Observability Enhancement of Smart Grid Based on Optimal Placement of PMUs

Alok Priyadarshi, **Vishal Rathore**[†], and **Krishna Bihari Yadav**, Non-members

ABSTRACT

This paper presents an efficient observation concerning the enhancement of smart grid (SG) based on the optimal placement of phasor measurement units (OPP) using nonlinear programming (NLP). The proposed algorithm tries to achieve two objectives: (i) to ascertain the minimum number of phasor measurement units (PMUs) and (ii) to increase the redundancy of the SG at all the buses. Synchronized current and voltage phasors are obtained to enhance the accuracy of the state estimation results—a minimum number of PMUs results in a lack of communication facilities at the substation. PMU losses will lead to unobservable buses at the SG. Therefore, PMU losses and communication constraints should be considered during the design process. Limited channel capacity, conventional measurement, and zero-injection bus measurements are also included in the proposed PMU formulation. The proposed algorithm is examined on IEEE 14-, 30-, 57-, 118-, and 300-bus test systems in MATLAB to verify its effectiveness. Furthermore, the results are compared with the simplex linear programming and mixed linear programming methods to prove the efficacy of the presented algorithm. The output thus obtained reveals that the NLP algorithm obtains approximately the same PMUs as other methods.

Keywords: Nonlinear Programming, Mixed Linear Programming, Simplex Linear Programming, Optimal PMU Placement, Observability

1. INTRODUCTION

Observability is essential for the state estimation of power systems. It provides a time-synchronized measurement of all electrical network elements [1]. Synchronized measurement is obtained with the reference signal received from the global positioning system [2]. The output of phasor measurement units (PMUs) is used for online monitoring and control. Power system observability ascertains whether the present state of measurement and its distribution is enough to solve the current state of the network. The weighted least squares (WLS) technique is conventionally used for state eliminators to solve system state problems. However, it is a nonlinear iterative method that takes more time in comparison to linear methods. Detailed data on state variables must be obtained for accurate results in controlled systems. However, due to the complexity of the power system, it is not easy to find exact data on state variables [3].

Traditionally, remote terminal units (RTUs) with supervisory control and data acquisition (SCADA) provide control and monitoring for different system networks [4]. In the 1980s, with the introduction of digital signal processing and the Global Positioning System (GPS), PMUs were developed. Nowadays, GPS is synchronized with the PMU signal to find the state of the power system. GPS can bring microsecond data in real time [5, 6], which it highly efficient for monitoring and rectifying the power system during unstable conditions [7]. PMUs have an advantage over conventional measurements since they can provide more precise results. Hence, PMUs improve the functionality of the power grid by introducing it at a particular bus to obtain current and voltage phasors [8].

PMUs consist of a clock synchronization unit, data transmission unit, and measurement unit. The clock synchronization unit provides standard sampling to the measurement unit. The control system generates an output signal to compare the phase with an input signal. An anti-aliasing filter, A/D converter, and processor are elements of the measuring unit. The anti-aliasing filter ensures that the signal phase difference and magnitudes are not changed. PMU works on discrete Fourier transform and is synchronized with a GPS signal. Therefore, the calculated voltage and current phasor are accurate with space-time coordination. PMUs measure the node voltage and incident branch current phasor of the system, making it possible to monitor the node where the PMU is connected. The dependence of system observability lies in the system, the change in number, and arrangement of measuring elements. The complete power system is observable by selecting PMUs with a subset of nodes [9].

PMUs enable observability of the whole grid. The complete power network is obtained by placing PMUs at optimal locations [10, 11]. PMUs cannot be put at all the network buses to make them utterly observable because such an arrangement would make the system expensive and uneconomical [12]. Therefore, optimum PMUs are used at specific buses to make the system observable and cost-effective. Initially, PMUs were connected to the nodes where most of the loads are connected. However,

Manuscript received on December 29, 2021; revised on March 8, 2022; accepted on April 19, 2022. This paper was recommended by Associate Editor Yuttana Kumsuwan.

The authors are with the Department of Electrical Engineering, National Institute of Technology Jamshedpur, Jharkhand, India.

[†]Corresponding author: vishalrathore01@gmail.com

Digital Object Identifier: 10.37936/ecti-eec.2022203.247514

such an arrangement did not give any option during a dynamic change in load. This is because PMUs are placed to monitor load changes, but when loading at buses, the position of PMUs has to be changed to monitor the system.

The optimization technique makes the system completely observable using the least number of PMUs. Many search methodologies have been suggested for the optimal placement of PMUs (OPP). The strategy for determining the OPP can be mainly classified into two categories: heuristic and mathematical programming [13]. The former approach is based on the continuous search for an optimal solution to a problem. Several heuristic methods have been studied in the literature. To determine the minimum number of PMUs and achieve complete system observability, a bisecting search method is proposed in [1].

