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# Observable Horizons in the Expanding Universe

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Considering the absorption of extragalactic radiation in an expanding medium, we investigated observable limits of distances, i.e. observable horizons. For each of various radiations, the universe was cloudy in the early stage filled with a dense absorbing medium, and it was thereafter cleared up at some critical time $t_c$. Photons emitted at $t_c$ suffer the reddening, being expressed in terms of red-shift parameter $z$; $z+1=10^{1.6}\sim10^{3.1}$ for $\gamma$-rays, $10^{0.9}\sim10^{2.3}$ for X-rays and $10^{0.9}\sim10^{3}$ for thermal photons corresponding to the Friedmann universe model with the present matter density $\rho_{m0}=10^{-29}\sim10^{-32}$ g/cm$^3$.

As the cosmic black-body radiation is a relict radiation emitted from matter at the observable horizon, it may carry the information on the structure in the early period of $10^5\sim10^7$ years since the birth of the universe. From this point, an inhomogeneous distribution of this radiation with the angular size of $10'\sim1''$ is likely to take place concerning the galaxy formation.

## § 1. Introduction

In the early stage of the expanding universe, a photon cannot propagate freely because of a dense absorbing medium. As the expansion proceeds, such a cloudy universe becomes transparent for the propagation of the photon. The cloudy universe was cleared up at some time of the cosmic evolution. A clear-up time may depend on the energy of photons.

Sometimes, it is said that the observable limit of the universe is attainable upto the so-called horizon of the universe, where the red-shift parameter $z$*) becomes infinite. However, such an observation of the horizon would be possible only if informations from the horizon were transported by some fanciful radiation with an infinite penetrability. Therefore, we must know a more realistic limit of observability, that is, an observable horizon.

The observable horizon is limited by the above-stated clear-up time: Information such as the distributions in direction and energy contained in some radiation cannot reach us if the starting time of a photon is prior to the clear-up time of the universe. Another limitation to the observation depends on the time of occurrence of the radiation sources. Then, the problem to clarify the observable horizon is closely connected with more general problems of the extragalactic matter and the formation of galaxies. In this paper, we try to explain the concept

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*) Here, the red-shift parameter $z$ is defined as $z=(\lambda_0-\lambda_e)/\lambda_e$, where $\lambda_0$ and $\lambda_e$ are the observed and emitted wave lengths respectively.

of the observable horizon under simple assumptions.

In § 2, we define the clear-up time and summarize the results. In § 3, the absorption and scattering mechanisms of photons are summarized for various energy regions. In § 4, the clear-up time is given in terms of a maximum red-shift papameter $z_c$. In § 5, we explain that the cosmic black-body radiation is the radiation from remotest observable sources. In § 6, we suggest that this radiation may have an inhomogeneous angular distribution with a small angular size such as $10' \sim 1''$, which may have relations to the formation of local objects such as galaxies.[1] In § 7, related problems are discussed, such as the isotropic component of X-rays and the ionization degree of extragalactic hydrogen.

Our discussion of the observable horizon is similar to Bahcall and Salpeter's discussion of the absorption line of the radiation from quasars.[2]

## § 2. Horizons in principle and in reality

In principle, we may take as a horizon a front of the radiation which starts from the observer's site at the outset time of the cosmic expansion; we call such a horizon a horizon in principle.*) As stated in § 1, however, this horizon is not attainable in a realistic observation and we define the observable horizon in a more realistic manner.

Now, we consider the energy flux $j(\varepsilon, t)$ of radiation generated from uniformly distributed sources with the average energy yield $Q(\varepsilon, t)$ and the density $\mathcal{N}(t)$, where $t$ is the world time measured from the outset of the expansion and $\varepsilon$ is a photon energy. Taking into account the geometrical dilution and the absorption effect of the radiation, we have

$$j(\varepsilon, t_0) = \frac{c}{4\pi}\int_0^{t_0} \frac{\mathcal{N}(t)\, Q(\varepsilon R_0/R(t),\, t)}{(R_0/R(t))^3} e^{-\tau_0(\varepsilon, t)} dt \tag{1}$$

and

$$\tau_0(\varepsilon, t) = c\int_t^{t_0} n(t')\,\sigma(\varepsilon R_0/R(t'))\,dt', \tag{2}$$

where $R(t)$ is the scale factor given in Appendix A, $n(t)$ is the density of the absorbing medium and $\sigma(\varepsilon)$ is the cross section of a photon with energy $\varepsilon$. Equation (1) shows that the contribution from time $t$ for which $\tau_0(\varepsilon, t) \gg 1$ is automatically cut off, thus giving the observable limit of the early universe. Such an observable limit appears also in the observation of a remote discrete source. This is a reason why we call the early universe "*a cloudy universe*".

*) A complete analysis of the horizon in principle was given by Rindler.[3] According to his classification, the above-stated horizon is a particle horizon different from an event horizon. In the steady state universe, the observable horizon may coincide with the horizon in principle, i.e. the event horizon in this model.

