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OBSERVABLES IN LOW-ENERGY SUPERSTRING MODELS

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ABSTRACT

We compile phenomenological constraints on the minimal low-energy effective theory which can be obtained from the superstring by Calabi-Yau compactification. Mixing with the single additional neutral gauge boson in this model reduces the mass of the conventional  $Z^0$ . Field vacuum expectation values are constrained by the experimental upper bound on this shift. Then, requiring the sneutrino mass squared to be positive constrains the scale of supersymmetry breaking more than do lower bounds on the masses of new charged particles and of sparticles. More model-dependent constraints follow from the "naturalness" requirement that observables do not depend sensitively on input parameters. We find a preference for the second neutral gauge boson to weigh  $\lesssim 320$  GeV,  $m_{\tilde{g}} \lesssim 250$  GeV and  $m_{\tilde{q}} \lesssim 500$  GeV. Dynamical generation of the gauge hierarchy is possible if  $m_t \lesssim 70$  GeV, with lower values of  $m_t$  being favoured.

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Many possibilities exist for the low-energy effective four-dimensional supersymmetric gauge theory<sup>1)</sup> obtained from the superstring<sup>2)</sup> after Calabi-Yau compactification<sup>3)</sup> and gauge symmetry breaking by Wilson loops<sup>4)</sup>. The only known source of soft supersymmetry breaking in the observable sector is a gaugino mass<sup>5)</sup>  $m_{\frac{1}{2}}$ . No-scale scenarios<sup>6)</sup> for the dynamical generation of the gauge hierarchy do not admit<sup>7)</sup> intermediate scales of gauge symmetry breaking. A unique low energy gauge group with a single well-defined additional gauge generator is then selected<sup>7)</sup>, along with an almost unique set of light matter fields<sup>7),8)</sup>. In this paper, we explore systematically the phenomenological constraints<sup>7)</sup> on the class of models with the general structure so selected.

The new neutral gauge boson  $Z_E$  expected in these models mixes with the conventional  $Z^0$ , pushing its mass down. Observations by UAl and UA2<sup>9)</sup> set an upper bound to this shift, which provides in turn a lower bound on the vacuum expectation value  $x$  of the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  singlet field  $N$  which gives most of the mass of the second neutral gauge boson. This constraint is more severe than bounds from low-energy neutral current data<sup>6),10)</sup> and from the non-observation of the second neutral gauge boson at the CERN  $\bar{p}p$  collider<sup>6),11)</sup>. The lightest charged scalar sparticle is then found to be the  $\tilde{e}_L$ , if  $m_{\frac{1}{2}}$  is small enough to yield sparticle masses of contemporary experimental interest. However, imposing  $m_{\tilde{e}_L} > 22$  GeV as required by PETRA limits<sup>12)</sup> is not so severe a constraint on the scale  $m_{\frac{1}{2}}$  of soft supersymmetry breaking as is the requirement that  $m_{\tilde{\nu}}^2 > 0$ . This bound is also more severe than  $\bar{p}p$  collider bounds on  $m_{\tilde{g}}$ ,  $m_{\tilde{q}}$ <sup>13)</sup>,  $m_{\tilde{e}_L}$ <sup>14)</sup>, as well as bounds on  $m_{\tilde{e}}$  and  $m_{\tilde{\gamma}}$  from searches for  $e^+e^- \rightarrow \gamma + \text{nothing}$ <sup>15)</sup>. Constraints on  $m_{\frac{1}{2}}$  from the masses of unseen charged Higgs-like particles are possibly more severe, but depend on their unknown Yukawa couplings. A model-dependent and permeable upper bound on  $m_{\frac{1}{2}}$  follows from requiring that the dynamically determined ratio  $x/m_W$  not be "unnaturally" sensitive to other unknown input Yukawa couplings. The possibility of such a naturalness problem was first emphasized in Ref. 16). A "no-scale" scenario<sup>6),7),17)</sup> for the dynamical generation of  $m_W$  which is consistent with all these phenomenological constraints is only possible if  $m_t \lesssim 70$  GeV. Arbitrarily low values of  $m_t$  are allowed, while values much above 50 GeV require very special values of the input parameters, so smaller values of  $m_t$  are preferred.

The mass matrix for the massive neutral gauge bosons ( $Z^0, Z_E$ ) in the minimal low-energy theory that can be obtained from the superstring after Calabi-Yau compactification and Wilson loop gauge symmetry breaking is<sup>6)</sup>:

$$m_{Z^0}^2 \begin{pmatrix} 1 & \sin\theta_W \frac{4v^2 - \bar{v}^2}{3(v^2 + \bar{v}^2)} \\ \sin\theta_W \frac{4v^2 - \bar{v}^2}{3(v^2 + \bar{v}^2)} & \sin^2\theta_W \frac{25x^2 + 16v^2 + \bar{v}^2}{9(v^2 + \bar{v}^2)} \end{pmatrix}, \quad (1)$$

where  $m_{Z^0}^2 = \frac{1}{2}(g_2^2 + g_1'^2)(v^2 + \bar{v}^2)$  is the unmixed  $Z^0$  mass, with  $v \equiv \langle 0 | H^0 | 0 \rangle$  and  $\bar{v} \equiv \langle 0 | \bar{H}^0 | 0 \rangle$  the conventional Higgs vacuum expectation values, and  $x \equiv \langle 0 | N | 0 \rangle$ . The mixing in (1) pushes the mass  $m_{Z_1}$  of the lighter neutral gauge boson eigenstate  $Z_1$  below  $m_{Z^0}$ , so that:

$$\sin^2\bar{\theta}_W \equiv 1 - \frac{m_W^2}{m_{Z_1}^2} < \sin^2\theta_W \equiv \frac{g_1'^2}{g_2^2 + g_1'^2} \Big|_{\mu=m_W}. \quad (2)$$

Taking  $\sin^2\theta_W = 0.224 \pm 0.011$  and  $\sin^2\bar{\theta}_W = 0.212 \pm 0.022$  from a recent compilation of UA1 and UA2 measurements of the  $W^\pm$  and  $Z^0$  (sorry,  $Z_1$ ) masses<sup>9)</sup>, we infer

$$\Delta \equiv \sin^2\theta_W - \sin^2\bar{\theta}_W = 0.012 \pm 0.023 \quad (3)$$

