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Observation of a Disordered Bosonic Insulator from Weak to Strong Interactions

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We employ ultracold atoms with controllable disorder and interaction to study the paradigmatic problem of disordered bosons in the full disorder-interaction plane. Combining measurements of coherence, transport and excitation spectra, we get evidence of an insulating regime extending from weak to strong interaction and surrounding a superfluidlike regime, in general agreement with the theory. For strong interaction, we reveal the presence of a strongly correlated Bose glass coexisting with a Mott insulator.

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The interplay of disorder and interaction in quantum matter is an open problem in physics. A paradigmatic system explored in theory is bosons at $T = 0$ in one [1] or in higher dimensions [2]. While disorder alone leads to the celebrated Anderson localization [3], a weak repulsive interaction can compete with disorder and progressively establish coherence, leading eventually to the formation of a superfluid. However, a stronger interaction brings the superfluid into a strongly correlated regime where disorder and interaction cooperate, leading to a new insulator. The overall phase diagram is, therefore, predicted to consist in a superfluid, surrounded by an insulator [1]. The two insulating regimes appearing at weak and strong interaction have been named Bose glass, due to the gapless nature of their excitations. There is an ongoing effort to establish whether they are two distinguishable quantum phases [4–7]. In lattices, the Bose glass at strong interaction is a distinct phase from the gapped Mott insulator appearing at commensurate densities and moderate disorder [2].

An experimental observation of the insulator extending from weak to strong interactions is still missing. Photonic systems and ultracold atoms have demonstrated that weak interactions tend to restore the coherence in Anderson insulators [8,9]. Magnetic systems with tunable density have provided evidence of the transition from a Mott insulator to a strongly correlated Bose glass [10–12], but they lack the possibility to control the interaction. An insulating regime at strong interaction and strong disorder has also been detected with ultracold atoms in tunable optical lattices [13–15]. However, so far, it was not possible to distinguish the Bose glass from the Mott insulator.

In this Letter, we study the full problem of disordered interacting bosons in a one-dimensional lattice. We employ ultracold atoms with independently tunable disorder and

interaction, which allow us to study systematically the whole disorder-interaction plane. We study several experimental observables and we make a close comparison with the theory. In particular, by means of coherence and transport measurements we identify an insulating regime extending from weak to strong interactions and surrounding a superfluidlike regime. Using a lattice modulation spectroscopy, we observe a different response of disordered insulators with weak or strong interactions. In the latter regime, we reveal spectral features that are consistent with a strongly correlated Bose glass coexisting with a Mott insulator. The comparison with theory indicates that the strongly correlated regime is only weakly affected by the finite temperature in the experiment.

We employ an array of quasi-1D samples of ^{39}K atoms, subjected to a quasiperiodic optical lattice [13,16], which provide a realization of the interacting Aubry-André model [17,18]. To a good approximation [19,20], the system is described by the Hubbard Hamiltonian $H = -J\sum_i (b_i^\dagger b_{i+1} + \text{H.c.}) + \Delta\sum_i \cos(2\pi\beta i)n_i + U/2\sum_i n_i(n_i - 1) + \alpha/2\sum_i (i - i_0)^2 n_i$, which is characterized by three energy scales: the tunneling energy J , the quasidisorder strength Δ , and the interaction energy U . A primary lattice with lattice constant $d = \lambda_1/2 = 0.532 \mu\text{m}$ fixes J [J/h is typically 110(5) Hz]. Δ is essentially the depth of a secondary lattice with an incommensurate wavelength λ_2 ($\beta = \lambda_1/\lambda_2 = 1.243$). U can be varied from about zero to large positive values thanks to a Feshbach resonance [21]. The fourth term represents a harmonic potential, while b_i^\dagger , b_i , and n_i are the creation, annihilation, and number operators at site i . For $U = 0$ all eigenstates are localized above a critical disorder strength $\Delta = 2J$ [16,17]. Accurate phase diagrams for $U > 0$ were obtained theoretically for homogeneous systems and $T = 0$ [19,20,22,23]. However, the unavoidable harmonic

confinement in the experiment results in finite-size, inhomogeneous systems and changes the nature of the problem, transforming the quantum phase transitions into crossovers and leading to a coexistence of different phases. The 1D systems are populated from an initially three-dimensional Bose-Einstein condensate (BEC), which is split into several quasi-1D tubes by a 2D lattice. About 500 such systems are initially created at $U \approx 40J$, then both U and Δ are slowly changed using almost isoentropic transformations. The mean site occupation n depends on U . For $\Delta = 0$, we calculate $n \approx 7$ for the smallest U and $n \approx 2$ for the largest U [24].