For definiteness, we take the observable condition as $\tau_0(\varepsilon, t)=1$ and define the clear-up time $t_c$ of a photon of energy $\varepsilon$ as*)

$$\tau_0(\varepsilon, t_c)=1. \tag{3}$$

The observable horizon $r_0$ in the comoving coordinate is defined as

$$\int_0^{r_0}\frac{dr}{\sqrt{1-kr^2}}=\int_{t_c}^{t_0}\frac{cdt}{R(t)} \tag{4}$$

in contrast to the horizon in principle, $r_p$, defined by

$$\int_0^{r_p}\frac{dr}{\sqrt{1-kr^2}}=\int_0^{t_0}\frac{cdt}{R(t)}, \tag{5}$$

where we have assumed the cosmological line element as

$$ds^2=c^2dt^2-R^2(t)\left(\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\omega^2\right). \tag{6}$$

In fact, $r_0$ is smaller than $r_p$ but nearly equal to $r_p$. It is, therefore, more convenient to express the observable horizon in terms of the observable red-shift limit defined by

$$z_c+1=R_0/R(t_c). \tag{7}$$

The value of $z_c$ is determined if we assume the absorption and scattering cross sections of the radiation and the universe model. For an illustration, we summarize these values in Table I, assuming the Friedmann model with the

Table I. Clear-up time $t_c$ and observable red-shift limit $z_c$ for the universe with the present matter density (pure hydrogen) $\rho_{m0}=6.7\cdot10^{-31}$ g/cm³ (or $2q_0=10^{-1}$). For electron neutrinos $\nu_e$, we have assumed the following interaction; $e+\nu_e\to e+\nu_e$. For thermal photons, the $z_c$ and $t_c$ depend sharply on the degree of ionization $x$ of hydrogen. For details, see § 4.

| Horizon | Radiation | $t_c$ | $z_c+1$ |
|---|---|---|---|
| Principal | — | 0 | |
| Observable | thermal[a] $\nu_\mu$ | $10^{-4.0}$ sec | $10^{11.2}$ |
| | thermal $\nu_e$ | $10^{-2.1}$ sec | $10^{10.2}$ |
| | $\gamma$-rays | $10^{7.1}$ years | $10^{2.2}$ |
| | relativistic protons | $10^{7.4}$ years | $10^{1.9}$ |
| | X-rays | $10^{8.3}$ years | $10^{1.4}$ |
| | thermal protons $x=0$ | $10^{5.8}$ years | $10^3$ |
| | thermal protons $x=1$ | $10^{8.3}$ years | $10^{1.4}$ |

a) By "thermal", we mean the radiation with energy $\varepsilon=kT_{r0}$, where $T_{r0}$ is the temperature of the cosmic black-body radiation.

*) If we define $\tau_0(\varepsilon, t_c)=a(>1)$, the $z_c$ in Eq. (7) must be increased; for the interactions as in Fig. 3, $z_c+1$ is multiplied by the factor $a^{1/2}$.

present matter density $\rho_{m0}=6.7\cdot10^{-31}\mathrm{g/cm^3}$. In the case of thermal photons, the values of $t_c$ and $z_c$ depend sharply on the ionization degree of extragalactic matter. For this we consider two extreme cases, completely neutral $x=0$ and fully ionized $x=1$. These values instruct us that the remotest quasars with $z\simeq2$ may lie very close to the observable horizons of X-rays and relativistic protons. Hence it seems feasible in the near future to attain the observable horizon.

## § 3. Interaction of photons with cosmic medium

In the present universe, the collision mean free path of photons with various energies is given in Fig. 1. For simplicity, we have assumed that the extragalactic matter is composed of pure hydrogen with relatively low temperature $T_m<10^4$ °K.*) The effective interaction mechanisms are summarized in Table II. The mean free path in the regions $A\sim D$ in Fig. 1 depends on the present average matter density $\rho_{m0}$ or $q_0$, and in the region $E$ it depends on the present black-body temperature $T_{r0}$, if we fix the Hubble constant $H_0$. In the regions $A$ and $B$, the ionization degree $x$ is a crucial factor.

This figure shows that the universe is now transparent for all the photons except for far-ultraviolet photons (region $B$) and extremely high energy $\gamma$-rays

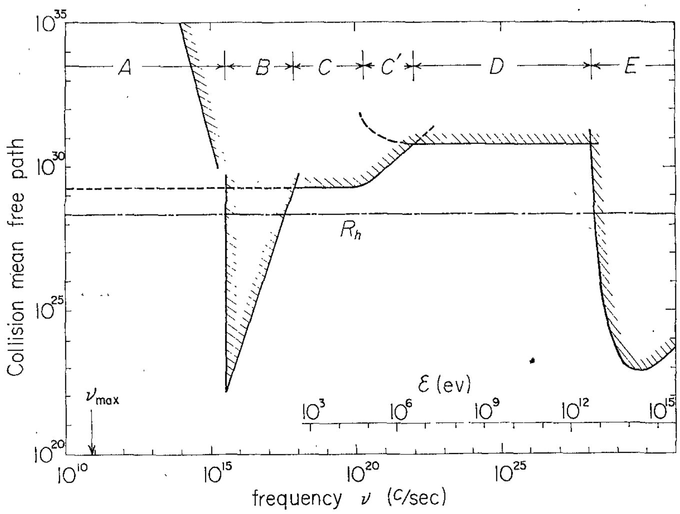


Fig. 1. Collision mean free path of a photon in the present extragalactic space with matter density (pure hydrogen) $\rho_{m0}=6.7\cdot10^{-30}$ g/cc (or $q_0=1/2$) and radiation temperature $T_{r0}=3.5$°K. The interaction mechanisms in the energy regions $A\sim E$ are summarized in Table II. In the regions $A$ and $B$, a solid line is for completely neutral hydrogen and a dashed line is for fully ionized one. $R_h$ denotes the distance of the horizon and $\nu_{\max}$ denotes the peak frequency of the black-body radiation, i.e. $\nu_{\max}=kT_r/h$. The dott-dashed line represents the Hubble radius of the universe.

