Observability and Decentralized Control of Fuzzy Discrete Event Systems

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Abstract— **Fuzzy discrete event systems as a generalization of (crisp) discrete event systems have been introduced in order that it is possible to effectively represent uncertainty, imprecision, and vagueness arising from the dynamic of systems. A fuzzy discrete event system has been modelled by a fuzzy automaton; its behavior is described in terms of the fuzzy language generated by the automaton. In this paper, we are concerned with the supervisory control problem for fuzzy discrete event systems with partial observation. Observability, normality, and co-observability of crisp languages are extended to fuzzy languages. It is shown that the observability, together with controllability, of the desired fuzzy language is a necessary and sufficient condition for the existence of a partially observable fuzzy supervisor. When a decentralized solution is desired, it is proved that there exist local fuzzy supervisors if and only if the fuzzy language to be synthesized is controllable and co-observable. Moreover, the infimal controllable and observable fuzzy superlanguage, and the supremal controllable and normal fuzzy sublanguage are also discussed. Simple examples are provided to illustrate the theoretical development.**

Index Terms— **Fuzzy discrete event systems, supervisory control, observability, normality, co-observability.**

I. Introduction

DISCRETE event systems (DES) are systems whose state space is discrete and whose state can only change as a result of asynchronously occurring instantaneous events over time. Such systems have been successfully applied to provide a formal treatment of many man-made systems such as communication systems, networked systems, manufacturing systems, and automated traffic systems. The behavior of a DES is described in terms of the sequences of events involved. Supervisory control of DES pioneered by Ramadge and Wonham [19] and subsequently extended by many other researchers (see, for example, [20], [2], and the bibliographies therein) provides a framework for designing supervisors for controlling the behavior of DES.

Usually, a DES is described by finite state automaton with events as input alphabets, and the behavior is thus the language accepted by the automaton. It is worth noting that such a model can only process crisp state transitions. In other words, no uncertainty arises in the state transitions of the model. There are, however, many situations such as mobile robots in an unstructured environment [14], intelligent vehicle control [21], and wastewater treatment [24], in which the state transitions of some systems are always somewhat imprecise, uncertain, and vague. A convincing example given in [12] and [13] is a patient's condition, where the change of the condition from a state, say "excellent", to another, say "fair", is obviously imprecise, since it is hard to measure exactly the change.

Vagueness, imprecision, and uncertainty are typical features of most of the complex systems. It is well known that the methodology of fuzzy sets first proposed by Zadeh [27] is a good tool for coping with imprecision, uncertainty, and vagueness. To capture significant uncertainty appearing in states and state transitions of DES, Lin and Ying have incorporated fuzzy set theory together with DES and thus have extended crisp DES to fuzzy DES by applying fuzzy finite automaton model [12], [13]. Under the framework of fuzzy DES, Lin and Ying have discussed state-based observability and some optimal control problems. Excitingly, the first application of fuzzy DES has recently been reported by Ying $et\ al.$ in [25], where fuzzy DES are used to handle treatment planning for HIV/AIDS patients. It is worth noting that fuzzy finite automata and fuzzy languages have some important applications to many other fields such as clinical monitoring and pattern recognition (see, for example, [15]).

As a continuation of the works [12] and [13], we have developed supervisory control theory for fuzzy DES modelled by (maxmin) fuzzy automata in [1]. The behavior of such systems is described by their generated fuzzy languages. Informally, a fuzzy language consists of certain event strings associated with membership grade. The membership grade of a string can be interpreted as the possibility degree to which the system in its initial state and with the occurrence of events in the string may enter another state. Although strings in a probabilistic language [16], [5], [8] are also endowed with weight, it should be pointed out that fuzzy languages are different from probabilistic languages in semantics: the weight in a fuzzy context describes the membership grade (namely, uncertainty) of a string, while the weight of a string in a probabilistic context reflects a frequency of occurrence. This difference appeals for distinct control action and can also satisfy diverse applications.

For control purposes, the set of events is partitioned into two disjoint subsets of controllable and uncontrollable events, as usually done in crisp DES. Control is exercised by a fuzzy supervisor that disables controllable events with certain degrees in the controlled system, also called a plant, so that the closed-loop system of supervisor and plant exhibits a pre-specified desired fuzzy language. The controllability of

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fuzzy languages has been introduced in [1] as a necessary and sufficient condition for the existence of a fuzzy supervisor that achieves a desired specification for a given fuzzy DES. A similar controllability condition based on different underlying constituents of fuzzy automata and uncontrollable event set has also been given independently by Qiu in [18].

In [1] and [18], both Qiu and we have restricted our attention to the problem of centralized control under full observation. There are, however, some systems in which supervisors cannot "see" or "observe" all the events. For example, in a diagnosis on a disease, if we regard the evolvement of the disease as a system with some physicians as supervisors, it is a fact that the physicians cannot see all the events that result in the evolvement of the disease. Unobservability of events arises principally from the limitations of the sensors attached to the systems and the distributed nature of some systems such as manufacturing systems and communication networks where events at some locations are not seen at other locations. For crisp DES, the issue of partial observation has been extensively studied in both centralized and decentralized control (see, for example, [3], [9]–[11], [22]). However, in terms of fuzzy DES, this issue is investigated only as a global property of systems in [12], [13]; a further study is desired.

The purpose of this paper is to develop these earlier works [1], [12], [13], [18] and address the supervisory control problem for fuzzy DES with partial observation. This requires that we have to deal with the presence of unobservable events in addition to the presence of uncontrollable events. We are mainly concerned with what controlled behavior can be achieved when controlling a fuzzy DES with a partially observable fuzzy supervisor and decentralized fuzzy supervisors, respectively.

The contribution of this paper is as follows.

1). To characterize the class of fuzzy languages achievable under the partially observable architecture, in Section III we introduce the notions of observability and strong observability of fuzzy languages which are generalized versions of observability of crisp languages. We show that there exists a partially observable fuzzy supervisor synthesizing a desired fuzzy language if and only if the fuzzy language is controllable and observable. The property that observable fuzzy languages are closed under arbitrary intersections leads to the existence of infimal controllable and observable fuzzy superlanguage.

2). Decentralized supervisory control with global specification has been investigated in Section IV. By generalizing co-observability of crisp languages to fuzzy languages, a necessary and sufficient condition for the existence of local fuzzy supervisors that achieve a given legal fuzzy language is derived.

3). In order to obtain an approximation for a given fuzzy language by using controllable and observable sublanguages, normality of fuzzy languages which is stronger than observability is defined in Appendix I. Some properties of normal fuzzy languages and the relation between normality, observability, and controllability are also presented.

Besides the sections mentioned above, Section II provides the necessary preliminaries; Section V addresses an illustrative example; Section VI concludes the work presented and identifies some future research directions. The paper also contains two appendices: Appendix I is devoted to the normality of fuzzy languages and Appendix II shows the proofs of theorems, propositions, and lemmas.

II. Preliminaries

In the first subsection, we will briefly recall a few basic facts on the supervisory control of crisp DES under partial observation. For a detailed introduction to the supervisory control theory, readers may refer to [2]. The second subsection is devoted to the background on the supervisory control of fuzzy DES.

A. Observability and Decentralized Control of Crisp DES

Let E denote the finite set of events, and E^* denote the set of all finite sequences of events, or stings, in E, including the empty string ϵ. A string $\mu \in E^*$ is a prefix of a string $\omega \in E^*$ if there exists $\nu \in E^*$ such that $\mu\nu = \omega$. In this case, we write $\mu \leq \omega$. The length of a string ω is denoted by $|\omega|$. Any subset of E^* is called a language over E. The prefix closure of a language L, denoted by $\overline{L}$, consists of the set of strings which are prefixes of strings in L. A language L is said to be prefix closed if $L = \overline{L}$.

A crisp DES, or plant, is usually described by a deterministic automaton: $G = (Q, E, \delta, q_0)$, where Q is a set of states with the initial state q_0, E is a set of events, and $\delta : Q\times E \to Q$ is a (partial) transition function. The function δ is extended to $\delta : Q \times E^* \to Q$ in the obvious way. In a logical model of a DES, we are interested in the strings of events that the system can generate. Thus the behavior of a DES is modelled as a prefix closed language $L(G) = \{s \in E^* : \delta(q_0, s) \text{ is defined}\}$ over the event set E.

Recall that associated with the system is a set E_c of events that can be disabled, and there is a set of events, E_o, that can be observed by partial observation supervisors. The sets of uncontrollable and unobservable events are denoted by $E_{uc} = E\backslash E_c$ and $E_{uo} = E\backslash E_o$, respectively. To represent the fact that a partial observation supervisor has only a partial observation of strings in $L(G)$, a natural projection operator $P : E^* \to E_o^*$ is used. Recall that $P(\epsilon) = \epsilon$ and $P(sa) = P(s)P(a)$ for any $s \in E^*$ and $a \in E$, where $P(a) = a$ if $a \in E_o$, and otherwise $P(a) = \epsilon$. A (partial observation) supervisor is a map $S_P : P[L(G)] \to 2^E$ such that $S_P[P(s)] \supseteq E_{uc}$ for any $s \in L(G)$.

The language generated by the controlled system, denoted by $L(S_P/G)$, is defined inductively as follows:

1) $\epsilon \in L(S_P/G)$;

2) $[s \in L(S_P/G),\ sa \in L(G), \text{ and } a \in S_P[P(s)]] \Leftrightarrow [sa \in L(S_P/G)]$.

Let us recall two key notions in crisp DES.

Definition 1: A prefix-closed language $K \subseteq L(G)$ is said to be *controllable* [19] (with respect to $L(G)$ and E_{uc}) if $KE_{uc} \cap L(G) \subseteq K$; it is called *observable* [9] (with respect to $L(G)$, P, and E_c) if for any $s, s' \in K$ and any $a \in E_c$,

$$[P(s) = P(s'),\ sa \in K, \text{ and } s'a \in L(G)] \Rightarrow [s'a \in K].$$

Given a nonempty and prefix-closed language $K \subseteq L(G)$, it has been shown in [3] and [9] that there exists a partial observation supervisor S_P such that $L(S_P/G) = K$ if and only if K is controllable and observable.

If the plant G is physically distributed, then it is desirable to have decentralized supervisors, where each supervisor is able to control a certain set of events and is able to observe certain other events. For the sake of simplicity, let us recall the decentralized supervisory control with only two local supervisors.

Let $E_{ic}, E_{io} \subseteq E, i = 1, 2$, be the local controllable and observable event sets, respectively. Let $P_i : E^* \to E_{io}^*$ be the corresponding natural projection. The local supervisor S_{iP} (or simply S_i), $i = 1, 2$, is given by the map $S_i : P_i[L(G)] \to 2^E$ that satisfies $S_i[P_i(s)] \supseteq E \backslash E_{ic}$ for any $s \in L(G)$.

The language $L(S_1 \wedge S_2/G)$ generated by the system G under the joint supervision of S_1 and S_2 is defined inductively by

1) $\epsilon \in L(S_1 \wedge S_2/G)$;
2) $[s \in L(S_1 \wedge S_2/G),\ sa \in L(G),$ and $a \in S_1[P_1(s)] \cap S_2[P_2(s)]] \Rightarrow [sa \in L(S_1 \wedge S_2/G)]$.

To state the existence condition for local supervisors, it is necessary to introduce co-observability, which is defined below.

Definition 2: A prefix-closed language $K \subseteq L(G)$ is *co-observable* [22] (with respect to $L(G)$, P_i, and $E_{ic}, i = 1, 2$), if for all $s, s', s'' \in K$ subject to $P_1(s) = P_1(s')$ and $P_2(s) = P_2(s'')$, the following hold:

1) if $a \in E_{1c} \cap E_{2c}$, $sa \in L(G)$, and $s'a, s''a \in K$, then $sa \in K$;
2) if $a \in E_{1c} \backslash E_{2c}$, $sa \in L(G)$, and $s'a \in K$, then $sa \in K$;
3) if $a \in E_{2c} \backslash E_{1c}$, $sa \in L(G)$, and $s''a \in K$, then $sa \in K$.

Given a nonempty and prefix-closed language $K \subseteq L(G)$, it has been shown by Rudie and Wonham in [22] that there exist local supervisors S_1 and S_2 such that $L(S_1 \wedge S_2/G) = K$ if and only if K is controllable and co-observable.

B. Fuzzy DES and Its Controllability

In this subsection, we recall the model of fuzzy DES and its supervisory control under full observation.

For later need, let us first review some notions and notation on fuzzy set theory. Each *fuzzy subset* (or simply *fuzzy set*), $\mathcal{A}$, is defined in terms of a relevant universal set X by a function assigning to each element x of X a value $\mathcal{A}(x)$ in the closed unit interval $[0, 1]$. Such a function is called a *membership function*, which is a generalization of the characteristic function associated to a crisp set; the value $\mathcal{A}(x)$ characterizes the degree of membership of x in $\mathcal{A}$.

The *support* of a fuzzy set $\mathcal{A}$ is a crisp set defined as supp($\mathcal{A}$) $= \{x : \mathcal{A}(x) > 0\}$. Whenever supp($\mathcal{A}$) is a finite set, say supp($\mathcal{A}$) $= \{x_1, x_2, \cdots, x_n\}$, then fuzzy set $\mathcal{A}$ can be written in Zadeh's notation as follows:

$$\mathcal{A} = \frac{\mathcal{A}(x_1)}{x_1} + \frac{\mathcal{A}(x_2)}{x_2} + \cdots + \frac{\mathcal{A}(x_n)}{x_n}.$$

We denote by $\mathcal{F}(X)$ the set of all fuzzy subsets of X. For any $\mathcal{A}, \mathcal{B} \in \mathcal{F}(X)$, we say that $\mathcal{A}$ is contained in $\mathcal{B}$ (or $\mathcal{B}$ contains $\mathcal{A}$), denoted by $\mathcal{A} \subseteq \mathcal{B}$, if $\mathcal{A}(x) \leq \mathcal{B}(x)$ for all $x \in X$. We say that $\mathcal{A} = \mathcal{B}$ if and only if $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}$. A fuzzy set is said to be *empty* if its membership function is identically zero on X. We use $\mathcal{O}$ to denote the empty fuzzy set.

