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Observational constraints on the model parameters of a class of emergent universe

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ABSTRACT

A class of emergent universe models is studied in the light of recent observational data. Significant constraints on model parameters are obtained from these observations. The density parameter for a class of models is also evaluated. Some of the models are in accordance with recent observations. Others are not of interest, yielding unrealistic present-day values of the density parameter.

Key words: cosmological parameters – dark energy – early Universe.

1 INTRODUCTION

It is generally believed that we live in an expanding Universe. After the discovery of cosmic microwave background (CMB) radiation (Penzias & Wilson 1965; Dicke et al. 1965), big bang cosmology became the standard model for cosmology, accommodating a beginning of the Universe at some point in the finite past. On its own, however, big bang cosmology faces some problems in both the early and late universe. A number of problems emerge when one describes the early Universe, namely the horizon problem, the flatness problem, etc. The above problems can be resolved by evoking a phase of inflation (Guth 1981; Sato 1981; Linde 1982; Albrecht & Steinhardt 1982) at a very early epoch. On the other hand, recent observations predict that our Universe is passing through a phase of acceleration (Riess et al. 1998). This phase of acceleration is believed to be a late-time phase of the Universe and may be accommodated in the standard model by a positive cosmological constant. Despite its overwhelming success, modern big bang cosmology still has some unresolved issues. The physics of the inflation and the introduction of a small cosmological constant for late acceleration are not clearly understood (Albrecht 2000; Carroll 2001). This is why there is sufficient motivation to search for an alternative cosmology. Emergent universe (EU) models are employed to accommodate the early inflationary phase and avoid the messy situation of the initial singularity (Ellis & Maartens 2004; Harrison 1967). EU scenarios can be realized in the framework of general relativity (Mukherjee et al. 2006), Gauss–Bonnet gravity (Paul & Ghose 2010), Brane world gravity (Banerjee, Bandyopadhyay & Chakraborty 2008; Debnath 2008), Brans–Dicke theory (del Campo, Herrera & Labrana 2007), etc. EUs are late-time de Sitter, and thus naturally incorporate the late-time accelerating phase. One such model was proposed by Mukherjee et al. (2006), in which a polytropic equation of state (EOS) in the form

$$p = A\rho - B\rho^{1/2}, \tag{1}$$

where A and B are constants, is used. This is a special case of the more general equation

$$p = A\rho - B\rho^{\alpha}, \tag{2}$$

with $\alpha = 1/2$. For such EOSs a phenomenological construction can be found in string theory, where most of these models interpolate between two phases of universe (Fabris et al. 2007). The universe in this model can stay large enough to avoid quantum gravitational effects, even in the very early universe. The model proposed by Mukherjee et al. (2006) admits an Einstein static universe in the asymptotic past, as the Hubble parameter and its derivatives, namely H, $\dot{H}$ and $\ddot{H}$, all vanish in the limit $t \to -\infty$. As the Einstein static universe solution obtained above is unstable, at some later time it transitions to a phase of rapid expansion, which is early inflation. Mukherjee et al. (2005) also showed that a successful inflation may be permitted in the EU scenario. On the other hand, as $t \to \infty$, the solution admits an asymptotically de Sitter universe. The paper noted that with a suitable choice of the parameters A, B with K observational data it may be possible to determine the onset of the recent accelerating phase. Recently, Paul, Thakur & Ghose (2010) studied the viability of the EU model in the light of recent observational data and established bounds on EOS parameters A and B. It was shown that the best-fitting value for A may be very small but negative, although a small positive value is allowed with 95 per cent confidence. For a viable cosmology, the bounds on A and B are determined for some fixed values of K. The parameter K, however, appears in the theory as an integration constant and may be fixed to some other value for a different initial configuration. Paul, Ghose & Thakur (2011) recently worked with a more specific model for a small value of A ($A \approx 0$). In the original work of Mukherjee et al. (2006) it was shown that the choice of A drastically changes the matter energy composition of the universe that equation (1) can mimic.

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In the present paper we obtain observational bounds on the model parameters B and K for the various choices of A considered in Mukherjee et al. (2006). These choices correspond to very different compositions of the cosmic fluid, and it will be interesting to determine whether realistic cosmologies are permitted for each case, as the theory itself puts some constraints on B and K (Mukherjee et al. 2006).

