ in Eq. (6), one can show that the existence of a complete and redundant imprint of observable $\sigma(\mu)$ in the environment requires $H_2(\cos^2\frac{\mu}{2}) \leq \delta$, where $H_2(p) = -[p\log p + (1-p) \times \log(1-p)]$. Inserting the actual values of the parameters chosen for our simulation, the above equation indicates that only observables with $|\mu| < 0.23$ leave a redundant imprint on the environment: the objective properties of the system are unique. This bound is in excellent agreement with our numerical results. Surprisingly, and as confirmed by our simulations, the interaction action a_k only plays a role in setting the maximum value of redundancy, but does not affect the selectivity of our criterion. Which observable becomes objective is largely decided by the structure of the interaction Hamiltonian (i.e., the set of pointer states), but not by its details such as strength and duration of the interaction. This ensures the stability of the pointer observable deduced from redundancy.

Robustness of information.—Objective information must be extractable through "realizable"—hence, not necessarily optimal—measurements for many observers to arrive at an operational consensus about the state of a system. For instance, human eyes can only measure photons separately, yet we can still learn about the position of objects. This issue is considered for our model in Fig. 1(c). Here, even local (i.e., spin by spin) *random* measurements eventually acquire the entire information available in $\mathcal{E}$ about the pointer states. Though surprising, this result naturally follows from quantum Darwinism and the fact that high redundancy protects information against a wide range of errors. Almost any observable of S is completely imprinted on the environment [see Fig. 1(a)]. However, as our theorem establishes and Fig. 1(b) illustrates, only the observables "close" to the maximally refined pointer observable $\pi = \sigma_z^S$ can be imprinted redundantly in the environment. Therefore only information about pointer states can tolerate errors, i.e., can be extracted by non-optimal measurements. In short, not only is the information about the pointer observable easy to extract from fragments of the environment, it is impossible to ignore.

Clearly, for the emergence of objective properties, it is much more important to know that $R_\delta(\sigma) \gg 1$ than to know its precise value: the whole idea of redundancy is that it allows one to be sloppy in decoding the message and still "get it right." Consequently, the essence of our conclusions is also largely unaffected by the assumption [see Eq. (3)] of even spreading of information in the environment.

The precise value of $R_\delta(\sigma)$ depends on the tensor decomposition of $\mathcal{E}$ into subsystems, which is *a priori* arbitrary. In the absence of well defined subsystems *locality* seems to be a useful guide: Our access to the information content of the environment is restricted by the fundamental Hamiltonians of nature that are local. Moreover, various observers occupy, and therefore monitor, different spatial regions. Hence, an operational notion of redundancy should reflect spreading of information in space.

Quantum Darwinism capitalizes on ideas related to decoherence and einselection, but goes further towards understanding of the quantum origins of objectivity. Existence of records in the environment was noted before [7–9,17], and the fact that it is easiest to find out about the pointer observable has been also appreciated [18]. Here we have described an even more dramatic turn of events—environment as a broadcast medium—which may seem fanciful until we realize that it describes rather accurately what happens in the real world (see also [15,19,20]). For instance, human observers acquire all of their visual data by intercepting a small fraction of their photon environment. An operational notion of objectivity emerges from redundant information as it enables many independent observers to find out the state of the system without disturbing it. Furthermore, *objective* observables are robust—insensitive to changes in the strategy through which the environment is interrogated, as well as to variations of the strength and duration of the interaction between S and $\mathcal{E}$, etc. One might regard quantum Darwinism as a fully quantum implementation of Bohr's idea [21] about the role of amplification in the transition from quantum to classical.

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