For any family $\lambda_i,\ i \in I$, of elements of $[0, 1]$, we write $\vee_{i \in I} \lambda_i$ or $\vee\{\lambda_i : i \in I\}$ for the supremum of $\{\lambda_i : i \in I\}$, and $\wedge_{i \in I} \lambda_i$ or $\wedge\{\lambda_i : i \in I\}$ for its infimum. In particular, if I is finite, then $\vee_{i \in I} \lambda_i$ and $\wedge_{i \in I} \lambda_i$ are the greatest element and the least element of $\{\lambda_i : i \in I\}$, respectively.

Now, we are able to introduce the model of fuzzy DES. A fuzzy DES is modelled by a fuzzy automaton which is known as maxmin automaton in some mathematical literature [23], [6] and is somewhat different from max-product automata used in [12], [13].

Definition 3: A *fuzzy automaton* is a four-tuple $G = (Q, E, \delta, q_0)$, where:

- Q is a crisp (finite or infinite) set of states;
- E is a finite set of events;
- $q_0 \in Q$ is the initial state;
- δ is a function from $Q \times E \times Q$ to $[0, 1]$, called a fuzzy transition function.

For any $p,\ q \in Q$ and $a \in E$, we can interpret $\delta(p, a, q)$ as the possibility degree to which the automaton in state p and with the occurrence of event a may enter state q. Contrast to crisp DES, an event in fuzzy DES may take the system to more than one states with different degrees. The concept of fuzzy automata is a natural generalization of nondeterministic automata. The major difference between fuzzy automata and nondeterministic automata is: in a nondeterministic automaton, $\delta(p, a, q)$ is either 1 or 0, so the possibility degrees of existing transitions are the same, but if we work with a fuzzy automaton, they may be different.

An *extended fuzzy transition function* from $Q \times E^* \times Q$ to $[0, 1]$, denoted by the same notation δ, can be defined inductively as follows:

$$\delta(p, \epsilon, q) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(p, \omega a, q) = \vee_{r \in Q} (\delta(p, \omega, r) \wedge \delta(r, a, q))$$

for all $\omega \in E^*$ and $a \in E$. The possibility degrees of transitions here are given by the operation $max\ min$; this is the main difference between maxmin automata and max-product automata used in [12] and [13].

Further, the language $\mathcal{L}(G)$ generated by G, called *fuzzy language*, is defined as a fuzzy subset of E^* and given by

$$\mathcal{L}(G)(\omega) = \vee_{q \in Q} \delta(q_0, \omega, q).$$

This means that a string ω of E^* is not necessarily either "in the fuzzy language $\mathcal{L}(G)$" or "not in the fuzzy language $\mathcal{L}(G)$"; rather ω has a membership grade $\mathcal{L}(G)(\omega)$, which measures its degree of membership in $\mathcal{L}(G)$.

If the state set of a fuzzy automaton G is empty, then it yields that $\mathcal{L}(G) = \mathcal{O}$, i.e., $\mathcal{L}(G)(\omega) = 0$ for all $\omega \in E^*$; otherwise, from the definition we see that $\mathcal{L}(G)$ has the following properties:

P1) $\mathcal{L}(G)(\epsilon) = 1$;
P2) $\mathcal{L}(G)(\mu) \geq \mathcal{L}(G)(\mu\nu)$ for any $\mu, \nu \in E^*$.

Conversely, given a fuzzy language $\mathcal{L}$ (over the event set E) satisfying the above properties, we can construct a fuzzy automaton $G = (Q_\mathcal{L}, E_\mathcal{L}, \delta_\mathcal{L}, q_\mathcal{L})$, where $Q_\mathcal{L} = \text{supp}(\mathcal{L})$, $E_\mathcal{L} = E$, $q_\mathcal{L} = \epsilon$, and

$$\delta_\mathcal{L}(\mu, a, \nu) = \begin{cases} \mathcal{L}(\nu), & \text{if } \nu = \mu a \\ 0, & \text{otherwise} \end{cases}$$

for all $\mu, \nu \in \text{supp}(\mathcal{L})$ and $a \in E$. There is no difficulty to verify that $\mathcal{L}(G) = \mathcal{L}$. Thus a fuzzy language and a fuzzy DES amount to the same thing in our context, which is analogous to the equivalence of regular expressions and finite automata due to Kleene [7].

Note that in the rest of the paper, by a fuzzy language we mean the empty fuzzy language $\mathcal{O}$ or a fuzzy language that satisfies the above properties P1) and P2), unless otherwise specified. In particular, the supports of such fuzzy languages are prefix closed by the property P2). We use $\mathcal{FL}$ to denote the set of all fuzzy languages over E. More explicitly, $\mathcal{FL} = \{\mathcal{A} \in \mathcal{F}(E^*) : \mathcal{A} = \mathcal{O} \text{ or } \mathcal{A} \text{ satisfies P1) and P2)}\}$.

Union and intersection of fuzzy languages can be defined as follows:

$$(\cup_{i\in I}\mathcal{L}_i)(\omega) = \vee_{i\in I}\mathcal{L}_i(\omega), \text{ for all } \omega \in E^*, \text{ and}$$

$$(\cap_{i\in I}\mathcal{L}_i)(\omega) = \wedge_{i\in I}\mathcal{L}_i(\omega), \text{ for all } \omega \in E^*.$$

It has been shown in [1] that fuzzy languages are closed under arbitrary unions and intersections, respectively.

The concatenation $\mathcal{L}_1\mathcal{L}_2$ of two fuzzy languages $\mathcal{L}_1$ and $\mathcal{L}_2$, which is again a fuzzy language [1], is defined by

$$(\mathcal{L}_1\mathcal{L}_2)(\omega) = \vee\{\mathcal{L}_1(\mu) \wedge \mathcal{L}_2(\nu) : \mu, \nu \in E^* \text{ and } \mu\nu = \omega\},$$

for all $\omega \in E^*$.

To model the control action of fuzzy DES, we partition the event set E into *controllable* and *uncontrollable* events: $E = E_c \dot{\cup} E_{uc}$, as usually done in crisp DES. However, unlike controllable events in crisp DES, a controllable event in fuzzy DES can be disabled with any degree.

Control is achieved by means of a fuzzy supervisor, which is allowed to disable any fuzzy sets of controllable events after having observed an arbitrary string $s \in \text{supp}(\mathcal{L}(G))$. Formally, a (fully observable) *fuzzy supervisor* for G is a map $S : \text{supp}(\mathcal{L}(G)) \to \mathcal{F}(E)$ such that $S(s)(a) = 1$ for any $s \in \text{supp}(\mathcal{L}(G))$ and $a \in E_{uc}$. The controlled system is denoted by S/G; the behavior of S/G is the fuzzy language $\mathcal{L}^S$ obtained inductively as follows:

1) $\mathcal{L}^S(\epsilon) = 1$;
2) $\mathcal{L}^S(sa) = \mathcal{L}(G)(sa) \wedge S(s)(a) \wedge \mathcal{L}^S(s)$ for any $s \in E^*$ and $a \in E$.

To state the controllability of fuzzy languages, let us first introduce a fuzzy subset $\mathcal{E}_{uc}$ of E^*, which is given by

$$\mathcal{E}_{uc}(\omega) = \begin{cases} 1, & \text{if } \omega \in E_{uc} \\ 0, & \text{if } \omega \in E^* \backslash E_{uc}. \end{cases}$$

Definition 4: A fuzzy language $\mathcal{K} \subseteq \mathcal{L}(G)$ is said to be *controllable* [1] (with respect to $\mathcal{L}(G)$ and E_{uc}) if

$$\mathcal{K}\mathcal{E}_{uc} \cap \mathcal{L}(G) \subseteq \mathcal{K},$$

or equivalently, $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(G)(sa)$ for any $s \in E^*$ and $a \in E_{uc}$.

We remark that a similar concept, called fuzzy controllability condition which is the same as the above when considered for the same underlying constituents of fuzzy automata and uncontrollable event set, has been introduced independently by Qiu in [18].

It is clear that the fuzzy languages $\mathcal{O}$ and $\mathcal{L}(G)$ are always controllable with respect to $\mathcal{L}(G)$ and any E_{uc}. A good property of controllable fuzzy languages is that they are closed under arbitrary unions and intersections, respectively [1]. Let $\mathcal{K} \subseteq \mathcal{L}(G)$ be a nonempty fuzzy language. It has been proved in [1] that there exists a (fully observable) fuzzy supervisor S for G such that $\mathcal{L}^S = \mathcal{K}$ if and only if $\mathcal{K}$ is controllable.

III. Observability

Up to this point it has been assumed that all of the events in a fuzzy DES can be directly observed by the fuzzy supervisor. However, in practice we usually only have local or partial observations.

To model a fuzzy DES with partial observations, we bring an additional event set $E_o \subseteq E$ of observable events that can be seen by the fuzzy supervisor, and a natural projection $P : E \to E_o \cup \{\epsilon\}$, as usual in crisp DES. More formally, the event set E is partitioned into two disjoint subsets: $E = E_o \cup E_{uo}$, where E_o and E_{uo} denote the observable event set and the unobservable event set, respectively. The natural projection P is defined in the same way as crisp DES, that is,

$$P(a) = \begin{cases} a, & \text{if } a \in E_o \\ \epsilon, & \text{if } a \in E_{uo}. \end{cases}$$

The action of the projection P is extended to strings by defining $P(\epsilon) = \epsilon$, and $P(sa) = P(s)P(a)$ for any $s \in E^*$ and $a \in E$.

Due to the presence of P, the fuzzy supervisor cannot distinguish between two strings s and s' that have the same projection, namely, $P(s) = P(s')$. For such pairs, the fuzzy supervisor will necessarily issue the same control action. To capture this fact, we define a *partially observable fuzzy supervisor* as a map $S_P : P[\text{supp}(\mathcal{L}(G))] \to \mathcal{F}(E)$ satisfying $S(s)(a) = 1$ for any $s \in P[\text{supp}(\mathcal{L}(G))]$ and $a \in E_{uc}$. This means that the control action may change unless an observable event occurs. We shall assume that the control action is instantaneously (i.e. before any unobservable event occurs) updated once an observable event occurs. The assumption is necessary as we may wish to update the control action for some of these unobservable events.

The behavior of S_P/G when S_P is controlling G is defined analogously to the case of full observation.

Definition 5: The fuzzy language $\mathcal{L}^{S_P}$ generated by S_P/G is defined inductively as follows:

1) $\mathcal{L}^{S_P}(\epsilon) = 1$;
2) $\mathcal{L}^{S_P}(sa) = \mathcal{L}(G)(sa) \wedge S_P[P(s)](a) \wedge \mathcal{L}^{S_P}(s)$ for any $s \in E^*$ and $a \in E$.

Consequently, the possibility of an event a following a string s under a partially observable fuzzy supervisor only depends on the physically possible degree of sa in the controlled system, the enabled degree of a after the occurrence of $P(s)$, and the physically possible degree of s in the closed-loop system. Clearly, $\mathcal{L}^{S_P} \subseteq \mathcal{L}(G)$ and $\mathcal{L}^{S_P} \in \mathcal{FL}$. For simplicity, we sometimes write $\mathcal{L}$ for $\mathcal{L}(G)$.

Before introducing the concepts of observability and strong observability from the view of event strings, we remark that based on state estimation, Lin and Ying have already introduced the same terms with different essence to fuzzy DES in [12] and [13]. They have used observability measure to measure the observability of a fuzzy DES, which is a good approach to describing the global property of the system. For our purpose, we need a characterization of the (strong) observability of fuzzy sublanguages.

Definition 6: A fuzzy language $\mathcal{K} \subseteq \mathcal{L}$ is said to be *observable* (respectively, *strongly observable*) with respect to $\mathcal{L}$, P, and E_c if for all $s, s' \in \text{supp}(\mathcal{K})$ with $P(s) = P(s')$ and for any $a \in E_c$ satisfying $sa \in \text{supp}(\mathcal{K})$, we have that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x$ for some (respectively, any) $x \in \{x \in [0,1] : \mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x\}$.

We interpret x as the enabled degrees of controllable events when $\mathcal{L}$ is restricted to $\mathcal{K}$. For simplicity, we shall omit the range $[0,1]$ of x in the sequel. Intuitively, for observable fuzzy language, we require that for each controllable event, there is an enabled degree which can be used after seeing the two strings that are the same under the supervisor.

Remark 1: The parameter E_c is included in the definition in order for the property of observability not to overlap with the property of controllability. As we will see, observability will only be used in conjunction with controllability, and the latter ensures that the requirements of the definition hold for $a \in E_{uc}$.

In what follows, (strong) observability will always be with respect to $\mathcal{L}$, P, and E_c; controllability is with respect to $\mathcal{L}$ and E_{uc}. We simply call $\mathcal{K}$ (strongly) observable or controllable if the context is clear. Note that we do not make any specific assumptions about the relation between the controllability and observability properties of an event. Thus, an unobservable event could be controllable, an uncontrollable event could be observable, and so forth.

Remark 2: In terms of crisp languages where membership grades are either 1 or 0, the above observability as well as strong observability is the same as the observability in [9].