Figure 1 shows contours of  $\Delta$  and the mass of the heavier eigenstate  $Z_2$  in the  $(\bar{v}/v, x/v)$  plane. Since  $H$  is coupled to the  $t$  quark, we expect  $\bar{v}/v < 1$  in any phenomenological model. Taking the  $1\sigma$  bound  $\Delta < 0.035$  from (3), and choosing as the experimentally preferred input values  $\sin^2\theta_W = 0.224$  and  $m_W = 82$  GeV, we infer

$$\frac{x}{v} > 2.55, \quad m_{Z_2} > 140 \text{ GeV} \quad (4a)$$

for  $\bar{v}/v < 1$ , which can be improved to

$$\frac{x}{v} > 3.20, \quad m_{Z_2} > 210 \text{ GeV} \quad (4b)$$

if we take  $\bar{v}/v < 0.6$  as in most of our "no-scale" models<sup>7)</sup>. The constraint (4) is more severe than our previous constraints from low-energy neutral currents<sup>6),10)</sup> and from non-observation of the  $Z_2$  at the CERN  $\bar{p}p$  collider<sup>6),11)</sup>, though less severe than the more uncertain cosmological lower bound on  $m_{Z_2}$  coming from the primordial nucleosynthesis bound on the number of effective species of light left-handed neutrinos<sup>18)</sup>.

We turn now to the sparticle masses. The gluino and photino<sup>\*)</sup> masses are

$$m_{\tilde{g}} = m_{1/2}, \quad m_{\tilde{\gamma}} \simeq \frac{1}{7} m_{1/2}. \quad (5)$$

When the corresponding Yukawa couplings are negligible, as for the first two generations, the spin-0 masses are given by:

$$m_i^2 = m_{S_i}^2(m_{1/2}) + m_{D_i}^2\left(\frac{x}{v}, \frac{\bar{v}}{v}\right), \quad (6)$$

with the soft SUSY-breaking terms:

$$m_{S_i}^2 = m_{1/2}^2 \cdot \sum_{a=3,2,1} C_a^i(m_{1/2}), \quad (7)$$

where

$$C_3^i(m_{1/2}) = c_3^i \frac{\alpha_U}{2\pi} \ln \frac{M_X}{m_{1/2}}, \quad (8a)$$

$$C_2^i(m_{1/2}) = \frac{c_2^i}{6} \left[ 1 - \frac{1}{\left(1 + 3 \frac{\alpha_U}{2\pi} \ln \frac{M_X}{m_{1/2}}\right)^2} \right], \quad (8b)$$

$$C_1^i(m_{1/2}) = \frac{c_1^i}{18} \left[ 1 - \frac{1}{\left(1 + 9 \frac{\alpha_U}{2\pi} \ln \frac{M_X}{m_{1/2}}\right)^2} \right], \quad (8c)$$

$\alpha_U$  is the gauge coupling at the grand unification scale  $M_X$ , and the numerical coefficients  $c_a^i$  are listed in Table 1. The general form of the D-terms in Eq. (6) is

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<sup>\*)</sup>We recall that the physical neutral fermion mass eigenstates are in general mixtures of the  $\tilde{\gamma}$  and other fields. We have checked that the lightest mass eigenstate in the models discussed later in this paper has a mass similar to that given in Eq. (5).

$$m_{D_i}^2 \left( \frac{x}{v}, \frac{\bar{v}}{v} \right) = m_W^2 \left\{ -T_3^i \left[ \frac{1 - (\bar{v}/v)^2}{1 + (\bar{v}/v)^2} \right] + Y^i \tan^2 \theta_W \cdot \right. \\ \left. \cdot \left[ \frac{1 - (\bar{v}/v)^2}{1 + (\bar{v}/v)^2} \right] + Y_E^i \tan^2 \theta_W \left[ \frac{5(x/v)^2 - (\bar{v}/v)^2 - 4}{3(1 + (\bar{v}/v)^2)} \right] \right\}, \quad (9)$$

where the numerical values of  $T_3^i$ ,  $Y^i$  and  $Y_E^i$  are also listed in Table 1. Bounds on the SUSY breaking parameter  $m_{\frac{1}{2}}$  resulting from low-energy SUSY search experiments in the simplifying case  $x = v = \bar{v}$  were quoted in Ref. 19). However, this special choice of vacuum expectation values is actually disfavoured by dynamical calculations<sup>7)</sup> and is excluded by the  $Z_1$  mass bound (4) shown in Fig. 1. Since one expects  $x/v > 1 > \bar{v}/v$ , it is clear from Eq. (9) that the D-terms are most negative if  $T_3^i > 0$ ,  $Y^i < 0$ ,  $Y_E^i < 0$ , and the quantum numbers in Table 1 therefore indicate that the  $\tilde{\nu}$ ,  $\bar{H}^0$  and  $H^+$  have the largest negative contributions  $m_{D_i}^2$  to their mass squared. One does not expect large supersymmetric masses for the  $\tilde{\nu}$  because their Yukawa couplings are negligible. One must require  $m_{\tilde{\nu}}^2 > 0$  in the physical vacuum, and this is shown in Fig. 2 as a constraint on  $m_{\frac{1}{2}}$  as a function of  $x/v$  for plausible values of  $\bar{v}/v$ . In addition to the Higgs doublets  $H, \bar{H}$  whose neutral components develop non-zero VEVs, the models considered in Refs. 6) and 7) contain also physical "unhiggses"  $H'_a, \bar{H}'_a$  ( $a = 1, 2$ ) with the same quantum numbers but no VEVs. To give to the fermionic partners of their charged components,  $\tilde{H}'_a{}^+$  and  $\tilde{\bar{H}}'_a{}^-$ , masses greater than 20 GeV, there must be in the superpotential couplings of the form  $\lambda'_a H'_a \bar{H}'_a N$ , which were assumed in Refs. 6) and 7) to be small compared to  $\lambda H \bar{H} N$ . In general, the "unhiggs" mass matrices will have diagonal contributions of the form  $m_S^2 + m_D^2 + \lambda_a'^2 x^2$  and off-diagonal contributions of the form  $\pm \lambda'_a (A_{\lambda_a} m_{\frac{1}{2}} x + \lambda v \bar{v})$ . If the terms involving the Yukawa couplings can be neglected (which need not be the case, since the validity of this approximation depends on the unknown value of  $\lambda'_a$ ), then we are left with Eq. (6), and in this case  $m_{H'_a} > 20$  GeV would give the more stringent constraint on  $m_{\frac{1}{2}}$  shown in Fig. 2 as a dashed line. Because of the previous lower bound on  $x/v$  shown in Fig. 1, we can derive an absolute lower bound on  $m_{\frac{1}{2}}$ :

$$m_{1/2} \geq \begin{cases} 140 & (210) \text{ GeV} \\ 100 & (150) \text{ GeV} \end{cases} \text{ for } \frac{\bar{v}}{v} = \begin{cases} 0.2 \\ 0.6 \end{cases} \quad (10)$$

if we use  $m_{\tilde{\nu}}^2 > 0$  ( $m_{H'_a} > 20$  GeV). We re-emphasize that the  $m_{H'_a}$  constraint is less certain because of the unknown Yukawa couplings  $\lambda'_a$ .