A graph theory approach and simulated annealing are implemented in [14], reducing the number of PMUs from one-fifth to one-third of their present values. Other heuristic algorithms like tabu search [15], genetic algorithm [16], particle swarm optimization [17], spanning tree [18], binary particle swarm optimization [19, 20], non-dominated sorting genetic [21], immunity genetic algorithm [22], recursive tabu search [23], and tabu search genetic [24] have been proposed to address the OPP problem. A heuristic approach for finding the optimal solution does not provide the global best [25]. Therefore, the linear programming (LP) approach to find the best global solution is developed in the literature. Mixed linear programming (MLP) and nonlinear programming (NLP) provide the best global solution, but the algorithm requires more time. This paper presents an NLP algorithm to find a solution to the optimal placement problem. The NLP method is discrete, so prior to implementing the proposed algorithm, simplex LP (SLP) and MLP methods are executed in a MATLAB/Simulink environment to assess the comparative efficiency of the proposed method and provide an accurate analysis.

In this paper, the enhancement of observability in the SG is based on three different methods and examined on IEEE 14-, 30-, 57-, 118-, and 300-bus test systems in MATLAB. The steps involved in determining the optimal solution are as follows. First, a linear objective is selected, for minimization, and the chosen objective then determined by nonlinear equality bus constraints. Second, these nonlinear bus constraints make the whole system observable. Third, the objective function minimizes the overall cost and is modified to obtain the PMU losses, limited communication facilities, channel capacity, and zero-injection measurement.

The remainder of this paper is divided into four sections. Section 2 elaborates on the design of the PMU placement problem. Section 3 presents the development of the optimized algorithm. Section 4 includes the simulation results and a discussion, followed by a conclusion in Section 5.

2. DESIGN OF THE OPP PROBLEM

PMU placement at a bus finds both the voltage and current phasor data for that bus. As a result, all the bus incidents on the PMU bus are observable. The observability of the whole system with the least number of PMUs is the primary function of the OPP. The PMU is initially considered to have enough channels for measuring the current phasor of all the bus incidents on a particular bus and the voltage phasor of that bus for the OPP solution [6]. As a result, using the observed current phasor with the known line characteristics, lines coming to that bus and the voltages of all nearby buses will be solved [26].

2.1 Design of OPP Measurement Problem

Firstly, the LP algorithm is designed to find the solution for the OPP. Next, the OPP is redesigned to minimize the cost function, and after that, the NLP algorithm solves this OPP problem. The constraint minimization objective function for the n-bus system is as follows [27]

$$\min_{x} \sum_{i=1}^{n} c_i x_i \tag{1}$$

subjected to $Ax \geq B$ and $x_i \in [0, 1]$, $i = 1, \ldots, n$, where the binary decision variable (x_i) indicates whether a PMU is installed at the i^{th} bus $(x_i = 1)$ or not $(x_i = 0)$, and c_i is the placement cost of PMU at bus i. In most cases, the cost of installing a PMU on each bus is the same. As a result, reducing the total number of PMUs is the same as minimizing $c_i x_i$ in the system. A and B matrices are given as

$$A(i, j) = \begin{cases} 1, & \text{bus } m \text{ and bus } n \text{ are connected} \\ 1, & i \text{ is equal to } j \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$B = [1 \;\; 1 \;\; 1 \;\; \ldots \;\; 1]^T \tag{3}$$

The linear observability constraint for the i^{th} bus is represented by the i^{th} entry of $f(x)$, which is defined as follows

$$f_i(x) = x_i + \sum_{j \in adj_i} x_j \geq 1 \tag{4}$$

where $f_i(x)$ is the observability function for the i^{th} bus x_i is the decision variable (binary) for installing PMU at the i^{th} bus, where x_j is the decision variable adjacent to bus i.

The sequential quadratic programming (SQP)-based NLP may be used to construct and solve the OPP issue. The NLP technique can yield several solutions to the OPP, but the LP approach can only give a single solution. Therefore, x_i maybe 0 or 1 by the constraints $x_i \left(x_j - 1\right) = 0$. The nonlinear constraint to enhance the observability of the i^{th} bus is represented by the i^{th} entry of $f(x)$, which is defined as

$$\min_x J(x) = x^T C x = \sum_{i=1}^{n} c_i x_i^2 \tag{5}$$

$$g_i(x) = (1 - x_i) \prod_{j \in adj_i} (1 - x_j) = 0 \tag{6}$$

where $0 \leq x_i \leq 1$, for all $i \in S$. The OPP objective function is represented by $J(x)$, adj_i indicates the adjacent buses of bus i, C is the weighted diagonal matrix, x^T is transpose vector, and S is the system bus set.

The observability criterion defined in Eq. (5) requires at least one PMU to be placed on bus i or j to determine the optimal solution and make the bus observable. On adding nonlinear observability constraints, the decision variables of LP [6] are turned into continuous variables. In this method, the equations of a consistent system are defined, with each equality constraint allowing for a solution in Eq. (6).