Remark 3: Clearly, $\mathcal{L}$ and $\mathcal{O}$ are always observable. It is obvious by definition that the concept of strong observability implies that of observability. But an observable fuzzy language need not to be strongly observable. For example, let

$$E = \{a, b\}, \quad E_c = E_o = \{b\},$$

and

$$\mathcal{L} = \frac{1}{\epsilon} + \frac{0.8}{a} + \frac{0.9}{b} + \frac{0.7}{ab}.$$

One can easily check that $\mathcal{L}$ is observable, but not strongly observable.

Intuitively, a strongly observable fuzzy language $\mathcal{K}$ allows the associated fuzzy supervisor to take more flexible control action than that of an observable fuzzy language, in order to exactly achieve $\mathcal{K}$. The cost of such flexibility is more constraint on $\mathcal{K}$ itself.

Let us provide some equivalent characterizations of observability.

Proposition 1: For a fuzzy language $\mathcal{K}$, the following are equivalent:

1) $\mathcal{K}$ is observable.

2) For any $s \in \text{supp}(\mathcal{K})$ and $a \in E_c$, there exists x such that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x$ for any $s' \in \{s' \in \text{supp}(\mathcal{K}) : P(s') = P(s)\}$.

3) For all $s, s' \in \text{supp}(\mathcal{K})$ with $P(s) = P(s')$, and any $a \in E_c$, there exists x such that both $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x$ and $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x$.

Proof: See Appendix II. ■

There is a difference between 2) and 3) above: in 2), we need for each controllable event, a common enabled degree for all the strings that are the same under the supervisor; in 3), we only need a common enabled degree for every two strings that are identical under the supervisor.

For strong observability, we have an equivalent characterization too.

Proposition 2: A fuzzy language $\mathcal{K}$ is strongly observable if and only if for any $s, s' \in \text{supp}(\mathcal{K})$ and $a \in E_c$ subject to $P(s) = P(s')$ and $sa, s'a \in \text{supp}(\mathcal{L})$, the following conditions are satisfied:

1) $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa)$ if and only if $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a)$;

2) $\mathcal{K}(sa) = \mathcal{K}(s'a)$.

Proof: See Appendix II. ■

The following results show us that (strongly) observable fuzzy languages are closed under intersection.

Proposition 3:

1) If $\mathcal{K}_1$ and $\mathcal{K}_2$ are observable, then so is $\mathcal{K}_1 \cap \mathcal{K}_2$.

2) If $\mathcal{K}_1$ and $\mathcal{K}_2$ are strongly observable, then so is $\mathcal{K}_1 \cap \mathcal{K}_2$.

Proof: See Appendix II. ■

The above proposition and proof remain true for arbitrary intersections.

Remark 4: Note that if $\mathcal{K}_1$ and $\mathcal{K}_2$ are strongly observable, we cannot obtain that $\mathcal{K}_1 \cup \mathcal{K}_2$ is (strongly) observable in general. The following counter-example serves:

Let $E = E_c = \{a, b\}$, $E_o = \{b\}$,

$$\mathcal{L} = \frac{1}{\epsilon} + \frac{0.9}{a} + \frac{0.8}{b} + \frac{0.7}{ab},$$

and

$$\mathcal{K}_1 = \frac{1}{\epsilon} + \frac{0.8}{a}, \ \mathcal{K}_2 = \frac{1}{\epsilon} + \frac{0.7}{b}.$$

Then

$$\mathcal{K}_1 \cup \mathcal{K}_2 = \frac{1}{\epsilon} + \frac{0.8}{a} + \frac{0.7}{b}.$$

It is evident that $\mathcal{K}_1$ and $\mathcal{K}_2$ are strongly observable; however, $\mathcal{K}_1 \cup \mathcal{K}_2$ is not (strongly) observable.

We are now ready to present the existence result for a partially observable fuzzy supervisor.

Theorem 1: Let $\mathcal{K} \subseteq \mathcal{L}(G)$, where $\mathcal{K}$ is a nonempty fuzzy language. Then there exists a partially observable fuzzy supervisor S_P for G such that $\mathcal{L}^{S_P} = \mathcal{K}$ if and only if $\mathcal{K}$ is controllable and observable.

Proof: See Appendix II. ■

The proof of Theorem 1 there is constructive in the sense that if the controllability and observability conditions are satisfied, it gives us a partially observable fuzzy supervisor that will achieve the specification. For illustrating the theorem and its proof, let us examine a simple example.

Example 1: Let $E = \{a, b, c, d\}$, $E_{uc} = \{d\}$, and $E_{uo} = \{c\}$. The fuzzy languages $\mathcal{L}$ and $\mathcal{K}$ are given by

$$\mathcal{L} = \frac{1}{\epsilon} + \frac{0.9}{a} + \frac{0.8}{ab} + \frac{0.8}{ad} + \frac{0.6}{ac} + \frac{0.4}{acb} + \frac{0.6}{acd},$$

$$\mathcal{K} = \frac{1}{\epsilon} + \frac{0.7}{a} + \frac{0.4}{ac} + \frac{0.7}{ad} + \frac{0.4}{acd}.$$

One can check by definitions that $\mathcal{K}$ is controllable and observable. The following partially observable fuzzy supervisor S_P that follows from the definition of $S_P[P(s)]$ given in the proof of Theorem 1 can achieve $\mathcal{K}$:

$$S_P(\epsilon) = S_P[P(\epsilon)] = \frac{0.7}{a} + \frac{0}{b} + \frac{0}{c} + \frac{1}{d},$$

$$S_P(a) = S_P[P(a)] = S_P[P(ac)] = \frac{0}{a} + \frac{0}{b} + \frac{0.4}{c} + \frac{1}{d},$$

$$S_P(ab) = S_P[P(ab)] = S_P[P(acb)] = \frac{0}{a} + \frac{0}{b} + \frac{0}{c} + \frac{1}{d},$$

$$S_P(ad) = S_P[P(ad)] = S_P[P(acd)] = \frac{0}{a} + \frac{0}{b} + \frac{0}{c} + \frac{1}{d}.$$

Let us check the membership grades of $S_P(a)$ as an example:
$S_P[a](d) = 1$ since $d \in E_{uc}$,
$S_P[a](a) = \vee\{\mathcal{K}(s'a) : P(s') = a\} = 0$,
$S_P[a](b) = \vee\{\mathcal{K}(s'b) : P(s') = a\} = \mathcal{K}(ab) \vee \mathcal{K}(acb) = 0$,
$S_P[a](c) = \vee\{\mathcal{K}(s'c) : P(s') = a\} = \mathcal{K}(ac) = 0.4$. ■

If a fuzzy language $\mathcal{K}$ does not satisfy the conditions of Theorem 1, then it is natural to consider the possibility of approximating $\mathcal{K}$. Let us first consider the existence of the "least" controllable and observable fuzzy superlanguage of $\mathcal{K}$. To this end, define

$$\mathscr{C}\mathscr{O}(\mathcal{K}) = \{\mathcal{M} \in \mathcal{FL} : \mathcal{K} \subseteq \mathcal{M} \subseteq \mathcal{L} \text{ and } \mathcal{M} \text{ is controllable and observable}\}.$$

Observe that $\mathcal{L} \in \mathscr{C}\mathscr{O}(\mathcal{K})$, so the class $\mathscr{C}\mathscr{O}(\mathcal{K})$ is not empty. Further, define $\mathcal{K}^{\downarrow(CO)} = \bigcap_{\mathcal{M} \in \mathscr{C}\mathscr{O}(\mathcal{K})} \mathcal{M}$. Then from Proposition 3 we have the following.

Proposition 4: The fuzzy language $\mathcal{K}^{\downarrow(CO)}$ is the least controllable and observable fuzzy superlanguage of $\mathcal{K}$.

Proof: Note that arbitrary intersections of controllable and observable fuzzy languages are again controllable and observable. As a result, the proposition evidently holds. ■

Since by definition $\mathcal{K}^{\downarrow(CO)} \subseteq \mathcal{M}$ for any $\mathcal{M} \in \mathscr{C}\mathscr{O}(\mathcal{K})$, we call $\mathcal{K}^{\downarrow(CO)}$ the *infimal controllable and observable fuzzy superlanguage* of $\mathcal{K}$. If $\mathcal{K}$ is controllable and observable, then $\mathcal{K}^{\downarrow(CO)} = \mathcal{K}$. In the "worst" case, $\mathcal{K}^{\downarrow(CO)} = \mathcal{L}$.

We end this section with the Supervisory Control Problem (SCP) for fuzzy DES with partial observation.

SCP: Given a fuzzy DES G with event set E, uncontrollable event set $E_{uc} \subseteq E$, observable event set $E_o \subseteq E$, and two fuzzy languages $\mathcal{L}_a$ and $\mathcal{L}_l$, where $\mathcal{O} \neq \mathcal{L}_a \subseteq \mathcal{L}_l \subseteq \mathcal{L}(G)$, find a partially observable fuzzy supervisor S_P such that $\mathcal{L}_a \subseteq \mathcal{L}^{S_P} \subseteq \mathcal{L}_l$.

Here, $\mathcal{L}_a$ describes the minimal acceptable behavior and $\mathcal{L}_l$ describes the maximal legal behavior. SCP requires to find a fuzzy supervisor such that the behavior of controlled system is both acceptable and legal. The following result provides an abstract solution to SCP.

Corollary 1: There exists a partially observable fuzzy supervisor S_P such that $\mathcal{L}_a \subseteq \mathcal{L}^{S_P} \subseteq \mathcal{L}_l$ if and only if $\mathcal{L}_a^{\downarrow(CO)} \subseteq \mathcal{L}_l$.

Proof: See Appendix II. ■

Although controllable fuzzy languages are closed under arbitrary unions, observable fuzzy languages are not closed under union in general. Consequently, it turns out that, in a given situation, a unique maximal controllable and observable sublanguage of $\mathcal{K}$ need not exist. To obtain an approximation using sublanguages in a reasonable manner, we will introduce a new subclass of fuzzy languages, called normal fuzzy languages, which is deferred to Appendix I since it is not necessary for subsequently discussing the decentralized control of fuzzy DES.

IV. Decentralized Control

In this section, we turn our attention to the decentralized supervisory control of a fuzzy DES that is physically distributed. Without loss of generality and for the sake of convenience, we consider the case that only two local partially observable fuzzy supervisors are used to realize the decentralized supervisory control.

The problem of decentralized control is formalized as follows. We have two partially observable fuzzy supervisors S_1 and S_2 (for simplicity, in the sequel we write S for S_P), each associated with a different projection $P_i, i = 1, 2$, jointly controlling the given system G with event set E. Associated with G are the four usual sets E_c, E_{uc}, E_o, and E_{uo}. Corresponding to fuzzy supervisors S_1 and S_2, we have:

- the sets of controllable events $E_{1c}, E_{2c} \subseteq E_c$ satisfying $E_{1c} \cup E_{2c} = E_c$;
- the sets of observable events $E_{1o}, E_{2o} \subseteq E_o$ satisfying $E_{1o} \cup E_{2o} = E_o$;
- the natural projection $P_i : E^* \to E_{io}^*$ corresponding to E_{io} for $i = 1, 2$.

Then the local partially observable fuzzy supervisor S_i, $i = 1, 2$, is given by $S_i : P_i[\text{supp}(\mathcal{L}(G))] \to \mathcal{F}(E)$ which satisfies $S_i(s)(a) = 1$ for any $s \in P_i[\text{supp}(\mathcal{L}(G))]$ and $a \in E \backslash E_{ic}$.

The behavior of the controlled system under the joint supervision of S_1 and S_2 is the fuzzy language $\mathcal{L}^{S_1 \wedge S_2}$ defined inductively by

1) $\mathcal{L}^{S_1 \wedge S_2}(\epsilon) = 1$;

2) $\mathcal{L}^{S_1 \wedge S_2}(sa) = \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s) \wedge S_1[P_1(s)](a) \wedge S_2[P_2(s)](a)$ for any $s \in E^*$ and $a \in E$.

Clearly, it follows from the definition of $\mathcal{L}^{S_1 \wedge S_2}$ that $\mathcal{L}^{S_1 \wedge S_2} \subseteq \mathcal{L}(G)$ and $\mathcal{L}^{S_1 \wedge S_2} \in \mathcal{FL}$.

Given a desired fuzzy language $\mathcal{K} \subseteq \mathcal{L}(G)$, our aim is to find the necessary and sufficient condition on $\mathcal{K}$ that will ensure the existence of S_1 and S_2 such that $\mathcal{L}^{S_1 \wedge S_2} = \mathcal{K}$. To this end, the concept of co-observability for fuzzy languages, as a generalization of co-observability for crisp languages [22], is required in place of observability appearing in the case of centralized control.

Definition 7: A fuzzy language $\mathcal{K} \subseteq \mathcal{L}$ is called *co-observable* (with respect to $\mathcal{L}$, P_i, and E_{ic}, $i = 1, 2$), if for all $s \in \text{supp}(\mathcal{K})$ and $a \in E_c = E_{1c} \cup E_{2c}$, the following hold:

1) if $a \in E_{1c} \cap E_{2c}$, then $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)} \mathcal{K}(s_1 a)] \wedge [\vee_{P_2(s_2)=P_2(s)} \mathcal{K}(s_2 a)]$;
2) if $a \in E_{1c} \backslash E_{2c}$, then $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)} \mathcal{K}(s_1 a)]$;
3) if $a \in E_{2c} \backslash E_{1c}$, then $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_2(s_2)=P_2(s)} \mathcal{K}(s_2 a)]$.

From the above definition, it is clear that the co-observability of fuzzy languages which takes the respective enabled degree of controllable events under two supervisors into account is a generation of observability.

Remark 5: Like observability, co-observability is also closed under arbitrary intersections; this can be easily verified by using the above definition. It is not difficult to check that co-observable fuzzy languages are not closed under union in general.