The paper is presented as follows. In the next section we describe the relevant field equations. In Section 3 we discuss the methods applied to constrain the parameters from (i) observed *Hubble* data (OHD) (Stern et al. 2010); (ii) SDSS data measuring a model-independent baryon acoustic oscillation (BAO) peak parameter (Eisenstein et al. 2005); and (iii) *WMAP*7 measurement of the CMB shift parameter. In Section 4 we study the density parameters (DPs) of the model (at the present epoch), and finally we discuss the results in Section 5.

2 FIELD EQUATIONS

The Friedmann equation in a flat universe reads as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}, \tag{3}$$

where H is the Hubble parameter and a is the scale factor of the universe.The usual conservation equation holds:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + 3H\,(p+\rho) = 0. \tag{4}$$

Using the EOS given by equation (1) in equation (3) and equation (4) one obtains

$$\rho\,(z) = \left(\frac{B}{A+1}\right)^2 + \frac{2BK}{(A+1)^2}\,(1+z)^{3(A+1)/2} + \left(\frac{K}{A+1}\right)^2 (1+z)^{3(A+1)}, \tag{5}$$

where z represents the cosmological redshift. The first term on the right-hand side of equation(5) is a constant, which can be interpreted as the cosmological constant and as describing dark energy. Equation (5) can be written as

$$\rho\,(z) = \rho_1 + \rho_2\,(1+z)^{3(A+1)/2} + \rho_3\,(1+z)^{3(A+1)}, \tag{6}$$

where $\rho_1 = (\frac{B}{A+1})^2$, $\rho_2 = \frac{2BK}{(A+1)^2}$ and $\rho_3 = (\frac{K}{A+1})^2$ represent densities at the present epoch. The Friedmann equation (equation 3) can now be written in terms of the redshift and density parameter as follows:

$$H^2\,(z) = H_0^2\left[\Omega_1 + \Omega_2\,(1+z)^{3(A+1)/2} + \Omega_3\,(1+z)^{3(A+1)}\right], \tag{7}$$

where we define the density parameter: $\Omega = 8\pi G\rho/3H_0^2 = \Omega(A, B, K)$. For a given $A = A_0$ (say), we note that the nature of evolution for the variable parts of the matter energy density may now be established. Hence, the choice of a suitable value for A leads to a known composition of fluids. For example, Paul et al. (2011) considered the case $A = 0$ with dark energy, dark matter and dust in the universe. Fixing A, one can rewrite equation (7) as

$$H^2\,(H_0, B, K, z) = H_0^2 E^2\,(B, K, z), \tag{8}$$

where

$$E^2\,(B, K, z) = \Omega_\Lambda + \Omega_2\,(1+z)^{3(A+1)/2} + \Omega_3\,(1+z)^{3(A+1)}. \tag{9}$$

Here we have replaced the constant part of the DP (Ω_1) by a new notation Ω_Λ.

3 ANALYSIS WITH OBSERVATIONAL DATA

3.1 Observed *Hubble* data (OHD)

Using the observed value of the Hubble parameter at different redshifts (12 data points listed in the observed *Hubble* data by Stern et al. (2010)) we analyse the model. For the analysis, we first define a chi-square function as follows:

$$\chi^2_{\mathrm{OHD}} = \sum \frac{\left[H_{\mathrm{Theory}}\,(H_0, B, K, z) - H_{\mathrm{Obs}}\right]^2}{2\sigma^2}, \tag{10}$$

where H_{Theory} and H_{Obs} are theoretical and observational values of the Hubble parameter at different redshifts, respectively, and σ is the corresponding error. Here, H_0 is a nuisance parameter and can be safely marginalized. We consider $H_0 = 72 \pm 8$ and a fixed prior distribution. A reduced chi-square function can be defined as follows:

$$\chi^2_{\mathrm{red}} = -2\ln \int \left[\mathrm{e}^{-\frac{\chi^2_{\mathrm{OHD}}}{2}}\,P\,(H_0)\right] \mathrm{d}H_0, \tag{11}$$

where $P(H_0)$ is the prior distribution. The regions of 68.3, 95.5 and 99.8 per cent confidence are shown in Figs 1(a) and 2(a) for $A = 1$ and $A = 1/3$ respectively. The best-fitting values are tabulated in Table 1.