*) The assumptions may be oversimplified. Recent studies show that the primordial cosmic matter may contain $He^4$ [5]) and the present extragalactic matter may be in a high temperature state such as $10^5\sim10^8$ °K.[6])

Table II.

| Energy region in Figs. 1 and 2 | Interaction mechanism in the extragalactic space |
|---|---|
| $A$ | Rayleigh scattering for the unionized case. (At the right end of region $A$, there may be several absorption lines, which are not shown in these figures.) Electron scattering for the ionized case. (In the high density case, there is free-free absorption on the left side $A'$.) |
| $B$ | Bound-free absorption for the unionized case. Electron scattering for the ionized case. |
| $C$ | Compton scattering. (In the region $C'$, the Klein-Nishina cross section decreases.) |
| $D$ | Pair electron creation in matter. |
| $E$ | Pair electron creation in the photon gas, i.e. $\gamma+\gamma\to e^{+}+e^{-}$.[4] |

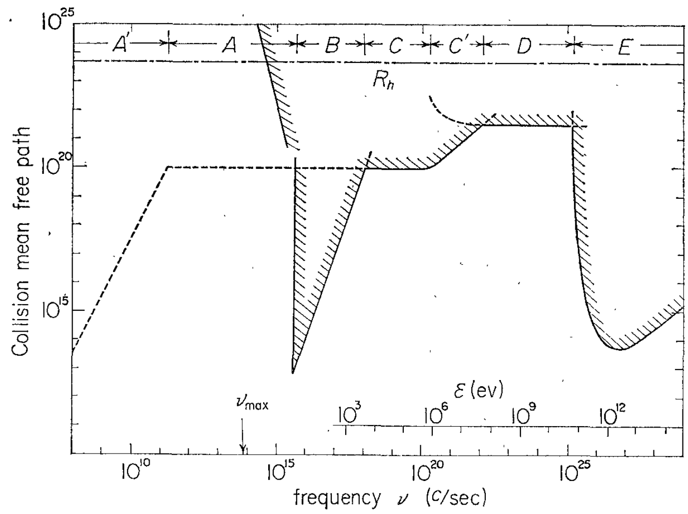


Fig. 2. Collision mean free path of a photon in the early universe with $R_0/R(t)=10^3$, where $R(t)$ is a scale factor of the expanding universe. The free-free absorption in region $A'$ in the ionized case is evaluated assuming that the electron temperature is equal to the radiation temperature. Other notations are the same as in Fig. 1.

(region $E$). For the early universe which is contracted by the factor $R_0/R(t)=10^3$, the same relation with Fig. 1 is now given in Fig. 2. This shows that this early universe is still transparent for the photons in the region $A$ if $x<10^{-3.5}$ but is already opaque for the photons in the other regions. This implies that this contracted stage of the universe is still earlier than the clear-up time of the photons except in the region $A$.

According to Gamow's hot universe model,[7] photons and matter were in a state of thermal equilibrium in the early universe. As the expansion proceeds, the temperature decreases and the ionization equilibrium shifts toward the neutral hydrogen at the temperature of about $T_r=4000\,°\mathrm{K}\sim3000\,°\mathrm{K}$ corresponding to

$2q_0=10\sim10^{-3}$.[8] The mean free path of the photons in the region $A$ suddenly becomes greater than the distance of the horizon: It clears up. Hence the clear-up time $t_c$ may be taken as equal to the neutralization time $t_n$. The exact value of $t_n$ will be estimated in a forthcoming paper and, here, we express the radiation temperature at $t_n$ as

$$T_r(t_n)=T_{1/2}/\xi\,, \tag{8}$$

where $T_{1/2}$ if the temperature where the ionization degree in thermal equilibrium equals 1/2 and $\xi$ is a numerical factor close to unity. The value of $t_n$ is of the order of $10^5\sim10^6$ years.

As the expansion further proceeds, galaxies will begin to be formed here and there in the homogeneous universe. As Gintzburg and Ozernoi[9] have advocated, these newly-born galaxies may be strong heating sources of the extragalactic matter and the neutral hydrogen may be ionized once again, so that the universe becomes cloudy again. Of course, it clears up once more as the density decreases. As the observable horizon, we should take the horizon related to the later clear-up time of the two. In this case, however, a radiation source behind the second observable horizon may be observable to us as a diffused source if the second cloudy stage is so short that the directivity of the radiation is not randomized completely by Thomson scattering.