The property of co-observability describes the class of fuzzy languages that can be achieved by decentralized supervisory control in the situation of partial observation, as shown in the following.

Theorem 2: Let $\mathcal{K} \subseteq \mathcal{L}(G)$ be a nonempty fuzzy language. Then there exist two local partially observable fuzzy supervisors S_1 and S_2 for G such that $\mathcal{L}^{S_1 \wedge S_2} = \mathcal{K}$ if and only if $\mathcal{K}$ is controllable and co-observable.

Proof: See Appendix II. ■

The above theorem and its proof can be easily extended for more than two local fuzzy supervisors. Analogous to that of Theorem 1, the proof of Theorem 2 is also constructive, and we can obtain two supervisors if the given fuzzy sublanguage is controllable and co-observable. We present an example to illustrate the theorem in the next section.

V. An Illustrative Example

In this section, we apply the previous results to an example arising from medical diagnosis and treatment.

Suppose that there is a patient infected by a new infectious disease. The director decides to hold a consultation first, and then appoints one or two physicians to the patient's physicians-in-charge. All the physicians have no complete knowledge about the disease, but the physicians by their experience believe that two antibiotic drugs, say penicillin and chlortetracycline, may be useful to the disease. It is well known among medical doctors that the drug combination of penicillin and chlortetracycline may be have as little effect against an infection as prescribing no antibiotic drug at all, event if the bacteria are susceptible to each of these drugs. So the physicians decide to use the drugs separately. Moreover, the physicians think that the effect of penicillin is better than that of chlortetracycline for a patient who is in a poor condition and vice versa for a patient who is in a fair condition, since penicillin is a bactericidal antimicrobial drug, while chlortetracycline is a bacteriostatic one. In addition, the drug-resistance of bacteria and some possible negative symptoms such as fever, high white blood cell count, and increased sedimentation rate of the blood have also been taken into account by the physicians.

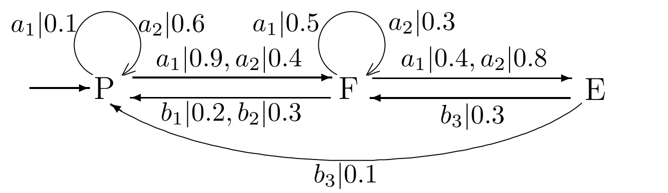

Fig. 1. Fuzzy automaton G to model a patient's condition.

Further, the physicians consider roughly the patient's condition to be three states: "poor", "fair", and "excellent", and agree that the present treatment must be stopped to finding other therapies once there are some negative symptoms indicating that the patient's condition reverts to the initial situation, i.e., the poor state. Such a status of diagnosis and treatment can be logically modelled via a fuzzy DES with supervisory control, in which the drugs and the negative symptoms are thought of as events. Suppose that for each event, the physicians by their experience have an estimation of the transition possibility among states (there are many methods for estimating membership grades; see, for example, pages 256-260 of [4]), and suppose that the dose of drug which is prescribed by the physician-in-charge can affect the transition possibility of the drug event. The drug events are considered to be controllable and observable, while the symptom events are uncontrollable and some of them are observable only by those physicians who are sensitive to or concerned about the symptoms.

We use a_1 and a_2 to denote the drug events, namely, penicillin and chlortetracycline. Denote by b_i, $i = 1, 2, 3$, the negative symptoms. According to the physicians' estimation, these events and their transition possibilities among states are depicted in Fig. 1, where the capital letters P, F, and E represent poor, fair, and excellent, respectively. This transition graph is a common knowledge among the physicians after consultation. Recall that the weight of a string is interpreted as the possibility of occurrence of the string and is obtained by using the operation $max\ min$. For example, the weight of the string $a_1 a_2$ is 0.8, which is the physical possibility to arrive at the excellent state after using penicillin and chlortetracycline in turn.

Any therapy is required to conform to the specification which is determined by the physicians according to their common knowledge and the patient's situation:

$$\mathcal{K} = \frac{1}{\epsilon} + \frac{0.9}{a_1} + \frac{0.8}{a_1 a_2} + \frac{0.2}{a_1 b_1} + \frac{0.3}{a_1 b_2} + \frac{0.2}{a_1 a_2 b_1} + \frac{0.3}{a_1 a_2 b_2}$$
$$+ \frac{0.3}{a_1 a_2 b_3} + \frac{0.2}{a_1 a_2 b_3 b_1} + \frac{0.3}{a_1 a_2 b_3 b_2} + \frac{0.2}{a_1 a_2 b_3 a_1} + \frac{0.2}{a_1 a_2 b_3 a_1 b_1}$$

$$+\frac{0.2}{a_1a_2b_3a_1b_2}+\frac{0.2}{a_1a_2b_3a_1b_3}+\frac{0.2}{a_1a_2b_3a_1b_3b_1}+\frac{0.2}{a_1a_2b_3a_1b_3b_2}.$$

Now, the director appoints a physician, say S_1, to the patient's physician-in-charge. We regard the physician-in-charge S_1 as a supervisor and suppose that $E_{1c} = \{a_1, a_2\}$ and $E_{1o} = \{a_1, b_1, a_2, b_3\}$. By definition, it is easy to check that $\mathcal{K}$ is controllable. But $\mathcal{K}$ is not observable by S_1. In fact, take $s = a_1, s' = a_1b_2 \in \text{supp}(\mathcal{K})$ and $a_2 \in E_{1c}$. We then see that $P_1(s) = P_1(s') = a_1$, but there is no x such that both $\mathcal{K}(sa_2) = \mathcal{K}(s) \wedge \mathcal{L}(sa_2) \wedge x$ and $\mathcal{K}(s'a_2) = \mathcal{K}(s') \wedge \mathcal{L}(s'a_2) \wedge x$, which contradicts 3) of Proposition 1. As a result, under the supervisory control of S_1, there is no hope of achieving the specification $\mathcal{K}$ by Theorem 1.

If the director takes possible medical errors into account and would like to appoint one more physician, say S_2, to the patient's physician-in-charge, and assume that $E_{2c} = \{a_1, a_2\}$ and $E_{2o} = \{a_1, b_2, a_2, b_3\}$. Then the specification $\mathcal{K}$ can be accomplished through the joint control of two physicians-in-charge, since $\mathcal{K}$ is now co-observable. From the proof of Theorem 2, we may take two supervisors S_1 and S_2 as follows:

$$S_1(\epsilon) = S_2(\epsilon) = \frac{0.9}{a_1} + \frac{0}{a_2} + \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3},$$

$$S_1(a_1) = S_2(a_1) = \frac{0}{a_1} + \frac{0.8}{a_2} + \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3},$$

$$S_1(a_1a_2b_3) = S_2(a_1a_2b_3) = \frac{0.2}{a_1} + \frac{0}{a_2} + \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3},$$

$$S_1(\omega_1) = S_2(\omega_2) = \frac{0}{a_1} + \frac{0}{a_2} + \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3}$$

for any $\omega_i \in P_i[\text{supp}(\mathcal{L}(G))]\backslash\{\epsilon, a_1, a_1a_2b_3\}, i = 1, 2$.

We compute the membership grades of $S_1(a_1a_2b_3)$ and $S_2(a_1a_2b_3)$ as an example. By the definition of $S_i[P_i(s)]$ given in the proof of Theorem 2, we get that $S_1(a_1a_2b_3)(b_j) = S_2(a_1a_2b_3)(b_j) = 1$ for $j = 1, 2, 3$, since $b_j \in E\backslash E_{1c}$, and that

$$\begin{aligned}
S_1(a_1a_2b_3)(a_1) &= \vee_{P_1(s_1)=a_1a_2b_3}\mathcal{K}(s_1a_1)\\
&= \mathcal{K}(a_1a_2b_3a_1) \vee \mathcal{K}(a_1a_2b_3b_2a_1)\\
&= 0.2 \vee 0 = 0.2,\\
S_1(a_1a_2b_3)(a_2) &= \vee_{P_1(s_1)=a_1a_2b_3}\mathcal{K}(s_1a_2)\\
&= \mathcal{K}(a_1a_2b_3a_2) \vee \mathcal{K}(a_1a_2b_3b_2a_2)\\
&= 0 \vee 0 = 0,\\
S_2(a_1a_2b_3)(a_1) &= \vee_{P_2(s_2)=a_1a_2b_3}\mathcal{K}(s_2a_1)\\
&= \mathcal{K}(a_1a_2b_3a_1) \vee \mathcal{K}(a_1a_2b_3b_1a_1)\\
&= 0.2 \vee 0 = 0.2,\\
S_2(a_1a_2b_3)(a_2) &= \vee_{P_2(s_2)=a_1a_2b_3}\mathcal{K}(s_2a_2)\\
&= \mathcal{K}(a_1a_2b_3a_2) \vee \mathcal{K}(a_1a_2b_3b_1a_2)\\
&= 0 \vee 0 = 0.
\end{aligned}$$

VI. CONCLUSION

In this paper, we have established and studied the centralized and decentralized supervisory control problem for fuzzy DES with partial observation. Observability, normality, and co-observability of crisp languages have been extended to fuzzy languages. We have elaborated on the necessary and sufficient conditions on a given fuzzy language for the existence of a partially observable fuzzy supervisor and local fuzzy supervisors, respectively. Moreover, we have discussed the infimal controllable and observable fuzzy superlanguage, and the supremal controllable and normal fuzzy sublanguage. These concepts and results are consistent with the existing theory in the framework of Ramadge–Wonham. Moreover, we have introduced the strong observability of fuzzy languages for comparison with the observability.

The supervisory control formalization presented here can be extended in many ways. Firstly, formulae and computation relating controllability, normality, and co-observability of fuzzy languages are yet to be established. Secondly, using non-conjunctive fusion rules [17], [26] to investigate decentralized control of fuzzy DES is feasible. Finally, decentralized supervisory control with local specification remains an interesting problem.

APPENDIX I
NORMALITY

In this appendix, we first introduce the concept of normal fuzzy languages which is a generalization of normality for crisp languages in the sense of Lin and Wonham [10]. Then we consider the existence of the "largest" controllable and normal fuzzy sublanguage, and also discuss the relation between normality, observability, and controllability.

Recall that we have defined the natural projection $P : E^* \to E_o^*$ whose effect on a string s is to erase the elements of s that are not in E_o. For later need, we extend P to $\mathcal{P} : \mathcal{F}(E^*) \to \mathcal{F}(E_o^*)$ via

$$\mathcal{P}(\mathcal{K})(\omega) = \vee_{P(\omega')=\omega}\mathcal{K}(\omega')$$

for any $\mathcal{K} \in \mathcal{F}(E^*)$ and $\omega \in E_o^*$.

We also define $\mathcal{P}^{-1} : \mathcal{F}(E_o^*) \to \mathcal{F}(E^*)$ as follows:

$$\mathcal{P}^{-1}(\mathcal{M})(\omega) = \mathcal{M}(P(\omega))$$

for any $\mathcal{M} \in \mathcal{F}(E_o^*)$ and $\omega \in E^*$.

Clearly, the definitions of $\mathcal{P}$ and $\mathcal{P}^{-1}$ are well-defined; furthermore, we have the following.

Proposition 5: Let $\mathcal{P}$ and $\mathcal{P}^{-1}$ be defined as above.

1) If $\mathcal{K}$ is a fuzzy language over E, then $\mathcal{P}(\mathcal{K})$ is a fuzzy language over E_o.

2) If $\mathcal{M}$ is a fuzzy language over E_o, then $\mathcal{P}^{-1}(\mathcal{M})$ is a fuzzy language over E.

Proof: See Appendix II. ■

Recall that in [10] (cf. the definition of recognizable languages in [3]) a crisp language $K \subseteq L$ is called *normal* (with respect to L and P) if $\overline{K} = P^{-1}[P(\overline{K})] \cap L$. For normality of fuzzy languages, we have to incorporate membership grades into the normality of their supports as crisp languages. Keeping the previous notation, we can now introduce the normality of fuzzy languages.

Definition 8: A fuzzy language $\mathcal{K} \subseteq \mathcal{L}$ is said to be *normal* (with respect to $\mathcal{L}$ and P) if

$$\mathcal{K} = \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})] \cap \mathcal{L}.$$

Intuitively, normality for a fuzzy language means that $\mathcal{K}$ can be exactly recovered from its projection $\mathcal{P}(\mathcal{K})$ and $\mathcal{L}$ itself. Observe that both $\mathcal{O}$ and $\mathcal{L}$ are normal with respect to $\mathcal{L}$ and P, so the above property is not vacuous.

The notion of fuzzy language normality reduces to normality of crisp languages if membership grades in fuzzy languages are only allowed to be either 0 or 1. The following relationship between them provides a necessary condition for a fuzzy language to be normal.

Proposition 6: If a fuzzy language $\mathcal{K} \subseteq \mathcal{L}$ is normal with respect to $\mathcal{L}$ and P, then the crisp language supp$(\mathcal{K})$ is normal with respect to supp$(\mathcal{L})$ and P in the sense of crisp language normality.

Proof: See Appendix II. ■

A desired property of normal fuzzy languages is that they are closed under arbitrary unions, as shown below.

Proposition 7: If for each $i \in I$, $\mathcal{K}_i \subseteq \mathcal{L}$ is normal with respect to $\mathcal{L}$ and P, then so is $\bigcup_{i\in I} \mathcal{K}_i$.

Proof: See Appendix II. ■

The following theorem shows us that normality is stronger than observability.

Theorem 3: If $\mathcal{K} \subseteq \mathcal{L}$ is normal with respect to $\mathcal{L}$ and P, then $\mathcal{K}$ is observable with respect to $\mathcal{L}$, P, and E_c for any $E_c \subseteq E$.