Since all $f(x)$ components are continuous variables, the mathematical equations (Eqs. (1)–(4)) have no problem converging into a locally optimal solution. The feasible set comprising nonlinear equality constraints in Eq. (4) is non-convex. As a result, the suggested model is non-convex and can produce several solutions to the OPP problem. Hence, there are various local minima for the optimization problem (Eqs. (1)–(4)). Therefore, the NLP algorithm is an efficient way to enhance the observability of the SG and find an optimum solution to the placement problem. To design the proposed algorithm, an IEEE 14-bus test system, illustrated in Fig. 1, is used and formulated by the following sets of equations

$$\min_x J(x) = \sum_{i=1}^{n} x_i \tag{7}$$

$$g_1(x) = (1 - x_5)(1 - x_2)(1 - x_1) = 0 \tag{8}$$

$$g_2(x) = (1 - x_5)(1 - x_4)(1 - x_3)(1 - x_2)(1 - x_1) = 0 \tag{9}$$

$$g_3(x) = (1 - x_4)(1 - x_3)(1 - x_2) = 0 \tag{10}$$

$$g_4(x) = (1 - x_9)(1 - x_7)(1 - x_5)(1 - x_4)(1 - x_3)(1 - x_2) = 0 \tag{11}$$

$$g_5(x) = (1 - x_6)(1 - x_5)(1 - x_4)(1 - x_2)(1 - x_1) = 0 \tag{12}$$

$$g_6(x) = (1 - x_{13})(1 - x_{12})(1 - x_{11})(1 - x_6)(1 - x_5) = 0 \tag{13}$$

$$g_7(x) = (1 - x_9)(1 - x_8)(1 - x_7)(1 - x_4) = 0 \tag{14}$$

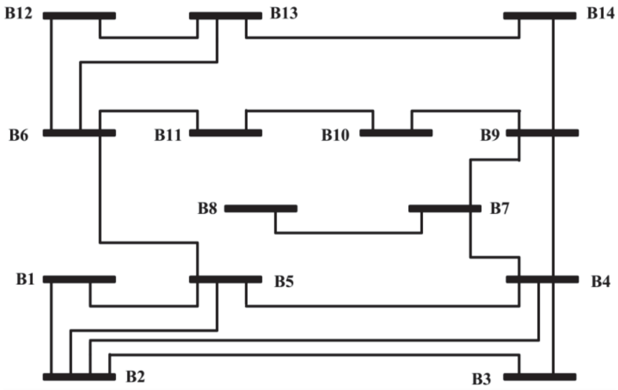

***Fig. 1**: IEEE 14-bus system.*

$$g_8(x) = (1 - x_8)(1 - x_7) = 0 \tag{15}$$

$$g_9(x) = (1 - x_{14})(1 - x_{10})(1 - x_9)(1 - x_7)(1 - x_4) = 0 \tag{16}$$

$$g_{10}(x) = (1 - x_{11})(1 - x_{10})(1 - x_9) = 0 \tag{17}$$

$$g_{11}(x) = (1 - x_{11})(1 - x_{10})(1 - x_6) = 0 \tag{18}$$

$$g_{12}(x) = (1 - x_{13})(1 - x_{12})(1 - x_6) = 0 \tag{19}$$

$$g_{13}(x) = (1 - x_{14})(1 - x_{13})(1 - x_{12})(1 - x_6) = 0 \tag{20}$$

$$g_{14}(x) = (1 - x_{14})(1 - x_{13})(1 - x_9) = 0 \tag{21}$$

2.2 Design of the OPP Measurement Problem Considering the Power Flow Measurement

Here, to examine the OPP measurement problem, the power flow measurement on lines $i - j$ is considered. If one of the voltages is known in the power flow measurement, the other can be estimated according to changes in the observability constraints. The observability constraints for the above-mentioned two buses are $g_i(x) = 0$ and $g_j(x) = 0$, with both constraints combined to form a new constraint represented as

$$g_{flow,i} = g_i g_j = 0 \tag{22}$$

where $g_{flow,i}(x)$ is obtained by a combination of two functions $g_{bus,i}(x)$ and $g_{bus,j}(x)$ associated with observability constraints. The proposed algorithm uses the existing power flow measurements on lines 4–5 and 5–6. Thus, the new simplified joint constraint g_{4new} obtained by the existence of the two power flow measurements is given by

$$g_{4new}(x) = g_4(x)\, g_5(x)\, g_6(x) = 0 \tag{23}$$

$$\begin{aligned} g_{4new}(x) = & \left(1-x_{13}\right)\left(1-x_{12}\right)\left(1-x_{11}\right)\left(1-x_9\right) \\ & \left(1-x_7\right)\left(1-x_6\right)\left(1-x_5\right)\left(1-x_4\right) \\ & \left(1-x_3\right)\left(1-x_2\right)\left(1-x_1\right) = 0 \end{aligned} \tag{24}$$

2.3 Design of the OPP Measurement Problem Considering a Zero-Injection Bus

The most general situation of the power system is a zero-injection bus (ZIB). During power system planning, ZIB refers to a bus with no load or injection. To control the voltage drop at the receiving end of the transmission line, the voltage profile needs to be regulated, and thus, voltage improvement devices are installed. A ZIB does not inject any current into the system. This method decreases the required number of PMUs and enhances system observability. Assuming that bus i is a ZIB, the joint observability constraint is given by

$$g_{inj,i}(x) = g_i(x) \prod_{j \in adj_i} g_j(x) = 0 \tag{25}$$

where $g_{inj,i}(x)$ is the combination of two functions $g_{inj,bus}(x)$ and $g_{bus}(x)$. It is advantageous to combine the two ZIB constraints with the observability constraints of all associated buses into one constraint equation. Fig. 1 shows a standard 14-bus system, which locates a ZIB at bus 7 and its augmented buses are 4, 6, and 8. The simplified observability constraints of the augmented bus $g_{inj,7}(x)$ are given as

$$g_{inj,7}(x) = g_7(x)\, g_4(x)\, g_8(x)\, g_9(x) = 0 \tag{26}$$

$$\begin{aligned} g_{inj,7}(x) = & \left(1-x_{14}\right)\left(1-x_{10}\right)\left(1-x_9\right)\left(1-x_8\right) \\ & \left(1-x_7\right)\left(1-x_5\right)\left(1-x_4\right)\left(1-x_3\right) \\ & \left(1-x_2\right) = 0 \end{aligned} \tag{27}$$