These constraints (10) are much more severe than those arising from unsuccessful sparticle searches. The  $\bar{p}p$  collider bound<sup>13)</sup> on  $m_{\tilde{g}}$  just translates, via Eq. (5), into:

$$m_{1/2} \gtrsim 45 \text{ GeV}, \quad (11)$$

which is also shown in Fig. 2. The  $\bar{p}p$  collider bound<sup>13)</sup>  $m_{\tilde{q}} \gtrsim 50 \text{ GeV}$  is even less interesting, since Eqs. (5) to (9) tell us<sup>5)</sup> that  $m_{\tilde{q}} \gtrsim 1.95 m_{\tilde{g}}$  before the inclusion of D-terms, which do not change the situation drastically. In the relevant range of  $x/v$  and  $m_{\frac{1}{2}}$ ,  $m_{\tilde{e}_L} < m_{\tilde{e}_R}$  and the  $e^+e^-$  constraint<sup>12)</sup>  $m_{\tilde{e}_L} > 22 \text{ GeV}$  is also shown in Fig. 2. The UA1 bound<sup>14)</sup> on  $(m_{\tilde{e}_L}, m_{\tilde{\nu}})$  from the absence of  $W \rightarrow \tilde{e}\tilde{\nu}$  decay is not interesting in our model, because Eqs. (6) to (9) and Table 1 tell us

$$m_{\tilde{e}_L}^2 = m_{\tilde{\nu}}^2 + m_W^2 \left[ \frac{1 - (\bar{v}/v)^2}{1 + (\bar{v}/v)^2} \right], \quad (12)$$

so that  $m_{\tilde{\nu}}^2 > 0$  provides a lower bound on  $m_{\tilde{e}_L}$  which is greater than the UA1 lower bound of 33 GeV if  $\bar{v}/v < 0.85$ , as expected in our models. The ASP bound<sup>15)</sup> on  $(m_{\tilde{e}}, m_{\tilde{\gamma}})$  is also uninteresting, since the constraints (10) due to  $m_{\tilde{\nu}}^2 > 0$  and the relation (5) tell us

$$m_{\tilde{\gamma}} \gtrsim 15 \text{ GeV}, \quad (13)$$

whereas the ASP experiment is only sensitive to  $m_{\tilde{\gamma}} \lesssim 13 \text{ GeV}$ .

We now supplement the above bounds with some more model-dependent considerations. We assume that the weak interaction scale is generated dynamically along the lines proposed in Refs. 5) and 6) and discussed in more detail in Ref. 7). There we discuss two possible variants of no-scale models for generating the gauge hierarchy  $m_W/M_P \ll 1$  by radiative corrections. In one "hybrid" variant<sup>6),7)</sup> the gaugino mass  $m_{\frac{1}{2}}$ , which is the seed of soft SUSY breaking, has a relation to the vacuum expectation values  $v$ ,  $\bar{v}$  and  $x$  which is determined by dynamics at the Planck scale:

$$m_{1/2} = (\text{const}) \times \frac{x^2 v^2 + x^2 \bar{v}^2 + v^2 \bar{v}^2}{x v \bar{v}}. \quad (14)$$

In the other variant<sup>5),6)</sup>, the constant in (14) is not fixed at the Planck scale, and  $m_{\frac{1}{2}}$  is determined independently of  $v$ ,  $\bar{v}$  and  $x$  by dynamics at the weak inter-

action scale. In the rest of this paper we will concentrate on "hybrid" models (14), but similar results would apply to models of the other variant.

In these models, a field vacuum expectation value can appear when radiative corrections drive the corresponding soft SUSY breaking mass squared  $m_{S_i}^2 < 0$  (20). This will occur at some renormalization scale  $\mu_i$ :

$$\mu_i = M_P \exp \left[ -O(1) / (\alpha_t \text{ or } \alpha_\lambda \text{ or } \alpha_k) \right], \quad (15)$$

where  $\alpha_t \equiv h_t^2/4\pi$ ,  $\alpha_\lambda \equiv \lambda^2/4\pi$  and  $\alpha_k \equiv k^2/4\pi$  are coupling strengths related to the following superpotential terms:

$$P = h_t Q_3 T^c H + \lambda H \bar{H} N + k D D^c N \quad (16)$$

with  $Q_3$  the third-generation quark doublet,  $T^c$  the right-handed  $t$  quark, and  $D, D^c$  the additional charge  $-1/3$  quark (and its antiquark) which is present in each  $E_6$  matter generation. In order to get  $x > v, \bar{v}$  as required by the phenomenological bounds obtained earlier in this paper, we must demand that  $m_N^2$  be driven negative at a higher scale than  $m_H^2$ . (This would have happened before  $m_H^2$  went negative, since  $m_t > m_b$  corresponds to  $h_t > h_b$ .) A generic sketch of the variation of  $m_N^2$ ,  $m_H^2$  and  $m_{\frac{1}{2}}$ ,  $x$ ,  $v$  and  $\bar{v}$  with the renormalization scale  $\mu$  is shown in Fig. 3. The vacuum expectation values  $v$  and  $\bar{v}$  develop simultaneously because of the trilinear term  $\lambda H \bar{H} N$  in the superpotential, which is reflected in a trilinear soft SUSY breaking term  $2A_\lambda m_{\frac{1}{2}} \text{Re} \lambda H \bar{H} N$  in the effective low energy potential. Close to the threshold at  $\mu_H$ , the ratio  $\bar{v}/v$  is essentially constant, taking a value between 0.2 and 0.6 which depends on details of the choice of model parameters. Clearly also  $x/v \rightarrow \infty$  at threshold, so that  $m_{\frac{1}{2}}(14)$  tends to become much larger than  $m_W = (g_2/\sqrt{2})(v^2 + \bar{v}^2)^{\frac{1}{2}}$ . This limit appears "unnatural" in at least two respects. One is that if  $m_{\frac{1}{2}} \gg m_W$ , the original motivation for SUSY, namely that it protects the Higgs mass against radiative corrections, seems to be lost. The other "unnatural" feature is that one must fine-tune the input Yukawa coupling parameters if one is to arrange that the physical vacuum is close to the threshold, rather than at some generic renormalization scale. The physical vacuum is found by stopping the evolution of the model parameters at some renormalization scale  $\mu_f = O(m_{\frac{1}{2}})$ , and the "natural" expectation is that  $|\ln \mu_H/\mu_f| = O(1)$ , not  $\ll 1$ .