Proof: See Appendix II. ■

Remark 6: The converse statement of the above theorem is not true in general. For instance, $\mathcal{K}_1$ in Remark 4 is observable; however, it is not normal.

We are now in the position to present an approximation of $\mathcal{K}$ using controllable and observable sublanguages, as promised. For this, let us introduce a new class of fuzzy sublanguages of $\mathcal{K}$ as follows:

$$\mathscr{C}\mathscr{N}(\mathcal{K}) = \{\mathcal{M} \subseteq \mathcal{K} : \mathcal{M} \text{ is controllable and normal}\}.$$

Observe that $\mathcal{O} \in \mathscr{C}\mathscr{N}(\mathcal{K})$, so the class is not empty.

Define $\mathcal{K}^{\uparrow(CN)} = \bigcup_{\mathcal{M}\in\mathscr{C}\mathscr{N}(\mathcal{K})} \mathcal{M}$. Next, using Proposition 7 we can establish the existence of the supremal controllable and normal fuzzy sublanguage of $\mathcal{K}$.

Proposition 8: The fuzzy language $\mathcal{K}^{\uparrow(CN)}$ is the largest controllable and normal fuzzy sublanguage of $\mathcal{K}$.

Proof: Note that arbitrary unions of controllable and normal fuzzy languages are again controllable and normal. Consequently, the proposition holds obviously. ■

It should be pointed out that $\mathcal{K}^{\uparrow(CN)}$ may not in general a maximal controllable and observable fuzzy sublanguage of $\mathcal{K}$. In other words, there may be controllable and observable fuzzy sublanguages that are strictly larger than the supremal controllable and normal fuzzy sublanguage.

As shown earlier, an observable fuzzy language is not normal in general; however, under certain conditions, observability does imply normality.

Theorem 4: Suppose that $E_c \subseteq E_o$. If $\mathcal{K} \subseteq \mathcal{L}$ is controllable (with respect to $\mathcal{L}$ and E_{uc}) and observable (with respect to $\mathcal{L}$, P, and E_c), then $\mathcal{K}$ is normal (with respect to $\mathcal{L}$ and P).

Proof: See Appendix II. ■

The importance of this theorem lies in the fact that when the assumption $E_c \subseteq E_o$ is fulfilled, that is, when all the controllable events are observable, or equivalently when all the unobservable events are uncontrollable, the theorem implies that the supremal controllable and observable fuzzy sublanguage does exist, which is given by $\mathcal{K}^{\uparrow(CN)}$.

APPENDIX II
PROOFS

For the need of proofs, we make an observation first.

Lemma 1: Let $\mathcal{K}$ be an observable fuzzy language, $s_1, s_2 \in$ supp$(\mathcal{K})$ with $P(s_1) = P(s_2)$, and $a \in E_c$.

1) If $\mathcal{K}(s_ia) < \mathcal{K}(s_i) \wedge \mathcal{L}(s_ia)$ for $i = 1, 2$, then $\mathcal{K}(s_1a) = \mathcal{K}(s_2a)$.

2) If $\mathcal{K}(s_1a) < \mathcal{K}(s_1) \wedge \mathcal{L}(s_1a)$ and $\mathcal{K}(s_2a) = \mathcal{K}(s_2) \wedge \mathcal{L}(s_2a)$, then $\mathcal{K}(s_2a) \leq \mathcal{K}(s_1a)$.

Proof:

1): If $s_1a \notin$ supp$(\mathcal{K})$ and $s_2a \notin$ supp$(\mathcal{K})$, then $\mathcal{K}(s_1a) = \mathcal{K}(s_2a) = 0$. Otherwise, there is s_ia, say s_2a, in supp$(\mathcal{K})$. By the definition of observability, there exists x such that $\mathcal{K}(s_1a) = \mathcal{K}(s_1) \wedge \mathcal{L}(s_1a) \wedge x$ and $\mathcal{K}(s_2a) = \mathcal{K}(s_2) \wedge \mathcal{L}(s_2a) \wedge x$. From the assumption that $\mathcal{K}(s_ia) < \mathcal{K}(s_i) \wedge \mathcal{L}(s_ia)$ for $i = 1, 2$, we see that $\mathcal{K}(s_1a) = x = \mathcal{K}(s_2a)$, as desired.

2): If $\mathcal{K}(s_2a) = 0$, then it is obvious that $\mathcal{K}(s_2a) \leq \mathcal{K}(s_1a)$. In the case of $\mathcal{K}(s_2a) > 0$, there exists x such that $\mathcal{K}(s_1a) = \mathcal{K}(s_1) \wedge \mathcal{L}(s_1a) \wedge x$ and $\mathcal{K}(s_2a) = \mathcal{K}(s_2) \wedge \mathcal{L}(s_2a) \wedge x$ by the definition of observability. Hence, it follows from the conditions of 2) that $\mathcal{K}(s_1a) = x$ and $\mathcal{K}(s_2a) \leq x$. Consequently, $\mathcal{K}(s_2a) \leq \mathcal{K}(s_1a)$. ■

Proof of Proposition 1:

1)⇒2): For any $s \in$ supp$(\mathcal{K})$ and $a \in E_c$, set $[s] = \{s' \in$ supp$(\mathcal{K}) : P(s') = P(s)\}$ and $x_{[s]a} = \vee_{s'\in[s]}\mathcal{K}(s'a)$. We claim that $x = x_{[s]a}$ satisfies $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x$ for any $s' \in [s]$. In fact, if $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a)$, then the claim evidently holds since $\mathcal{K}(s'a) \leq x_{[s]a}$. If $\mathcal{K}(s'a) < \mathcal{K}(s') \wedge \mathcal{L}(s'a)$, then by Lemma 1 we have that $x_{[s]a} = \vee_{s'\in[s]}\mathcal{K}(s'a) = \mathcal{K}(s'a)$, and the claim holds too.

Observe that 2)⇒3) and 3)⇒1) are obvious, so the proposition is true. ■

Proof of Proposition 2: We first prove the necessity. Let $s, s' \in$ supp$(\mathcal{K})$ and $a \in E_c$ such that $P(s) = P(s')$ and $sa, s'a \in$ supp$(\mathcal{L})$.

For condition 1), we only prove the 'only if' part; the 'if' part is symmetric. Suppose that $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa)$. From the assumptions that $s \in$ supp$(\mathcal{K})$ and $sa \in$ supp$(\mathcal{L})$, we see that $\mathcal{K}(sa) > 0$. Thus by the definition of strong observability, we know that for any x, if $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x$, then $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x$. In particular, taking $x = 1$, we get that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge 1 = \mathcal{K}(s') \wedge \mathcal{L}(s'a)$.

To verify condition 2), let us first show the fact that $\mathcal{K}(sa) = 0$ if and only if $\mathcal{K}(s'a) = 0$. We prove the 'if' part; the 'only if' part is similar. Suppose that $\mathcal{K}(s'a) = 0$. By contradiction, assume that $\mathcal{K}(sa) > 0$. Then by the definition of strong observability, for any $x \in \{x : \mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x\}$, we must have that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x$. However, taking $x = \mathcal{K}(sa)$, we see that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge \mathcal{K}(sa) > 0$ since $s' \in \text{supp}(\mathcal{K})$ and $s'a \in \text{supp}(\mathcal{L})$ by the assumptions. It is a contradiction. Next, we consider the case that both $\mathcal{K}(sa) > 0$ and $\mathcal{K}(s'a) > 0$. By the foregoing property P2) we see that $\mathcal{K}(sa) \leq \mathcal{K}(s)$ and $\mathcal{K}(s'a) \leq \mathcal{K}(s')$. Note that $\mathcal{K} \subseteq \mathcal{L}$ which means that $\mathcal{K}(\omega) \leq \mathcal{L}(\omega)$ for any ω. Consequently, $\mathcal{K}(sa) \leq \mathcal{K}(s) \wedge \mathcal{L}(sa)$ and $\mathcal{K}(s'a) \leq \mathcal{K}(s') \wedge \mathcal{L}(s'a)$. By 1), there are only two cases: The first is that both $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa)$ and $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a)$; the second is that both $\mathcal{K}(sa) < \mathcal{K}(s) \wedge \mathcal{L}(sa)$ and $\mathcal{K}(s'a) < \mathcal{K}(s') \wedge \mathcal{L}(s'a)$. For the first case, by taking $x = \mathcal{K}(sa)$ in the definition of strong observability, we have that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge \mathcal{K}(sa)$, which yields that $\mathcal{K}(s'a) \leq \mathcal{K}(sa)$. By interchanging s and s', and taking $x = \mathcal{K}(s'a)$, we get that $\mathcal{K}(sa) \leq \mathcal{K}(s'a)$. Hence $\mathcal{K}(sa) = \mathcal{K}(s'a)$ in the first case. For the second case, noting that a strongly observable fuzzy language is observable, we obtain that $\mathcal{K}(sa) = \mathcal{K}(s'a)$ from Lemma 1, thus finishing the proof of the necessity.

Now, let us show the sufficiency. Suppose that $s, s' \in \text{supp}(\mathcal{K})$ with $P(s) = P(s')$, $a \in E_c$ satisfying $sa \in \text{supp}(\mathcal{K})$, and $x_0 \in \{x : \mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x\}$. It needs to verify that $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x_0$.

If $\mathcal{L}(s'a) = 0$, then it forces that $\mathcal{K}(s'a) = 0$, and thus $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x_0$.

In the case of $\mathcal{L}(s'a) > 0$, conditions 1) and 2) can be applicable. Further, if $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa)$, then we get that $\{x \in [0, 1] : \mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x\} = [\mathcal{K}(sa), 1]$ and $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a)$ by condition 1). Therefore, $x_0 \geq \mathcal{K}(s'a)$ by condition 2), and thus $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x_0$ holds. If $\mathcal{K}(sa) < \mathcal{K}(s) \wedge \mathcal{L}(sa)$, then $\{x : \mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge x\} = \{\mathcal{K}(sa)\}$ and $\mathcal{K}(s'a) < \mathcal{K}(s') \wedge \mathcal{L}(s'a)$ by condition 1). Hence, $x_0 = \mathcal{K}(s'a)$ by condition 2) and $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(s'a) \wedge x_0$ holds. This completes the proof of the proposition. ■

Proof of Proposition 3:

1) Suppose that $s, s' \in \text{supp}(\mathcal{K}_1 \cap \mathcal{K}_2)$ and $a \in E_c$ such that $P(s) = P(s')$ and $sa \in \text{supp}(\mathcal{K}_1 \cap \mathcal{K}_2)$. Since $\mathcal{K}_i$, $i = 1, 2$ is observable, there exists x_i such that $\mathcal{K}_i(sa) = \mathcal{K}_i(s) \wedge \mathcal{L}(sa) \wedge x_i$ and $\mathcal{K}_i(s'a) = \mathcal{K}_i(s') \wedge \mathcal{L}(s'a) \wedge x_i$. Setting $x = x_1 \wedge x_2$, we can easily check that $(\mathcal{K}_1 \cap \mathcal{K}_2)(sa) = (\mathcal{K}_1 \cap \mathcal{K}_2)(s) \wedge \mathcal{L}(sa) \wedge x$ and $(\mathcal{K}_1 \cap \mathcal{K}_2)(s'a) = (\mathcal{K}_1 \cap \mathcal{K}_2)(s') \wedge \mathcal{L}(s'a) \wedge x$. It follows from definition that $\mathcal{K}_1 \cap \mathcal{K}_2$ is observable.

2) By contradiction, suppose that $\mathcal{K}_1 \cap \mathcal{K}_2$ is not strongly observable. Then by definition there exist $s, s' \in \text{supp}(\mathcal{K}_1 \cap \mathcal{K}_2)$ with $P(s) = P(s')$, $a \in E_c$, and x such that $sa \in \text{supp}(\mathcal{K}_1 \cap \mathcal{K}_2)$ and

$$\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) = \mathcal{K}_1(s) \wedge \mathcal{K}_2(s) \wedge \mathcal{L}(sa) \wedge x, \quad (1)$$

but

$$\mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a) \neq \mathcal{K}_1(s') \wedge \mathcal{K}_2(s') \wedge \mathcal{L}(s'a) \wedge x. \quad (2)$$

The last inequality implies that $s'a \in \text{supp}(\mathcal{L})$; otherwise, both sides of inequality (2) are zero. As $sa \in \text{supp}(\mathcal{K}_1 \cap \mathcal{K}_2)$, we get that $sa \in \text{supp}(\mathcal{L})$. Thus Proposition 2 is applicable to $\mathcal{K}_1$ and $\mathcal{K}_2$. As a result, we have the following: for $i = 1, 2$,

$$\mathcal{K}_i(sa) = \mathcal{K}_i(s) \wedge \mathcal{L}(sa) \Leftrightarrow \mathcal{K}_i(s'a) = \mathcal{K}_i(s') \wedge \mathcal{L}(s'a), \quad (3)$$

and

$$\mathcal{K}_i(sa) = \mathcal{K}_i(s'a). \quad (4)$$

Four cases need to be discussed:

Case 1: $\mathcal{K}_i(s'a) = \mathcal{K}_i(s') \wedge \mathcal{L}(s'a)$, $i = 1, 2$. Using (3), we see that $\mathcal{K}_i(sa) = \mathcal{K}_i(s) \wedge \mathcal{L}(sa)$. Applying them to (1) yields that $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) = \mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) \wedge x$. However, we find from (2) and (4) that $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) \neq \mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) \wedge x$. This is a contradiction.