2.4 Limited Communication Facilities at a Substation

PMUs must connect with the control center to synchronize voltage and current phasor readings via data links at the substations to provide the PMU data to the SCADA system. However, the limited communication capabilities act as a major impediment to synchronizing the data for wider implementation and real-time monitoring. As a result, the cost of PMU placement for SLP, MLP, and NLP will increase for each bus with a lack of communication capability [15]. The optimum solution may eliminate the restricted communication buses due to the high installation cost. For example, assuming that buses 2 and 9 in the IEEE 14-bus system have limited communication capability (Fig. 1), the placement cost for these two buses will increase to $c_i = 10^9$.

2.5 Limited Channel Capacity

PMUs have an unlimited number of channels, according to most of the studies in the literature. However, the manufacturer often gives PMUs a predefined channel capacity, so that an efficient algorithm must be implemented to obtain the best solution for PMU placement. Let C be the channel capacity; if the number of buses adjacent to i^{th} bus (m_i) is more than C, then the combination of buses (Δ_i) out of m_i lines is given by [28]

$$\Delta_i = \frac{m_i!}{C!\left(m_i - C\right)!} \tag{28}$$

First, the observability constraint corresponding to the i^{th} bus is fixed to ensure that only three buses correspond to each constraint. The channel capacity must be equal to or less than the number of buses neighboring to the i^{th} bus. The observability requirements are then modified to accommodate the various line combinations. Taking buses {1, 3, 4, 5} of an IEEE 14-bus system adjacent to bus 2 with a channel capacity of $\Delta_i = 2$, as illustrated in Fig. 1, corresponding to the channel capacity, the combination of buses (Δ_i) is given as

$$\Delta_i = \frac{4!}{2!\,(4-2)!} \tag{29}$$

In particular, the line combinations are {2-4, 2-5}, {2-3, 2-5}, {2-3, 2-4}, {2-1, 2-5}, {2-1, 2-4}, and {2-1, 2-3}. The observability constraint for bus 2 is then changed as follows

$$g_{21}(x) = \left(1-x_2\right)\left(1-x_1\right)\left(1-x_3\right) = 0 \tag{30}$$

$$g_{22}(x) = \left(1-x_2\right)\left(1-x_1\right)\left(1-x_4\right) = 0 \tag{31}$$

$$g_{23}(x) = \left(1-x_2\right)\left(1-x_1\right)\left(1-x_5\right) = 0 \tag{32}$$

$$g_{24}(x) = \left(1-x_2\right)\left(1-x_3\right)\left(1-x_4\right) = 0 \tag{33}$$

$$g_{25}(x) = \left(1-x_2\right)\left(1-x_3\right)\left(1-x_5\right) = 0 \tag{34}$$

$$g_{26}(x) = \left(1-x_2\right)\left(1-x_4\right)\left(1-x_5\right) = 0 \tag{35}$$

The process is then repeated for the remaining buses to make sure that each constraint has only three adjacent buses.

2.6 PMU Failure

Despite the excellent dependability and reliability of PMUs, a single-PMU failure is a possibility. Therefore, primary and backup sets are collected to ensure total observability of the system. In the primary set, the optimal solution is obtained without considering the PMU failure, whereas in the backup set, the optimal solution is obtained considering the case of PMU failure. For example, to allow the monitoring of each bus by two PMUs, one side of the LP restrictions might be changed to two. Instead, the main set terms x_j and x_i of LP restrictions can be deleted to produce a new set. In addition, the backup set, provided by the terms $(1 - x_j)$ and $(1 - x_i)$ of the NLP restrictions, is also eliminated. As a result, the primary set buses will not be picked again, and the backup set will ensure that the system is completely observable even if one of the PMUs fails.

3. DESIGN OF THE OPTIMIZATION ALGORITHM

The minimization algorithm proposed in Section 2 is utilized to solve the OPP in this section. In addition, the LP model [26] is utilized as a comparison tool to show the proposed model's accuracy in terms of the number of PMUs necessary for complete system observability. In the simulation part, the two algorithms are tested on the IEEE test bus system. Determining the number of PMUs is critical because maintaining system observability is not economical for addressing the OPP problem.

3.1 Design of the Proposed Minimization Algorithm

The proposed minimization algorithm contains the following elements:

1. Dataset comprising λ sets of buses with n buses, cost of the buses (c_i), and sets of buses adjacent to bus i (adj_i).
2. Binary decision variable x is bounded by the lower x_l and upper limit x_u as:

$$x_l \leq x \leq x_u \tag{36}$$

where $x_l = [0 \;\; 0 \;\; \ldots \;\; 0]^T$ and $x_u = [1 \;\; 1 \;\; \ldots \;\; 1]^T$.