3.2 Joint analysis with BAO peak parameter

In the previous analysis, we used the standard value for H_0. In this section we consider analysis that is independent of the measurement of H_0 and does not consider any particular dark energy model. We use here a method proposed by Eisenstein et al. (2005), and for this part of our analysis we follow their approach. A model-independent BAO peak parameter can be defined for low-redshift (z_1) measurements in a flat universe:

$$\mathcal{A} = \frac{\Omega_{\mathrm{m}}}{E\,(z_1)}\,\frac{\int_0^{z_1} \frac{\mathrm{d}z}{E(z)}}{z_1}, \tag{12}$$

where Ω_{m} is the matter density parameter for the universe. Now the chi-square function can be defined as follows:

$$\chi^2_{\mathrm{BAO}} = \frac{(\mathcal{A} - 0.469)^2}{2\,(0.017)^2}, \tag{13}$$

where we have used the measured value for $\mathcal{A}$ (0.469 ± .0.017) as obtained by Eisenstein et al. (2005) from the SDSS data for the LRG (luminous red galaxies) survey. Now we can define a total chi-square function for our joint analysis as

$$\chi^2_{\mathrm{tot}} = \chi^2_{\mathrm{red}} + \chi^2_{\mathrm{BAO}}. \tag{14}$$

The 68.3, 95.5 and 99.8 per cent regions obtained from this joint analysis are given in Fig. 1(b) for $A = 1$ and in Fig. 2(b) for $A = 1/3$. Best-fitting values are shown in Table 2.

3.3 Joint analysis with OHD, BAO peak parameter and CMB shift parameter ($\mathcal{R}$)

The CMB shift parameter ($\mathcal{R}$) is given by

$$\mathcal{R} = \sqrt{\Omega_{\mathrm{m}}} \int_0^{z_{\mathrm{ls}}} \frac{\mathrm{d}z'}{H(z')/H_0}, \tag{15}$$

where z_{ls} is the z at the surface of last scattering. The *WMAP*7 data give $\mathcal{R} = 1.726 \pm 0.018$ at $z = 1091.3$ (Komatsu et al. 2010). Thus we consider