## § 4. Observable red-shift limit

The observable red-shift limit $z_c$ defined by Eq. (7) can be calculated using Eqs. (2), (3) and (A·1) and referring to the interaction mechanisms of Table II. In the special cases where a cross section does not depend on $\varepsilon$ over the wide energy range, we have a formula for $z_c$ assuming a dust universe model:

$$(2z_cq_0+1)^{1/2}\{(z_c+3)q_0-1\}=\left(\frac{3H_0}{2\sigma n_c c}+3\right)q_0-1\,, \tag{9}$$

where $n_c$ is the present density of the absorbing medium for $q_0=1/2$. For the models of $q_0\ll1$, Eq. (9) reduces to

$$z_c+1\simeq\left(\frac{3H_0}{\sigma n_c c}+6\right)^{1/2}(2q_0)^{-1/2}. \tag{10}$$

This relation is shown in Fig. 3*) for the cases of Thomson scattering of X-rays, electron pair creation of $\gamma$-rays and nuclear reactions by relativistic protons. For the other ranges of photon energy, we can also calculate $z_c$ in a similar way. These relations vary with $q_0$ and, especially in the ranges $A$ and $B$, with $x$. We show this relation for some choices of these parameters in Figs. 4 and 5.

*) Some values of $z_c$ in Table I are taken from this Figure.

In Figs. 4 and 5(a), we have assumed that once neutralized atoms remain unionized thereafter. In Figs. 5(b), (c) and (d), once neutralized atoms are assumed to be reionized at an age of $10^7$ years; this epoch may be identified with the epoch of formation of galaxies. Under these circumstances, the diffusing-out of the discrete images of the sources will occur as stated in the last part of § 3; optical photons emitted in the first transparent stage (the period from $10^5$ years to $10^7$ years) suffer the scattering by free electrons in the period of the second cloudy stage (the period from $10^7$ years to $10^8$ years). After $10^8$ years, photons propagate straightly to the observer and will be observed as some diffuse sources or isotropic background radiation like the blue sky in our atmosphere.

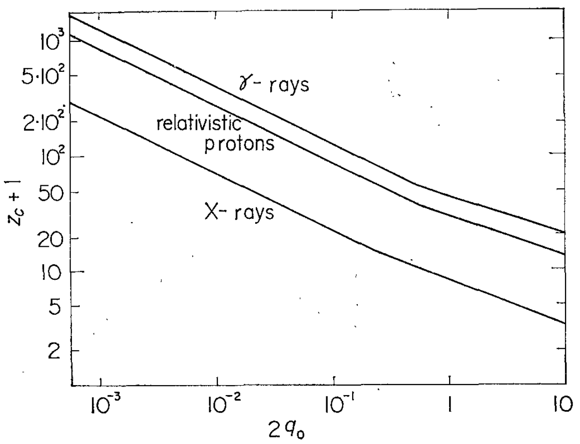


Fig. 3. The observable red-shift limit $z_c$ versus $q_0$, that specifies the present matter density of the universe such as $\rho_{m0}=6.7\cdot10^{-3}$ $(2q_0)$ g/cm$^3$. X-rays and $\gamma$-rays in this figure represent the photons in the energy regions $C$ and $D$ in Fig. 1 respectively. For a relativistic proton, the cross section is taken as $\sigma=4.8\cdot10^{-26}$ cm$^2$.

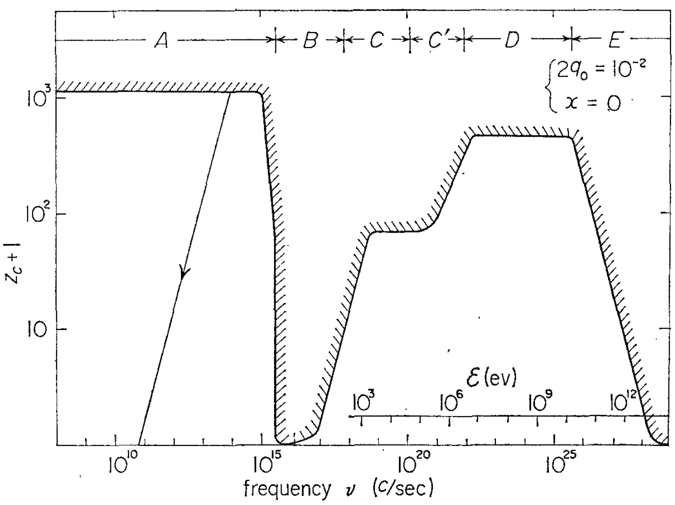


Fig. 4. The observable red-shift limit $z_c$ versus the photon energy for the universe model of $2q_0=10^{-2}$ and the ionization degree $x=0$. The energy regions $A\sim E$ correspond to those of Fig. 1. The line with an arrow denotes the decreasing of the peak frequency $\nu_{\max}$ of the black-body radiation.