3. The observability constraints are

$$g_i(x) = \left(1 - x_i\right) \prod_{j \in adj_i} \left(1 - x_j\right) = 0 \tag{37}$$

4. The cost function to be minimized is

$$J(x) = \sum_{i=1}^{n} c_i x_i \tag{38}$$

In NLP, the sequential programming approach is the most successful problem-solving tool. In similarity to Newton's method and unconstrained optimization, the proposed NLP method simulates the algorithm to optimize the problem. First, a quasi-Newton iteration method is used to approximate the Hessian of the Lagrangian function at each major iteration. It is then utilized to construct a quadratic programming sub problem, which is used to form the direction of the line search procedure. The main concept involves the formulation of a solution to the quadratic programming problem of the Lagrangian function, with the description given as

$$\min_x f(x) \tag{39}$$

$$g_i(x) = 0, \; i = 1, 2, \ldots, m \tag{40}$$

$$g_i(x) \leq 0, \; i = m + 1, \ldots, m \tag{41}$$

$$H(x, \Delta) = f(x) + \sum_{i=1}^{m} \Delta_i g_i(x) \tag{42}$$

The NLP algorithm is given by

$$x_{k+1} = x_k + a_k d_k \tag{43}$$

A suitable line search approach is used to calculate the step length parameter a_k, resulting in a satisfactory drop in the merit function. H_k is a positive approximation of the Lagrangian function. A quasi-Newton algorithm can be used to update H_k. Two m-files are written to implement MATLAB's NLP-based on SQP. The first m-file after successive iteration returns the function's current value, which can be used to call the objective function using fmincon. Another m-file provides the solution for vector x by the present value of the observability constraints. The decision variable x is bounded within the specific limit by supplying simple restrictions to the constrained optimizer procedure. After that, fmincon is run with a particular starting point.

3.2 Design of the Proposed SLP Minimization Algorithm

The elements of the SLP minimization algorithm are as follows:

1. Dataset comprising λ sets of buses with n buses, cost of the buses (c_i), and sets of buses adjacent to bus i (adj_i).
2. Binary decision variable x is bounded by the lower x_l and upper limit x_u as:

$$x_i = \begin{cases} 1, & \text{bus } m \text{ and bus } n \text{ are connected} \\ 1, & i \text{ is equal to } j \\ 0, & \text{otherwise} \end{cases} \tag{44}$$

3. The observability constraints are

$$\sum_{adj_i}^{n} x_i \geq 1 \tag{45}$$

4. The cost function to be minimized is

$$J(x) = \sum_{i=1}^{n} c_i x_i \tag{46}$$

The branch-and-bound (BB) approach can solve the SLP algorithm (Eqs. (45), (46)). The BB approach is highly efficient and more commonly used to solve LP problems by the bintprog solver.

***Table 1**: Results using MLP, SLP, and NLP.*

IEEE Test System	PMU			NLP Solution
	MLP	SLP	NLP	
14-bus	4	4	4	5
30-bus	10	10	10	7
57-bus	17	17	17	19
118-bus	32	32	32	10
300-bus	87	87	87	8

***Table 2**: Location of PMUs using MLP, SLP, and NLP.*

IEEE Test System	PMU			PMU Position
	MLP	SLP	NLP	
14-bus	4	4	4	2, 6, 7, 9
30-bus	10	10	10	1, 7, 9, 10, 12, 18, 24, 25, 27, 28
57-bus	17	17	17	1, 4, 6, 13, 19, 22, 25, 27, 29, 32, 36, 39, 41, 45, 47, 51, 54
118-bus	32	32	32	3, 7, 9, 11, 12, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

4. RESULTS AND DISCUSSION

Different IEEE test systems are used to compare LP (MLP and SLP) with NLP. These methods are employed to study five different cases: power flow measurement, ZIB measurement, PMU failure, lack of communication facilities, and limited channel capacity. A zero-injection formulation based on NLP is introduced and tested. LP is solved with MATLAB's intlinprog function, while fmincon is used to solve the NLP. The data for the power system may be found in [23]. MATLAB optimization solvers were used to carry out the calculations. The suggested algorithm's performance is evaluated in terms of network size and the CPU time required to obtain the desired optimum.

The NLP approach achieves two goals: (i) the enhancement of observability by minimizing the number of PMUs; and (ii) increasing measurement redundancy. A dependable power network necessitates a high level of measurement redundancy. As a result, the best outcome will be chosen based on the highest SORI. The fmincon NLP method is used to optimize the provided model to obtain the OPP solution. The upper and lower limits of the decision variables are also provided in the solver solution. The objective function and observability restrictions are referred to as m-files by the optimizer solver. Any point within the bounds [0,1] can be used as the starting point. bintprog, on the other hand, demands a realistic starting point to optimize the SLP model. Therefore, the optimizer solver utilizes the initial default point if the predefined initial point is not an integer variable.

To find the optimum of number of PMUs and pace of convergence, the NLP results are compared to those achieved by the MLP and SLP methods. For each minimization model, the PMU cost is set at $c_i = 1$.

Table 1 compares the proposed approach to MLP and SLP algorithms employed for typical operating conditions without considering any contingencies. As can be observed, all the optimization algorithms provide the same minimum number of PMUs to enhance the system's observability, validating the correctness and effectiveness of the proposed algorithm. However, the NLP method can produce more than one solution to the OPP problem, while the MLP and SLP formulations can only provide a single solution. Thus, NLP is another effective algorithm for solving the OPP problem by obtaining various optimal solutions for selection.