A first indication of the nature of the system comes from a measurement of the momentum distribution $P(k)$, achieved through absorption imaging after a free flight. The root-mean-square width Γ of $P(k)$ is a measure of the coherence of the system. The evolution of Γ in the disorder-interaction plane is reported in Fig. 1(a). It clearly shows a coherent regime (blue) for small Δ and moderate U , surrounded by an incoherent regime extending from weak to strong U (orange), with a smooth change of coherence between the two regimes. In the superfluid (SF) regime of moderate U and no disorder, we can compare $P(k)$ to calculations, concluding that our system is consistent with being in thermal equilibrium at a temperature $k_B T \approx 3J$ [24]. This temperature is below the 1D degeneracy temperature for our mean tube, $k_B T_D \approx 8J$ [31]. Calculating the temperature dependence of $P(k)$ in all the other regimes in the $\Delta - U$ plane is challenging because of the coexistence of different phases. Therefore, a measurement of temperature or even of the presence of thermal equilibrium is not possible.

The insulating nature of the incoherent regions is confirmed by transport measurements. These are performed by applying a sudden shift to the harmonic confinement, and detecting the momentum δp accumulated in a fixed time interval of 0.9 ms. Since the mean force arising from the shift is constant, δp is a measure for the mobility of the system. Figure 1(b) shows $\delta p(U)$ for three different values of Δ . In the nondisordered case, the motion is almost ballistic for small U , there is a progressive reduction of the mobility moving to larger U , and finally, the system reaches a strongly insulating regime where the mobility is very small and less dependent from U [32]. For finite disorder, the mobility at small U is strongly reduced; for increasing U , however, it increases and finally decreases again. This behavior confirms the coherence measurement showing the presence of a disorder-driven insulator at small U and of another insulating regime at large U dominated by the interaction. An additional measurement at a larger T , also shown in Fig. 1(b), indicates that the mobility for intermediate Δ is essentially T independent in the accessible range of temperatures.

The overall shape of the incoherent regime in Fig. 1(a) is reminiscent of the Bose glass (BG) found in theory at $T = 0$ for homogeneous systems [19,20,22,23]. The