Case 2: $\mathcal{K}_i(s'a) < \mathcal{K}_i(s') \wedge \mathcal{L}(s'a)$, $i = 1, 2$. Using (3) again, we have that $\mathcal{K}_i(sa) < \mathcal{K}_i(s) \wedge \mathcal{L}(sa)$. Hence $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) < \mathcal{K}_1(s) \wedge \mathcal{K}_2(s) \wedge \mathcal{L}(sa)$, which means that $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) = x$ by (1). Using (4) and $\mathcal{K}_i(s'a) < \mathcal{K}_i(s') \wedge \mathcal{L}(s'a)$, we have that $x = \mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) = \mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a) < \mathcal{K}_1(s') \wedge \mathcal{K}_2(s') \wedge \mathcal{L}(s'a)$. This forces that $\mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a) = \mathcal{K}_1(s') \wedge \mathcal{K}_2(s') \wedge \mathcal{L}(s'a) \wedge x$, which contradicts with (2).

Case 3: $\mathcal{K}_1(s'a) = \mathcal{K}_1(s') \wedge \mathcal{L}(s'a)$ and $\mathcal{K}_2(s'a) < \mathcal{K}_2(s') \wedge \mathcal{L}(s'a)$. By (3), we also have that $\mathcal{K}_1(sa) = \mathcal{K}_1(s) \wedge \mathcal{L}(sa)$ and $\mathcal{K}_2(sa) < \mathcal{K}_2(s) \wedge \mathcal{L}(sa)$. In the subcase of $\mathcal{K}_1(sa) \leq \mathcal{K}_2(sa)$, we also get that $\mathcal{K}_1(s'a) \leq \mathcal{K}_2(s'a)$ by (4). Thus $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) = \mathcal{K}_1(s) \wedge \mathcal{K}_2(s) \wedge \mathcal{L}(sa)$ and $\mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a) = \mathcal{K}_1(s') \wedge \mathcal{K}_2(s') \wedge \mathcal{L}(s'a)$. From the former and (1), we see that $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) = \mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) \wedge x$; from the latter and (2), we see that $\mathcal{K}_1(s'a) \wedge \mathcal{K}_2('sa) \neq \mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a) \wedge x$. This is absurd by (4). In the other subcase, i.e., $\mathcal{K}_1(sa) > \mathcal{K}_2(sa)$, we see that $\mathcal{K}_1(s'a) > \mathcal{K}_2(s'a)$ from (4). Further, we have that $\mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa) < \mathcal{K}_1(s) \wedge \mathcal{K}_2(s) \wedge \mathcal{L}(sa)$ and $\mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a) < \mathcal{K}_1(s') \wedge \mathcal{K}_2(s') \wedge \mathcal{L}(s'a)$. The former and (1) force that $x = \mathcal{K}_1(sa) \wedge \mathcal{K}_2(sa)$. Therefore $x = \mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a)$, and thus $\mathcal{K}_1(s') \wedge \mathcal{K}_2(s') \wedge \mathcal{L}(s'a) \wedge x = \mathcal{K}_1(s'a) \wedge \mathcal{K}_2(s'a)$, which contradicts with (2).

Case 4: $\mathcal{K}_1(s'a) < \mathcal{K}_1(s') \wedge \mathcal{L}(s'a)$ and $\mathcal{K}_2(s'a) = \mathcal{K}_2(s') \wedge \mathcal{L}(s'a)$. This case is symmetric to Case 3, so we omit its proof. ■

Proof of Theorem 1: We prove the sufficiency first. Suppose that $\mathcal{K}$ is both controllable and observable. For any $P(s) \in P[\text{supp}(\mathcal{L}(G))]$, let us define

$$S_P[P(s)](a) = \begin{cases} 1, & \text{if } a \in E_{uc} \\ \vee\{\mathcal{K}(s'a) : P(s') = P(s)\}, & \text{if } a \in E_c. \end{cases}$$

Clearly, with the above definition S_P is a partially observable fuzzy supervisor.

It remains to verify that $\mathcal{L}^{S_P} = \mathcal{K}$. We prove this by using induction on the length of strings. The base case is for strings of length 0. By the definition of $\mathcal{L}^{S_P}$, we see that $\mathcal{L}^{S_P}(\epsilon) = 1 = \mathcal{K}(\epsilon)$. So the base case holds. The induction hypothesis is that $\mathcal{L}^{S_P}(s) = \mathcal{K}(s)$ for all strings s with length n. We now prove the same for strings sa, where $a \in E$. In the case of $a \in E_{uc}$, we obtain by definition that

$$\begin{aligned} \mathcal{L}^{S_P}(sa) &= \mathcal{L}(G)(sa) \wedge S_P[P(s)](a) \wedge \mathcal{L}^{S_P}(s) \\ &= \mathcal{L}(G)(sa) \wedge \mathcal{L}^{S_P}(s) \text{ (by definition of } S_P) \\ &= \mathcal{L}(G)(sa) \wedge \mathcal{K}(s) \text{ (using induction hypothesis)} \\ &= \mathcal{K}(sa), \text{ (by controllability of } \mathcal{K}) \end{aligned}$$

i.e., $\mathcal{L}^{S_P}(sa) = \mathcal{K}(sa)$. Now, let us consider the other case $a \in E_c$. In this case,

$$\begin{aligned}\mathcal{L}^{S_P}(sa) &= \mathcal{L}(G)(sa) \wedge S_P[P(s)](a) \wedge \mathcal{L}^{S_P}(s)\\ &= \mathcal{L}(G)(sa) \wedge S_P[P(s)](a) \wedge \mathcal{K}(s)\\ &= \mathcal{L}(G)(sa) \wedge [\vee\{\mathcal{K}(s'a) : P(s') = P(s)\}]\\ &\quad \wedge \mathcal{K}(s). \qquad (5)\end{aligned}$$

Thus, it is clear that $\mathcal{L}^{S_P}(sa) \geq \mathcal{K}(sa)$. By contradiction, suppose that $\mathcal{L}^{S_P}(sa) > \mathcal{K}(sa)$. Then from (5) we get that $\mathcal{L}(G)(sa) \wedge \mathcal{K}(s) > \mathcal{K}(sa)$ and $\vee\{\mathcal{K}(s'a) : P(s') = P(s)\} > \mathcal{K}(sa)$. So there exists s' with $P(s') = P(s)$ such that $\mathcal{K}(s'a) > \mathcal{K}(sa)$.

If $\mathcal{K}(sa) > 0$, then it is necessary that $s, s' \in \text{supp}(\mathcal{K})$. Furthermore, by the observability of $\mathcal{K}$, there exists x such that both $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(G)(sa) \wedge x$ and $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(G)(s'a) \wedge x$. The first equality, together with the previous argument that $\mathcal{L}(G)(sa) \wedge \mathcal{K}(s) > \mathcal{K}(sa)$, forces that $x = \mathcal{K}(sa)$; the second equality implies that $x \geq \mathcal{K}(s'a)$. Consequently, $\mathcal{K}(sa) \geq \mathcal{K}(s'a)$, which contradicts with $\mathcal{K}(s'a) > \mathcal{K}(sa)$.

In the case of $\mathcal{K}(sa) = 0$, if $\vee\{\mathcal{K}(s'a) : P(s') = P(s)\} = 0$, then we see from (5) that $\mathcal{L}^{S_P}(sa) = 0 = \mathcal{K}(sa)$; otherwise, there exists s' such that $\mathcal{K}(s'a) > 0$ and $P(s') = P(s)$. Clearly, $s' \in \text{supp}(\mathcal{K})$. Note also that $s \in \text{supp}(\mathcal{K})$ since $\mathcal{L}(G)(sa) \wedge \mathcal{K}(s) > \mathcal{K}(sa)$. Again, by the observability of $\mathcal{K}$, there exists x such that both $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(G)(s'a) \wedge x$ and $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(G)(sa) \wedge x$. The second equality, together with the fact that $\mathcal{L}(G)(sa) \wedge \mathcal{K}(s) > \mathcal{K}(sa)$, forces that $x = \mathcal{K}(sa) = 0$. Therefore $\mathcal{K}(s'a) = \mathcal{K}(s') \wedge \mathcal{L}(G)(s'a) \wedge x = 0$, which contradicts with $\mathcal{K}(s'a) > 0$. This completes the proof of the induction step.

Next, to see the necessity, suppose that there exists a partially observable fuzzy supervisor S_P for G such that $\mathcal{L}^{S_P} = \mathcal{K}$. For controllability, by definition it suffices to show that $\mathcal{L}^{S_P}(sa) = \mathcal{L}^{S_P}(s) \wedge \mathcal{L}(G)(sa)$ for any $s \in E^*$ and $a \in E_{uc}$. In fact, by definition we have that

$$\begin{aligned}\mathcal{L}^{S_P}(sa) &= \mathcal{L}^{S_P}(s) \wedge S_P[P(s)](a) \wedge \mathcal{L}(G)(sa)\\ &= \mathcal{L}^{S_P}(s) \wedge \mathcal{L}(G)(sa),\end{aligned}$$

since $S_P[P(s)](a) = 1$ for each $a \in E_{uc}$. To prove the observability of $\mathcal{K}$, take any $s, s' \in \text{supp}(\mathcal{K})$ with $P(s) = P(s')$ and $a \in E_c$ such that $\mathcal{K}(sa) > 0$. Selecting $x = S_P[P(s)](a)$, we have that

$$\begin{aligned}\mathcal{K}(sa) &= \mathcal{L}^{S_P}(sa)\\ &= \mathcal{L}^{S_P}(s) \wedge \mathcal{L}(G)(sa) \wedge S_P[P(s)](a)\\ &= \mathcal{K}(s) \wedge \mathcal{L}(G)(sa) \wedge x\end{aligned}$$

and

$$\begin{aligned}\mathcal{K}(s'a) &= \mathcal{L}^{S_P}(s'a)\\ &= \mathcal{L}^{S_P}(s') \wedge \mathcal{L}(G)(s'a) \wedge S_P[P(s')](a)\\ &= \mathcal{K}(s') \wedge \mathcal{L}(G)(s'a) \wedge S_P[P(s)](a)\\ &= \mathcal{K}(s') \wedge \mathcal{L}(G)(s'a) \wedge x.\end{aligned}$$

Hence, $\mathcal{K}$ is observable. The proof of the theorem is completed. ∎

Proof of Corollary 1: Suppose that there is a partially observable fuzzy supervisor S_P such that $\mathcal{L}_a \subseteq \mathcal{L}^{S_P} \subseteq \mathcal{L}_l$. Then by Theorem 1 we know that $\mathcal{L}^{S_P} \supseteq \mathcal{L}_a$ is both controllable and observable. Note that $\mathcal{L}_a^{\downarrow(CO)}$ is the infimal controllable and observable fuzzy superlanguage of $\mathcal{L}_a$ by Proposition 4, so $\mathcal{L}_a^{\downarrow(CO)} \subseteq \mathcal{L}^{S_P} \subseteq \mathcal{L}_l$.

Conversely, assume that $\mathcal{L}_a^{\downarrow(CO)} \subseteq \mathcal{L}_l$. Clearly, the controllable and observable fuzzy language $\mathcal{L}_a^{\downarrow(CO)}$ is not the empty fuzzy language. Therefore by Theorem 1 there is a partially observable fuzzy supervisor S_P such that $\mathcal{L}^{S_P} = \mathcal{L}_a^{\downarrow(CO)}$, and thus $\mathcal{L}_a \subseteq \mathcal{L}^{S_P} \subseteq \mathcal{L}_l$. ∎

Proof of Theorem 2: We first prove the necessity. Suppose that there exist two local partially observable fuzzy supervisors S_1 and S_2 for G such that $\mathcal{L}^{S_1 \wedge S_2} = \mathcal{K}$. To see the controllability of $\mathcal{K}$, let $s \in E^*$ and $a \in E_{uc}$. Then we obtain that

$$\begin{aligned}\mathcal{K}(sa) &= \mathcal{L}^{S_1 \wedge S_2}(sa) \quad \text{(by assumption)}\\ &= \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s) \wedge S_1[P_1(s)](a)\\ &\quad \wedge S_2[P_2(s)](a) \quad \text{(by definition)}\\ &= \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s). \quad (\text{since } a \in E_{uc})\end{aligned}$$

Hence, $\mathcal{K}$ is controllable by the definition of controllability.

Now we consider the co-observability. Let $s \in \text{supp}(\mathcal{K})$ and $a \in E_c$. The assumption $\mathcal{K} = \mathcal{L}^{S_1 \wedge S_2}$ gives rise to

$$\mathcal{K}(sa) = \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s) \wedge S_1[P_1(s)](a) \wedge S_2[P_2(s)](a). \qquad (6)$$

From this equality, we always have that $\mathcal{K}(sa) \leq \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s)$. Note that if $\mathcal{K}(sa) = \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s)$, then all the conditions 1), 2), and 3) in the definition of co-observability are satisfied since $\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1 a) \geq \mathcal{K}(sa)$ and $\vee_{P_2(s_2)=P_2(s)}\mathcal{K}(s_2 a) \geq \mathcal{K}(sa)$. It remains to discuss the case of $\mathcal{K}(sa) < \mathcal{L}(sa) \wedge \mathcal{L}^{S_1 \wedge S_2}(s)$. In this case, we see from (6) that

$$\mathcal{K}(sa) = S_1[P_1(s)](a) \wedge S_2[P_2(s)](a). \qquad (7)$$

Let us consider all possible subcases:

Subcase 1: $a \in E_{1c} \cap E_{2c}$. According to Definition 7, we need to show that $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1 a)] \wedge [\vee_{P_2(s_2)=P_2(s)}\mathcal{K}(s_2 a)]$. Observe that $\mathcal{K}(sa) \leq \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1 a)] \wedge [\vee_{P_2(s_2)=P_2(s)}\mathcal{K}(s_2 a)]$. Assume that $\mathcal{K}(sa) < \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1 a)] \wedge [\vee_{P_2(s_2)=P_2(s)}\mathcal{K}(s_2 a)]$. Then we obtain that $\mathcal{K}(sa) < \vee_{P_i(s_i)=P_i(s)}\mathcal{K}(s_i a)$ for $i = 1, 2$. So there exists s_i' with $P_i(s_i') = P_i(s)$, $i = 1, 2$, such that $\mathcal{K}(s_i'a) > \mathcal{K}(sa)$.