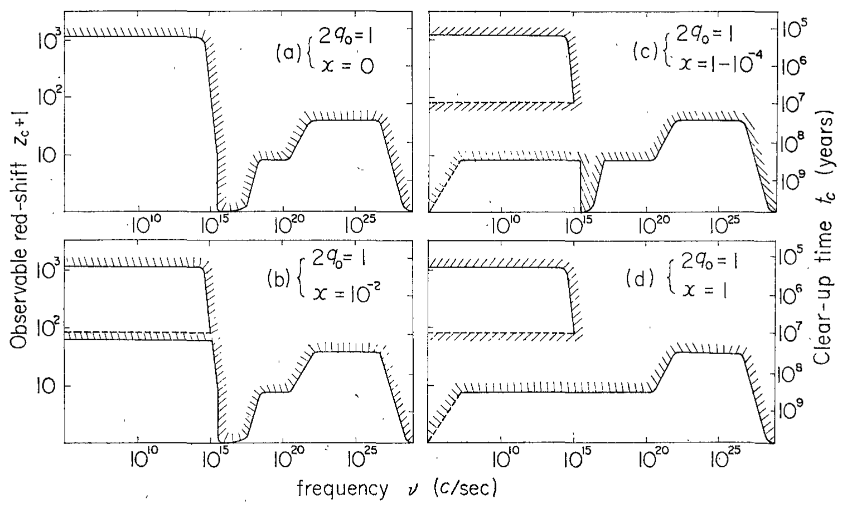


Fig. 5. The same relations as in Fig. 4 for the universe model $2q_0=1$ and for various values of ionization degree $x$. In the cases of (b)$\sim$(d), once neutralized atoms are assumed to be re-ionized at the time of $10^7$ years. The dotted lines in the left end of Figs. (c) and (d) indicate the absorption by free-free transition at an electron temperature of $10^4$ °K.

## § 5. The local condensation of matter and the spatial inhomogeneity of cosmic black-body radiation[1)]

The cosmic black-body radiation[10)] is thought to be a relict radiation which had been emitted from the uniform cosmic matter.[8)] Thus the uniform matter itself is a kind of radiation source in the early period. This radiation, therefore, carries information on the state of the cosmic matter at the decoupling time when this radiation ceased to interact with matter.*)

If the spatial distribution of temperature at this decoupling time was not perfectly homogeneous for some reasons such as due to the local heating of matter by condensation, the radiation flux observed at present should have an inhomogeneous angular distribution, though it may be observed as isotropic with a poor angular resolution, as indicated by observations.[11)] However, it has not been rejected that the distribution has the inhomogeneity with small angular size of $10'\sim1''$.**) Since such a small scale inhomogeneity will give us some information on the local structure of the early universe at the time of $10^5\sim10^6$ years, and consequently on the origin of galaxies, an attempt at detecting the non-uniformity will be of great value in cosmology.

The angular size of the inhomogeneity can be estimated as follows. We assume that appreciable part of cosmic matter within the observable horizon is divided into $N$ individual condensed objects. Then, the mean angular distance $\Delta\theta$ between the neighbouring objects near the horizon is estimated as

*) In §§ 5 and 6, we assume that once neutralized atoms remain unionized; the second cloudy stage does not appear.

**) Epstein seems to have an interest to detect such a small inhomogeneity.[12)]

$$\Delta\theta \sim 1/N^{1/3}, \tag{11}$$

neglecting the curvature of the space. For several values of the condensed mass $M_c$, $\Delta\theta$ is given in Table III.

Table III. Angular distance $\Delta\theta$ between the centers of the neighbouring objects with mass $M_c$. $N$ is the number of such objects within the observable horizon.

| $M_c\,(M_\odot)$ | $2q_0=1$ | | $2q_0=10^{-2}$ | |
|---|---|---|---|---|
| | $N$ | $\Delta\theta$ | $N$ | $\Delta\theta$ |
| $10^7$ | $10^{16.3}$ | $0''.65$ | $10^{14.3}$ | $3''.1$ |
| $10^{11}$ | $10^{12.3}$ | $13''$ | $10^{10.3}$ | $1'.0$ |
| $10^{15}$ | $10^{8.3}$ | $5'.6$ | $10^{6.3}$ | $26'$ |

There are a number of causes which give rise to the inhomogeneity. The condensation results in the variation of the optical depth and consequently in the variation of temperature, with which radiation is emitted. The condensation further results in the inhomogeneous motion of matter and the inhomogeneous gravitational potential which modify the energy spectrum of the radiation, as was first mentioned by Sacks and Wolfe.[13] Taking these effects into account, the observed flux distribution $T(\theta, \omega)$ can be given in terms of the intrinsic distribution $T_i(\theta, \omega)$ due to the first cause and the modification factor due to the second as

$$T(\theta, \omega) = T_i(\theta, \omega)\, f(\theta, \omega), \tag{12}$$

where $\theta$ and $\omega$ are angular parameters and $f(\theta, \omega)$ is given in Eq. (A·8) in Appendix A.

There may be another effect of the local condensation; photons suffer the gravitational deflection by the inhomogeneous field. This effect may restrict the measurement of such a small angular size as in Table III, as suggested by Zeldovich and Gunn.[14] This problem will be discussed elsewhere.

## § 6. Local condensation of matter

As the cause of the matter condensation, we consider the following two possibilities:

(a) Primordial condensation

By some reasons, there have been primordially many condensed regions with temperatures higher than in surrounding medium; Ambartzmian's $D$-object may be the case.[15]

(b) Fluctuational condensation

Density fluctuations in a uniform medium grows to form local condensed objects, as a result of gravitational instability, thermal instability, compression by turbulent motion or other causes.