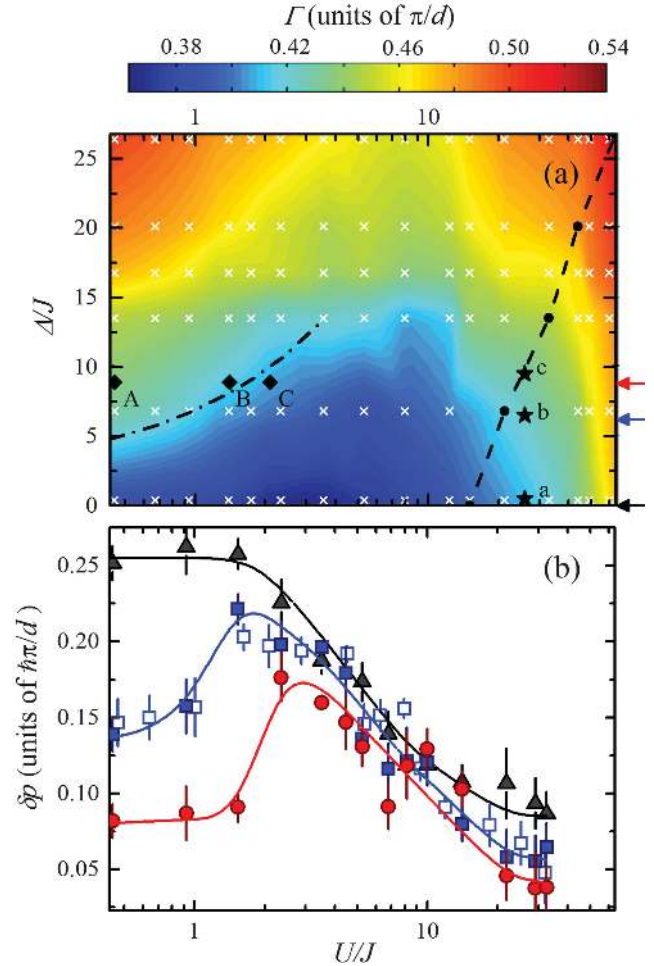


FIG. 1 (color). Coherence and mobility. (a) Measured width of the momentum distribution. The diagram is built with 94 data points (crosses), with a standard deviation between 2% and 5%. $T = 0$ calculations reveal a MI only on the right of the dashed line, see text. The dashed-dotted line is calculated as $\Delta - 2J = nU$. Stars (diamonds) refer to the measurements shown in Fig. 2 (Fig. 3). (b) Momentum acquired after an applied impulse for varying interaction strength and three disorder strengths: $\Delta = 0$ (triangles), $\Delta = 6.2J$ (squares), and $\Delta = 8.8J$ (circles), also shown by the arrows to the right of (a). The measured SF temperature is $k_B T = 3.1(4)J$ or $k_B T = 4.5(7)J$ (empty squares). The lines are a guide to the eye. The uncertainties are the standard deviation of, typically, five measurements.

prediction is, indeed, of a weakly interacting BG appearing for vanishing U and $\Delta > 2J$, which is turned into a SF when the interaction energy nU becomes comparable to the disorder strength, although there is not yet consensus on the exact shape of the transition line in the $\Delta - U$ plane [33–35]. A stronger interaction is, instead, expected to lead to a new insulating regime approximately when $U > 2nJ$, i.e., when the interaction energy becomes larger than the kinetic energy available in the lattice band. Here, theory predicts a strongly correlated BG for an

incommensurate density (noninteger n) and $\Delta > 2J$. There is also a Mott insulator (MI) for a commensurate density (integer n), which can survive in the disorder only up to approximately $2\Delta < U$, since U controls the MI gap. From a numerical study with a density-matrix renormalization group (DMRG) technique [20,36,37], we calculate the region of existence of MI domains in our inhomogeneous system at $T = 0$, which is delimited by the dashed line in Fig. 1(a) [24]. The calculation gives a critical U at $\Delta = 0$ that is larger than the homogeneous result for $n = 2$, $U_c \approx 5.5J$, and increases for increasing Δ . The opposite slope of the crossover, from the superfluid to the insulating region we observe experimentally, cannot be interpreted in terms of MI physics alone, and suggests the appearance of a BG regime.

To probe the nature of the insulating regimes, we perform a lattice modulation spectroscopy [38]. This consists in measuring the energy absorbed by the system when the amplitude of the main lattice, and therefore J , is modulated with a sinusoid of variable frequency ν . We start the discussion from the less intuitive large- U regime, summarized in Fig. 2. Here, the absorption is measured as a decrease of the condensed fraction once the system is transferred back into a 3D trap; we show three characteristic spectra for $U = 26J$ and increasing Δ . In the non-disordered case, one notices the standard MI spectrum with a first excitation peak centered at the MI gap, $h\nu \approx U$, due to excitations within individual MI domains with $n = 1-3$, and a second peak centered at $h\nu \approx 2U$, due to excitations between different MI domains [38,39]. The MI domains are connected by incommensurate SF components, which show little response for $h\nu < U$ [39].

For finite disorder the spectrum changes radically. First, we observe a broadening of the MI peaks by approximately Δ that indicates an inhomogeneous broadening of the Mott gap, as already observed in previous experiments at strong disorder [13,15,40]. Second, we observe a striking extra peak appearing in the MI gap, around Δ . This new observation cannot be explained in terms of MI physics, but agrees, instead, with the expected behavior of a strongly correlated BG, which can be seen as a weakly interacting

fermionic insulator with a response at about the characteristic disorder energy Δ [1]. As shown in Fig. 2(d), the peak shape is, indeed, in good agreement with the excitation spectrum of the BG calculated with a fermionized-boson model [41,42], using the same parameters of the experiment and the temperature measured for the SF. There is a systematic shift of the theory curve to larger frequencies, which might be due to the $U = \infty$ assumption in the model. The smooth decrease of the theoretical response towards zero frequency is associated to the gapless nature of the BG [41]. The experiment is compatible with such a prediction, but further investigation is necessary to draw conclusions in this direction. In the range where we can detect the extra peak, we observe that its center shifts linearly with Δ , as expected in the fermionic picture [41]. From other data, not shown [24], we see that the peak is no longer detectable for $U/J < 25$ as it overlaps with the one at U ; for very large U , the Mott plateaus are, instead, extending to most of the system, thus, lowering the incommensurate fraction and the weight of the peak at $h\nu \approx \Delta$. In the strong disorder limit, we observe, instead, a very broad and essentially featureless spectral response, in agreement with previous experiments [13,15,40].