On the other hand, from the assumption $\mathcal{K} = \mathcal{L}^{S_1 \wedge S_2}$ and $P_i(s_i') = P_i(s)$, we have that

$$\begin{aligned}\mathcal{K}(s_i'a) &= \mathcal{L}^{S_1 \wedge S_2}(s_i'a)\\ &= \mathcal{L}(s_i'a) \wedge \mathcal{L}^{S_1 \wedge S_2}(s_i') \wedge S_1[P_1(s_i')](a)\\ &\quad \wedge S_2[P_2(s_i')](a)\\ &\leq S_1[P_1(s_i')](a) \wedge S_2[P_2(s_i')](a)\\ &\leq S_i[P_i(s)](a),\end{aligned}$$

that is, $\mathcal{K}(s_i'a) \leq S_i[P_i(s)](a)$ for $i = 1, 2$. Further, if $S_1[P_1(s)](a) \leq S_2[P_2(s)](a)$, then (7) forces that $\mathcal{K}(sa) =$

$S_1[P_1(s)](a)$. Consequently, $\mathcal{K}(s_1'a) \leq \mathcal{K}(sa)$, which contradicts with the foregoing argument $\mathcal{K}(s_i'a) > \mathcal{K}(sa)$. If $S_1[P_1(s)](a) > S_2[P_2(s)](a)$, then (7) yields that $\mathcal{K}(sa) = S_2[P_2(s)](a)$. This, together with $\mathcal{K}(s_i'a) \leq S_i[P_i(s)](a)$, leads to $\mathcal{K}(s_2'a) \leq \mathcal{K}(sa)$, which is again a contradiction. So the previous assumption that $\mathcal{K}(sa) < \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)] \wedge [\vee_{P_2(s_2)=P_2(s)}\mathcal{K}(s_2a)]$ does not work, and we have thus proved Subcase 1.

Subcase 2: $a \in E_{1c}\backslash E_{2c}$. Now we need to verify that $\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)]$. Since $a \notin E_{2c}$, by definition we have that $S_2[P_2(s)](a) = 1$. Accordingly, we see from (7) that $\mathcal{K}(sa) = S_1[P_1(s)](a)$. Clearly, $\mathcal{K}(sa) \leq \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)]$. Similar to the proof of Subcase 1, we assume that $\mathcal{K}(sa) < \mathcal{K}(s) \wedge \mathcal{L}(sa) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)]$, which means that $\mathcal{K}(sa) < \vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)$. Therefore there exists s_1' with $P_1(s_1') = P_1(s)$ such that $\mathcal{K}(s_1'a) > \mathcal{K}(sa)$. Notice that

$$\begin{aligned}\mathcal{K}(s_1'a) &= \mathcal{L}^{S_1\wedge S_2}(s_1'a)\\ &= \mathcal{L}(s_1'a) \wedge \mathcal{L}^{S_1\wedge S_2}(s_1') \wedge S_1[P_1(s_1')](a)\\ &\quad \wedge S_2[P_2(s_1')](a)\\ &\leq S_1[P_1(s_1')](a)\\ &= S_1[P_1(s)](a),\end{aligned}$$

i.e., $\mathcal{K}(s_1'a) \leq S_1[P_1(s)](a)$. This, together with $\mathcal{K}(sa) = S_1[P_1(s)](a)$, gives rise to $\mathcal{K}(s_1'a) \leq \mathcal{K}(sa)$, which contradicts with $\mathcal{K}(s_1'a) > \mathcal{K}(sa)$. Thus Subcase 2 is proved.

Subcase 3: $a \in E_{2c}\backslash E_{1c}$. It is similar to Subcase 2, so we can omit its proof. So far we have completed the proof of the necessity.

Next, we show the sufficiency by constructing two partially observable fuzzy supervisors $S_i : P_i[\text{supp}(\mathcal{L}(G))] \rightarrow \mathcal{F}(E)$. Suppose that $\mathcal{K}$ is controllable and co-observable. For any $s \in \text{supp}(\mathcal{L}(G))$, define $S_i[P_i(s)]$ according to

$$S_i[P_i(s)](a) = \begin{cases} 1, & \text{if } a \in E\backslash E_{ic}\\ \vee_{P_i(s_i)=P_i(s)}\mathcal{K}(s_ia), & \text{if } a \in E_{ic}.\end{cases}$$

Obviously, S_i, $i = 1, 2$ is a fuzzy supervisor.

We claim that $\mathcal{L}^{S_1\wedge S_2} = \mathcal{K}$, which will be proved by induction on the length of string s.

Basis step: $s = \epsilon$. Clearly, $\mathcal{L}^{S_1\wedge S_2}(\epsilon) = 1 = \mathcal{K}(\epsilon)$ by the definition of $\mathcal{L}^{S_1\wedge S_2}$ and the condition $\mathcal{K} \neq \mathcal{O}$. So the basis step is valid.

Induction step: Assume that for all strings s with $|s| \leq n$, we have that $\mathcal{L}^{S_1\wedge S_2}(s) = \mathcal{K}(s)$. We now prove the same for strings of the form sa, where $a \in E$. Four possible cases need to be considered.

Case 1: $a \in E_{uc}$. By the definition of $\mathcal{L}^{S_1\wedge S_2}$, we have that

$$\begin{aligned}\mathcal{L}^{S_1\wedge S_2}(sa) &= \mathcal{L}(sa) \wedge \mathcal{L}^{S_1\wedge S_2}(s) \wedge S_1[P_1(s)](a)\\ &\quad \wedge S_2[P_2(s)](a)\\ &= \mathcal{L}(sa) \wedge \mathcal{L}^{S_1\wedge S_2}(s) \quad (\text{since } a \in E_{uc})\\ &= \mathcal{L}(sa) \wedge \mathcal{K}(s) \text{ (by induction hypothesis)}\\ &= \mathcal{K}(sa), \quad (\text{by controllability of } \mathcal{K})\end{aligned}$$

i.e., $\mathcal{L}^{S_1\wedge S_2}(sa) = \mathcal{K}(sa)$, as desired.

Case 2: $a \in E_{1c} \cap E_{2c}$. In this case, the co-observability of $\mathcal{K}$ can be applicable, and we obtain that

$$\begin{aligned}\mathcal{L}^{S_1\wedge S_2}(sa) &= \mathcal{L}(sa) \wedge \mathcal{L}^{S_1\wedge S_2}(s) \wedge S_1[P_1(s)](a)\\ &\quad \wedge S_2[P_2(s)](a)\\ &= \mathcal{L}(sa) \wedge \mathcal{K}(s) \wedge S_1[P_1(s)](a)\\ &\quad \wedge S_2[P_2(s)](a) \text{ (by induction hypothesis)}\\ &= \mathcal{L}(sa) \wedge \mathcal{K}(s) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)]\\ &\quad \wedge[\vee_{P_2(s_2)=P_2(s)}\mathcal{K}(s_2a)] \text{ (by definition of } S_i)\\ &= \mathcal{K}(sa), \quad (\text{by co-observability of } \mathcal{K})\end{aligned}$$

i.e., $\mathcal{L}^{S_1\wedge S_2}(sa) = \mathcal{K}(sa)$.

Case 3: $a \in E_{1c}\backslash E_{2c}$. In this case, the co-observability of $\mathcal{K}$ can also be applicable, and we have that

$$\begin{aligned}\mathcal{L}^{S_1\wedge S_2}(sa) &= \mathcal{L}(sa) \wedge \mathcal{L}^{S_1\wedge S_2}(s) \wedge S_1[P_1(s)](a)\\ &\quad \wedge S_2[P_2(s)](a)\\ &= \mathcal{L}(sa) \wedge \mathcal{K}(s) \wedge S_1[P_1(s)](a) \text{ (since } a \notin E_{2c})\\ &= \mathcal{L}(sa) \wedge \mathcal{K}(s) \wedge [\vee_{P_1(s_1)=P_1(s)}\mathcal{K}(s_1a)]\\ &\qquad\qquad \text{(by definition of } S_1)\\ &= \mathcal{K}(sa), \quad (\text{by co-observability of } \mathcal{K})\end{aligned}$$

i.e., $\mathcal{L}^{S_1\wedge S_2}(sa) = \mathcal{K}(sa)$.

Case 4: $a \in E_{2c}\backslash E_{1c}$. It is similar to Case 3, and the proof is omitted. This finishes the proof of the theorem. ■

Proof of Proposition 5:

1): If $\mathcal{K}$ is the empty fuzzy language, then it is evident that $\mathcal{P}(\mathcal{K})$ is the empty fuzzy language. For $\mathcal{K} \neq \mathcal{O}$, we need to check the properties P1) and P2) in the definition of fuzzy languages. Obviously, $\mathcal{P}(\mathcal{K})(\epsilon) = \vee_{P(\omega')=\epsilon}\mathcal{K}(\omega') = 1$ since $\mathcal{K}(\epsilon) = 1$. Hence, P1) holds. To see P2), it is sufficient to show that $\mathcal{P}(\mathcal{K})(sa) \leq \mathcal{P}(\mathcal{K})(s)$ for any $s \in E_o^*$ and $a \in E_o$. By contradiction, assume that $\mathcal{P}(\mathcal{K})(sa) > \mathcal{P}(\mathcal{K})(s)$ for some $s \in E_o^*$ and $a \in E_o$. Note that $\mathcal{P}(\mathcal{K})(sa) = \vee_{P(\omega')=sa}\mathcal{K}(\omega')$ and $\mathcal{P}(\mathcal{K})(s) = \vee_{P(\mu')=s}\mathcal{K}(\mu')$. Thereby, there exists $\omega_0' \in E^*$ with $P(\omega_0') = sa$ such that $\mathcal{K}(\omega_0') > \mathcal{K}(\mu')$ for any $\mu' \in E^*$ satisfying $P(\mu') = s$. Since $P(\omega_0') = sa$, we can write ω_0' as $\omega_1'\omega_2'$ which satisfies $P(\omega_1') = s$ and $P(\omega_2') = a$. Thus, we get that $\mathcal{K}(\omega_1'\omega_2') > \mathcal{K}(\omega_1')$, a contradiction. We therefore conclude that P2) holds.

2): If $\mathcal{M} = \mathcal{O}$, then it is clear that $\mathcal{P}^{-1}(\mathcal{M}) = \mathcal{O}$. For $\mathcal{M} \neq \mathcal{O}$, we check the properties P1) and P2). By definition, we see that $\mathcal{P}^{-1}(\mathcal{M})(\epsilon) = \mathcal{M}(P(\epsilon)) = \mathcal{M}(\epsilon) = 1$. So P1) holds. For P2), it suffices to verify that $\mathcal{P}^{-1}(\mathcal{M})(sa) \leq \mathcal{P}^{-1}(\mathcal{M})(s)$, namely $\mathcal{M}(P(sa)) \leq \mathcal{M}(P(s))$, for any $s \in E^*$ and $a \in E$. If $a \in E_{uo}$, then $P(sa) = P(s)$, and thus $\mathcal{M}(P(sa)) = \mathcal{M}(P(s))$. If $a \in E_o$, then $P(sa) = P(s)a$, and thus $\mathcal{M}(P(sa)) = \mathcal{M}(P(s)a) \leq \mathcal{M}(P(s))$. This completes the proof. ■

Proof of Proposition 6: By definition and the fact that supp($\mathcal{K}$) is closed, we need to show that supp($\mathcal{K}$) $= P^{-1}[P(\text{supp}(\mathcal{K}))] \cap \text{supp}(\mathcal{L})$. It is obvious that supp($\mathcal{K}$) $\subseteq P^{-1}[P(\text{supp}(\mathcal{K}))] \cap \text{supp}(\mathcal{L})$, so we need only to prove that $P^{-1}[P(\text{supp}(\mathcal{K}))] \cap \text{supp}(\mathcal{L}) \subseteq \text{supp}(\mathcal{K})$. Suppose that $s \in P^{-1}[P(\text{supp}(\mathcal{K}))] \cap \text{supp}(\mathcal{L})$. Then $\mathcal{L}(s) > 0$ and $P(s) \in P(\text{supp}(\mathcal{K}))$. So there exists $s' \in \text{supp}(\mathcal{K})$ such that $P(s') =$

$P(s)$. Thus, from the assumption that $\mathcal{K}$ is normal, we have that

$$\begin{aligned}\mathcal{K}(s) &= [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](s)\\ &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})](s)\wedge\mathcal{L}(s)\\ &= [\vee_{P(\omega')=P(s)}\mathcal{K}(\omega')]\wedge\mathcal{L}(s)\\ &\geq \mathcal{K}(s')\wedge\mathcal{L}(s)\\ &> 0,\end{aligned}$$

namely, $s\in \text{supp}(\mathcal{K})$. Thus, $P^{-1}[P(\text{supp}(\mathcal{K}))]\cap\text{supp}(\mathcal{L})\subseteq \text{supp}(\mathcal{K})$, finishing the proof. ∎