In the above two cases, respectively, the following conditions must be satisfied

so that they are observable as the inhomogeneity of the black-body radiation:
(a) In order that the primordial object can be discriminated against the surrounding black-body radiation, it will be a sufficient condition that a higher temperature state had been maintained up to the clear-up time. For the radius and the density of the object of $r$ and $n'$, the cooling time $t_{\rm cool}$ by radiation flow may be of the order of

$$t_{\rm cool} \simeq \sigma_T r^2 n'/c\,. \tag{13}$$

The observability condition

$$t_{\rm cool} > t_c \tag{14}$$

can be rewritten as

$$M_p > 10^{13.0}\left(\frac{n}{n'}\right)^{1/2}\left(\frac{10^3 R(t_c)}{R_0}\right)^{15/4}\frac{\Psi_c^{-3/4}}{(2q_0)^{1/2}}M_\odot\,, \tag{15}$$

where $M_p$ is the mass of the condensed object defined as $M_p = 4\pi\, m_p n' r^3/3$; $m_p$ is the proton mass and $\Psi_c$ is the value at $t_c$ of $\Psi$ given by Eq. (A·2)
(b) In the fluctuational condensation, there must be two additional conditions, a sufficient amount of available energy for heating and a long enough period of time to establish the thermal equilibrium between matter and radiation. In the ionized stage, the energy is exchanged between matter and radiation through the Compton interaction,[6] and the time scale $t_{\rm ex}$ of this exchange is given as

$$t_{\rm ex} \simeq (n'\sigma_T k T_r/m_e c^2)^{-1}. \tag{16}$$

Therefore, the second condition above is written as

$$\eta \equiv t_{\rm ex}/t_c < 1\,. \tag{17}$$

This condition imposes the lower limit on the density contrast $n'/n$ in terms of $\eta$. If we take

$$t_c = \lambda t_{\rm cool}\,, \tag{18}$$

we have

$$M_f \simeq 10^{11.5}\lambda^{-3/2}\eta^{1/2}(10^3 R(t_c)/R_0)^{5/2}\Psi_c^{-1}M_\odot\,. \tag{19}^{*)}$$

If we assume $\lambda^{-3/2}\eta^{1/2} > 1$ rather arbitrarily, the observable condition becomes

$$M_f > 10^{11.5}(10^3 R(t_c)/R_0)^{5/2}\Psi_c^{-1}M_\odot\,. \tag{20}$$

*) It is remarkable that $M_f$ takes a value close to the mass of a galaxy in the case $\lambda^{-3/2}\,\eta^{1/2} \sim 1$ and does not sharply depend on $q_0$. $M_f$ is also written as

$$M_f \sim m_p \frac{(c/H(t_c))^2}{\sigma_T}\left(\frac{kT_{rc}}{m_e c^2}\right)^{1/2}.$$

Here, we do not discuss whether this coincidence is meaningful or accidental for the galaxy formation.

For the available energy for heating to be $\Delta E$ per nucleon, the deviation of radiation temperature $\Delta T_r$ from the surrounding temperature is given as

$$\frac{\Delta T_r}{T_r} < 10^{0.84}\left(\frac{\Delta E}{m_p c^2}\right)^{1/4}\left(\frac{10^3 R(t_c)}{R_0}\right)^{7/8}\frac{\Psi_c^{1/8}}{\eta^{1/4}}. \tag{21}$$

The numerical results are given in Table IV using Eqs. (15), (20) and (21). It is remarkable that the critical mass values in this table lie in the range between the mass of a galaxy and that of a cluster of galaxies. Although this result is only a preliminary one and may change even qualitatively, there will be always a lower limit of the observable mass as a discreate source or as an inhomogeneous background radiation.

Table IV. Observable lower mass limit for $M_p$ and $M_f$. For the fluctuational condensation, the degree of density contrast $(n'/n)$ and the deviation of temperature $\Delta T_r/T_r$ are shown. The parameters $\xi$, $\eta$ and $\lambda$ defined by Eqs. (8), (17) and (18) are taken unity for simplicity.

| $2q_0$ | $(n'/n)^{1/2}M_p(M_\odot)$ | $M_f(M_\odot)$ | $(n'/n)$ | $\Delta T_r/T_r$ [a] |
|---|---|---|---|---|
| 1 | $10^{12.7}$ | $10^{11.4}$ | $10^{2.7}$ | $10^{-0.22}$ |
| $10^{-1}$ | $10^{13.5}$ | $10^{11.7}$ | $10^{3.6}$ | $10^{-0.24}$ |
| $10^{-2}$ | $10^{14.2}$ | $10^{11.9}$ | $10^{4.6}$ | $10^{-0.26}$ |
| $10^{-3}$ | $10^{14.8}$ | $10^{12.0}$ | $10^{5.7}$ | $10^{-0.21}$ |

[a] Assuming $\Delta E/m_p c^2 = 10^{-4}$.

## § 7. Some related problems

The local condensed objects such as in § 6 may evolve into galaxies, in which the star formation from the gaseous matter commences. Such a process of galaxy formation is usually considered as a very violent phenomenon and a young galaxy may be a strong source of various radiations. For example, Peebles and Partridge[16] have considered a young galaxy as a strong source of ultra-violet radiation, which however may be observed in the infra-red region because of a larger red-shift. Another possibility has been suggested that the isotropic component of the background X-rays may be due to these young galaxies.

Recently, the observed flux of the background X-rays has been found to be larger, by the factor $10\sim10^2$, than the value expected from the summation of all the contributions from observable galaxies including quasars. It is a natural attitude to assume the contribution of probable powerful sources which have not been observed directly up to now.[17),18)] It is a crucial point with regard to this interpretation whether the early violent phase appears after the clear-up time of X-rays or not.