Table 2 illustrates a comparison of the OPP problem using three algorithms without considering ZIB.

In addition to the above analysis, five cases are simulated. In the first case, power flow measurement was analyzed, whereas in the second case, ZIB measurements were examined. Furthermore, the effect of PMU failure is considered in the third cases. Finally, the fourth case addresses the lack of communication facilities, whereas the fifth case considers the limited channel capacity.

***Table 3**: OPP Solution using the power flow results.*

IEEE Test System	Branch	PMU			NLP Solution
		MLP	SLP	NLP	
14-bus	5	3	3	3	11
30-bus	20	5	5	5	6
57-bus	40	6	6	6	5
118-bus	31	24	24	24	5
300-bus	43	81	81	81	4

A. Case 1: OPP solution using power flow measurements

The placement problem is resolved with additional constraints such as power flow measurements. The IEEE 14-, 30-, 57-, 118-, and 300-bus systems are used in the simulations. Table 3 presents a comparison of the power flow case results using three different algorithms.

B. Case 2: OPP solution by considering ZIB

ZIB is used as an additional constraint. The simulation is performed on the standard 14-, 30-, 57, 118-, and 300-bus systems with ZIB. Table 4 illustrates the location of ZIB, while the case results are presented in Table 5.

C. Case 3: PMU failure

An OPP technique can be adjusted to be resilient enough to maintain or enhance the system's observability in the event of PMU failure. In addition, an appropriate backup set for the PMUs is used to improve the accuracy of measurements in the event of a breakdown. The NLP results are comparable to the MLP and SLP model's single-PMU outage situation, as illustrated in Table 6.

***Table 4**: Location of ZIB.*

IEEE Test System	Location of ZIB
14-bus	7
30-bus	11, 28, 6, 25, 9
57-bus	4, 21, 11, 7, 22, 26, 24, 39, 36, 34, 37, 48, 45, 40, 46
118-bus	64, 9, 5, 38, 30, 81, 37, 63, 71, 68
300-bus	58, 17, 294, 233, 256

***Table 5**: ZIB results.*

IEEE Test System	ZIB	PMU			NLP Solution
		MLP	SLP	NLP	
14-bus	1	3	3	3	1
30-bus	6	7	7	7	3
57-bus	15	11	11	11	6
118-bus	10	28	28	28	4
300-bus	5	82	82	82	2

***Table 6**: PMU failure results.*

IEEE Test System	PMU			NLP Solution
	MLP	SLP	NLP	
14-bus	9	9	9	4
30-bus	21	21	21	4
57-bus	35	35	35	3
118-bus	75	75	75	2
300-bus	221	221	221	2

***Table 7**: Location under a lack of communication facilities.*

IEEE Test System	Branch	PMU			NLP Solution
		MLP	SLP	NLP	
14-bus	5	3	3	3	11
30-bus	20	5	5	5	6
57-bus	40	6	6	6	5
118-bus	31	24	24	24	5
300-bus	43	81	81	81	4

***Table 8**: Location under limited channel capacity.*

IEEE Test System	Channels	PMU			NLP Solution
		MLP	SLP	NLP	
14-bus	3	4	4	4	1
30-bus	4	10	10	10	2
57-bus	5	17	17	17	4
118-bus	6	32	32	32	3
300-bus	7	87	87	87	3

***Table 9**: Comparison of the time required by the CPU.*

Case Type	IEEE Test System	CPU Time (s)		
		MLP	SLP	NLP
None	14-bus	0.0313	0.0571	0.1563
	30-bus	0.0386	0.0632	0.7535
	57-bus	0.0469	0.0824	0.9375
	118-bus	0.0781	0.0954	9.9063
	300-bus	0.0821	0.0988	42.7654
Power flow measurement	14-bus	0.0313	0.0451	0.0781
	30-bus	0.0386	0.1725	0.2871
	57-bus	0.0469	0.2387	0.4844
	118-bus	0.0625	0.8764	6.7500
	300-bus	0.0824	0.9541	41.2354
Limited communication	14-bus	0.0313	0.0571	0.0781
	30-bus	0.0386	0.0632	0.2871
	57-bus	0.0469	0.0824	0.6406
	118-bus	0.0625	0.0954	6.3906
	300-bus	0.0824	0.0954	41.2466
Limited channel capacity	14-bus	0.0313	0.0571	0.0938
	30-bus	0.0386	0.0722	3.2589
	57-bus	0.0469	0.0823	13.2188
	118-bus	0.0781	0.1254	25.6875
	300-bus	0.0854	0.0571	43.2276
PMU failure	14-bus	0.0313	0.0571	0.0781
	30-bus	0.0386	0.0632	0.4573
	57-bus	0.0479	0.0824	0.8438
	118-bus	0.0635	0.2931	6.7969
	300-bus	0.0854	0.4256	44.2876
ZIB	14-bus	0.0313	0.0571	0.0938
	30-bus	0.0386	0.0632	0.4573
	57-bus	0.0479	0.0824	0.8423
	118-bus	0.0743	0.0954	6.7969
	300-bus	0.1824	0.9541	58.2466

D. Case 4: Lack of communication facilities

The placement site of PMUs is also affected by the lack of communication facilities and their cost [28]. This example extends the modeling to account for limited communication facilities being available in power network substations. The NLP algorithm is used in this study to calculate the restricted communication problems of PMU placement. The results of the simulation on different IEEE test bus systems are summarized in Table 7.