This requirement of "naturalness" is rather imprecise, and largely a matter of taste. Nevertheless, we have tried to quantify the concept as follows. We should worry that in a model with  $m_W \ll m_{\frac{1}{2}}$ , a small variation in the input

parameters would produce a large change in the ratio  $m_{\frac{1}{2}}/m_w$ . We can replace this ratio by the alternative and essentially equivalent sensitivity indicator  $x/v$ . As input parameters which largely determine  $x/v$  we have  $\alpha_k$  and  $\alpha_\lambda$ . Therefore, we choose  $S_{k,\lambda} \equiv \left| \frac{\partial \ln(x/v)}{\partial \ln k, \lambda} \right|_{\mu=\mu_f}$  as our measure of sensitivity, and require

$$S_{k,\lambda} \equiv \left| \frac{\partial \ln(x/v)}{\partial \ln k, \lambda} \right| \leq 5 \quad (17)$$

as our criterion of "naturalness".

For any given choice of input parameters  $(h_t, \lambda, k) \Big|_{\mu=M_X}$ , we tend to find two solutions to the consistency condition  $\mu_f = 0(m_{\frac{1}{2}})$ , where  $m_{\frac{1}{2}}$  is given by Eq. (14) with  $x$ ,  $v$  and  $\bar{v}$  evaluated at  $\mu = \mu_f$  in Fig. 3. One of these is close to the threshold  $\mu = \mu_H$  and typically has  $x/v \gg 1$ , while the other one is further from threshold and has  $x/v$  smaller but still  $>1$ . We find that there is a finite region in the parameter space where the latter solution is compatible with the "naturalness" requirement (17), while the threshold solution is almost never compatible and generally corresponds to a large value of  $\left| \frac{\partial \ln m_w}{\partial \ln \mu} \right|_{\mu=\mu_f}$ . The threshold solutions will be disregarded in the following, while the properties of the other ones are illustrated in Fig. 4, where domains of the input parameters  $(k, \lambda) \Big|_{\mu=M_X}$  compatible with the previous phenomenological constraints are delineated. Fig. 4a shows the allowed domain for  $h_t(M_X)$  chosen to get  $m_t \approx 40$  GeV: we see that a sizeable fraction of the area allowed by purely phenomenological considerations is also compatible with the "naturalness" constraint (5). In the case  $m_t \approx 55$  GeV shown in Fig. 4b, we see that the area of the  $[k(M_X), \lambda(M_X)]$  plane allowed by the phenomenology is larger, but a smaller fraction is "natural", so that the fully acceptable domain has shrunk considerably. Further investigation shows that the acceptable domain does not disappear completely until  $m_t \sim 70$  GeV, which is therefore the absolute upper bound on the top quark mass, but this is rather an exceptional case and smaller values of  $m_t$  are more generic. If one takes seriously our "naturalness" constraint (17), one can also give upper bounds on  $m_{\frac{1}{2}}$  and  $x/v$ , and therefore on the masses of the new particles predicted by the model. The allowed ranges for the different particles are given in Table 2: some could well be detectable at the FNAL  $p\bar{p}$  collider. Note also that because of the bounds on  $x/v$  and  $\bar{v}/v$  in our model, there would also be a lower bound on the  $Z^0$  mass shift:  $\Delta \gtrsim 0.02$  in this case.

In this paper we have explored systematically the constraints on the masses of the new particles appearing in the minimal low energy theory which could be obtained from the superstring by Calabi-Yau compactification. Some of our bounds are quite model-independent, since they follow from the renormalization group



equations applied to supersymmetry breaking parameters initiated by a universal gaugino mass. Other bounds are more model-dependent, since they emerge from a particular no-scale scenario for generating the weak interaction scale. Our results suggest that though sparticles and a new neutral gauge boson have not yet been found, their discovery cannot be long delayed.

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Field	$T_3$	$Y$	$Y_E$	$c_3$	$c_2$	$c_1$
u	1/2	1/6	1/3	16/3	3	1/3
$u^c$	0	-2/3	1/3	16/3	0	4/3
d	-1/2	1/6	1/3	16/3	3	1/3
$d^c, D^c$	0	1/3	-1/6	16/3	0	1/3
D	0	-1/3	-2/3	16/3	0	4/3
$e, \bar{H}^-$	-1/2	-1/2	-1/6	0	3	2/3
$e^c$	0	1	1/3	0	0	8/3
$\nu, \bar{H}^0$	1/2	-1/2	-1/6	0	3	2/3
$\nu^c, N$	0	0	5/6	0	0	5/3
$H^+$	1/2	1/2	-2/3	0	3	5/3
$H^0$	-1/2	1/2	-2/3	0	3	5/3

Table 1

Quantum numbers with respect to  $SU(2)_L \times U(1)_Y \times U(1)_E$  of the fields contained in a 27 of  $E_6$ . A conventional normalization has been adopted for the hypercharges  $Y$  and  $Y_E$ : the properly normalized quantities, such that  $\text{Tr}(T_3^2) = \text{Tr}(\hat{Y}^2) = \text{Tr}(\hat{Y}_E^2)$  on the full 27, are given by  $\hat{Y} = \sqrt{3/5} Y$  and  $\hat{Y}_E = \sqrt{3/5} Y_E$ . In the last three columns we show the numerical values of the coefficients  $c_a^i$  appearing in Eq. (8), needed for the computation of the scalar masses.

Particle	Lower Limit	Upper limit
$t$	—	70
$Z_2$	185	320
$\tilde{g}$	100	250
$\tilde{\gamma}$	15	35
$\tilde{e}_L$	55	150
$\tilde{e}_R$	90	170
$\tilde{\nu}$	0	150
$\tilde{q}_{1,2}$	180	500

Table 2

Limits on the masses of unseen particles in the minimal low-energy superstring models considered in the text. The first column gives lower limits, using experimental data, the assumption that supersymmetry breaking in the observable sector is triggered by a universal primordial gaugino mass and the result  $0.2 \lesssim \bar{v}/v \lesssim 0.6$  of our dynamical calculations. The second column gives more tentative and model-dependent upper limits, obtained by imposing on our hybrid no-scale models the "naturalness" constraint (17). All the masses are expressed in GeV units.