If extragalactic matter contains helium or heavier elements, the collision mean free path is reduced, especially in the region $B$ in Table II. The absorption of soft X-rays is very sensitive to the composition of extragalactic matter and we might be possible to obtain some information on the composition from this absorption.[18] The degree of ionization $x$ is also a critical factor for this problem as may be anticipated from Fig. 5. Moreover, a temporal change of $x$ is a very sensitive factor to determine the behaviour of the observable horizon in the regions $A$ and $B$ in Table II. Up to now, no definite conclusion on $x$ seems to be given from observations.[19] From theoretical considerations, hydrogen may become essentially neutral, i.e. $x=10^{-5.5}\sim10^{-2.5}$ for $q_0=10\sim10^{-3}$,[20] if matter is spatially uniform and no heating mechanism is operative. The heating associated with the galaxy formation should, however, be seriously investigated.*)

In this paper, we have assumed that appreciable part of cosmic matter remains in extragalactic space even after the galaxy formation. If we give up this assumption and assume that almost all of the cosmic matter has condensed into local objects, the universe may clear up owing to this condensation (see Appendix B). However, the fraction of condensed matter into local objects remains to be solved.

Recently, Rees and Sciama[21] have conjectured that there may be such large scale density inhomogeneities like quasar clusters that may modify the angular distribution of cosmic black-body radiation by the mechanism pointed out by Sacks and Wolfe.[13] Such a large scale structure of the universe is also a critical factor of the observable horizon, because the density distribution and our site in this universe are very sensitive to the absorption of radiation.

Finally, the remark should be made that we have considered in this paper the absorption or scattering only in extragalactic space but not in more dense circumstances such as interstellar space and interplanetary space. Therefore, the observable limit in practice may be more restricted than the observable horizon in this paper. If we use photons of various energies, the detection of the observable horizon seems to be feasible in near future.


## Acknowledgements

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## Appendix A

*Red-shift parameter in the expanding universe*

In the isotropic uniform universe model, a scale factor $R(t)$ changes according to the relation

$$\left(\frac{1}{R(t)}\frac{dR(t)}{dt}\right)^2=H_0{}^2\left(\frac{R_0}{R(t)}\right)^3\Psi(R(t)/R_0,\, q_0,\, \beta_0), \qquad (\mathrm{A}\cdot 1)$$

*) If the heating is operative in the near past, i.e. $R(t)/R_0\simeq1$, the matter temperature must be limited by the available energy $W$ per galaxy with density $\mathfrak{N}$ as $kT_m\simeq W\mathfrak{N}/n$. For the nuclear energy $W\simeq10^{62}$ erg and the gravitational energy $W\simeq10^{58}$ erg, we have $T_m\simeq10^8$ °K and $T_m\simeq10^4$ °K respectively.

where

$$\Psi = \frac{2q_0 + q_0\beta_0 R_0/R(t) + \{(1+\beta_0) - 2q_0(1+\beta_0/2)\}R(t)/R_0}{(1+\beta_0)},$$

$$H_0 = \left(\frac{1}{R(t)}\frac{dR(t)}{dt}\right)_0, \quad \beta_0 = \frac{2bT_{r0}^4}{\rho_{m0}c^2} \tag{A·2}$$

and

$$q_0 = -\left(\frac{d^2R(t)}{dt^2}/R(t)\right)/H_0^2 = \frac{\rho_{m0}(1+\beta_0)}{(4\pi G/3H_0^2)},$$

$\rho_m$ being the matter density and $bT_r^4$ being the radiation energy density. The suffix 0 denotes the value at the present $t=t_0$.

The observable quantities are $H_0$ and $T_{r0}$ and we take the values in this paper as

$$H_0 = 100 \text{ km/sec Mpc} \quad \text{and} \quad T_{r0} = 3.5°\text{K}.$$

The present matter density is rewritten in terms of $q_0$ as

$$\rho_{m0} = 6.7\ 10^{-30}(2q_0)\ g/\text{cm}^3. \tag{A·3}$$

$q_0$ has not yet been determined from observation.

The red-shift parameter $z$ in the expanding universe is given as

$$z+1 = R_0/R(t_e), \tag{A·4}$$

where $t_e$ is the time of emission. In the case of the statistically uniform model with small inhomogeneity, this parameter can be calculated as given in reference 22). In the Newtonian approximation for inhomogeneous gravitational field, we have

$$z+1 = (R_0/R(t_e))f(\theta, \omega) \tag{A·5}$$

and

$$f(\theta, \omega) = \frac{1-(v_e^2-\varphi_e)/c^2}{1-(v_0^2-\varphi_0)/c^2}\ \frac{1-\boldsymbol{e}\cdot\boldsymbol{v}_0/c+(\varphi_0-\psi)/c^2}{1-\boldsymbol{e}\cdot\boldsymbol{v}_e/c+\varphi_e/c^2}, \tag{A·6}$$

where $\boldsymbol{v}_e$, $\boldsymbol{v}_0$, $\varphi_e$ and $\varphi_0$ are the peculiar velocities and the local gravitational fields of the emitter (suffix $e$) and of the observer (suffix 0), respectively, $\boldsymbol{e}$ is the unit vector from the emitter to the observer whose direction is represented by $\theta$ and $\omega$, and $\psi$ is given as