Proof of Proposition 7: For simplicity, we only prove the case that I has two elements. There is no difficulty to generalize the proof to infinite index set. Suppose that $\mathcal{K}_1$ and $\mathcal{K}_2$ are normal fuzzy languages, namely, $\mathcal{K}_i=\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K}_i)]\cap\mathcal{L}$ for $i=1,2$. To prove the normality of $\mathcal{K}_1\cup\mathcal{K}_2$, it suffices to show that $(\mathcal{K}_1\cup\mathcal{K}_2)(s)=\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K}_1\cup\mathcal{K}_2)](s)\wedge\mathcal{L}(s)$ for any $s\in E^*$. By the normality of $\mathcal{K}_i$, we obtain that

$$\begin{aligned}\mathcal{K}_i(s) &= [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K}_i)]\cap\mathcal{L}](s)\\ &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K}_i)](s)\wedge\mathcal{L}(s)\\ &= [\vee_{P(\omega')=P(s)}\mathcal{K}_i(\omega')]\wedge\mathcal{L}(s)\end{aligned}$$

for $i=1,2$. Therefore, for any $s\in E^*$ we have the following:

$$\begin{aligned}(\mathcal{K}_1\cup\mathcal{K}_2)(s) &= \mathcal{K}_1(s)\vee\mathcal{K}_2(s)\\ &= [(\vee_{P(\omega')=P(s)}\mathcal{K}_1(\omega'))\wedge\mathcal{L}(s)]\vee\\ &\qquad [(\vee_{P(\omega')=P(s)}\mathcal{K}_2(\omega'))\wedge\mathcal{L}(s)]\\ &= [(\vee_{P(\omega')=P(s)}\mathcal{K}_1(\omega'))\vee\\ &\qquad (\vee_{P(\omega')=P(s)}\mathcal{K}_2(\omega'))]\wedge\mathcal{L}(s)\\ &= [\vee_{P(\omega')=P(s)}(\mathcal{K}_1(\omega')\vee\mathcal{K}_2(\omega'))]\wedge\mathcal{L}(s)\\ &= [\vee_{P(\omega')=P(s)}(\mathcal{K}_1\cup\mathcal{K}_2)(\omega')]\wedge\mathcal{L}(s)\\ &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K}_1\cup\mathcal{K}_2)](s)\wedge\mathcal{L}(s),\end{aligned}$$

i.e., $(\mathcal{K}_1\cup\mathcal{K}_2)(s)=\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K}_1\cup\mathcal{K}_2)](s)\wedge\mathcal{L}(s)$, thus finishing the proof of the proposition. ∎

Proof of Theorem 3: We use the statement 3) of Proposition 1 to show the observability of $\mathcal{K}$. Let $s,s'\in\text{supp}(\mathcal{K})$ with $P(s)=P(s')$, and $a\in E_c$. It suffices to prove the claim that $\mathcal{K}(sa)=\mathcal{K}(s)\wedge\mathcal{L}(sa)\wedge x$ and $\mathcal{K}(s'a)=\mathcal{K}(s')\wedge\mathcal{L}(s'a)\wedge x$ have a common solution. By the definition of normality, we have the following:

$$\begin{aligned}\mathcal{K}(sa) &= [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](sa)\\ &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})](sa)\wedge\mathcal{L}(sa)\\ &= [\vee_{P(\omega')=P(sa)}\mathcal{K}(\omega')]\wedge\mathcal{L}(sa)\\ &=: \lambda\wedge\mathcal{L}(sa)\ (\text{write }\lambda\text{ for }\vee_{P(\omega')=P(sa)}\mathcal{K}(\omega'))\end{aligned}$$

and

$$\begin{aligned}\mathcal{K}(s'a) &= [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](s'a)\\ &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})](s'a)\wedge\mathcal{L}(s'a)\\ &= [\vee_{P(\omega')=P(s'a)}\mathcal{K}(\omega')]\wedge\mathcal{L}(s'a)\\ &= [\vee_{P(\omega')=P(sa)}\mathcal{K}(\omega')]\wedge\mathcal{L}(s'a)\\ &= \lambda\wedge\mathcal{L}(s'a).\end{aligned}$$

To give a common solution x, let us consider all possible cases.

Case 1: $\mathcal{K}(sa)=\mathcal{K}(s)\wedge\mathcal{L}(sa)$ and $\mathcal{K}(s'a)=\mathcal{K}(s')\wedge\mathcal{L}(s'a)$. In this case, the claim evidently holds by taking $x=1$.

Case 2: $\mathcal{K}(sa)=\mathcal{K}(s)\wedge\mathcal{L}(sa)$ and $\mathcal{K}(s'a)<\mathcal{K}(s')\wedge\mathcal{L}(s'a)$. In this case, we first verify that $\mathcal{K}(sa)\leq\mathcal{K}(s'a)$. From the previous arguments, we find that $\lambda\wedge\mathcal{L}(s'a)=\mathcal{K}(s'a)<\mathcal{K}(s')\wedge\mathcal{L}(s'a)\leq\mathcal{L}(s'a)$. We thus get that $\lambda<\mathcal{L}(s'a)$, and furthermore, $\lambda=\mathcal{K}(s'a)$. Note that $\mathcal{K}(sa)=\lambda\wedge\mathcal{L}(sa)\leq\lambda$. Therefore, $\mathcal{K}(sa)\leq\mathcal{K}(s'a)$. Taking $x=\mathcal{K}(s'a)$, we see that the claim holds.

Similarly, the claim holds for $\mathcal{K}(sa)<\mathcal{K}(s)\wedge\mathcal{L}(sa)$ and $\mathcal{K}(s'a)=\mathcal{K}(s')\wedge\mathcal{L}(s'a)$.

Case 3: $\mathcal{K}(sa)<\mathcal{K}(s)\wedge\mathcal{L}(sa)$ and $\mathcal{K}(s'a)<\mathcal{K}(s')\wedge\mathcal{L}(s'a)$. In this case, we have that $\lambda\wedge\mathcal{L}(sa)=\mathcal{K}(sa)<\mathcal{K}(s)\wedge\mathcal{L}(sa)\leq\mathcal{L}(sa)$. Consequently, $\lambda<\mathcal{L}(sa)$, and thus $\lambda=\mathcal{K}(sa)$. In the same way, we can get that $\lambda=\mathcal{K}(s'a)$. Thus the claim holds by taking $x=\lambda$. The proof of the theorem is finished. ∎

Proof of Theorem 4: If $\mathcal{K}$ is the empty fuzzy language, then the result is trivial. Now, let us consider the case of $\mathcal{K}\neq\mathcal{O}$. Observe that $\mathcal{K}\subseteq\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}$, so we need only to prove that $\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}\subseteq\mathcal{K}$. It is sufficient to verify that $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](s)\leq\mathcal{K}(s)$ for any $s\in E^*$. By contradiction, assume that there exists $t\in E^*$ such that $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](t)>\mathcal{K}(t)$. It is clear that $t\neq\epsilon$, since $\mathcal{K}\neq\mathcal{O}$. Let sa be the shortest such t, that is, $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](sa)>\mathcal{K}(sa)$ but $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](\omega)=\mathcal{K}(\omega)$ for any $\omega\in E^*$ with $|\omega|\leq|s|$. Two cases need to be discussed.

Case 1: $a\in E_{uc}$. By the controllability of $\mathcal{K}$, we get that $\mathcal{K}(sa)=\mathcal{K}(s)\wedge\mathcal{L}(sa)$. From the previous assumption, we obtain that

$$\begin{aligned}\mathcal{K}(s) &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})](s)\wedge\mathcal{L}(s)\\ &= [\vee_{P(\mu')=P(s)}\mathcal{K}(\mu')]\wedge\mathcal{L}(s).\end{aligned}$$

Hence, we get by $\mathcal{L}(s)\geq\mathcal{L}(sa)$ that

$$\begin{aligned}\mathcal{K}(sa) &= \mathcal{K}(s)\wedge\mathcal{L}(sa)\\ &= [\vee_{P(\mu')=P(s)}\mathcal{K}(\mu')]\wedge\mathcal{L}(s)\wedge\mathcal{L}(sa)\\ &= [\vee_{P(\mu')=P(s)}\mathcal{K}(\mu')]\wedge\mathcal{L}(sa).\end{aligned}$$

In the subcase of $a\in E_{uo}$, we thus have that

$$\begin{aligned}[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](sa) &= \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})](sa)\wedge\mathcal{L}(sa)\\ &= [\vee_{P(\omega')=P(sa)}\mathcal{K}(\omega')]\wedge\mathcal{L}(sa)\\ &= [\vee_{P(\omega')=P(s)}\mathcal{K}(\omega')]\wedge\mathcal{L}(sa)\\ &= \mathcal{K}(sa).\end{aligned}$$

This contradicts with the assumption that $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](sa)>\mathcal{K}(sa)$. In the other subcase $a\in E_o$, we get that $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](sa)=[\vee_{P(\omega')=P(s)a}\mathcal{K}(\omega')]\wedge\mathcal{L}(sa)$. Therefore, the assumption $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})]\cap\mathcal{L}](sa)>\mathcal{K}(sa)$ implies that $[\vee_{P(\omega')=P(s)a}\mathcal{K}(\omega')]\wedge\mathcal{L}(sa)>[\vee_{P(\mu')=P(s)}\mathcal{K}(\mu')]\wedge\mathcal{L}(sa)$, which forces that $\vee_{P(\omega')=P(s)a}\mathcal{K}(\omega')>\vee_{P(\mu')=P(s)}\mathcal{K}(\mu')$. Whence, there exists $\omega_0'\in E^*$ with $P(\omega_0')=P(s)a$ such that $\mathcal{K}(\omega_0')>\mathcal{K}(\mu')$ for all $\mu'\in E^*$ satisfying $P(\mu')=P(s)$. As a result, we can write ω_0' as $\omega_{01}'\omega_{02}'$, where

$P(\omega'_{01}) = P(s)$ and $P(\omega'_{02}) = a$, and furthermore, we see that $\mathcal{K}(\omega'_{01}\omega'_{02}) > \mathcal{K}(\omega'_{01})$. This is absurd.

Case 2: $a \in E_c$. From the condition $E_c \subseteq E_o$, we see that $a \in E_o$. Hence the assumption $[\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})] \cap \mathcal{L}](sa) > \mathcal{K}(sa)$ implies that $[\vee_{P(\omega')=P(s)a}\mathcal{K}(\omega')] \wedge \mathcal{L}(sa) > \mathcal{K}(sa)$. So there exists $\omega' \in E^*$ with $P(\omega') = P(s)a$ such that $\mathcal{K}(\omega') \wedge \mathcal{L}(sa) > \mathcal{K}(sa)$. Since $P(\omega') = P(s)a$, we can set $\omega' = s'a\omega''$ such that $P(s') = P(s)$ and $P(\omega'') = \epsilon$. We thus see that $\mathcal{K}(s'a) \geq \mathcal{K}(\omega')$, and moreover, $\mathcal{K}(s'a) \wedge \mathcal{L}(sa) > \mathcal{K}(sa)$. The latter forces that both $\mathcal{K}(s'a) > \mathcal{K}(sa)$ and $\mathcal{L}(sa) > \mathcal{K}(sa)$.

Because we need Lemma 1 in the later development, let us pause to check its conditions. We have known that $P(s') = P(s)$ and $a \in E_c$, so it remains only to show that $s, s' \in \text{supp}(\mathcal{K})$. In fact, by the previous arguments we have that $\mathcal{K}(s) = [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})] \cap \mathcal{L}](s) \geq [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})] \cap \mathcal{L}](sa) > \mathcal{K}(sa) \geq 0$, i.e., $\mathcal{K}(s) > 0$, and $\mathcal{K}(s') \geq \mathcal{K}(s'a) > \mathcal{K}(sa) \geq 0$, i.e., $\mathcal{K}(s') > 0$.

From $\mathcal{K}(s'a) > \mathcal{K}(sa)$ and Lemma 1, we obtain that

$$\mathcal{K}(sa) = \mathcal{K}(s) \wedge \mathcal{L}(sa) \tag{8}$$

and

$$\mathcal{K}(s'a) \leq \mathcal{K}(s') \wedge \mathcal{L}(s'a). \tag{9}$$

From (8) and the proven fact $\mathcal{L}(sa) > \mathcal{K}(sa)$, we find that $\mathcal{K}(s) = \mathcal{K}(sa) < \mathcal{L}(sa) \leq \mathcal{L}(s)$, i.e., $\mathcal{K}(s) < \mathcal{L}(s)$. Note that $\mathcal{K}(s) = [\vee_{P(\mu')=P(s)}\mathcal{K}(\mu')] \wedge \mathcal{L}(s)$, so we get that $\mathcal{K}(s) = \vee_{P(\mu')=P(s)}\mathcal{K}(\mu')$. On the other hand, we see from $\mathcal{K} \subseteq \mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})] \cap \mathcal{L}$ that

$$\begin{aligned}
\mathcal{K}(s') &\leq [\mathcal{P}^{-1}[\mathcal{P}(\mathcal{K})] \wedge \mathcal{L}](s') \\
&= [\vee_{P(\omega')=P(s')}\mathcal{K}(\omega')] \wedge \mathcal{L}(s') \\
&= [\vee_{P(\omega')=P(s)}\mathcal{K}(\omega')] \wedge \mathcal{L}(s') \\
&\leq \vee_{P(\omega')=P(s)}\mathcal{K}(\omega') \\
&= \mathcal{K}(s),
\end{aligned}$$

i.e., $\mathcal{K}(s') \leq \mathcal{K}(s)$. Accordingly, $\mathcal{K}(s'a) \leq \mathcal{K}(s') \leq \mathcal{K}(s) = \mathcal{K}(sa)$, that is, $\mathcal{K}(s'a) \leq \mathcal{K}(sa)$, which contradicts with the proven fact $\mathcal{K}(s'a) > \mathcal{K}(sa)$. Thus the proof of the theorem is completed. ∎

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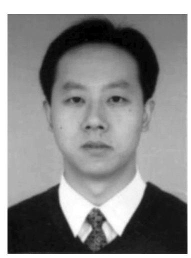

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