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FIGURE CAPTIONS

Fig. 1 : Contours of  $\Delta \equiv \sin^2\theta_W - \sin^2\bar{\theta}_W$  (solid lines) and  $m_{Z_2}$  (dashed lines) in the  $(\bar{v}/v, x/v)$  plane, calculated for the input values  $m_W = 82$  GeV and  $\sin^2\theta_W = 0.224$ .

Fig. 2 : Bounds in the  $(x/v, m_{\frac{1}{2}})$  plane coming from various phenomenological constraints. The two extreme values of the remaining parameter  $\bar{v}/v$ , allowed by our dynamical calculations, have been considered: (a)  $\bar{v}/v = 0.2$ ; (b)  $\bar{v}/v = 0.6$ . The dashed line, representing the constraint  $m_{H_a^+} > 20$  GeV on the masses of charged unhiggses, has been derived neglecting the Yukawa couplings  $\lambda'_a$  which give masses to the corresponding fermions, which might not be a good approximation. On the horizontal axis the values of  $m_{Z_2}$  (in GeV) associated to the different values of  $x/v$  are also shown. As input values we have assumed  $m_W = 82$  GeV and  $\sin^2\theta_W = 0.224$  as before.

Fig. 3 : Sketch of the generation of the vacuum expectation values  $x$ ,  $v$  and  $\bar{v}$ , as an effect of the evolution of the relevant parameters in the scalar potential, according to the renormalization group equations. The evolution of the soft supersymmetry breaking masses  $m_N^2$ ,  $m_H^2$  and  $m_{\bar{H}}^2$  is also shown. At the scale  $\mu_N$  where  $m_N^2$  becomes negative a non-zero  $x$  is generated, while  $v$  and  $\bar{v}$  become non-zero at the lower scale  $\mu_H$  where the determinant of the mass matrix  $M_{H, \bar{H}}^2$  for the fields  $H^0$  and  $\bar{H}^0$  also becomes negative.

Fig. 4: Allowed regions in the  $[k(M_X), \lambda(M_X)]$  plane for two different values of the top Yukawa coupling:  
 (a)  $h_t(M_X) = 0.025$  (corresponding to  $m_t \approx 40$  GeV);  
 (b)  $h_t(M_X) = 0.035$  (corresponding to  $m_t \approx 55$  GeV).  
 Above the solid lines, it is not possible to generate an electroweak breaking scale  $m_W$  as large as the experimental value  $\approx 82$  GeV. Dashed lines correspond to the experimental constraints  $\Delta < 0.05$  and  $m_{H_a^+} > 20$  GeV: the latter is imposed considering the full mass matrix for the charged unhiggses, but assuming that their Yukawa couplings satisfy the condition  $\lambda'_a < \frac{1}{2}\lambda$ . It might also be that  $\lambda'_a \sim \lambda$ : in this case, the R.G.E.s of Ref. 7) must be modified. Finally, dotted lines correspond to the theoretical bounds associated with our "naturalness" constraints (17). The shaded area is the surviving region.