E. Case 5: Limited channel capacity

The required number of PMUs and their effect on channel capacity in suitable locations are studied here. For example, let us assume that the channel capacity for the installation of PMUs is equal to 3. The results are presented in Table 8, and the CPU time required to compute the OPP problem for different IEEE bus test systems in Table 9.

The nonlinear constraint tolerance can be varied from 10^{-4} to 10^{-12} to get the least number of PMUs. From Table 1, it is evident that NLP obtains the least number of PMUs, which is the same as MLP and SLP. NLP can also provide several solutions to the OPP problem. One of the NLP optimal sets matches the MLP solution. On the other hand, the computational time of the MLP is less than the NLP. Table 9 presents the average CPU time for

***Table 10**: Comparison of ZIB results with the proposed MLP, SLP, and NLP.*

IEEE Test System	SA [1]	IPL [6]	PSO [15]	GA [16]	ILP [22], [30]	TS [24]	NLP [29]	MLP	SLP	NLP
14-bus	3	3	3	3	3	3	3	3	3	3
30-bus	8	–	7	7	7	10	7	7	7	7
57-bus	11	12	11	12	11	13	11	11	13	11
118-bus	29	29	28	29	28	–	28	28	29	28

MLP, SLP, and NLP on different IEEE systems.

From Tables 3 and 5, it can be observed that the number of PMUs in these methods is reduced to less than the general case due to the measurement of power flow and zero-injection, respectively. On the contrary, the restricted communication and PMU failure cases result in more PMUs, as shown in Tables 6 and 7. A backup set is generated for the single-PMU failure case, which would increase the PMU installation cost. It should be noted that the number of PMUs would be reduced if power flow and zero injections are considered. To validate the effectiveness of the NLP zero-injection formulation, a comparison of several algorithms' results for zero-injection case is presented in Table 10.

5. CONCLUSION

The OPP problem is addressed using various algorithms to improve the observability of the SG model. To address the placement problem, the observability of MLP, SLP, and NLP formulations is proven. In contrast to the LP method, NLP has the advantage of presenting numerous optimal solutions. Furthermore, NLP, MLP, and SLP require less CPU time to execute the designed algorithm than NLP. To solve the redundancy and optimal difficulties, the NLP is considered with the ZIB measurement. Furthermore, NLP is examined using five different cases: power flow measurement, ZIB measurement, PMU failure, lack of communication facilities, and limited channel capacity.

REFERENCES

[1] T. Baldwin, L. Mili, M. Boisen, and R. Adapa, "Power system observability with minimal phasor measurement placement," *IEEE Transactions on Power Systems*, vol. 8, no. 2, pp. 707–715, May 1993.

[2] S. Shankar, K. B. Yadav, A. Priyadarshi, and V. Rathore, "Study of phasor measurement unit and its applications," in *Recent Advances in Power Systems. Lecture Notes in Electrical Engineering*. Singapore: Springer, 2021, vol. 699, pp. 247–257.

[3] H. Morais, P. Vancraeyveld, A. H. B. Pedersen, M. Lind, H. Johannsson, and J. Ostergaard, "SOSPO-SP: Secure operation of sustainable power systems simulation platform for real-time system state evaluation and control," *IEEE Transactions on Industrial Informatics*, vol. 10, no. 4, pp. 2318–2329, Nov. 2014.

[4] Y. Wang, C. Wang, W. Li, J. Li, and F. Lin, "Reliability-based incremental PMU placement," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 2744–2752, Nov. 2014.

[5] V. Basetti and A. K. Chandel, "Simultaneous placement of PMUs and communication infrastructure in WAMS using NSGA-II," *IETE Technical Review*, vol. 33, no. 6, pp. 621–637, 2016.

[6] B. Xu and A. Abur, "Observability analysis and measurement placement for systems with PMUs," in *IEEE PES Power Systems Conference and Exposition, 2004.*, 2004, pp. 943–946.

[7] S. Shankar, V. Rathore, K. Yadav, and A. Priyadarshi, "Observability of system using optimal PMUs location," in *2021 7th International Conference on Electrical Energy Systems (ICEES)*, 2021, pp. 414–419.

[8] B. Gou and R. G. Kavasseri, "Unified PMU placement for observability and bad data detection in state estimation," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 2573–2580, Nov. 2014.

[9] R. Bottura and A. Borghetti, "Simulation of the volt/var control in distribution feeders by means of a networked multiagent system," *IEEE Transactions on Industrial Informatics*, vol. 10, no. 4, pp. 2340–2353, Nov. 2014.

[10] K. B. Yadav, A. Priyadarshi, S. Shankar, and V. Rathore, "Study of fault current limiter—a survey," in *Innovations in Electrical and Electronic Engineering. Lecture Notes in Electrical Engineering*. Singapore: Springer, 2021, vol. 661, pp. 97–113.

[11] T. Zabaiou, L.-A. Dessaint, and I. Kamwa, "Preventive control approach for voltage stability improvement using voltage stability constrained optimal power flow based on static line voltage stability indices," *IET Generation, Transmission & Distribution*, vol. 8, no. 5, pp. 924–934, May 2014.