$$\chi^2_{\mathrm{CMB}} = \frac{(\mathcal{R} - 1.726)^2}{(0.018)^2}, \tag{16}$$

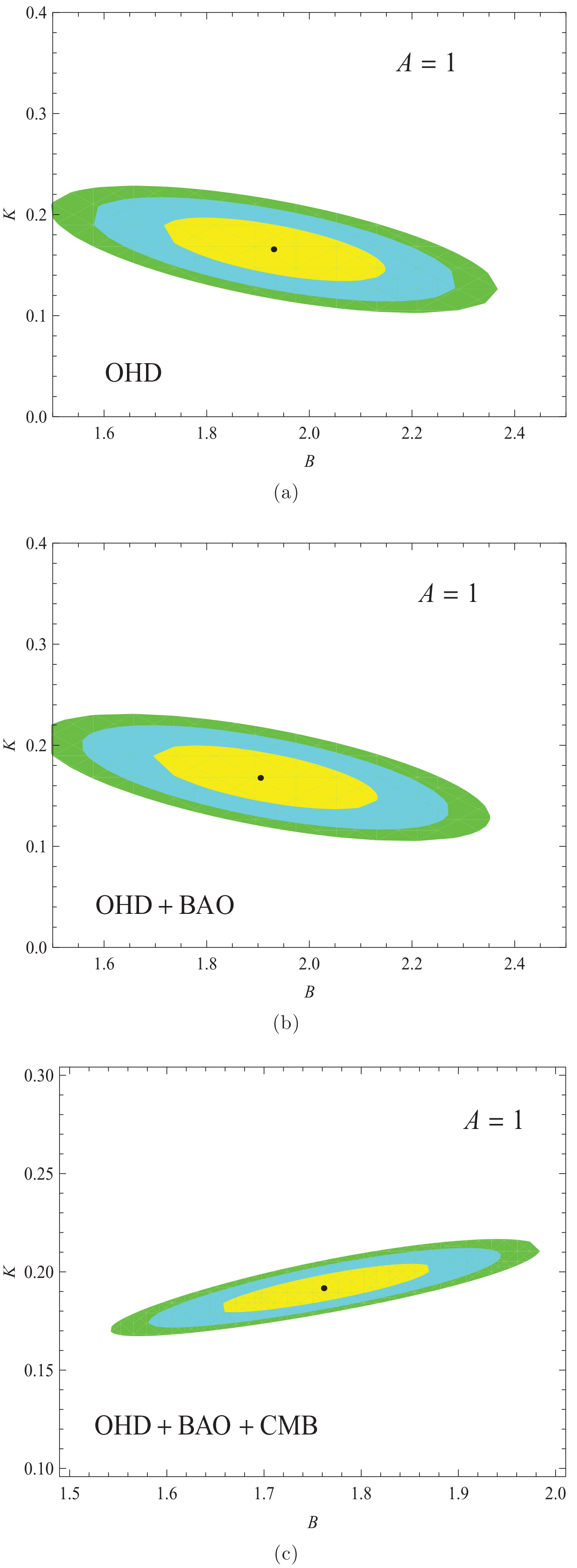

Figure 1. Constraints from (a) observed *Hubble* data, (b) SDSS (baryon acoustic oscillation) and (c) *WMAP*7 (CMB shift) data for an emergent universe with $A = 1$: 68.3, 95.5 and 99.8 per cent regions are shown.

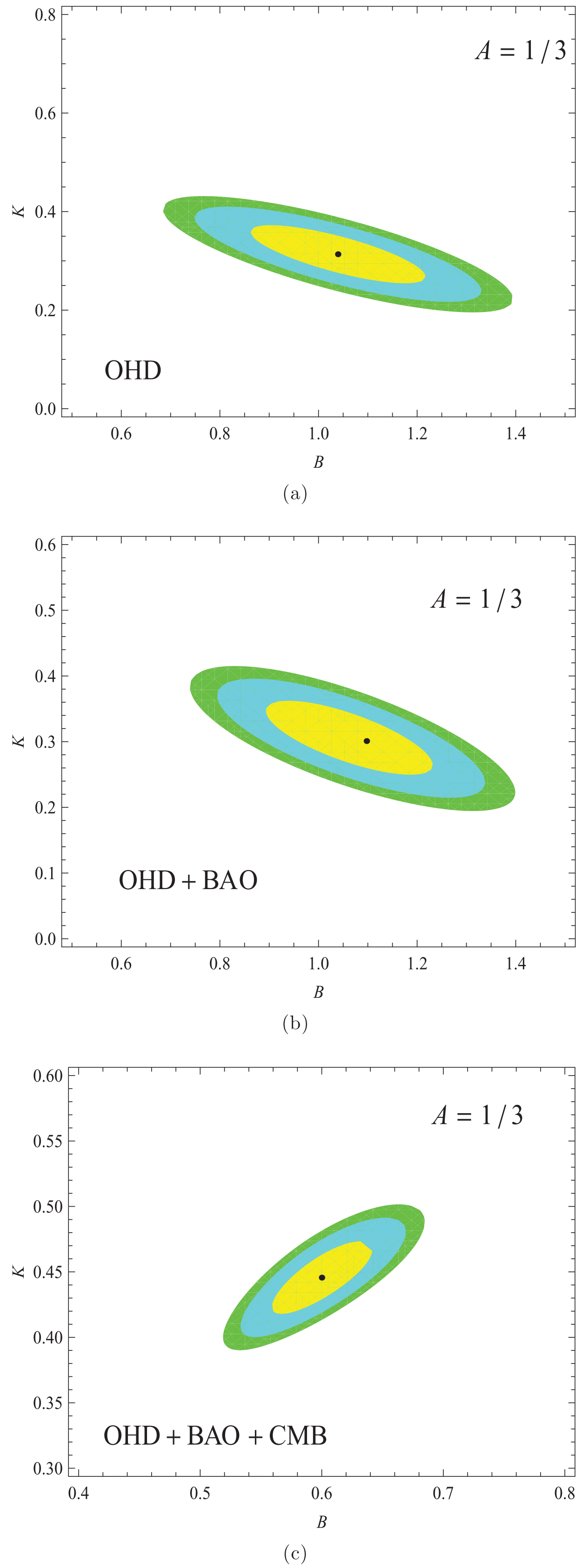

Figure 2. Constraints from (a) observed *Hubble* data, (b) SDSS (baryon acoustic oscillation) and (c) *WMAP*7 (CMB shift)data for an emergent universe with $A = 1/3$: 68.3, 95.5 and 99.8 per cent regions are shown

Table 1. Findings: observed *Hubble* data.