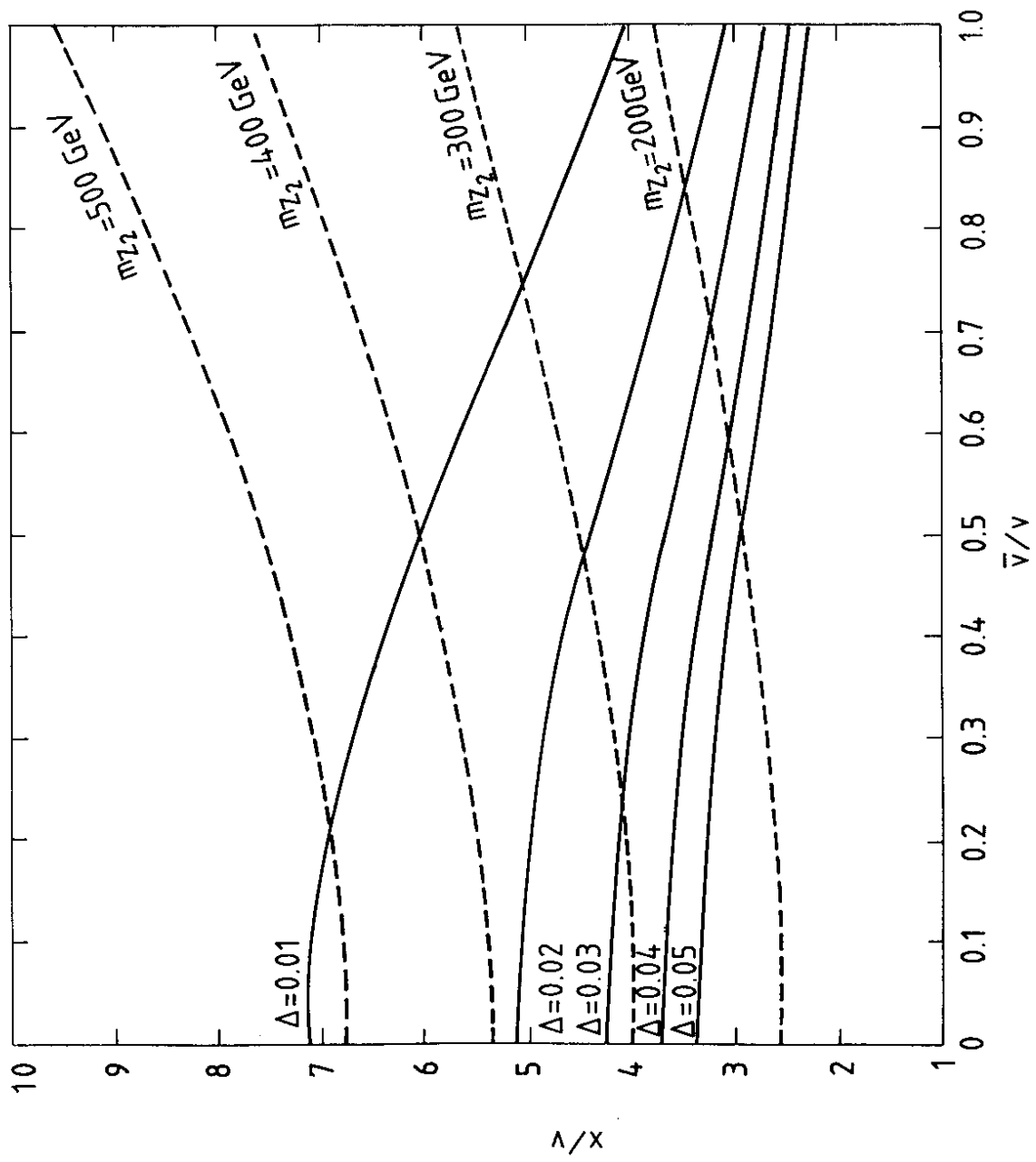


Fig. 1

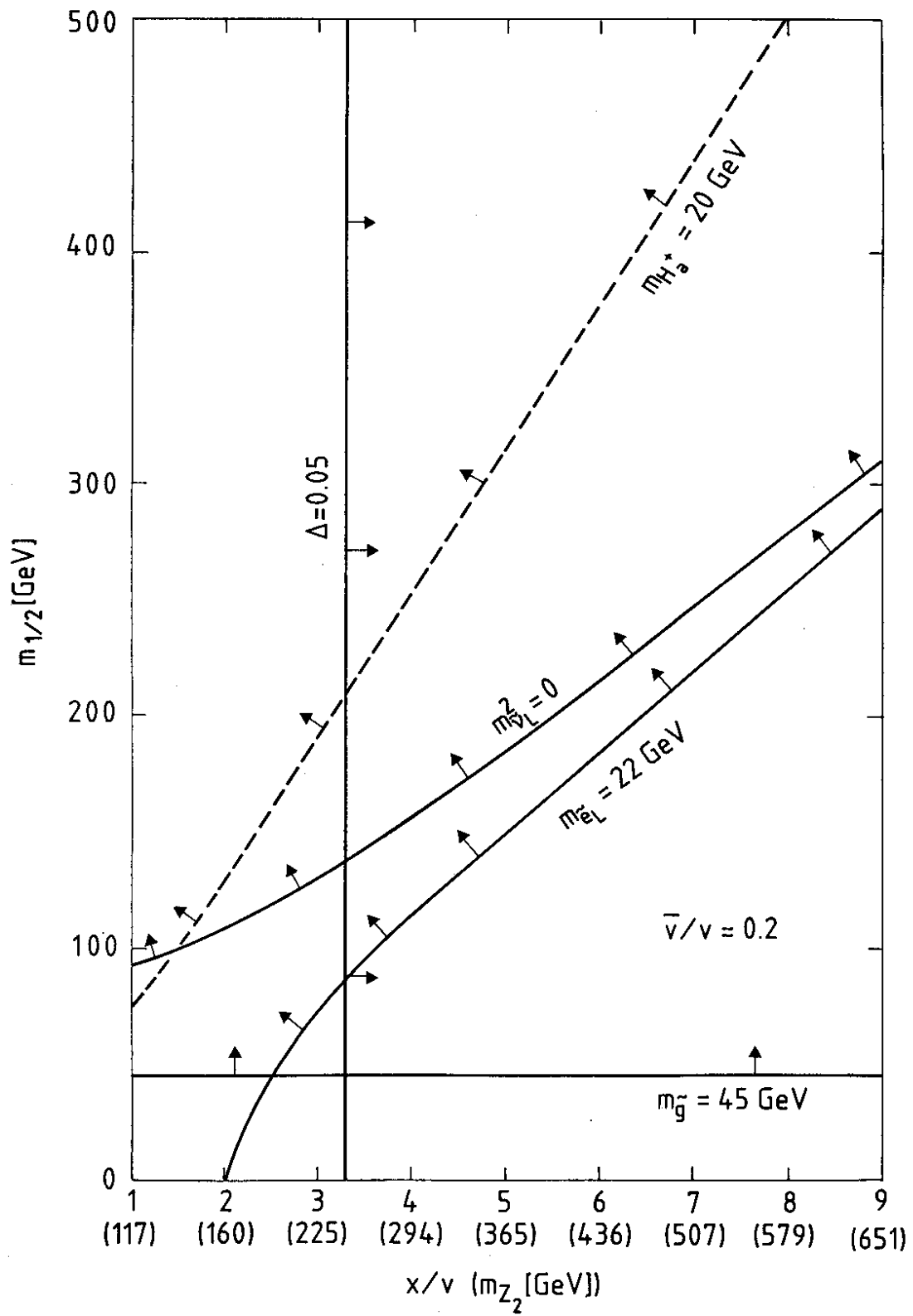


Fig. 2a