[12] R. Salgado, "Determination of critical reactive power points in power systems loadability studies," *IFAC Proceedings Volumes*, vol. 45, no. 21, pp. 393–398, 2012.

[13] N. M. Manousakis, G. N. Korres, and P. S. Georgilakis, "Taxonomy of PMU placement methodologies," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 1070–1077, May 2012.

[14] R. Nuqui and A. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," *IEEE Transactions on Power Delivery*, vol. 20, no. 4, pp. 2381–2388, Oct. 2005.

[15] M. Hajian, A. M. Ranjbar, T. Amraee, and A. R. Shi-

rani, "Optimal placement of phasor measurement units: Particle swarm optimization approach," in *2007 International Conference on Intelligent Systems Applications to Power Systems*, 2007.

[16] F. Marín, F. García-Lagos, G. Joya, and F. Sandoval, "Genetic algorithms for optimal placement of phasor measurement units in electrical networks," *Electronics Letters*, vol. 39, no. 19, pp. 1403–1405, Sep. 2003.

[17] K.-S. Cho, J.-R. Shin, and S. H. Hyun, "Optimal placement of phasor measurement units with GPS receiver," in *2001 IEEE Power Engineering Society Winter Meeting*, 2001, pp. 258–262.

[18] G. Denegri, M. Invernizzi, and F. Milano, "A security oriented approach to PMU positioning for advanced monitoring of a transmission grid," in *International Conference on Power System Technology*, 2002, pp. 798–803.

[19] S. Shankar, V. Rathore, K. Yadav, and A. Priyadarshi, "Comparative analysis for optimal positioning of PMU," in *2021 International Conference on Communication, Control and Information Sciences (ICCISc)*, 2021.

[20] A. Ahmadi, Y. Alinejad-Beromi, and M. Moradi, "Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy," *Expert Systems with Applications*, vol. 38, no. 6, pp. 7263–7269, Jun. 2011.

[21] B. Milosevic and M. Begovic, "Nondominated sorting genetic algorithm for optimal phasor measurement placement," *IEEE Transactions on Power Systems*, vol. 18, no. 1, pp. 69–75, Feb. 2003.

[22] F. Aminifar, C. Lucas, A. Khodaei, and M. Fotuhi-Firuzabad, "Optimal placement of phasor measurement units using immunity genetic algorithm," *IEEE Transactions on Power Delivery*, vol. 24, no. 3, pp. 1014–1020, Jul. 2009.

[23] N. C. Koutsoukis, N. M. Manousakis, P. S. Georgilakis, and G. N. Korres, "Numerical observability method for optimal phasor measurement units placement using recursive tabu search method," *IET Generation, Transmission & Distribution*, vol. 7, no. 4, pp. 347–356, Apr. 2013.

[24] J. Peng, Y. Sun, and H. Wang, "Optimal PMU placement for full network observability using tabu search algorithm," *International Journal of Electrical Power & Energy Systems*, vol. 28, no. 4, pp. 223–231, May 2006.

[25] S. Azizi, A. S. Dobakhshari, S. A. N. Sarmadi, and A. M. Ranjbar, "Optimal PMU placement by an equivalent linear formulation for exhaustive search," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 174–182, Mar. 2012.

[26] S. Chakrabarti, E. Kyriakides, and D. Eliades, "Placement of synchronized measurements for power system observability," *IEEE Transactions on Power Delivery*, vol. 24, no. 1, pp. 12–19, Jan. 2009.

[27] B. Gou, "Generalized integer linear programming formulation for optimal PMU placement," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1099–1104, Aug. 2008.

[28] N. P. Theodorakatos, N. M. Manousakis, and G. N. Korres, "A sequential quadratic programming method for contingency-constrained phasor measurement unit placement," *International Transactions on Electrical Energy Systems*, vol. 25, no. 12, pp. 3185–3211, Dec. 2015.

Alok Priyadarshi received his B.Sc. degree in Electrical Engineering from Muzaffarpur Institute of Technology, Bihar, India, in 1991. He had worked with different national industries for six years. Currently, he is an Assistant Professor in the Department of Electrical Engineering, National Institute of Technology Jamshedpur, India. His fields of interest are grid automation, synchrophasor technology, and power generation.

Vishal Rathore received his B.E. degree in Electrical and Electronics Engineering from Laxmi Narayan College of Technology (LNCT), Bhopal, India, in 2007, M.Tech in Electrical Drives from Maulana Azad National Institute of Technology (MANIT), Bhopal, India, in 2012. Currently, he is working towards his Ph.D. degree at National Institute of Technology (NIT) Jamshedpur, Jharkhand, India. His fields of interest include multilevel inverters based multiphase drives, real-time controllers for power electronics based drives systems, and usage of power electronics in transit systems.

Krishna Bihari Yadav received his B.Sc. in Electrical Engineering from Regional Institute of Technology presently National Institute of Technology Jamshedpur, India, in 1989, M.Sc. in power electronics and Ph.D. from Indian Institute of Technology Roorkee, India, in 2007. Currently, he is an Associate Professor in the Department of Electrical Engineering at the National Institute of Technology Jamshedpur, India. His fields of interest include multiphase electrical machines, power electronics & electric drives, power system economics, renewable power generation, and smart grid.