Such a response at large U contrasts with the one at small U , as shown in Fig. 3. Here, the energy absorption is measured as an increase of the thermal width of the system. For vanishing U , where the system is still globally insulating, we observe a weak excitation peak centered at Δ , which is, however, already broader than the one predicted for noninteracting bosons [24], especially towards small ν . This suggests the formation of large coherent regions due to the coupling of single-particle states by the interaction, leading to the possibility of long-distance, small- ν excitations. This behavior recalls the prediction for the $T = 0$ weakly interacting BG being composed by large, disconnected SF regions. A moderate increase of $U < \Delta$ leads, indeed, to a rapid broadening of the response, with a strong enhancement for small ν . This behavior is in clear contrast with that of the strongly interacting insulator, where the weak response at small ν indicates a strong fragmentation. A further increase of U

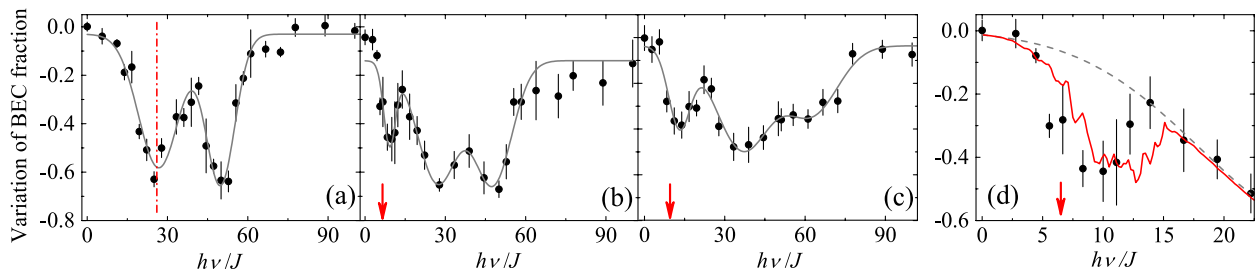


FIG. 2 (color online). Excitation spectrum for strong interactions. Experimental spectra for $U = 26J$ [indicated by the dashed-dotted line in (a)] and $\Delta = 0$ (a), $\Delta = 6.5J$ (b), and $\Delta = 9.5J$ (c). These sets of parameters are also shown in Fig. 1(a) (stars). The arrows are at $h\nu = \Delta$, the dashed line in (a) is at $h\nu = U$, and the continuous lines are fits with multiple Gaussians. (d) Comparison of the low-frequency peak for $\Delta = 6.5J$ with theory (continuous line). The theory includes the Gaussian tail of the Mott peak (dashed line).

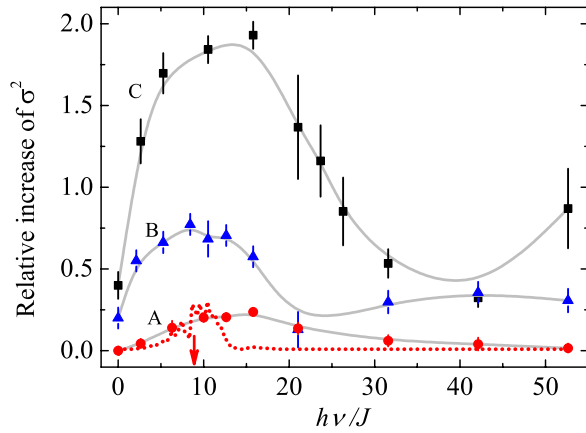


FIG. 3 (color online). Excitation spectrum for weak interactions. Spectra for $\Delta = 8.9J$, and $U = 0.35J$ (circles), $U = 1.4J$ (triangles), or $U = 2.1J$ (squares). These sets of parameters are also shown in Fig. 1(a) (diamonds). Successive data sets have been shifted upwards by 0.2 for clarity. The dotted red line is the theory for noninteracting bosons, the arrow is at $h\nu = \Delta$ and the continuous lines are a guide to the eye.

eventually brings the system into a regime spectrally indistinguishable from a clean SF, confirming the delocalizing role of the interactions in this regime [9,32].