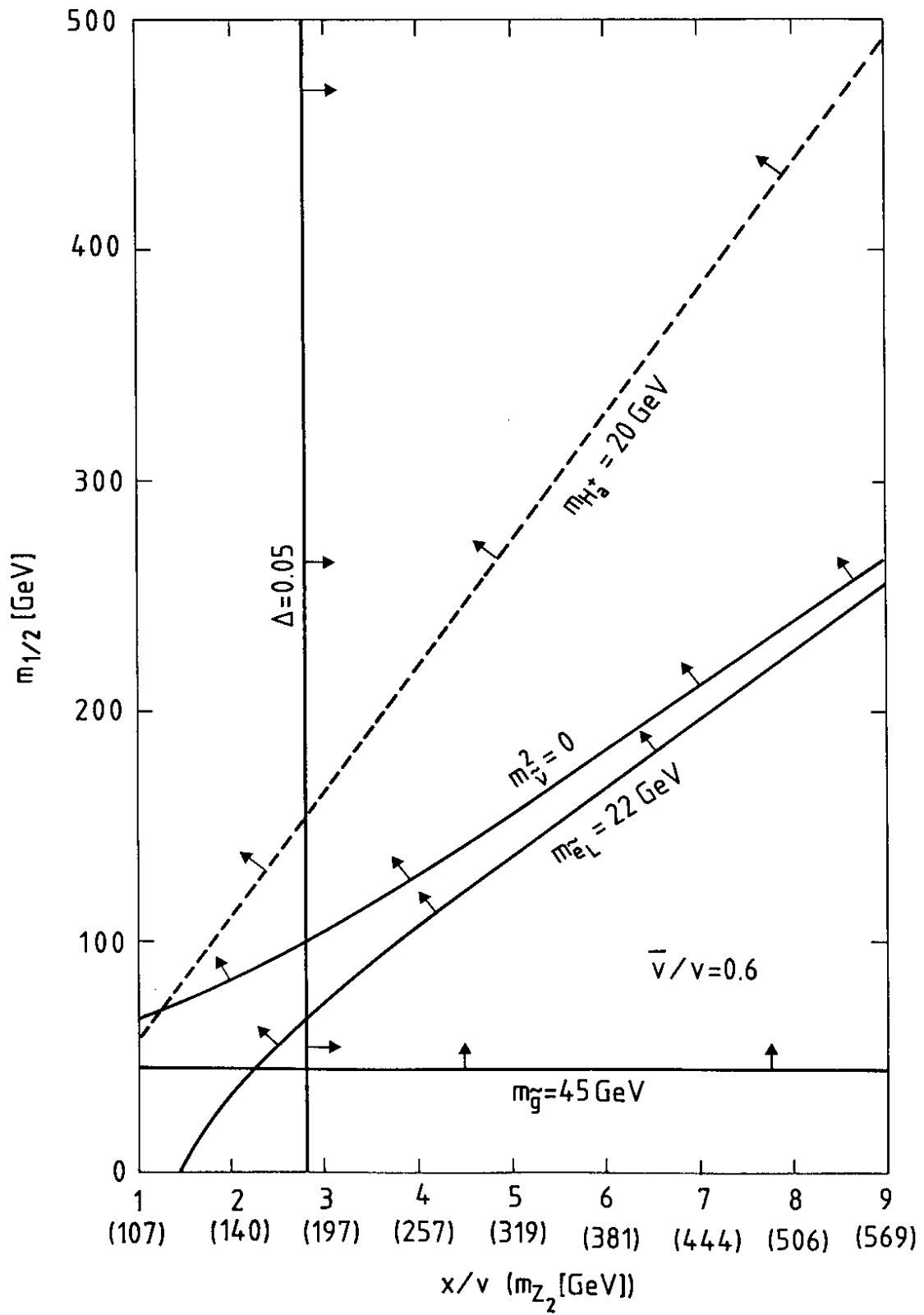


Fig. 2b

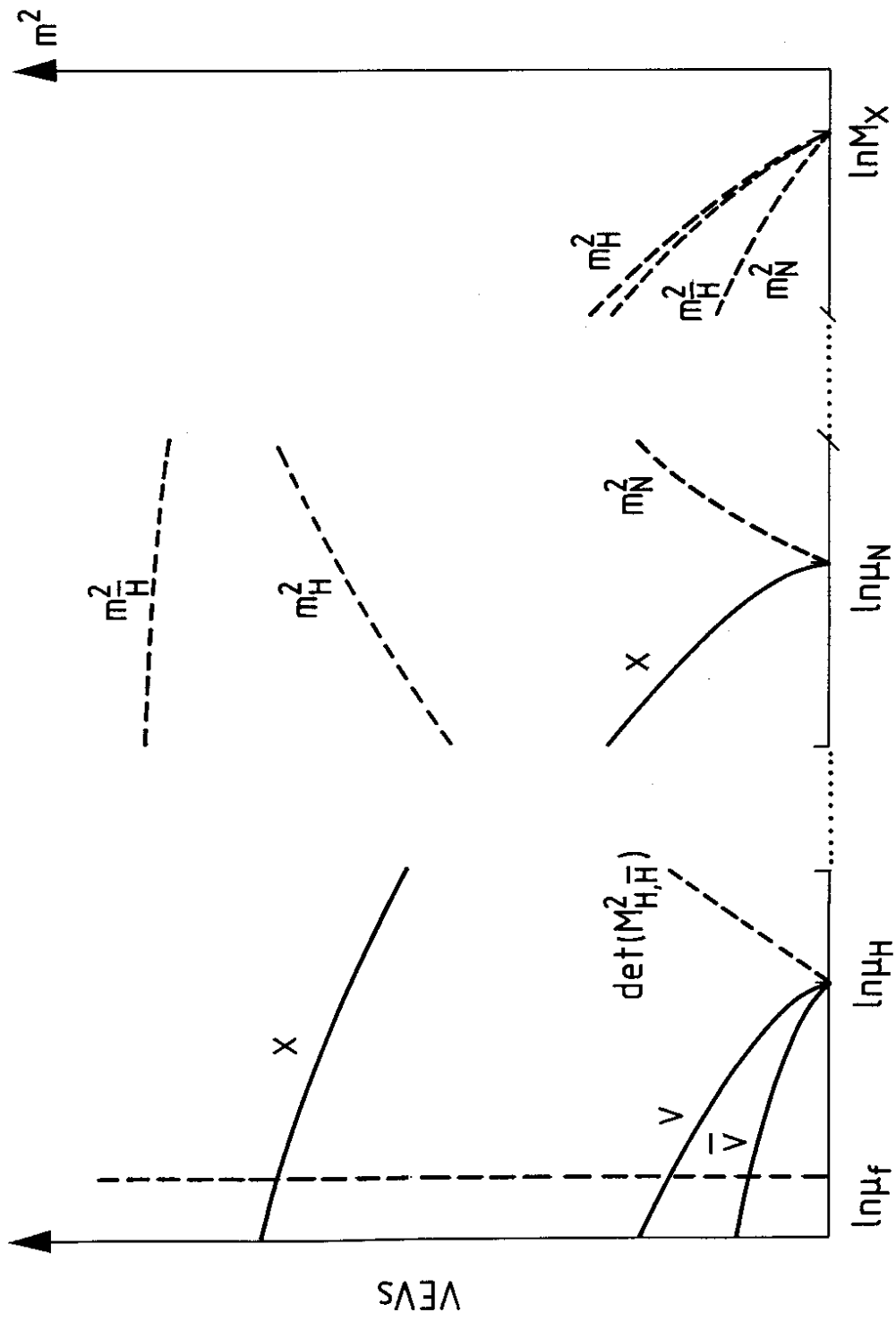


Fig. 3