$$\psi = -\frac{1}{2}\int_{t_e}^{t_0}\left(\frac{d\varphi}{dt}\right)dt.$$

Considering that $\varphi \sim v^2$, $\psi \sim \varphi\, v/c$, and $v/c \ll 1$, Eq. (A·6) is expressed to the order of $v^2/c^2$ as

$$f(\theta, \omega) = 1 + \frac{(\boldsymbol{v}_e - \boldsymbol{v}_0) \cdot \boldsymbol{e}}{c} - \frac{(\varphi_e - \varphi_0)}{c^2} - \frac{v_e^2 - v_0^2}{c^2}. \tag{A·8}$$

## Appendix B

*Clearing up by the local condensation of matter*

Taking the absorption cross section of a local object as its geometrical area, the condition of the clearing up is given as

$$\pi r^2 \mathcal{N} < H/c, \tag{B·1}$$

where $\mathcal{N}$ is the number density of the local objects with radius $r$ and matter density $n'$, and the mean density in the universe is $n = 4\pi r^3 n' \mathcal{N}/3$. This condition can be rewritten as

$$(\mathcal{M}/M_c)^{1/2} < n'/n, \tag{B·2}$$

where $\mathcal{M} = 4\pi\rho_m (c/H)^3/3$ and $M_c$ is the mass of an object. If we take $q_0 = 0.5$ Eq. (B·2) becomes

$$(\mathcal{M}_0/M_c)^{1/2} (R(t)/R_0)^{3/4} < n'/n, \tag{B·3}$$

where

$$\mathcal{M}_0 = 4\pi\rho_{m0} (c/H_0)^3/3.$$

We further assume a condition that the density of matter $n_r$, remaining in extragalactic space is small enough to satisfy the condition such as

$$n_r/n < H/(\sigma n c). \tag{B·4}$$

For $M_c = 10^{11} M_\odot$, $R_0/R(t) = 10$, $q_0 = 0.5$ and $\sigma = \sigma_T$ as an example, Eqs. (B·3) and (B·4) give

$$n'/n > 10^{5.5} \quad \text{and} \quad n_r/n < 10^{-0.5}. \tag{B·6}$$

### References

1) H. Satō, Prog. Theor. Phys. **37** (1967), 1032.
2) J. N. Bahcall and E. E. Salpeter, Astrophys. J. **142** (1965), 1677.
3) W. Rindler, Month. Notice R.A.S. **116** (1956), 662.
4) R. J. Gould and G. Schreder, Phys. Rev. Letters **16** (1966), 252.
J. V. Jelley, Phys. Rev. Letters **16** (1966), 469.
5) R. V. Wagoner, W. A. Fowler and F. Hoyle, Astrophys. J. **148** (1967), 3.
H. Satō, Prog. Theor. Phys. **38** (1967), 1083.
6) R. Weymann, Astrophys. J. **145** (1966), 560; **147** (1967), 887.
7) G. Gamow, Rev. Mod. Phys. **21** (1949), 367.
8) R. H. Dicke, P. J. E. Peebles, P. G. Roll and D. T. Wilkinson, Astrophys. J. **142** (1965), 414.
9) V. L. Gintzburg and L. M. Ozernoi, Soviet Astron.-AJ **9** (1966), 726.
10) A. A. Penzias and R. W. Wilson, Astrophys. J. **142** (1965), 419.
As to the references of other observations, see the paper: D. T. Wilkinson, Phys. Rev.

Letters **19** (1967), 1195.
11) R. B. Partridge and D. T. Wilkinson, Phys. Rev. Letters **18** (1967), 557; Nature **215** (1967). 719.
E. K. Conklin and R. N. Bracewell, Phys. Rev. Letters **18** (1967), 557; Nature **216** (1967), 777.
12) E. E. Epstein, Astrophys. J. Letters **148** (1967), 157.
13) R. L. Sacks and A. M. Wolfe, Astrophys. J. 147 (1967), 73.
14) Ya. B. Zeldovich, Soviet Astron. AJ 8 (1964), 13.
J. E. Gunn, Astrophys. J. **147** (1967), 1.
15) For example, Ya. B. Zeldovich and I. D. Novikov, Soviet Astron.-AJ **10** (1967), 602.
16) R. B. Partridge and P. J. E. Peebles, Astrophys. J. **147** (1967), 868; **148** (1967), 377.
17) M. Matsuoka, H. Satō and H. Tabara, Uchusen Kenkyu (mimeographed circular in Japanese) **13** (1968), 403.
J. Silk, Astrophys. J. **151** (1968), L. 19.
18) M. Matsuoka, M. Oda, Y. Ogawara, S. Hayakawa and T. Kato, Space Research VIII (1968).
19) J. Gunn and B. Peterson, Astrophys. J. **142** (1965), 1633.
J. Koehler, Astrophys. J. **146** (1966), 504.
20) H. Takeda and H. Satō, unpublished.
21) M. J. Rees and D. W. Sciama, Nature **217** (1968), 511.
22) J. Kristian and R. K. Sacks, Astrophys. J. **143** (1966), 379.