Although the temperature cannot be measured for finite Δ , we can still gain an insight in thermal effects by comparing the experimental $P(k)$ to the exact $T = 0$ theory supplemented by a phenomenological account of a finite temperature. To do this, we apply the DMRG technique to simulate an ensemble of trapped systems with the same parameters as in the experiment. Figures 4(a)–4(d) show a few representative comparisons of the experimental $P(k)$ and the theoretical ones at $T = 0$. In the low- U region, the calculated $P(k)$ is definitely narrower than the measured

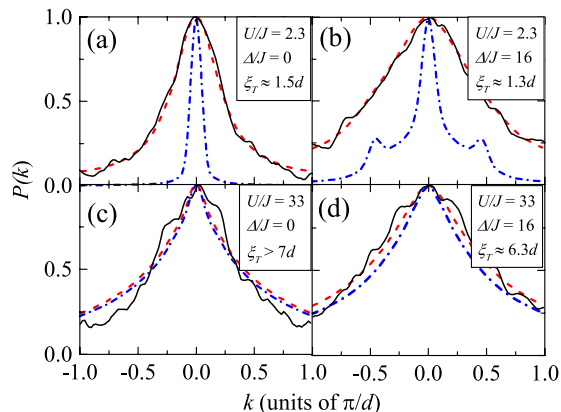


FIG. 4 (color online). Finite-temperature effects. (a)–(d) Comparison of the experimental momentum distributions (black, continuous line) with $T = 0$ DMRG calculations (blue, dashed-dotted line) supplemented by the introduction of a thermal correlation length ξ_T (red, dashed line), for four points in the $\Delta - U$ plane.

one, whereas in the strongly correlated region, the broadening is less relevant. Assuming thermal equilibrium, we quantify this thermal broadening by convolving the calculated $P(k)$ with a Lorentzian distribution corresponding to an exponential decay of the correlations with a thermal length ξ_T , an approach known to be valid for the SF [43]. The dashed red lines in Figs. 4(a)–4(d) are the best fit of the theory to the experiment with ξ_T as the only fitting parameter. There is a good agreement in all regimes, except for the one at small Δ and large U [Fig. 4(c)]. For small U , the thermal length is short ($\xi_T \approx d$), revealing a relevant thermal excitation of both SF and insulating regimes. In the large- U region, ξ_T is, instead, large, suggesting that the strongly correlated phases are only weakly affected by the finite T . This different impact of temperature for small and large U seems to persist in all our accessible range of disorder strengths [24]. It is interesting to note that for small U , while $P(k)$ shows a relevant thermal broadening, the mobility does not show an apparent variation with temperature [see Fig. 1(b)]. In the future, it will be interesting to study the possible relation of this persisting insulating behavior at finite T with the proposed many-body localization [44,45].

To confirm the role of thermal excitations, we also performed a numerical study of the finite- T problem by exact diagonalization. We find in particular that the correlation length of the strongly correlated BG stays almost unaffected until the thermal energy becomes of the order of Δ . This behavior is analogous to the one known for the MI, which starts to be affected only by thermal energies of the order of U [46], and supports the indications of the analysis in Fig. 4. A detailed report on this study goes beyond the scope of the present Letter and will be presented in a future publication.

In conclusion, we have shown evidences of the insulator extending from weak to strong interactions predicted for disordered bosons. The strongly interacting regime shows excitation properties as predicted by the $T = 0$ theory for the Bose glass. The general shape of the insulating regimes, and in particular, the reappearing at large U of an incommensurate insulator with an excitation spectrum dominated by disorder, is not specific to the quasiperiodic lattice we have explored here, but is expected to apply to a wider range of disorder types [20,41]. It is possible to apply our techniques to further studies of disordered bosons, such as in the absence of a lattice, to study the Bose glass in one dimension without overlap with the Mott physics, or in lattices of higher dimensionality, where the superfluid is expected to be much more resistant to disorder [2,47].

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