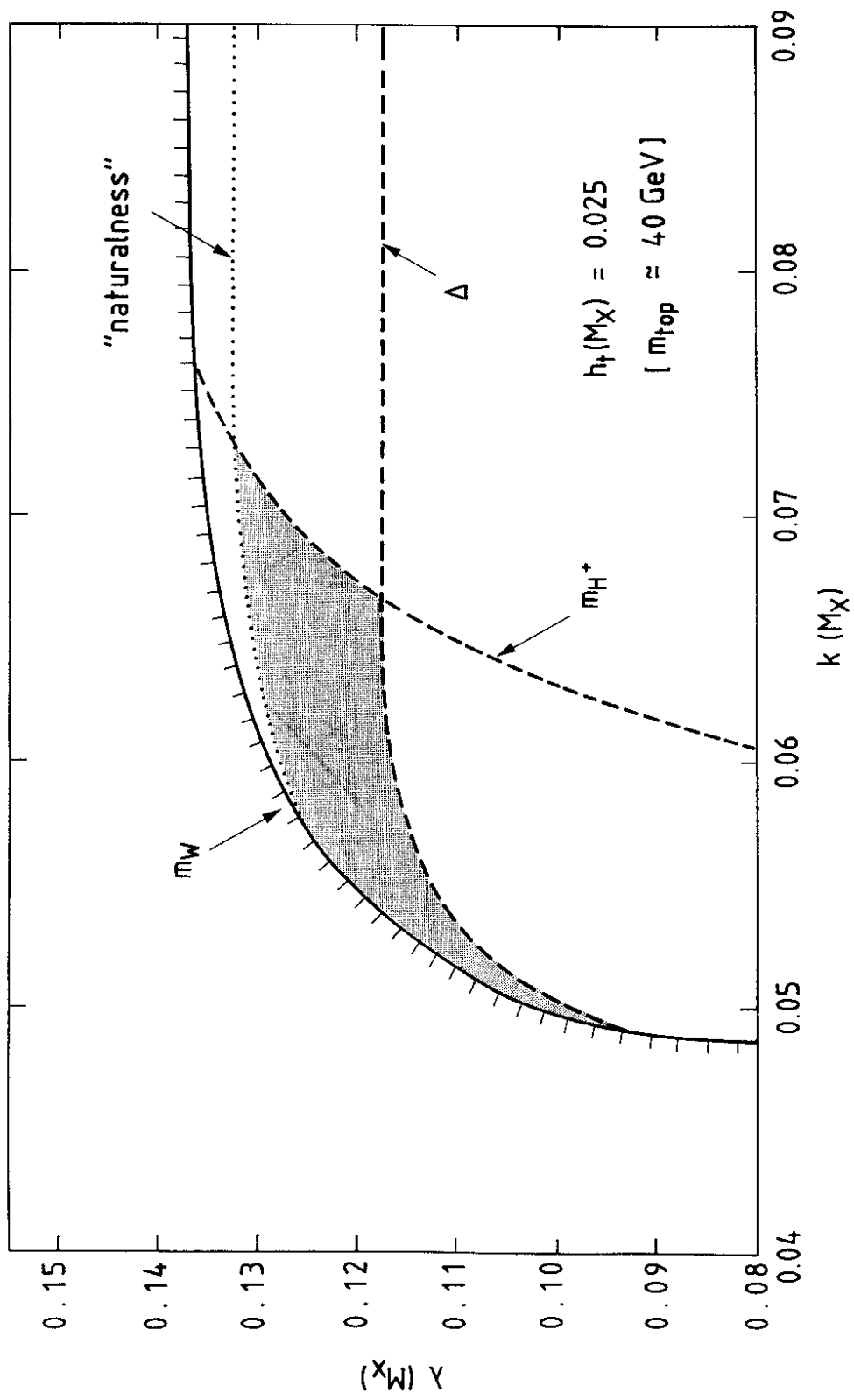


Fig. 4a

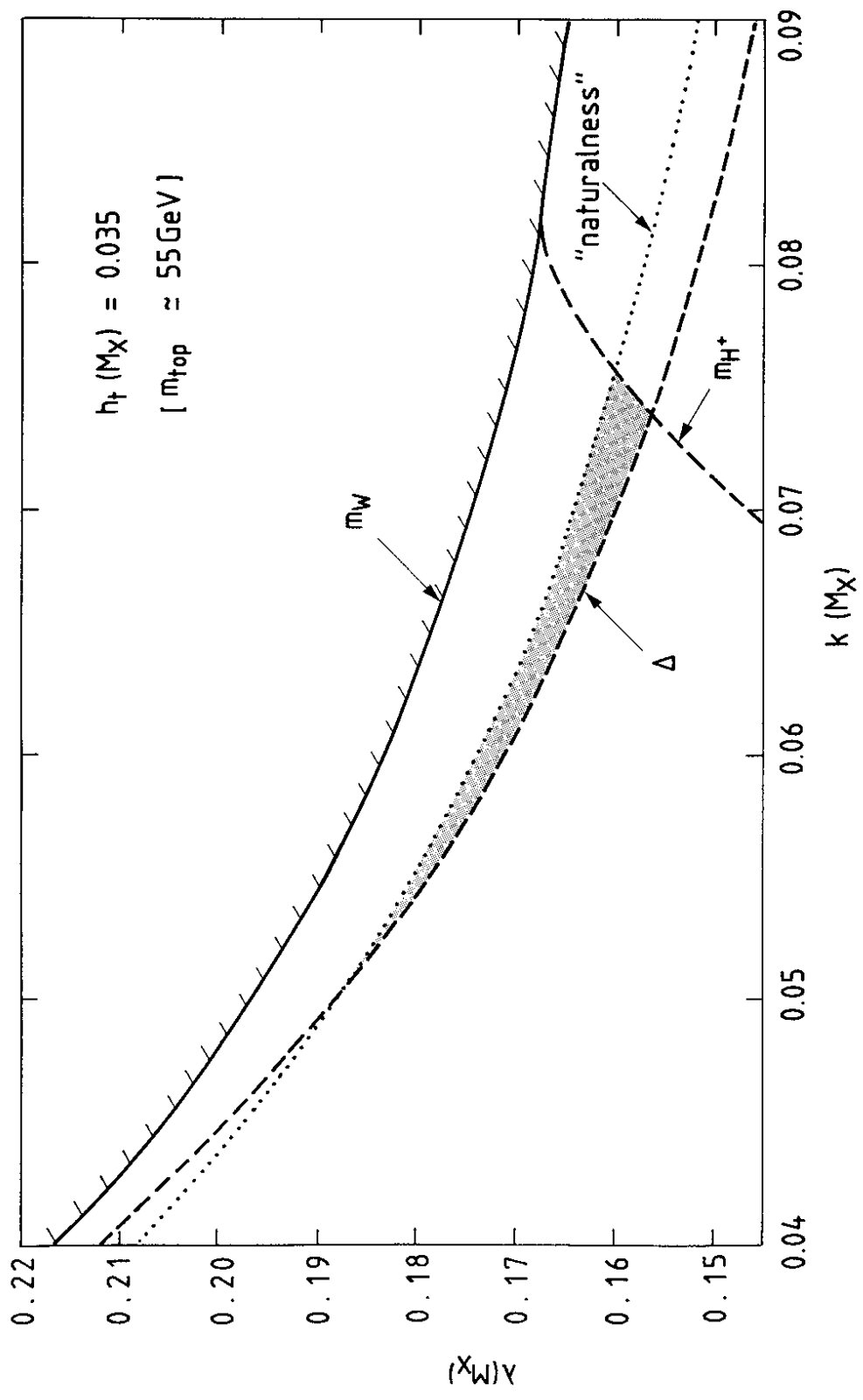


Fig. 4b