Model	B	K	$\chi^2_{\rm min}$ (d.o.f)
$A = 1$	1.931	0.166	0.818
$A = 1/3$	1.5600	0.470	0.737

Table 2. Findings: observed *Hubble* data+SDSS (baryon acoustic oscillation).

Model	B	K	$\chi^2_{\rm min}$ (d.o.f)
$A = 1$	1.905	0.168	0.875
$A = 1/3$	1.646	0.451	0.707

Table 3. Findings: observed *Hubble* data+SDSS (baryon acoustic oscillation)+*WMAP*7 (CMB shift)

Model	B	K	$\chi^2_{\rm min}$ (d.o.f)
$A = 1$	1.762	0.192	0.925
$A = 1/3$	1.174	0.128	0.925

Table 4. Goodness of fit.

Model	P(OHD)	P(BAO)	P(CMB)
$A = 1$	0.5721	0.562	0.0.520
$A = 1/3$	0.689	0.733	0.004
$A = -1/3$	0.715	0.004	$0 \ll .001$

with $\chi^2_{\rm Tot} = \chi^2_{H-z} + \chi^2_{\rm BAO} + \chi^2_{\rm CMB}$, which imposes additional constraints on the model parameters. The statistical analysis with $\chi^2_{\rm Tot}$ further tightens the bounds on B and K. Figs 1(c) and 2(c) show different confidence regions for $A = 1$ and $A = 1/3$, respectively. Best-fitting values are tabulated in Table 3.

3.4 Goodness of fit

In all the cases of our previous analysis we calculated χ^2 per degree of freedom. Generally, if this value is not far from 1 the fit is considered good. However, as discussed in Viswakarma & Narlikar (2010), a better qualitative assessment can be made if one calculates the χ^2-probability. If the fitted model yields a χ^2-value of x for n degrees of freedom, the probability is given by

$$P(n, x) = \frac{1}{\Gamma\left(\frac{n}{2}\right)} \int_{x/2}^{\infty} \mathrm{e}^{-u} u^{n/2-1}\, \mathrm{d}u. \tag{17}$$

However, this strictly holds for normally distributed errors. Any non-Gaussianity decreases the probability P. Generally, models with $P > 0.001$ are considered acceptable. We have tabulated (Table 4) the P-values found when we fitted different EU models with different data. Note that the model with $A = -1/3$ yields a very poor fit with *WMAP*7 data; this model thus fails the credibility test and it is not worth constraining its parameters.

4 DENSITY PARAMETERS IN DIFFERENT EU MODELS

In the previous section we determined the best-fitting values for B and K corresponding to different models evoked by different choices of A. We now plot contours on the Ω_1–Ω_2 plane. The 68.3 (solid),

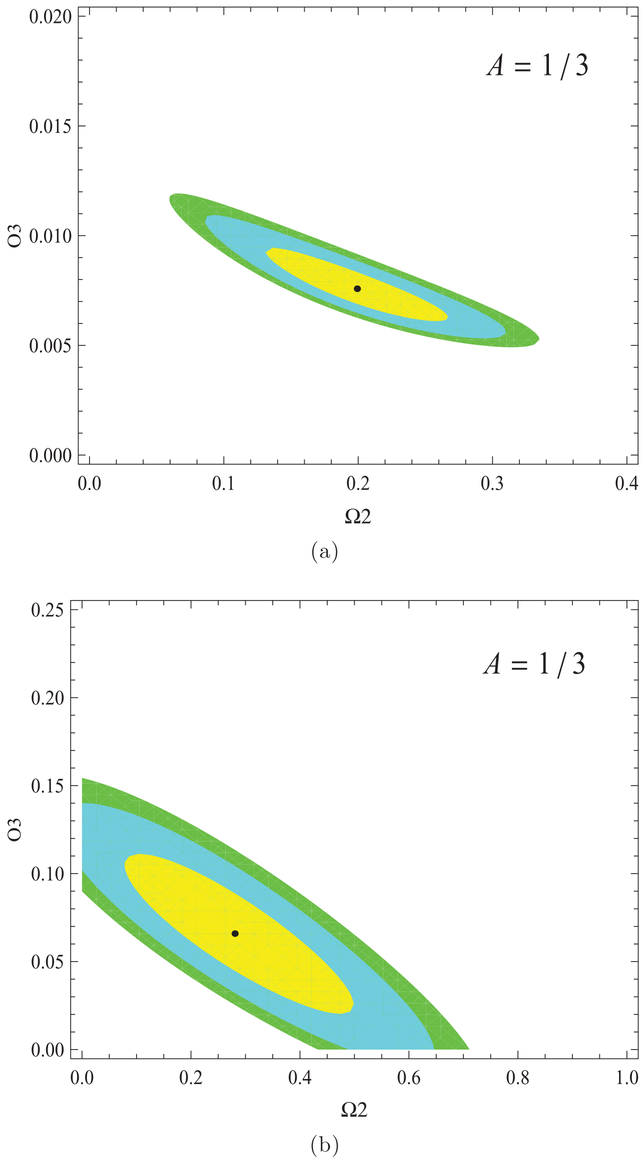

Figure 3. Contours on the Ω_1–Ω_2 plane for (a) $A = 1$, (b) $A = 1/3$: 68.3, 95.5 and 99.8 per cent confidence regions are shown.

95.5 (dashed) and 99.8 per cent (dotted) contours are shown in Figs 3(a) and (b). The EU model with $A = 1$ permits a composition of dark energy (Ω_Λ), dust (Ω_1) and stiff matter (Ω_2) (Mukherjee et al. 2006). For $A = 1/3$, Ω_1 represents the DP for cosmic strings, and Ω_2 represents the DP for radiation. The best-fitting values for Ω_1 and Ω_2 for different models are obtained, which in turn determines the best-fitting values for Ω_Λ in the corresponding model as

$$\Omega_\Lambda = 1 - \Omega_1 - \Omega_2. \tag{18}$$

The best-fitting values for the parameters of a EU are given in Table 5. With *WMAP*7 data, however, the case with $A = -1/3$

Table 5. Findings: analysis of density parameters.

Model	Ω_1	Ω_2	Ω_Λ
$A = 1$	0.200	0.008	0.792
$A = 1/3$	0.281	0.066	0.653

gives a very poor fit (P-value much lower than acceptable), so it is disregarded.

5 DISCUSSION

In this paper we determined observational constraints on EOS parameters for a class of EU models. It was shown by Mukherjee et al.(2006) that an EU model may be admitted for different values of the A parameter for matter of various compositions. The analysis with observational data was carried out here with different values of A belonging to a class of EU given by Mukherjee et al. (2006). The model parameters of the EU were estimated using the OHD as well as using a joint analysis with the measurement of a BAO peak parameter. We used the BAO peak parameter, as proposed by Eisenstein et al. (2005), which is independent of the dark energy model. We also determined observational constraints on EOS parameters from the measurement of the CMB shift parameter ($\mathcal{R}$) determined by *WMAP*7. It was shown by Mukherjee et al. (2006) that a consistent theoretical EU model with $A = -1/3$ corresponds to a cosmic fluid that behaves as a composition of dark energy (cosmological constant), domain walls and cosmic strings. It was found that the case $A = -1/3$ cannot be fitted well with *WMAP*7 data. In the above analysis, it was found that the case $A = -1/3$ is not realistic, as the present-day value of the DP does not agree with observations. Thus, the EU model with $A = -1/3$ can be ruled out. In the other two cases, namely $A = 1$ and $A = 1/3$, one obtains cosmological models with a physically realistic DP. The best-fitting values for the model parameters B and K for a given A were determined. It was found that the model admits a dark energy density close to that predicted by observations in a ΛCDM cosmology. The analysis we adopted here involves kinematics only, and it would be interesting to analyse and determine the model constraints using dynamical aspects such as structure formation etc. A more stringent constraint on the EU can be obtained from such dynamical consideration. All these issues will be considered elsewhere.

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